**For some reasons, there are large gaps between pages. Please move to the next page if you cannot find my response.**

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| **SCI - 201:** | **Student ID:** |
| **Assignment 3** | **Last Name:** |
| Due Saturday, February 9, 11:59 AM,<br>electronic submission (drop box A3) | **Score:** **/27** |

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**1. [8] Short answer questions:**

a) [1] How grey body differs from black body? (2-3 sentences)

**Grey body does not absorb all radition and emit a portion of them based on a constant ε less than 1, while for black body the constant is 1.**

b) [1] What is emissivity?

**The emissivity of a material is its surface's relative ability to emit radiation energy.**

c) [1] What is expression for energy flux from grey body?

**It is $\epsilon \sigma T^4$ , where ε is the emissivity constant, σ is the Stefan-Boltzmann constant, and T is the temperature.**

d) [1] Calculate the maximum wavelength of radiation for Sun and then for Earth. [1] What are corresponding frequencies?

**By Wien's Law, the wavelength of radiation for**

- **the Sun:** $\lambda_{max} = \frac{3000}{5780} \mu m = 519\, nm$ **, frequency is**

$$f = \frac{\text{speed of light}}{\lambda_{max}} = \frac{2.9979\text{e+}17}{519} = 5.77\text{e+}14\, Hz = 577\, \text{Terahertz}$$

- **the Earth:** $\lambda_{max} = \frac{3000}{288} \mu m = 10.4\, \mu m$ **, frequency is**

$$f = \frac{\text{speed of light}}{\lambda_{max}} = \frac{2.9979\text{e+}14}{10.4} = 2.88\text{e+}13\, Hz = 28.8\, \text{Terahertz}$$

e) [1] What is difference in passing Earth atmosphere by solar and earth radiation?

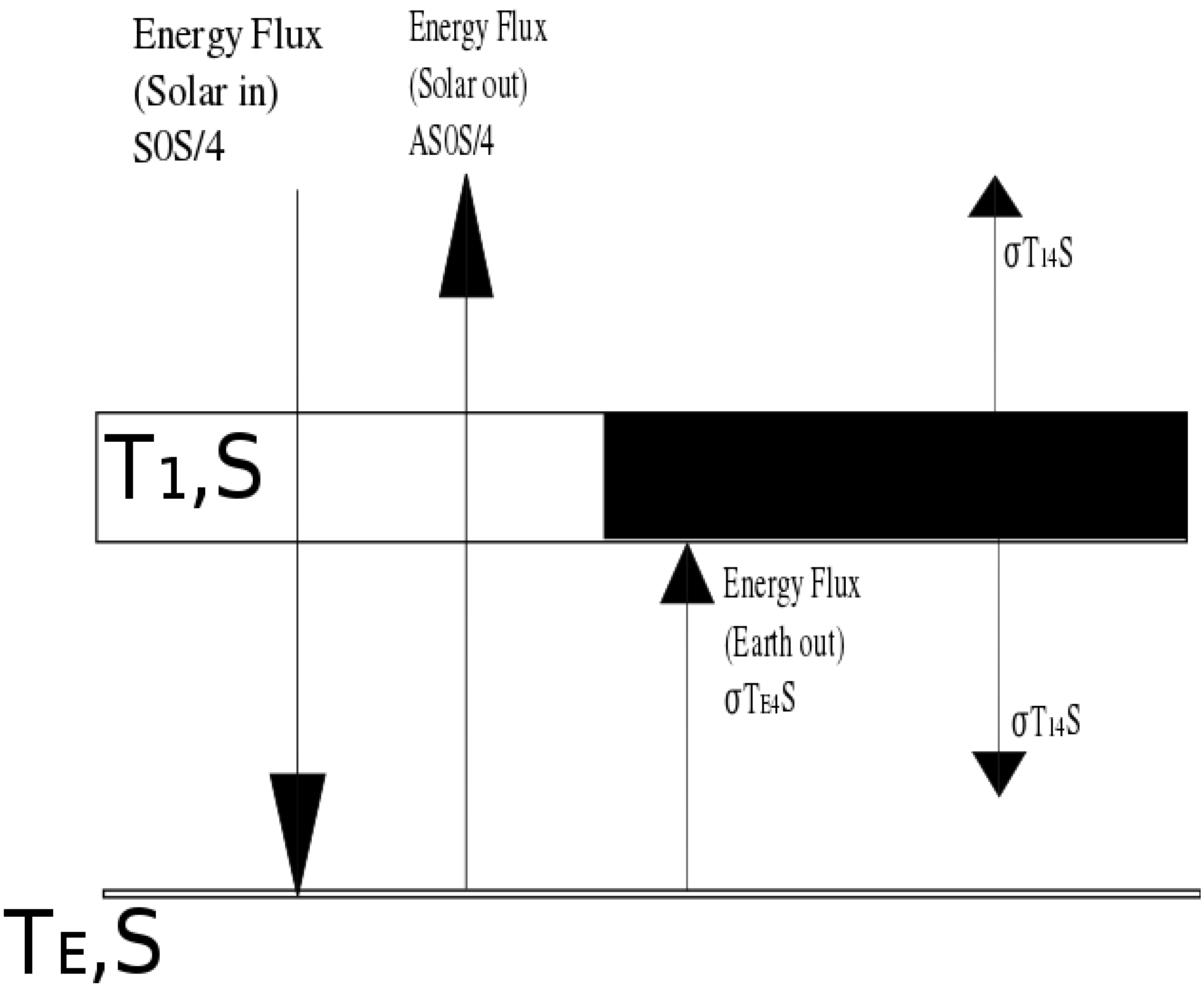
**Solar radiation is being reflected off by objects on Earth, which is about 30% of the incoming solar flux (Albedo). Since the atmosphere is a grey body, the Earth radiation is being partly absorbed by the atmostphere.**

