M311 Calculus 3

Fall 2025

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Homework 1

2.6, 1) $4x = y^2 + z^2$

[3 var, 2 sq², 1 sq² are (+)
constant = 0
directed along x-axis

Paraboloid, $x=k$, $y=k$, $z=k$

if $x=0$, $(0,0,0)$

if $y=0$, $x = \frac{1}{4}z^2$

if $z=0$, $x = \frac{1}{4}y^2$

if $x=1$, $1 = \frac{y^2}{4} + \frac{z^2}{4}$

if $x=-1$, $-4 \neq y^2+z^2$

if $x=4$, $1 = \frac{y^2}{16} + \frac{z^2}{16}$

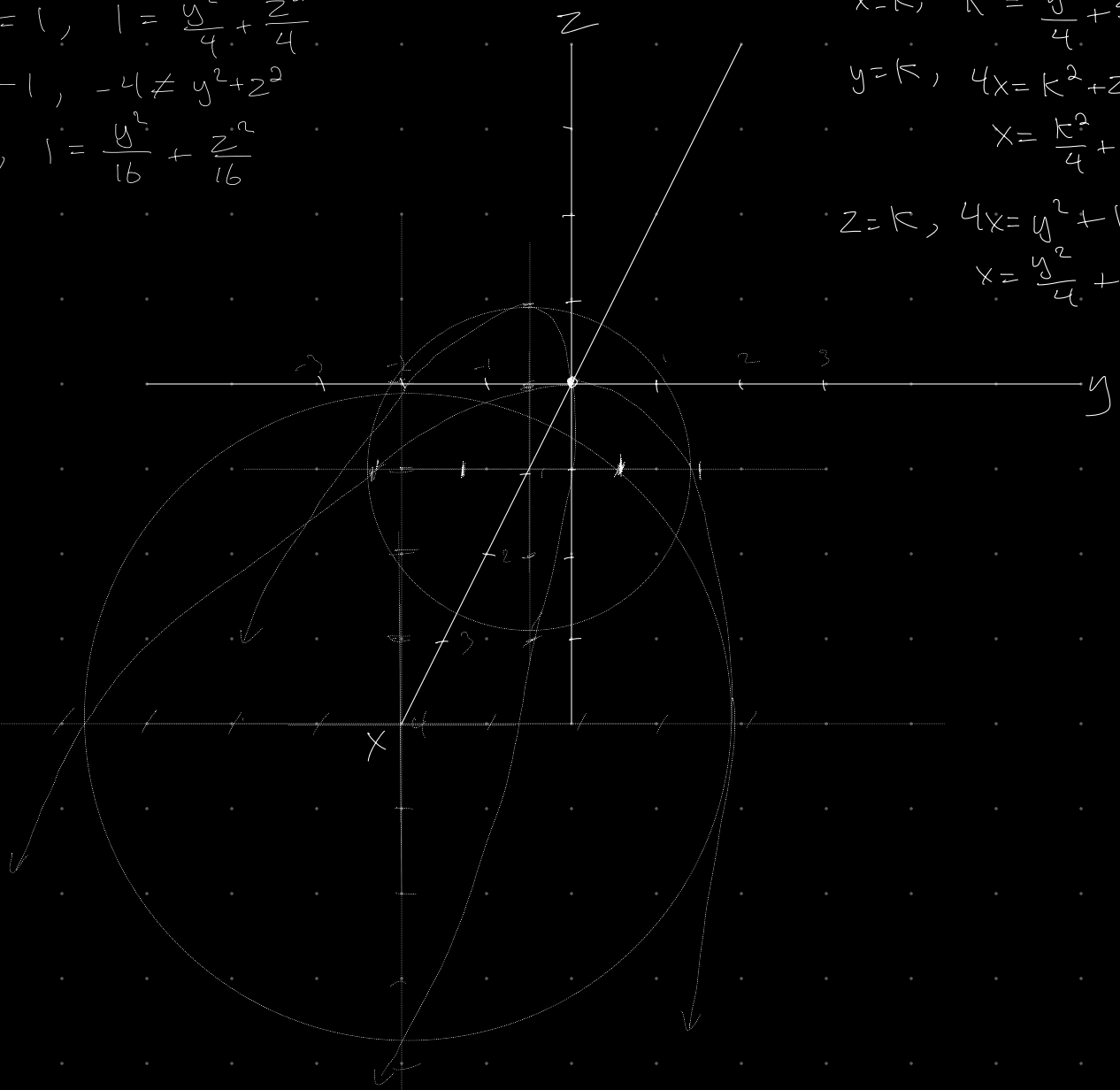
$x=k$, $k = \frac{y^2}{4} + \frac{z^2}{4}$

