

Pawan Acharya
Convex Hull
Project Report

Convex hull

The Convex Hull is the line completely enclosing a set of points in a plane so that there are no concavities in the line.

Algorithmic pseudocode:

findHull(points):

Sort points

div and conquer(points):

LH points - find leftmost point

RH points - find rightmost point

Find Upper tangent

Find lower tangent

Empty list() -for storing convex hull points

Merge the two halls:

 From leftmost point, move clockwise until upper leftmost point

 Add the points

 Counterclockwise check

 Add the points from upper right to lower right

Return the convex list

Time Complexity:

The major subpart complexities included the following:

Sorting the points by X value : $O(n \log n)$

Dividing the list into n sublists : $O(n)$

Merging the lists to find the convex hull : $O(n \log n)$

Using the master theorem we are able to compute the overall complexity of the algorithm.

Given $T(n) = aT(n/b) + O(n^d)$ we see that the problem divides the problem into n subproblems of size $n/2$. Merging all of the problems back together is constant time and thus results in $O(n)$.

The master theorem then comes out to be $T(n) = nT(n/2) + O(n)$. Thus the overall time complexity for solving this problem is $O(n \log n)$.

Space complexity: I think that the worst case for space complexity will be $O(n \log n)$ assuming that all points are in the convex hull. This means that the algorithm will compute $\log n$ memory cells deep at most n times.

Theoretical analysis:

The divide step and conquer steps require $O(n \log n)$ time. So, the recurrence relation for the divide and conquer part is: $T(n) = 2T(n/2) + O(n)$. Which gives the complexity of $O(n \log n)$. The initial pre-sort requires extra $O(n \log n)$ time. The overall complexity is, therefore, $O(n \log n)$.

Data Analysis of the plot :

The data plot about the algorithm that I got is included in the excel file.

From the data, the plot appears to be a polynomial function of order 2 (approx. analysis).

I think that there is negligible difference between the approximated data and the experimental data. There might be differences due to the bad coding practices and time taken to draw the lines by the GUI.

The source code, screenshot of demo and the excel plot is included in the project submission zip file.