f) [2] Describe greenhouse effect. (2-4 sentences)

**Since Earth's radiation wavelengths are longer than that of the Sun, the atmostphere absorbs radiation and then emit part of them back to the Earth. Hence the greenhouse effect.**

**2. [10] A two-layer model.**

a) [1] First, repeat one layer modeling of atmosphere (as it was done in class). The layer is transparent to visible light but a blackbody for infrared. Also, assume albedo for visible light is 0.3. Solution should include diagram of energy flow and energy balances of your choice as well as numerical values for mean Earth surface temperature and layer temperature.




**(note: The 4 in $T_{E4}$ and $T_{14}$ above should be exponents)**
**Balance of Earth surface and Layer (solve for Layer temperature):**

$$E_{in} = E_{out} + E_{\text{out from atmosphere}}$$

$$\frac{S_0 S}{4} = \frac{A S_0 S}{4} + \sigma T_1^4 S$$

$$T_1 = \sqrt[4]{\frac{S_0(1-A)}{4\sigma}}$$

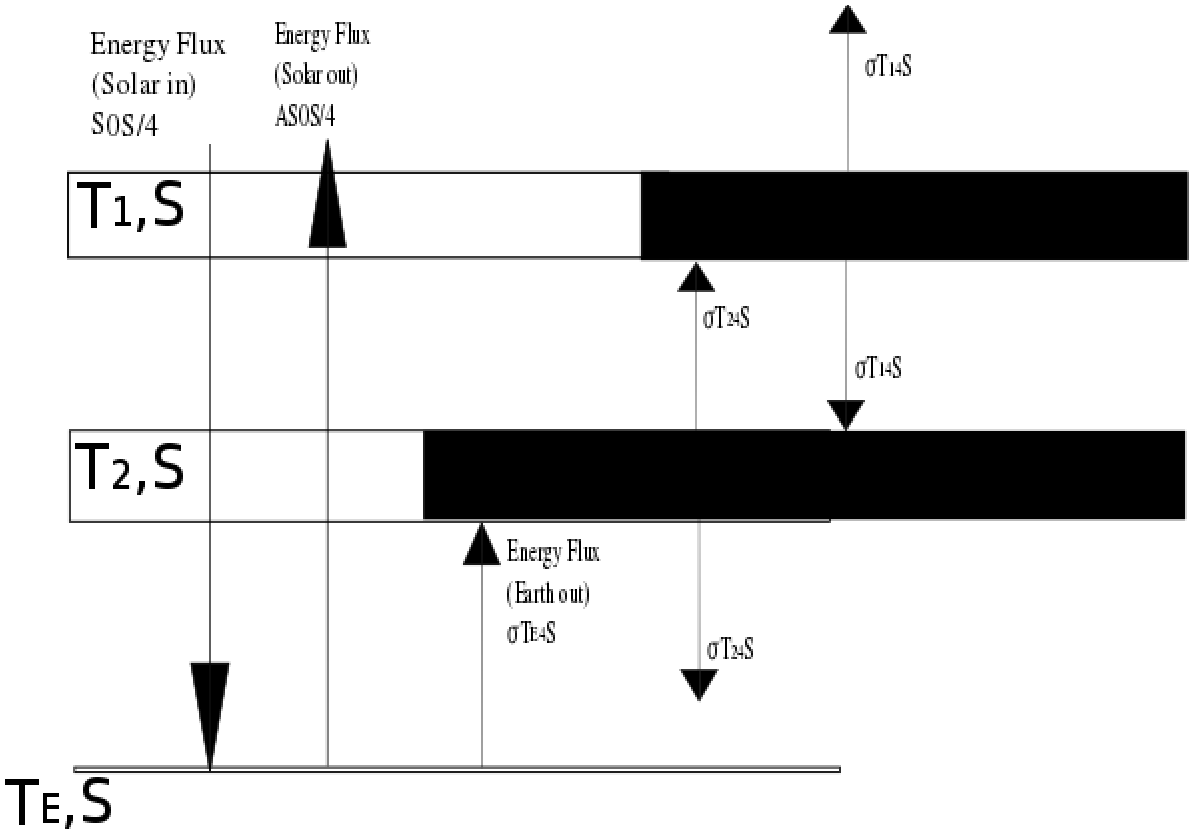
$T_1 = 254\text{K}$ (by the solar constant from A2)

**Balance of layer (solve for Average temperature of Earth):**

$$E_{\text{in}} = E_{\text{out}}$$
$$\sigma T_E^4 S = 2\sigma T_1^4 S$$
$$T_E = \sqrt[4]{2}\, T_1$$
$$T_E = 302\text{K}$$