$y=k$, $4x = k^2 + z^2$

$x = \frac{k^2}{4} + \frac{z^2}{4}$

$z=k$, $4x = y^2 + k^2$

$x = \frac{y^2}{4} + \frac{k^2}{4}$



2.6, 2)

$$x^2 + 4y^2 + z^2 - 2x + 8y + 4 = 0$$

$$x^2 - 2x + 4y^2 + 8y + z^2 + 4 = 0$$

$$(x^2 - 2x + 1 - 1) + 4(y^2 + 2y) + z^2 + 4 = 0$$

$$((x-1)^2 - 1) + 4(y^2 + 2y + 1 - 1) + z^2 + 4 = 0$$

$$(x-1)^2 - 1 + 4(y+1)^2 - 4 + z^2 + 4 = 0$$

$$(x-1)^2 + 4(y+1)^2 + z^2 = 1$$

$$\frac{(x-1)^2}{4} + \frac{(y+1)^2}{1} + \frac{z^2}{4} = \frac{1}{4}$$

3 var, 3 sq^2, all (+)

Ellipsoid

$z = 0$, $\frac{(x-1)^2}{1} + \frac{(y+1)^2}{1/4} = 1$

$(1, -1)$
center

$x = 1$, $\frac{(y+1)^2}{1/4} + \frac{z^2}{1} = 1$

2.7, 1)

