Homework 5

1]

$$f(x,y) = x^2 + 5xy + y^2 \quad , \quad \frac{\partial f}{\partial y} = ?$$

$$f_y = \emptyset + (1)5x + (2y)$$

$$\boxed{f_y = 5x + 2y}$$

2] $f(x,y) = \tan^{-1}(y/x)$ $\qquad$ $u = y/x$

$$\frac{d}{dx} \tan^{-1} = \frac{1}{1+x^2}$$

$$\frac{\partial f}{\partial x} = f_x = \frac{1}{1+(y/x)^2} \cdot \left(\frac{y}{x}\right)'$$

$$= \frac{1}{\frac{x^2+y^2}{x^2}} \cdot y(-x^{-2})$$

$$\frac{\partial f}{\partial x} = \frac{x^2}{x^2+y^2} \cdot -\frac{y}{x^2} = \boxed{-\frac{y}{x^2+y^2}}$$

$$\frac{\partial f}{\partial y} = f_y = \frac{1}{1+(y/x)^2} \cdot \left(\frac{y}{x}\right)'$$

$$= \frac{1}{\frac{x^2+y^2}{x^2}} \cdot \frac{1}{x}$$

$$= \frac{x^2}{x^2+y^2} \cdot \frac{1}{x}$$

$$\frac{\partial f}{\partial y} = \boxed{\frac{x}{x^2+y^2}}$$

3|

$$u = 2x^3 + 3x^2y - 6xy^2 - y^3$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

1) find $\frac{\partial u}{\partial x}$

$$\frac{\partial u}{\partial x} = 6x^2 + 6xy - 6y^2$$

2) find $\frac{\partial^2 u}{\partial x^2}$

$$\frac{\partial^2 u}{\partial x^2} = 12x + 6y$$

3) find $\frac{\partial u}{\partial y}$

$$\frac{\partial u}{\partial y} = 0 + 3x^2 - 12xy - 3y^2$$

$$= 3x^2 - 12xy - 3y^2$$

4) find $\frac{\partial^2 u}{\partial y^2}$

$$\frac{\partial^2 u}{\partial y^2} = 0 - 12x - 6y$$

5) Verify Laplace's Equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$(12x + 6y) + (-12x - 6y) = 0$$

$$0 = 0 \checkmark$$

|3

4) $z = x^3 + 2xy^2$ $(1, 2, 9)$

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$(x_0, y_0, z_0) = $ point of tangency

$f_x = 3x^2 + 2y^2$

$f_x(1,2) = 3(1) + 2(4) = 11$

$f_y = 4xy$

$f_y(1,2) = 8$

$z_0 = 9$

$z - 9 = 11(x-1) + 8(y-2)$

$z - 9 = 11x - 11 + 8y - 16$

$\boxed{z = 11x + 8y - 18}$

5] $z = x^2 + 5xy$ $X, 3 \rightarrow 3.02$

                                  $y, 4 \rightarrow 4.01$

                                      $\Delta z, dz$

1) find dz

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

$$\frac{\partial z}{\partial x} = 2x + 5y \qquad\qquad \frac{\partial z}{\partial y} = 5x$$

$$\left.\frac{\partial z}{\partial x}\right|_{(3,4)} = 6 + 20 \qquad\qquad \left.\frac{\partial z}{\partial y}\right|_{(3,4)} = 15$$
$$\qquad\qquad\qquad = 26$$

$$dx = 3.02 - 3 = 0.02 \qquad\qquad dy = 4.01 - 4 = 0.01$$

$$dz = 26(0.02) + (15)(0.01)$$
$$= 0.52 + 0.15$$
$$= 0.67$$

$$\Delta z = z(3.02, 4.01) - z(3,4)$$

$$\Delta z = \left[(3.02)^2 + 5(3.02)(4.01)\right] - \left[3^2 + 5(3)(4)\right]$$

$$= (9.120 + 60.551) - (9 + 60)$$

$$= 69.671 - 69$$

$$\Delta z = 0.671$$

$$\boxed{dz = 0.67 \quad , \quad \Delta z = 0.671}$$

5