Lab 1

Indiana University | M303 Linear Algebra | Fall 2025

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1) System 1

$$\begin{aligned} x + 3y &= -1 \\ 4x - y &= 2 \end{aligned}$$

```
A1 = [1 3 -1; 4 -1 2];
rrefA1 = rref(A1)
```

```
rrefA1 =
       1              0              5/13
       0              1             -6/13
```

System 1 is **consistent**, with **one solution**. The solution is

$$x = \frac{5}{13}, \quad y = -\frac{6}{13}$$

2) System 2

$$\begin{aligned} 2x - 3y + z &= 10 \\ -x - y + 4z &= 2 \\ -5x + 11z &= -4 \end{aligned}$$

```
A2 = [2 -3 1 10; -1 -1 4 2; -5 0 11 -4];
rrefA2 = rref(A2)
```

```
rrefA2 =
       1              0            -11/5            4/5
       0              1             -9/5          -14/5
       0              0              0              0
```

System 2 is **consistent**, with **infinitely many solutions**. We have two independent equations with three unknowns. One variable is free. Let $z = t$.

$$x = \frac{11}{5}t + \frac{4}{5}, \quad y = \frac{9}{5}t - \frac{14}{5}, \quad z = t$$

3) System 3

$$\begin{aligned} 2x_1 + x_2 - x_3 + 4x_4 &= 5 \\ 0x_1 - x_2 + 4x_3 - 3x_4 &= 11 \\ x_1 + 2x_2 + 3x_3 + 4x_4 &= -10 \\ 3x_1 + 2x_2 + 6x_3 + 5x_4 &= 12 \end{aligned}$$

```
A3 = [2 1 -1 4 5; 0 -1 4 -3 11; 1 2 3 4 -10; 3 2 6 5 12];
rrefA3 = rref(A3)
```

```
rrefA3 =
       1              0              0          17/19            0
       0              1              0          37/19            0
       0              0              1          -5/19            0
       0              0              0            0              1
```

System 3 is **inconsistent**, with **no solution** as $0 \neq 1$.

4)

```
A4 = [2 1 5 3; 1 0 1 2; -3 -1 1 4]
```

```
A4 =
       2              1              5              3
       1              0              1              2
      -3             -1              1              4
```

$R_1 \leftrightarrow R_2$

```
A4([1, 2],:) = A4([2, 1],:)
```

```
A4 =
       1              0              1              2
       2              1              5              3
      -3             -1              1              4
```

$R_2 - 2R_1 \rightarrow R_2$

```
A4(2,:) = A4(2,:) - 2*A4(1,:)
```

```
A4 =
       1              0              1              2
       0              1              3             -1
      -3             -1              1              4
```

$R_3 + 3R_1 \rightarrow R_3$

```
A4(3,:) = A4(3,:) + 3*A4(1,:)
```

```
A4 =
       1              0              1              2
       0              1              3             -1
       0             -1              4             10
```

$R_3 + R_2 \rightarrow R_3$

```
A4(3,:) = A4(3,:) + A4(2,:)
```

```
A4 =
       1              0              1              2
       0              1              3             -1
       0              0              7              9
```

$\frac{1}{7}R_3 \to R_3$

```
A4(3,:) = 1/7 * A4(3,:)
```

```
A4 =
       1              0              1              2
       0              1              3             -1
       0              0              1              9/7
```

$R_2 - 3R_3 \to R_2$

```
A4(2,:) = A4(2,:) - 3 * A4(3,:)
```

```
A4 =
       1              0              1              2
       0              1              0            -34/7
       0              0              1              9/7
```

$R_1 - R_3 \to R_1$

```
A4(1,:) = A4(1,:) - A4(3,:)
```

```
A4 =
       1              0              0              5/7
       0              1              0            -34/7
       0              0              1              9/7
```