$(-4, 4\sqrt{3}, 4)$, $R(x,y,z) \rightarrow C(r,\theta,z)$

$r^2 = x^2 + y^2$

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{16 + 48}$

$r = \sqrt{64}$

$r = 8$

$\theta = \arctan \frac{y}{x}$

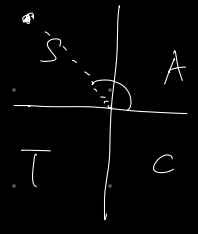
$\theta = \arctan \frac{4\sqrt{3}}{-4}$

$\theta = \arctan \frac{-\sqrt{3}}{1}$

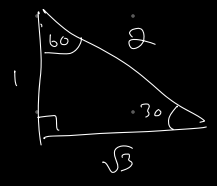
$\theta = \frac{2\pi}{3}$

$z = z$

$z = 4$



$\therefore (8, \frac{2\pi}{3}, 4)$



$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

2.7, 2) $z^2 = 1 + x^2 + y^2$

$z^2 = 1 + r^2$

$R \rightarrow C$

$r^2 = x^2 + y^2$

2.7, 3) $z = 4r\cos\theta$

$\boxed{z = 4x}$

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

2.7, 4) $R(0, 1, \sqrt{3})$ $\quad R \to S\ (\rho, \theta, \psi)$

$x = 0$

$y = 1$

$z = \sqrt{3}$

$r^2 = 0 + 1$

$r^2 = 1$

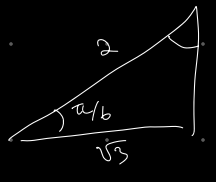
$r = 1$

$\rho^2 = 1^2 + (\sqrt{3})^2$

$\rho^2 = 4$

$\underline{\rho = 2}$

$\psi = \arctan \frac{1}{\sqrt{3}}$

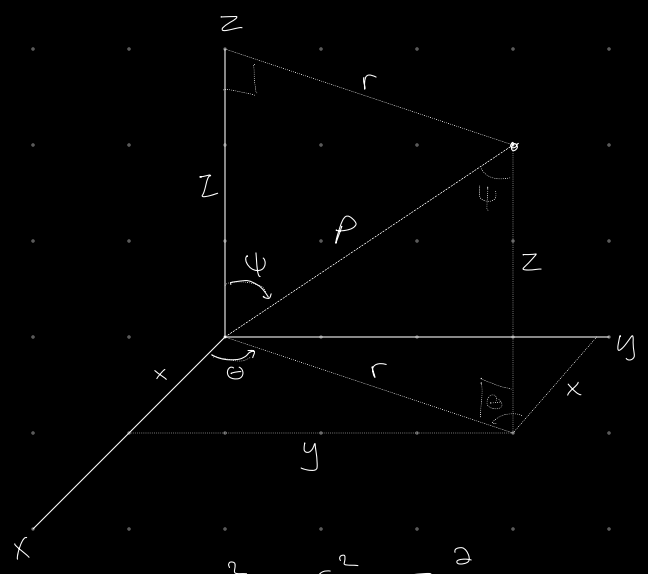


$\underline{\psi = \pi/6}$

$\theta = \arcsin \frac{y}{r} = \arcsin \frac{1}{1}$

$= \pi/2$

$\therefore \left(2, \frac{\pi}{2}, \frac{\pi}{6}\right)$



$\rho^2 = r^2 + z^2$

$\psi = \arctan \frac{r}{z}$

$r^2 = x^2 + y^2$

$\theta = \arctan \frac{y}{x}$

$\theta = \arcsin \frac{y}{r}$

27, 5) $\rho = 2\sin(\theta)\sin(\psi)$ in R?

(x, y, z)

$\rho = 2\left(\frac{y}{r}\right)\left(\frac{r}{\rho}\right)$

$\rho = 2\left(\frac{y}{\rho}\right)$

$\rho^2 = 2y$

$\rho^2 = r^2 + z^2$

$r^2 = x^2 + y^2$

$\rho^2 = x^2 + y^2 + z^2$

$x^2 + y^2 + z^2 = 2y$

$x = r\cos\theta \quad y = r\sin\theta$

$x^2 + y^2 - 2y + z^2 = 0$

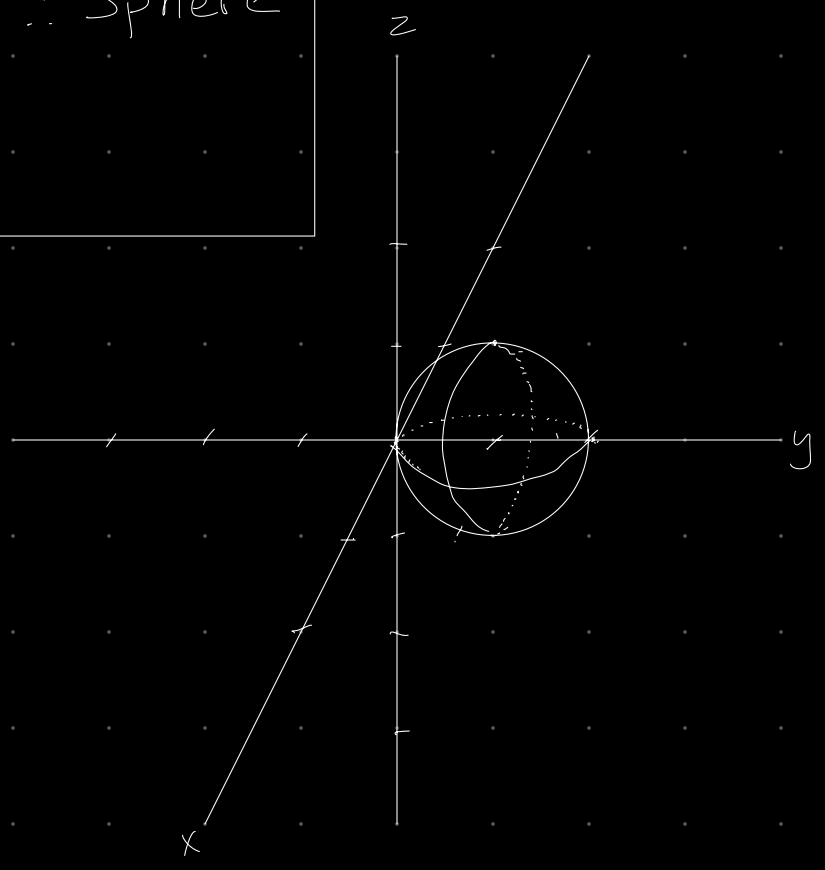
$\sin\psi = r/\rho \quad \cos\psi = \frac{z}{\rho}$

$x^2 + (y-1)^2 + z^2 = 1$

$r = \rho\sin\psi \quad z = \rho\cos\psi$

$$\frac{x^2}{1} + \frac{(y-1)^2}{1} + \frac{z^2}{1} = 1$$

All (+)
All sq^2
has constant r all equal
} ∴ Sphere



xy
zy
zx