Exam 1

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2025-09-21

M311

Fall 2025

Calculus 3

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1. $x = 3z^2 \pm 3y^2$

elliptic paraboloid along x-axis

2 var sylinder

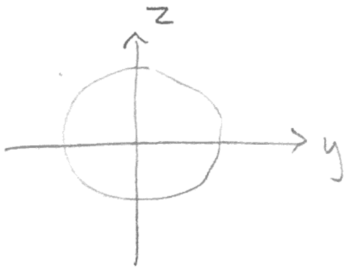
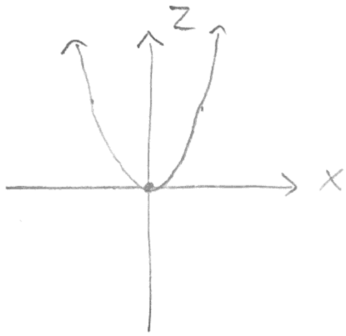
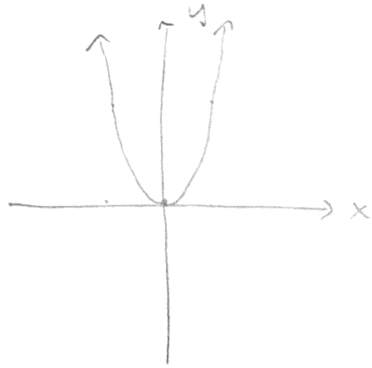
3 var 3 sq all + ellipsoid

3 var 3 sq (1 or 1 −) hyperboloid (1 or 2)

no constant cone

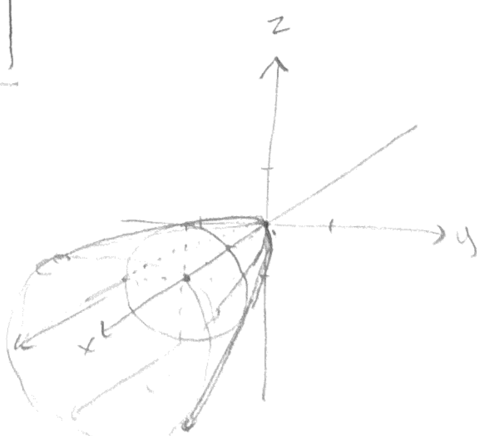
(+) 3v2sq elliptic paraboloid

(−) 3v2sq hypelbolic paraboloid

$x = k$	$y = k$	$z = k$
$k = 3z^2 + 3y^2$	$x = 3z^2 + 3k^2$	$x = 3k^2 + 3y^2$
$\frac{k}{3} = z^2 + y^2$	$x = 3z^2 \quad (k=0)$	$x = 3y^2 \quad (k=0)$
		
circle	parabola	parabola

$\boxed{\text{elliptic paraboloid (circular)}}$

directed along x-axis



2

$$4x^2 + 4y^2 - z^2 = 9$$

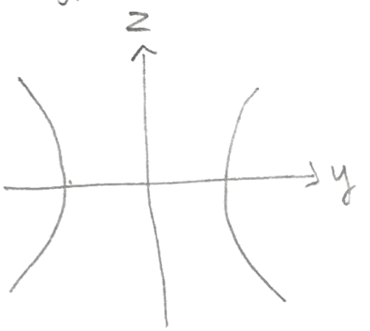
3 var, 3 sqrd
1 negative
hyperboloid of one sheet

$$\frac{x^2}{9/4} + \frac{y^2}{9/4} - \frac{z^2}{9} = 1$$

$x = k$

$$\frac{y^2}{9/4} - \frac{z^2}{9} = 1$$

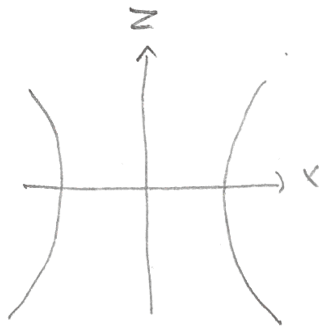
hyperbola



$y = k$

$$\frac{x^2}{9/4} - \frac{z^2}{9} = 1$$

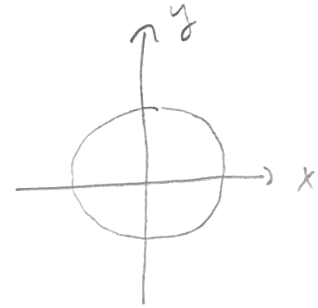
hyperbola



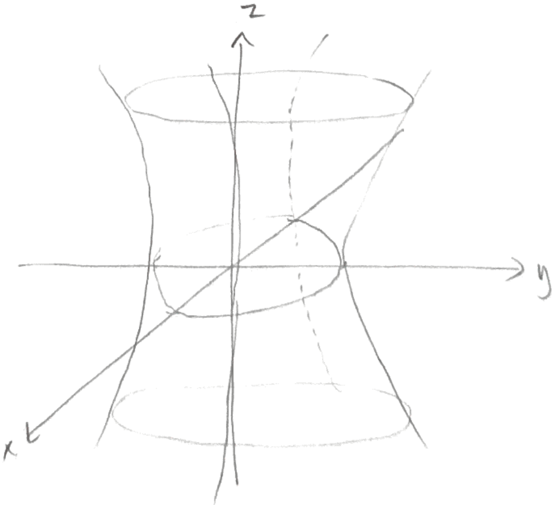
$z = k$

$$\frac{x^2}{9/4} + \frac{y^2}{9/4} = 1$$

circle



$\boxed{\text{hyperboloid of one sheet directed along z-axis}}$



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3] Rec $(1, \sqrt{3}, 2) \longrightarrow$ Cyl (r, θ, z)

