

# Exam 1

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M311

Fall 2025

Calculus 3

$$1) \quad x = 3z^2 \pm 3y^2$$

elliptic paraboloid  
along x-axis

2 var cylinder  
3 var ellipsoid  
3 var 1 pos 1 neg hyperboloid (1 or 2)  
no constant cone  
(+) 3v2sq elliptic paraboloid  
(-) 3v2sq hyperbolic paraboloid

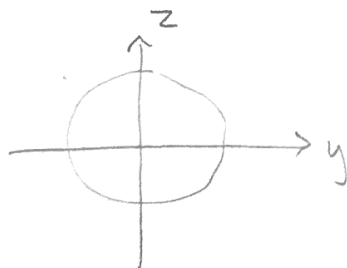
$$x = k$$

$$y = k$$

$$z = k$$

$$k = 3z^2 + 3y^2$$

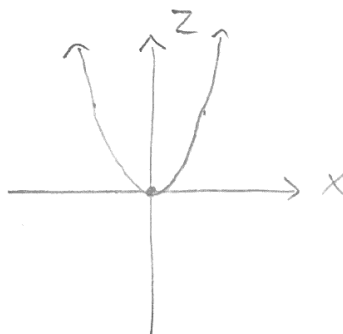
$$\frac{k}{3} = z^2 + y^2$$



circle

$$x = 3z^2 + 3k^2$$

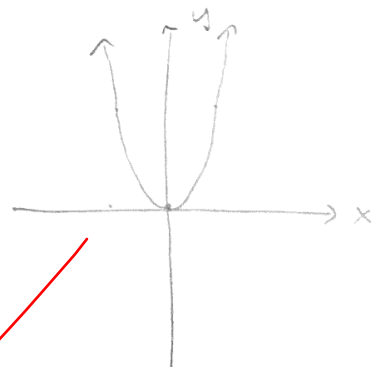
$$x = 3z^2 \quad (k=0)$$



parabola

$$x = 3k^2 + 3y^2$$

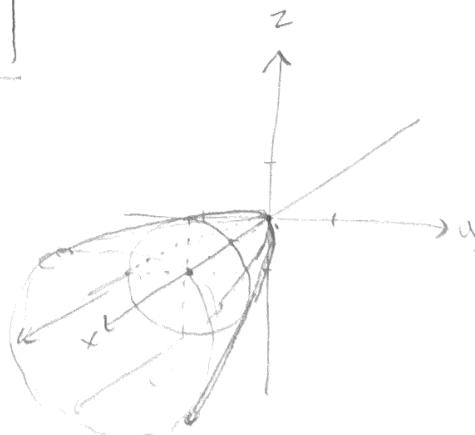
$$x = 3y^2 \quad (k=c)$$



parabola

elliptic paraboloid (circular)

directed along x-axis



2

$$4x^2 + 4y^2 - z^2 = 9$$

3 var, 3 std  
1 negative

hyperboloid of one sheet

$$\frac{x^2}{9/4} + \frac{y^2}{9/4} - \frac{z^2}{9} = 1$$

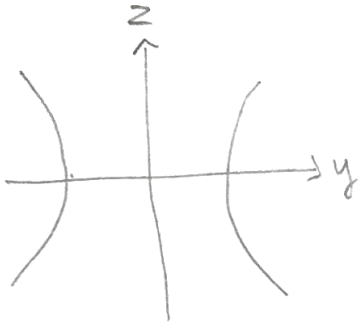
$$x = k$$

$$y = k$$

$$z = k$$

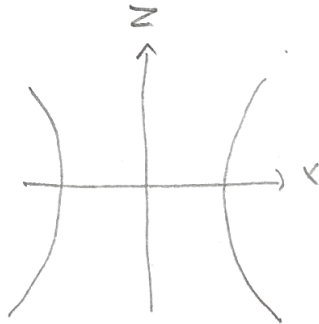
$$\frac{y^2}{9/4} - \frac{z^2}{9} = 1$$

hyperbola



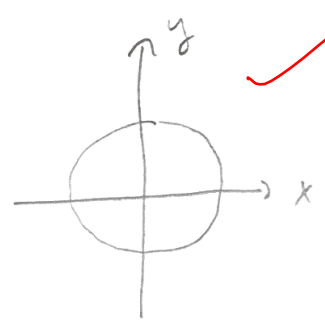
$$\frac{x^2}{9/4} - \frac{z^2}{9} = 1$$

hyperbola

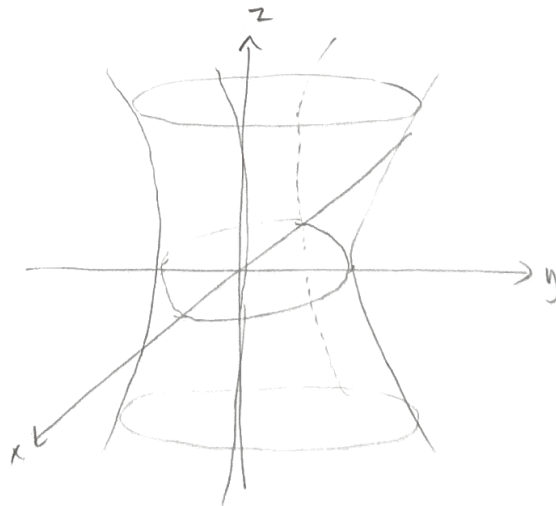


$$\frac{x^2}{9/4} + \frac{y^2}{9/4} = 1$$

circle



hyperboloid of one sheet  
directed along z-axis



$$\underline{3} \quad \text{Rec}(1, \sqrt{3}, 2) \rightarrow \text{Cyl}(r, \theta, z)$$

