M311 Calculus 3 2025-09-14
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3.3-1) $x = 4\cos(t)$, $y = 4\sin(t)$, $z = 3t$

$$r(t) = 4\cos(t)\hat{i} + 4\sin(t)\hat{j} + 3t\hat{k}$$

$$L = \int_a^b |v|\,dt \qquad v = r'(t)$$

$$v(t) = r'(t) = -4\sin t\,\hat{i} + 4\cos t\,\hat{j} + 3\hat{k}$$

$$|v| = \sqrt{16\sin^2 t + 16\cos^2 t + 9} = \sqrt{16(\sin^2 t + \cos^2 t) + 9}$$

$$= \sqrt{16 + 9} = \sqrt{25} = 5$$

$$L = \int_a^b 5\,dt = 5t\Big|_a^b = 5(b-a)$$

... hmm, find xy trace.

$$\cos(t) = \frac{x}{4} \qquad \sin(t) = \frac{y}{4}$$

$$\cos^2(t) + \sin^2(t) = 1$$

$$\frac{x^2}{16} + \frac{y^2}{16} = 1$$

, circle w/ radius 4
2 var, cylinder directed along z-axis

As $x = 4\cos t$ and $y = 4\sin t$ have the same period (2π), as t increases by 2π, both x & y return to starting values. Helix completes one full revolution around cylinder every 2π units of t. $z = 3t$, so in 2π units of t, z "climbs up" 6π units in the (+) z-direction. So we can use $t \in [0, 2\pi]$ to calculate arc length of one revolution.

$$L = \int_0^{2\pi} 5\,dt = 5t\Big|_0^{2\pi} = 5(2\pi) - 5(0) = \boxed{10\pi}$$

3.3-2)

$$s(t) = \int_{t_0}^{t} |v(\tau)| d\tau$$

$$= \int_0^t 5 \, d\tau$$

$$= 5t$$

$$t = 5/s$$

$$\boxed{r(t(s)) = 4\cos\left(\tfrac{5}{s}\right)\hat{i} + 4\sin\left(\tfrac{5}{s}\right)\hat{j} + \tfrac{15}{s}\hat{k}}$$

3.3-3) Find $\kappa = \left|\frac{dT}{ds}\right|$

unit tangent vector

arc length

kappa/curvature

Curvature is defined as the change in the Unit tangent vector w.r.t. arc length

$$\kappa = \left|\frac{dT}{dt} \cdot \frac{dt}{ds}\right|$$ — if parameterized by t and not arclength s

Recall $\frac{ds}{dt} = |v|$

$$T = \frac{v}{|v|} \qquad \kappa = \left|\frac{dT}{dt} \cdot \frac{1}{ds/dt}\right| = \frac{1}{|v|}\left|\frac{dT}{dt}\right|$$

$|v| = 5$ (from 3.3-1)

$$r = \langle 4\cos t, 4\sin t, 3t\rangle$$

$$r'(t) = v = \langle -4\sin t, 4\cos t, 3\rangle$$

$$|v| = \sqrt{16\sin^2 t + 16\cos^2 t + 9}$$

$$= \sqrt{16(\sin^2 t + \cos^2 t) + 9}$$

$$= \sqrt{16(1) + 9} = \sqrt{25} = 5$$

$$T = \frac{v}{|v|} = \langle -\tfrac{4}{5}\sin t, \tfrac{4}{5}\cos t, \tfrac{3}{5}\rangle$$

$$\frac{dT}{dt} = \langle -\tfrac{4}{5}\cos t, -\tfrac{4}{5}\sin t, 0\rangle$$

$$\left|\frac{dT}{dt}\right| = \sqrt{\tfrac{16}{25}\cos^2 t + \tfrac{16}{25}\sin^2 t + 0}$$

$$= 4/5$$

$$\kappa = \frac{1}{|v|}\left|\frac{dT}{dt}\right| = \frac{1}{5} \cdot \frac{4}{5} = \boxed{\frac{4}{25}}$$