$r^2 = x^2 + y^2$

$r^2 = (1)^2 + (\sqrt{3})^2$

$r^2 = 1 + 3$

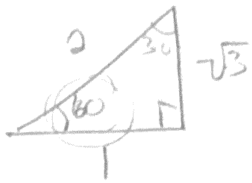
$r^2 = 4$

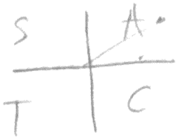
$r = 2$

$\theta = \tan^{-1} \frac{y}{x}$

$\theta = \tan^{-1} \frac{\sqrt{3}}{1}$

$\theta = \frac{\pi}{3}$



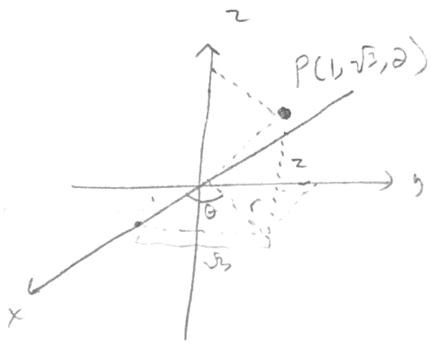


$$\boxed{\left(2, \frac{\pi}{3}, 2\right)}$$

$x = 1$ $\quad r = 2$

$y = \sqrt{3}$ $\quad \theta = \pi/3$

$z = 2$ $\quad z = 2$



4

$$\rho = 2\cos\phi \longrightarrow \text{Rec}$$

$$\rho^2 = 2\rho\cos\phi$$

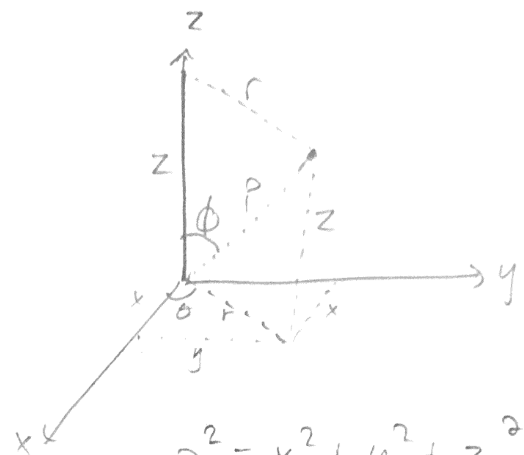
$$x^2+y^2+z^2 = 2z$$

$$x^2+y^2+z^2-2z = 0$$

$$x^2+y^2+z^2-2z+1-1 = 0$$

$$\boxed{x^2+y^2+(z-1)^2 = 1}$$

sphere of radius 1
center at $(0,0,1)$



$$\rho^2 = x^2+y^2+z^2$$

$$\cos\phi = \frac{z}{\rho}$$

$$\rho\cos\phi = z$$

√ 4

5]