$$r^2 = x^2 + y^2$$

$$\theta = \tan^{-1} \frac{y}{x}$$

$$r^2 = (1)^2 + (\sqrt{3})^2$$

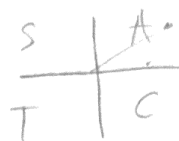
$$\theta = \tan^{-1} \frac{\sqrt{3}}{1}$$

$$r^2 = 1 + 3$$

$$\theta = \frac{\pi}{3}$$

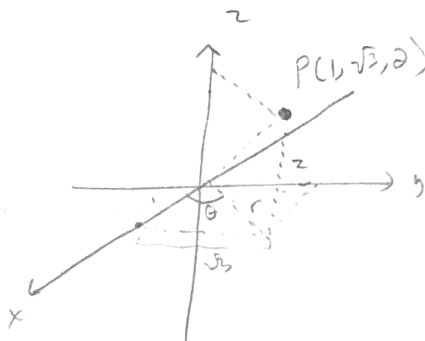
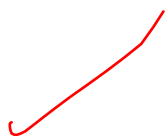
$$r^2 = 4$$

$$r = 2$$



$$\boxed{(2, \frac{\pi}{3}, 2)}$$

$$\begin{array}{ll} x = 1 & r = 2 \\ y = \sqrt{3} & \theta = \pi/3 \\ z = 2 & z = 2 \end{array}$$



4]

$$\rho = 2 \cos \phi \quad \rightarrow \text{Rec.}$$

$$\rho^2 = 2 \rho \cos \phi$$

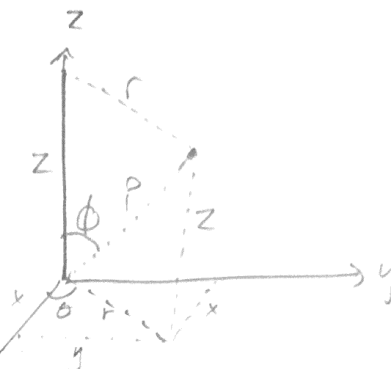
$$x^2 + y^2 + z^2 = 2z$$

$$x^2 + y^2 + z^2 - 2z = 0$$

$$x^2 + y^2 + z^2 - 2z + 1 - 1 = 0$$

$$\boxed{x^2 + y^2 + (z-1)^2 = 1}$$

Sphere of radius 1  
center at (0, 0, 1)



