Anthony Chao
M 311 - Week 5

Week 5 Homework 4

Problem 4

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x^2+y^2+1}$$

As $(x,y)\to(0,0)$: $x^2+y^2 \longrightarrow 0$

$x^2+y^2+1 \longrightarrow 1$

$\longrightarrow \frac{0}{1}$

$$\boxed{\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x^2+y^2+1} = 0}$$

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Problem 5

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$$

Along $y=mx$

$$\frac{x^2-(mx)^2}{x^2+(mx)^2} = \frac{x^2-m^2x^2}{x^2+m^2x^2} = \frac{x^2(1-m^2)}{x^2(1+m^2)}$$

$$= \frac{1-m^2}{1+m^2}$$

if $m=0$, $\frac{(1-0)}{(1+0)} = 1$

if $m=1$ $\frac{1-1}{1+1} = \frac{0}{2} = 0$

if $m=2$ $\frac{1-4}{1+4} = -\frac{3}{5}$

Limit does not exist because we get different limit values along linear paths.

Problem 6

$$f(x,y) = \frac{x+y}{y-2x+1}$$ where is continuous?

$y - 2x + 1 = 0$

$\boxed{y = 2x - 1}$ ——— this is the equation of a line where the function is undefined.

The function is continuous at all points (x,y) except those on the line $y = 2x - 1$.

$$\boxed{\{(x,y) \in \mathbb{R}^2 : y \neq 2x - 1\}}$$