Second, insert another atmospheric layer into the model, just like the first one and:

b) [2] Draw diagram of all energy fluxes for this case.




**(note: The 4 in $T_{E4}$ , $T_{14}$ and $T_{24}$ above should be exponents)**

c) [2] Write energy balance for the system Earth +Atmosphere, then obtain the temperature the top (layer one) atmospheric layer. Does this part of the exercise seem familiar in any way?

$$E_{in} = E_{out} + E_{\text{out from atmosphere}}$$

$$\frac{S_0 S}{4} = \frac{A S_0 S}{4} + \sigma T_1^4 S$$

$$T_1 = \sqrt[4]{\frac{S_0(1-A)}{4\sigma}}$$

$T_1 = 254\text{K}$ (by the solar constant from A2)

**The answer is the same in 2a).**

d) [2] Write energy balance for the top layer (layer 1). Insert the value you found in c into the energy balance for layer 1, and solve for the temperature of layer 2 in terms of layer 1 (don't calculate the number).

**The balance is**

$$E_{\text{in}} = E_{\text{out}}$$
$$\sigma T_2^4 S = 2\sigma T_1^4 S$$
$$T_2 = \sqrt[4]{2}\, T_1$$

e) [2] Write energy balance for the layer 2. Insert the value you found in d into the energy balance for layer 2, and solve for the temperature of Earth in terms of layer 1 (don't calculate the number).