$$\rho^2 = x^2 + y^2 + z^2$$

$$\cos \phi = \frac{z}{\rho}$$

$$\rho \cos \phi = z$$

5

$$r(t) = \langle 3 \cos t, 3 \sin t \rangle$$

$$x = 3 \cos t$$

$$y = 3 \sin t$$

$$\frac{x}{3} = \cos t$$

$$\frac{y}{3} = \sin t$$

$$\cos^2 t + \sin^2 t = 1$$

$$\frac{x^2}{9} + \frac{y^2}{9} = 1$$

circle w/  $r=3$

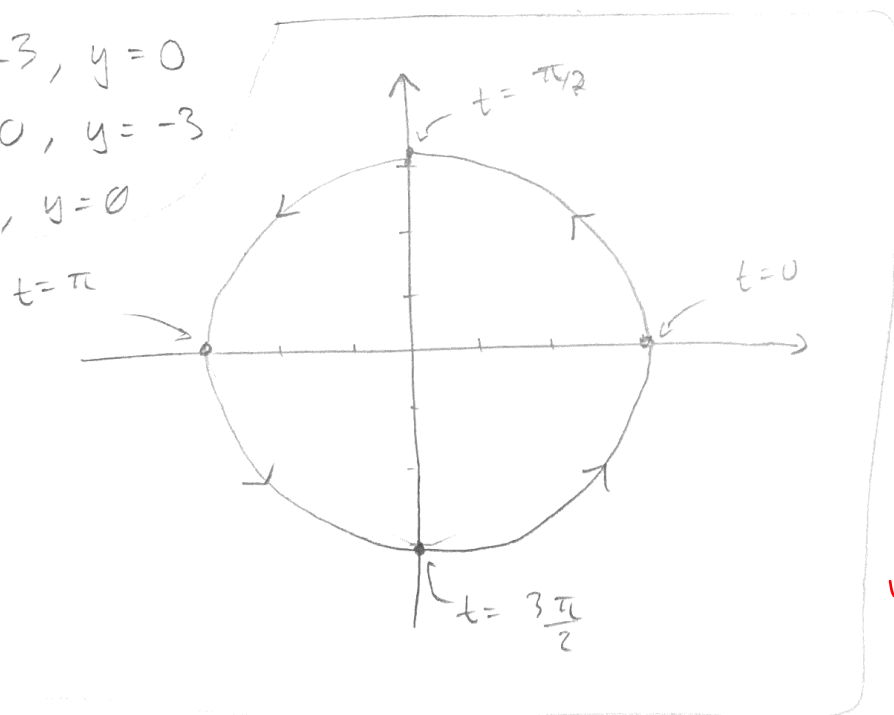
$$t=0, x=3, y=0$$

$$t=\frac{\pi}{2}, x=0, y=3$$

$$t=\pi, x=-3, y=0$$

$$t=\frac{3\pi}{2}, x=0, y=-3$$

$$t=2\pi, x=3, y=0$$



6

$$r(t) = \langle t^2, e^t, t^6 \rangle$$

$$a) \quad \frac{d}{dt} t^2 = 2t$$

$$\frac{d}{dt} e^t = e^t \cdot t' = e^t \cdot 1 = e^t$$

$$\frac{d}{dt} t^6 = 6t^5$$

$$r'(t) = \langle 2t, e^t, 6t^5 \rangle$$

$$b) \quad T(t) \quad T = \frac{v}{|v|} \quad v = r'(t)$$

$$|v| = \sqrt{(2t)^2 + (e^t)^2 + (6t^5)^2}$$

$$|v| = \sqrt{4t^2 + e^{2t} + 36t^{10}}$$

$$T(t) = \frac{1}{|v|} v = \frac{1}{\sqrt{4t^2 + e^{2t} + 36t^{10}}} \langle 2t, e^t, 6t^5 \rangle$$

$$= \frac{1}{\sqrt{40 + e^2}} \langle 2, e, 6 \rangle$$

$$T(t) = \left\langle \frac{2}{\sqrt{40 + e^2}}, \frac{e}{\sqrt{40 + e^2}}, \frac{6}{\sqrt{40 + e^2}} \right\rangle$$

7

$$r(t) = 3 \cos t \, i + 3 \sin t \, j + 4t \, k$$

$$0 \leq t \leq 2\pi$$

$$s = \int_a^b |v| \, dt$$

$$v = r'(t) = \langle -3 \sin t, 3 \cos t, 4 \rangle$$

$$|v| = \sqrt{9 \sin^2 t + 9 \cos^2 t + 16}$$

$$|v| = \sqrt{9(\sin^2 t + \cos^2 t) + 16}$$

$$|v| = \sqrt{9 + 16}$$

$$|v| = \sqrt{25} = 5$$

$$s = \int_0^{2\pi} 5 \, dt = 5t \Big|_0^{2\pi} = 5(2\pi) - 5(0) = \boxed{10\pi}$$

$$\underline{8]} \quad S = \int_0^t |v(u)| du$$

$$r(t) = 3 \cos t \mathbf{i} + 3 \sin t \mathbf{j} + 4t \mathbf{k}$$

$$v = r'(t) = \langle -3 \sin t, 3 \cos t, 4 \rangle \quad \dots \text{from } \underline{7]}$$

$$|v| = 5$$

$$S = \int_0^t 5 du = 5u \Big|_0^t = 5t - 5(0) = 5t$$

$$S = 5t$$

$$t = \frac{S}{5}$$

$$r(t(S)) = \left\langle 3 \cos\left(\frac{S}{5}\right), 3 \sin\left(\frac{S}{5}\right), 4\left(\frac{S}{5}\right) \right\rangle$$



$$9] \quad k = \frac{1}{|v|} \left| \frac{dT}{dt} \right|$$

$$r(t) = \langle 3 \cos t, 3 \sin t, 4t \rangle$$

$$v = \langle -3 \sin t, 3 \cos t, 4 \rangle$$

$$|v| = 5$$

$$T = \frac{v}{|v|} = \langle -\frac{3}{5} \sin t, \frac{3}{5} \cos t, \frac{4}{5} \rangle$$

$$\frac{dT}{dt} = \langle -\frac{3}{5} \cos t, -\frac{3}{5} \sin t, 0 \rangle$$

$$\begin{aligned} \left| \frac{dT}{dt} \right| &= \sqrt{\frac{9}{25} \cos^2 t + \frac{9}{25} \sin^2 t} \\ &= \sqrt{\frac{9}{25} (\cos^2 t + \sin^2 t)} \end{aligned}$$

$$\left| \frac{dT}{dt} \right| = \sqrt{\frac{9}{25} (1)} = \frac{3}{5}$$

$$k = \left( \frac{1}{5} \right) \left( \frac{3}{5} \right)$$

$$\boxed{k = \frac{3}{25}}$$