Lab 2

Indiana University | M303 Linear Algebra | Fall 2025

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```
format rat
format compact
```

```
A = [1  5 -1;
     3  1  5];

B = [1 -2;
     3  1;
     5  2];

L = [1 2 3;
     4 5 6];

R = [3 2;
     4 5;
     6 7];
```

1a) AR

```
A * R
```

```
ans = 2×2
    17    20
    43    46
```

1b) A **x (first column of** R**)**

```
A * R(:, 1)
```

```
ans = 2×1
    17
    43
```

1c) (second row of A**) x** R

```
A(2, :) * R
```

```
ans = 1×2
    43    46
```

1d) How do the answers in b.) and c.) relate to a.)?

The solution for b.) maps to the first column of solution a.). The solution for c.) maps to the second row of solution a.).

2a) LB

```
L * B
```

```
ans = 2×2
    22     6
    49     9
```

2b) Using your answer to a.), what should L x **(first column of** B**) be equal to? (You should not do any calculation for this part.)**

$L * B(:, 1) = [22; 49]$

2c) 1**x (first col of** L**) + 3 x (second col of** L**) + 5 x (third col of** L**)**

```
1 * L(:, 1) + 3 * L(:, 2) + 5 * L(:, 3)
```

```
ans = 2×1
    22
    49
```

2d) How does the answer to c.) correspond to the answer to a.)? Why is this?

The solution to 2c.) matches the first column of the solution for 2a.) This is true because each column of the product LB is a linear combination of the columns of L where the coefficients come from the corresponding column of B. In this case, the coefficients [1, 3, 5] from the first column of B determine the linear combination.

2e) Using the text w/ math symbol functionality in your LiveScript, write the second column of LB **as a linear combination of the columns of** L**. (Again, no calculations here.)**

-2 × (first col of L) + 1 × (second col of L) + 2 × (third col of L)

2f) Do the same, writing the second row of LB **as a linear combination of the rows of** B**.**

4 × (first row of B) + 5 × (second row of B) + 6 × (third row of B)

3.) Using A **and** B**,**

```
A = [1  5 -1;
     3  1  5];

B = [1 -2;
     3  1;
     5  2];
```

3a.) How can you tell, without doing any calculations, that the matrices A and B are not invertible?

A and B are not invertible because they are not square matrices. A matrix must be square (same number of rows and columns) to be invertible.

3b.) Use MATLAB to find AB.

```
A * B
```

```
ans = 2×2
    11     1
    31     5
```

3c.) Find the inverse to AB in two different ways. First, use `rref()` on the appropriate matrix, mimicking our strategy when we compute inverses by hand. (Don't do the row operations yourself! `rref()` does it for you.)

```
rref([11 1 1 0;
      31 5 0 1])
```

```
ans = 2×4
    1.0000         0    0.2083   -0.0417
         0    1.0000   -1.2917    0.4583
```

3d.) Now compute the inverse using the function `inv()`. Store the output of this as the matrix D. You should see that this agrees with the result from c.).

```
D = inv(A * B)
```

```
D = 2×2
    0.2083   -0.0417
   -1.2917    0.4583
```

4.) Using S and T.

```
S = [3 -1 5;
     2  0 1];
```

```
T = [1 -1;
     3  5;
     2 -4];
```

first find ST

```
S * T
```

```
ans = 2×2
    10   -28
     4    -6
```

```
rref([10 -28 1 0;
       4  -6 0 1])
```

```
ans = 2×4
    1.0000         0   -0.1154    0.5385
         0    1.0000   -0.0769    0.1923
```

```
D = inv(S * T)
```

```
D = 2×2
   -0.1154    0.5385
   -0.0769    0.1923
```

right inverse to S, e.g. $S(TD) = I$

```
T * D
```

```
ans = 3×2
   -0.0385    0.3462
   -0.7308    2.5769
    0.0769    0.3077
```

left inverse to T, e.g. $(DS)T = I$

```
D * S
```

```
ans = 2×3
    0.7308    0.1154   -0.0385
    0.1538    0.0769   -0.1923
```

5) Use *the above technique* to find a right-inverse of S. What do you notice about your result versus your answer in 4?

Find SS^T

```
S * S'
```

```
ans = 2×2
    35    11
    11     5
```

Confirm SS^T is invertible

```
D = inv(S * S')
```

```
D = 2×2
    0.0926   -0.2037
   -0.2037    0.6481
```

Therefore $SS^TD = I$, and S^TD is the right inverse for S.

```
S' * D
```

```
ans = 3×2
   -0.1296    0.6852
   -0.0926    0.2037
    0.2593   -0.3704
```

The right inverse found in problem 5 is **different** from the right inverse found in problem 4, even though both are valid right inverses. This shows that **non-square matrices can have multiple (infinitely many) right inverses**, unlike square invertible matrices which have exactly one unique inverse.

6) Use *the above technique* to find a right-inverse of:

```
M = [ 3 1  3 -4  3;
      1 0  0 -1  1;
      2 4 -1 -3 -4;
     -3 1  3 -1 -1];
```

Find MM^T

```
M * M'
```

```
ans = 4×4
    44    10     7     2
    10     3     1    -3
     7     1    46     2
     2    -3     2    21
```

Confirm MM^T is invertible

```
D = inv(M * M')
```

```
D = 4×4
    0.3839   -1.5283   -0.0142   -0.2535
   -1.5283    6.4795    0.0453    1.0669
   -0.0142    0.0453    0.0227    0.0057
   -0.2535    1.0669    0.0057    0.2236
```

Therefore $MM^TD = I$, and M^TD is the right inverse for M.

```
M' * D
```

```
ans = 5×4
    0.3555   -1.2155    0.0312   -0.3533
    0.0737   -0.2801    0.0822   -0.0072
    0.4051   -1.4297   -0.0482   -0.0954
```

0.2890	-1.5691	-0.0623	-0.2934
-0.0666	0.6464	-0.0935	0.0600