**The balance is**

$$E_{\text{in}} = E_{\text{out}}$$
$$\sigma T_E^4 S + \sigma T_1^4 S = 2\sigma T_2^4 S$$
$$T_E^4 + T_1^4 = 2T_2^4$$
$$T_E^4 + T_1^4 = 2\left(\sqrt[4]{2}\, T_1\right)^4$$
$$T_E^4 + T_1^4 = 2\left(\sqrt[4]{2}\, T_1\right)^4$$
$$T_E^4 + T_1^4 = 2 \times 2T_1^4$$
$$T_E^4 = 3T_1^4$$
$$T_E = \sqrt[4]{3}\, T_1$$

f) [1] Calculate numerical values for T2 and TE

**$T_2$ =** $\sqrt[4]{2}\, T_1 = 302\text{K}$

**$T_E$ =** $\sqrt[4]{3}\, T_1 = 334\text{K}$

**3. [5] Nuclear Winter.** [3] Let's go back to the 1-layer model (2(a)), but let's change it so that the atmospheric layer absorbs visible light rather than allowing to pass through. This could happen if the upper atmosphere was filled with dust. Also, let's assume that there is reflection of incoming solar radiation (atmospheric albedo) equal to 0.3. What is the temperature of the ground in this case? (shortcut hint: first balance is for E+A boundary; the second one is for E surface). [2] For the sake of exercise, instead of E surface balance write the second one for the layer and find the earth temperature from it. Any surprises?

**The question implies that no radiation will go through the atmostphere. So the balance for the E+A boundary is**

$$E_{\text{in}} = E_{\text{out}}$$
$$\frac{S_0 S}{4} = \frac{A S_0 S}{4} + \sigma T_1^4 S$$

**The energy for Earth surface is just the energy coming from the atmosphere:**

$$E_{\text{in}}=E_{\text{out}}$$
$$\sigma T_1^4 S=\sigma T_E^4 S$$

**So, we get**

$$\frac{S_0 S}{4}=\frac{A S_0 S}{4}+\sigma T_1^4 S$$
$$\frac{S_0 S}{4}=\frac{A S_0 S}{4}+\sigma T_E^4 S$$
$$T_E=\sqrt[4]{\frac{(1-A) S_0}{4\sigma}}$$
$$T_E=255\,K$$

**Now consider the atmosphere instead:**

$$E_{\text{in}}=E_{\text{out}}$$
$$\frac{S_0 S}{4}+\sigma T_E^4 S=A\frac{S_0 S}{4}+2\sigma T_1^4 S$$
$$\frac{1}{2}\left(\frac{S_0 S}{4}-A\frac{S_0 S}{4}+\sigma T_E^4 S\right)=\sigma T_1^4 S$$
$$\frac{S_0 S}{8}-A\frac{S_0 S}{8}+\frac{\sigma T_E^4 S}{2}=\sigma T_1^4 S$$

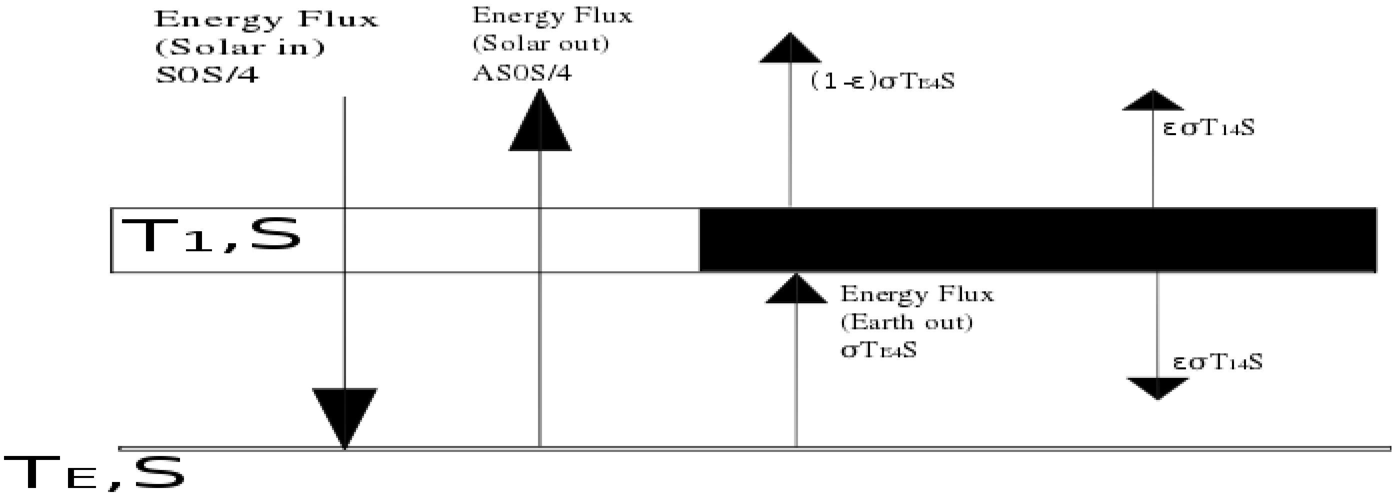
**Substitute the above into the E+A balance:**