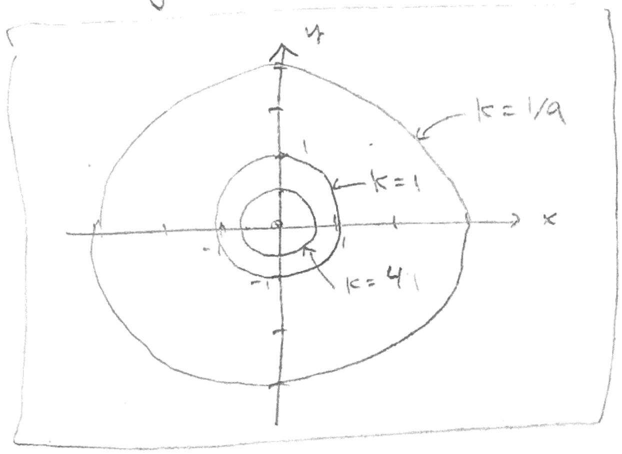
4.1-1

$$f(x,y) = \frac{1}{x^2+y^2}, \quad k = \frac{1}{9}, 1, 4$$

$$k = \frac{1}{x^2+y^2}$$

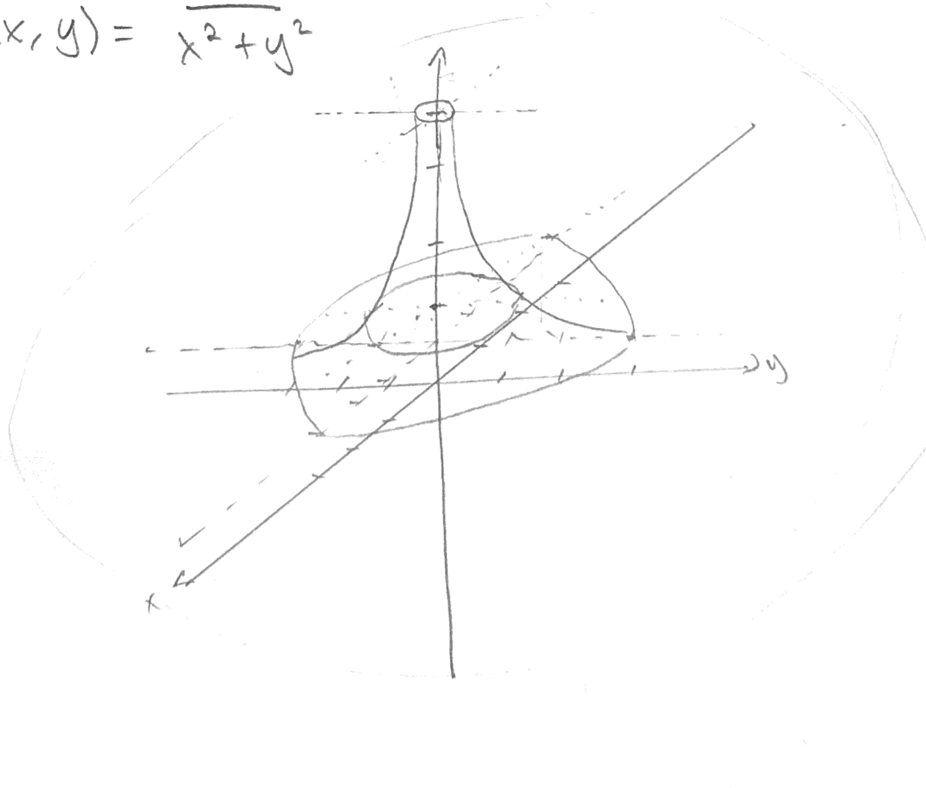
$$\frac{1}{k} = x^2+y^2 \quad \text{circles!}$$

if $k = 1/9$, $x^2+y^2 = 9$, $r = 3$
if $k = 1$, $x^2+y^2 = 1$, $r = 1$
if $k = 4$, $x^2+y^2 = 1/4$, $r = 1/2$



4.1-2

$$f(x,y) = \frac{1}{x^2+y^2}$$



4.1-3

$f(x,y,z) = x^2 + y^2 - z$, $k = 1$

3 var, 2 sq. @ neg,

$1 = x^2 + y^2 - z$

$z = x^2 + y^2 - 1$

$\boxed{\text{Elliptic (Circular) Paraboloid}}$

vertex $(0,0,-1)$

at $z = 0$, $x^2+y^2 = 1$

at $z = 3$, $x^2+y^2 = 4$

For $k = 1$, $1 = x^2 + y^2 - z$

$z = x^2 + y^2 - 1$