Practice Test

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1) $$\begin{aligned} x + 2y - z &= 3 \\ 3x - y + 4z &= -1 \\ -x + 5y - 6z &= 7 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 3 & -1 & 4 & -1 \\ -1 & 5 & -6 & 7 \end{array}\right] \xrightarrow[R_3 + R_1]{R_2 - 3R_1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -7 & 7 & -10 \\ 0 & 7 & -7 & 10 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -7 & 7 & -10 \\ 0 & 7 & -7 & 10 \end{array}\right] \xrightarrow{-\frac{1}{7}R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & -1/7 & 10/7 \\ 0 & 7 & -7 & 10 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & -1/7 & 10/7 \\ 0 & 7 & -7 & 10 \end{array}\right] \xrightarrow[R_3 - 7R_2]{R_1 - 2R_2} \left[\begin{array}{ccc|c} 1 & 0 & -5/7 & 1/7 \\ 0 & 1 & -1/7 & 10/7 \\ 0 & 0 & -6 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5/7 & 1/7 \\ 0 & 1 & -1/7 & 10/7 \\ 0 & 0 & -6 & 0 \end{array}\right] \xrightarrow{-\frac{1}{6}R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 0 & -5/7 & 1/7 \\ 0 & 1 & -1/7 & 10/7 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5/7 & 1/7 \\ 0 & 1 & -1/7 & 10/7 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow[R_2 + \frac{1}{7}R_3]{R_1 + \frac{5}{7}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/7 \\ 0 & 1 & 0 & 10/7 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\boxed{\begin{aligned} x &= 1/7 \\ y &= 10/7 \\ z &= 0 \end{aligned}}$$

$1/7 + 20/7 - 0 = 3$ ✓

$3/7 - 10/7 + 0 = -1$ ✓

$-1/7 + 50/7 - 0 = 7$ ✓

2) $A = \begin{bmatrix} 1 & 3 & 7 \\ 2 & -1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -1 \\ 3 & 4 \\ 1 & -5 \end{bmatrix}$

2×3 $\quad 3 \times 2$

$AB = \begin{bmatrix} 2+9+7 & -1+12-35 \\ 4-3+0 & -2-4+0 \end{bmatrix}$

(2×2)

$= \boxed{\begin{bmatrix} 18 & -24 \\ 1 & -6 \end{bmatrix}}$

3) $A = \begin{bmatrix} a-2 & 3 \\ 5 & a-4 \end{bmatrix}$ if $\det(A) = 0$, A is not invertible

$(a-2)(a-4) = 15$

$a^2 - 6a + 8 = 15$

$a^2 - 6a - 7 = 0$

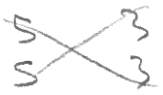
$(a-7)(a+1) = 0$

$\boxed{a = -1, 7}$

Check

$\begin{bmatrix} -1-2 & 3 \\ 5 & -5 \end{bmatrix}$

$15 - 15 = 0$ ✓



$15 - 15 = 0$ ✓

4)

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -2 & 3 \\ 2 & 1 & 0 \end{bmatrix} \qquad [A \mid I] \xrightarrow{\text{rref}} [I \mid A^{-1}]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 1 & -2 & 3 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\substack{R_2 - R_1 \\ R_3 - 2R_1}} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & -2 & 4 & -1 & 1 & 0 \\ 0 & 1 & +2 & -2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & -2 & 4 & -1 & 1 & 0 \\ 0 & 1 & 2 & -2 & 0 & 1 \end{array}\right] \xrightarrow{-1/2 R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1/2 & -1/2 & 0 \\ 0 & 1 & 2 & -2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1/2 & -1/2 & 0 \\ 0 & 1 & 2 & -2 & 0 & 1 \end{array}\right] \xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1/2 & -1/2 & 0 \\ 0 & 0 & 4 & -5/2 & 1/2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1/2 & -1/2 & 0 \\ 0 & 0 & 4 & -5/2 & 1/2 & 1 \end{array}\right] \xrightarrow{1/4 R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1/2 & -1/2 & 0 \\ 0 & 0 & 1 & -5/8 & 1/8 & 1/4 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1/2 & -1/2 & 0 \\ 0 & 0 & 1 & -5/8 & 1/8 & 1/4 \end{array}\right] \xrightarrow{\substack{R_1 + R_3 \\ R_2 + 2R_3}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/8 & 1/8 & 1/4 \\ 0 & 1 & 0 & -3/4 & 1/4 & 1/2 \\ 0 & 0 & 1 & -5/8 & 1/8 & 1/4 \end{array}\right]$$

$2/4 - 5/4 \quad -1/2 + 1/4 \quad 0 + 1/2$

$-\frac{1}{2} + \frac{1}{4}$

$$AA^{-1} = I$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & -2 & 3 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3/8 & 1/8 & 1/4 \\ -3/4 & 1/4 & 1/2 \\ -5/8 & 1/8 & 1/4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\frac{3}{8} + \frac{12}{8} - \frac{15}{8}$

$$A^{-1} = \begin{bmatrix} 3/8 & 1/8 & 1/4 \\ -3/4 & 1/4 & 1/2 \\ -5/8 & 1/8 & 1/4 \end{bmatrix}$$