$$\frac{S_0 S}{4}=\frac{A S_0 S}{4}+\sigma T_1^4 S$$
$$\frac{S_0 S}{4}=\frac{A S_0 S}{4}+\left(\frac{S_0 S}{8}-A\frac{S_0 S}{8}+\frac{\sigma T_E^4 S}{2}\right)$$
$$\frac{S_0 S}{8}=\frac{A S_0 S}{8}+\frac{\sigma T_E^4 S}{2}$$
$$\frac{S_0 S}{4}=\frac{A S_0 S}{4}+\sigma T_E^4 S$$
$$T_E=\sqrt[4]{\frac{(1-A) S_0}{4\sigma}}$$
$$T_E=255\,K$$

**The result is the same. Therefore the second balance can be either for the Earth's surface or for the atmosphere.**

4. **[4] Grey Layer**. One more time let's go back to the 1-layer model (2(a)). Modify the model assuming everything the same, but layer is grey body for IR with emissivity ε=0.6. Solution

should include diagram of energy flows and energy balances of your choice as well as numerical values for mean Earth surface temperature and layer temperature.




**Balance for layer:**

**The energy flux from Earth is** $\epsilon\sigma T_E^4 S$ **because the remaining flux emits through the atmosphere.**

$$E_{\text{in}} = E_{\text{out}}$$

$$\epsilon\sigma T_E^4 S = 2\epsilon\sigma T_1^4 S$$

$$T_1 = \sqrt[4]{\frac{1}{2}}\, T_E$$

**Surface + Layer balance:**

$$E_{\text{in}}=E_{\text{out}}+E_{\text{out from atmosphere}}+E_{\text{out light passing through atmosphere}}$$

$$\frac{S_0 S}{4}=\frac{A S_0 S}{4}+\epsilon\sigma T_1^4 S+(1-\epsilon)\sigma T_E^4 S$$

$$\frac{(1-A)S_0}{4}=\sigma(\epsilon T_1^4+T_E^4-\epsilon T_E^4)$$

$$\frac{(1-A)S_0}{4\sigma}=\epsilon\frac{T_E^4}{2}+T_E^4-\epsilon T_E^4$$

$$\frac{(1-A)S_0}{4\sigma}=T_E^4(\frac{\epsilon}{2}+1-\epsilon)$$

$$\frac{(1-A)S_0}{4\sigma}=T_E^4(\frac{2-\epsilon}{2})$$

$$\frac{(1-A)S_0}{2\sigma(2-\epsilon)}=T_E^4$$

$$T_E=\sqrt[4]{\frac{(1-A)S_0}{2\sigma(2-\epsilon)}}$$

$$T_E=279\text{K}$$

**Now substitue $T_E$ into the layer balance:**

$$T_1=\sqrt[4]{\frac{1}{2}}T_E$$

$$T_1=235\text{K}$$