Week 3 Work Set

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1 $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

$$w_1 = 8x_1 + 2x_2 - x_3$$
$$w_2 = 9x_1 - x_2 + x_3$$
$$w_3 = 6x_1 + 5x_2 - x_3$$

$T(-6,5,6) = ?$

$$\underbrace{\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} 8 & 2 & -1 \\ 9 & -1 & 1 \\ 6 & 5 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x}$$

by direct substitution

$$w_1 = 8(-6) + 2(5) - 6 = -48 + 10 - 6 = -44$$
$$w_2 = 9(-6) - 1(5) + 6 = -54 - 5 + 6 = -53$$
$$w_3 = 6(-6) + 5(5) - 6 = -36 + 25 - 6 = -17$$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 8 & 2 & -1 \\ 9 & -1 & 1 \\ 6 & 5 & -1 \end{bmatrix}}_{3\times 3} \underbrace{\begin{bmatrix} -6 \\ 5 \\ 6 \end{bmatrix}}_{3\times 1} = -6\begin{bmatrix} 8 \\ 9 \\ 6 \end{bmatrix} + 5\begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} + 6\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -48 \\ -54 \\ -36 \end{bmatrix} + \begin{bmatrix} 10 \\ -5 \\ 25 \end{bmatrix} + \begin{bmatrix} -6 \\ 6 \\ -6 \end{bmatrix}$$

by matrix multiplication

$$T(-6,5,6) = \begin{bmatrix} -44 \\ -53 \\ -17 \end{bmatrix}$$

2

$$\det(A) = \begin{vmatrix} 3 & 3 & 1 \\ 1 & 0 & -3 \\ 1 & -3 & 5 \end{vmatrix}$$

$$= a_{21}C_{21} + a_{22}C_{22} + a_{23}C_{23}$$

$$= a_{21}(-1)^3 M_{21} + a_{22}(-1)^4 M_{22} + a_{23}(-1)^5 M_{23}$$

$$= -\begin{vmatrix} 3 & 1 \\ -3 & 5 \end{vmatrix} + 0 + 3\begin{vmatrix} 3 & 3 \\ 1 & -3 \end{vmatrix}$$

$$= -(15+3) + 3(-9-3)$$

$$= -(18) + 3(-12)$$

$$= -18 - 36$$

$$= \boxed{-54}$$

3]

$$\det(A) = \begin{vmatrix} 4 & 1 & 4 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 4 & 1 & 0 \\ 0 & 2 & 3 & -3 \end{vmatrix} \quad R_1 \leftrightarrow R_2$$

$$(-1)\begin{vmatrix} 1 & 0 & 1 & 1 \\ 4 & 1 & 4 & 1 \\ 0 & 4 & 1 & 0 \\ 0 & 2 & 3 & -3 \end{vmatrix} \quad R_2 - 4R_1 \qquad 4-4 \;\; 1-0 \;\; 4-4 \;\; 1-4$$

$$(-1)\begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 4 & 1 & 0 \\ 0 & 2 & 3 & -3 \end{vmatrix} \quad \begin{matrix} R_3 - 4R_2 \\ R_4 - 2R_2 \end{matrix} \qquad \begin{matrix} 0\;\; 4-4\;\; 1-0\;\; 0+12 \\ 0\;\; 0\;\; 3\;\; -3+6 \end{matrix}$$

$$(-1)\begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 12 \\ 0 & 0 & 3 & 3 \end{vmatrix} \quad R_4 - 3R_3 \qquad 3-3 \;\; 3-3(12) = -33$$

$$(-1)\begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 12 \\ 0 & 0 & 0 & -33 \end{vmatrix}$$

$$(-1)(1)(1)(1)(-33) = \boxed{33}$$