The solution is

$$x = \frac{5}{7}, \quad y = -\frac{34}{7}, \quad z = \frac{9}{7}$$

5)

$$\begin{aligned} 2x_1 - x_2 + 3x_3 + x_4 - 4x_5 &= \\ x_1 + x_3 + 2x_4 - x_5 &= \\ -3x_1 - 2x_2 + x_3 + x_5 &= \\ -3x_2 + 5x_3 + 3x_4 - 5x_5 &= \\ 8x_1 + 6x_3 + 4x_4 - 10x_5 &= \end{aligned}$$

The constant vectors are $v_1 = [1, 0, 1, 2, 1]$, $v_2 = [1, -3, 1, 5, 4]$, $v_3 = [2, -1, 1, 2, 6]$, $v_4 = [2, 3, 1, 2, -2]$, and $v_5 = [7, 1, 5, 13, 10]$.

The augmented matrix is

```
C5 = [2 -1 3 1 -4; 1 0 1 2 -1; -3 -2 1 0 1; 0 -3 5 3 -5; 8 0 6 4 -10];
V1 = [1; 0; 1; 2; 1];
V2 = [1; -3; 1; 5; 4];
V3 = [2; -1; 1; 2; 6];
V4 = [2; 3; 1; 2; -2];
V5 = [7; 1; 5; 13; 10];
A5 = [C5, V1, V2, V3, V4, V5]
```

```
A5 =
```

```
     2          -1           3           1          -4           1           1
     1           0           1           2          -1           0          -3
    -3          -2           1           0           1           1           1
     0          -3           5           3          -5           2           5
     8           0           6           4         -10           1           4
```

```
rrefA5 = rref(A5)
```

```
rrefA5 =
     1           0           0          -4           0          1/2          0
     0           1           0           9           0         -3/2          0
     0           0           1           6           0         -1/2          0
     0           0           0           0           1          0            0
     0           0           0           0           0          0            1
```

```
V1_solution = rrefA5(:,[1:5,6])
```

```
V1_solution =
     1           0           0          -4           0          1/2
     0           1           0           9           0         -3/2
     0           0           1           6           0         -1/2
     0           0           0           0           1          0
     0           0           0           0           0          0
```

V1_solution is **consistent**, with **infinitely many solutions**, where x_4 is the free variable. The solutions are

$$x_1 = \frac{1}{2} + 4t, \quad x_2 = -\frac{3}{2} - 9t, \quad x_3 = -\frac{1}{2} - 6t, \quad x_4 = t, \quad x_5 = 0$$

```
V2_solution = rrefA5(:,[1:5,7])
```

```
V2_solution =
     1           0           0          -4           0           0
     0           1           0           9           0           0
     0           0           1           6           0           0
     0           0           0           0           1           0
     0           0           0           0           0           1
```

V2_solution is **inconsistent**, with **no solution** as $0 \neq 1$.

```
V3_solution = rrefA5(:,[1:5,8])
```

```
V3_solution =
     1           0           0          -4           0         20/3
     0           1           0           9           0         -9
     0           0           1           6           0         -5/3
     0           0           0           0           1          4
     0           0           0           0           0          2/3
```

V3_solution is **inconsistent**, with **no solution** as $0 \neq \frac{2}{3}$.

```
V4_solution = rrefA5(:,[1:5,9])
```

```
V4_solution =
```

```
        1              0              0             -4              0             -10/3
        0              1              0              9              0               1
        0              0              1              6              0              -5/3
        0              0              0              0              1              -4
        0              0              0              0              0              -4/3
```

V4_solution is **inconsistent**, with **no solution** as $0 \neq -\frac{4}{3}$.

```
V5_solution = rrefA5(:,[1:5,10])
```

```
V5_solution =
        1              0              0             -4              0               2
        0              1              0              9              0              -6
        0              0              1              6              0              -1
        0              0              0              0              1               0
        0              0              0              0              0               0
```

V5_solution is **consistent**, with **infinitely many solutions**, where $\mathbf{x_4}$ is the free variable. The solutions are

$$x_1 = 2 + 4t, \quad x_2 = -6 - 9t, \quad x_3 = -1 - 6t, \quad x_4 = t, \quad x_5 = 0$$