$r(t) = \langle 3\cos t, 3\sin t \rangle$

$x = 3\cos t$ $\qquad$ $y = 3\sin t$

$\frac{x}{3} = \cos t$ $\qquad$ $\frac{y}{3} = \sin t$

$$\cos^2 t + \sin^2 t = 1$$

$$\frac{x^2}{9} + \frac{y^2}{9} = 1 \qquad \text{circle w/ } r = 3$$

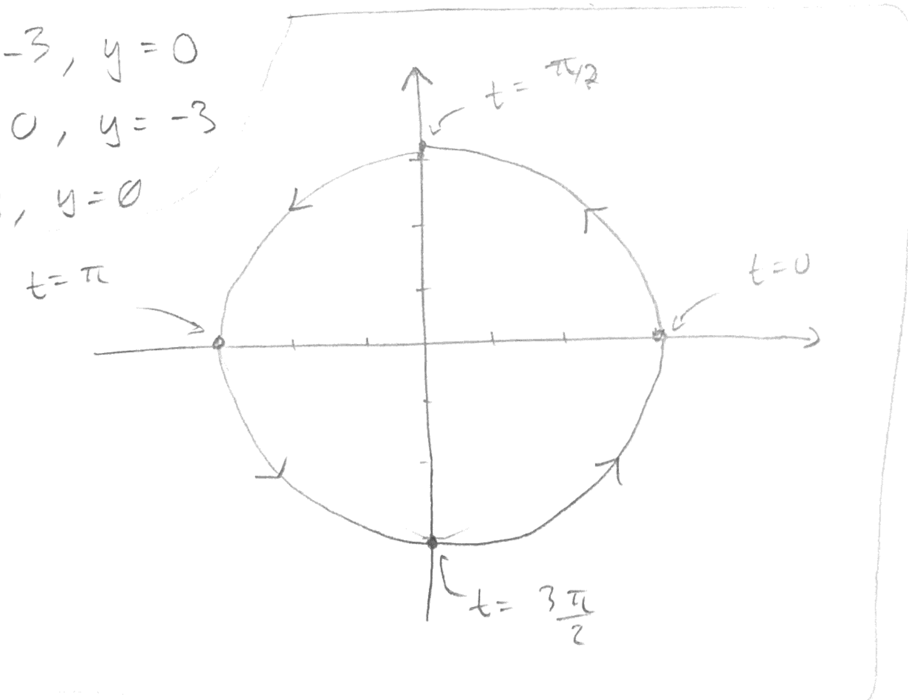
$t = 0, \; x = 3, \; y = 0$

$t = \frac{\pi}{2}, \; x = 0, \; y = 3$

$t = \pi, \; x = -3, \; y = 0$

$t = \frac{3\pi}{2}, \; x = 0, \; y = -3$

$t = 2\pi, \; x = 3, \; y = 0$



6]

$$r(t) = \langle t^2, e^t, t^6 \rangle$$

a) $\frac{d}{dt} t^2 = 2t$

$\frac{d}{dt} e^t = e^t \cdot t' = e^t \cdot 1 = e^t$

$\frac{d}{dt} t^6 = 6t^5$

$$\boxed{r'(t) = \langle 2t, e^t, 6t^5 \rangle}$$

b) $T(1)$ $\qquad T = \frac{v}{|v|} \qquad v = r'(t)$

$|v| = \sqrt{(2t)^2 + (e^t)^2 + (6t^5)^2}$

$|v| = \sqrt{4t^2 + e^{2t} + 36t^{10}}$

$T(1) = \frac{1}{|v|} v = \frac{1}{\sqrt{4+e^2+36}} \langle 2, e, 6 \rangle$

$= \frac{1}{\sqrt{40+e^2}} \langle 2, e, 6 \rangle$

$$\boxed{T(1) = \left\langle \frac{2}{\sqrt{40+e^2}}, \frac{e}{\sqrt{40+e^2}}, \frac{6}{\sqrt{40+e^2}} \right\rangle}$$

7

$r(t) = 3\cos t\, i + 3\sin t\, j + 4t\, k$

$0 \le t \le 2\pi$

$S = \int_a^b |v|\, dt$

$v = r'(t) = \langle -3\sin t, 3\cos t, 4 \rangle$

$|v| = \sqrt{9\sin^2 t + 9\cos^2 t + 16}$

$|v| = \sqrt{9(\sin^2 t + \cos^2 t) + 16}$

$|v| = \sqrt{9+16}$

$|v| = \sqrt{25} = 5$

$S = \int_0^{2\pi} 5\, dt = 5t \Big|_0^{2\pi} = 5(2\pi) - 5(0)$

$= \boxed{10\pi}$

8] $s = \int_0^t |v(u)|\,du$

$r(t) = 3\cos t\, i + 3\sin t\, j + 4t\, k$

$v = r'(t) = \langle -3\sin t, 3\cos t, 4 \rangle$ ... from 7]

$|v| = 5$

$s = \int_0^t 5\,du = 5u\Big|_0^t = 5t - 5(0) = 5t$

$s = 5t$

$t = \frac{s}{5}$

$$\boxed{r(t(s)) = \left\langle 3\cos\left(\frac{s}{5}\right), 3\sin\left(\frac{s}{5}\right), 4\left(\frac{s}{5}\right) \right\rangle}$$

9) $k = \frac{1}{|v|}\left|\frac{dT}{dt}\right|$

$r(t) = \langle 3\cos t,\ 3\sin t,\ 4t \rangle$

$v = \langle -3\sin t, 3\cos t, 4 \rangle$

$|v| = 5$

$T = \frac{v}{|v|} = \langle -\frac{3}{5}\sin t, \frac{3}{5}\cos t, \frac{4}{5} \rangle$

$\frac{dT}{dt} = \langle -\frac{3}{5}\cos t, -\frac{3}{5}\sin t, 0 \rangle$

$\left|\frac{dT}{dt}\right| = \sqrt{\frac{9}{25}\cos^2 t + \frac{9}{25}\sin^2 t}$

$= \sqrt{\frac{9}{25}(\cos^2 t + \sin^2 t)}$

$\left|\frac{dT}{dt}\right| = \sqrt{\frac{9}{25}(1)} = \frac{3}{5}$

$k = \left(\frac{1}{5}\right)\left(\frac{3}{5}\right)$

$\boxed{k = \frac{3}{25}}$