Design Experience 5

Travelling Salesperson Problem

Problem Let's say we have a graph with cost matrix:

$$\begin{bmatrix} inf & 20 & 30 & 10 & 11 \\ 15 & inf & 16 & 4 & 2 \\ 3 & 5 & inf & 2 & 4 \\ 19 & 6 & 18 & inf & 3 \\ 16 & 4 & 7 & 16 & inf \end{bmatrix}$$

$$\begin{bmatrix} inf & 10 & 20 & 0 & 1 \\ 13 & inf & 14 & 2 & 0 \\ 1 & 3 & inf & 0 & 2 \\ 16 & 3 & 15 & inf & 0 \\ 12 & 0 & 3 & 12 & inf \end{bmatrix} \begin{matrix} 10 \\ 2 \\ 2 \\ 3 \\ 4 \end{matrix}$$

$$\begin{bmatrix} inf & 10 & 17 & 0 & 1 \\ 12 & inf & 11 & 2 & 0 \\ 0 & 3 & inf & 0 & 2 \\ 15 & 3 & 12 & inf & 0 \\ 11 & 0 & 0 & 12 & inf \end{bmatrix}$$

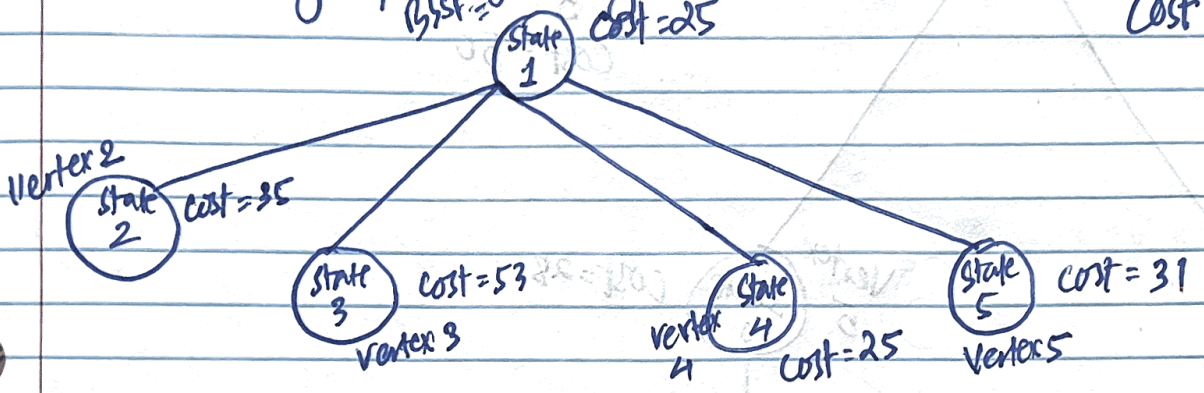
1 3

Now,

Constructing space tree using branch and bound

cost = 10+2+2+3+4+1+3

Cost = 25



For state 2 : cost = 25 + 0 + 10 = 35 [∵ Since the matrix is reduced → 0]

So, Resulting matrix is :

$$\begin{bmatrix} inf & inf & inf & inf & inf \\ inf & inf & 11 & 2 & 0 \\ 0 & inf & inf & 0 & 2 \\ 15 & inf & 12 & inf & 0 \\ 11 & inf & 0 & 12 & inf \end{bmatrix}$$

For state 3 : Reduce column 1 by 11. , cost = 25 + 11 + 17 = 53

Resulting matrix :

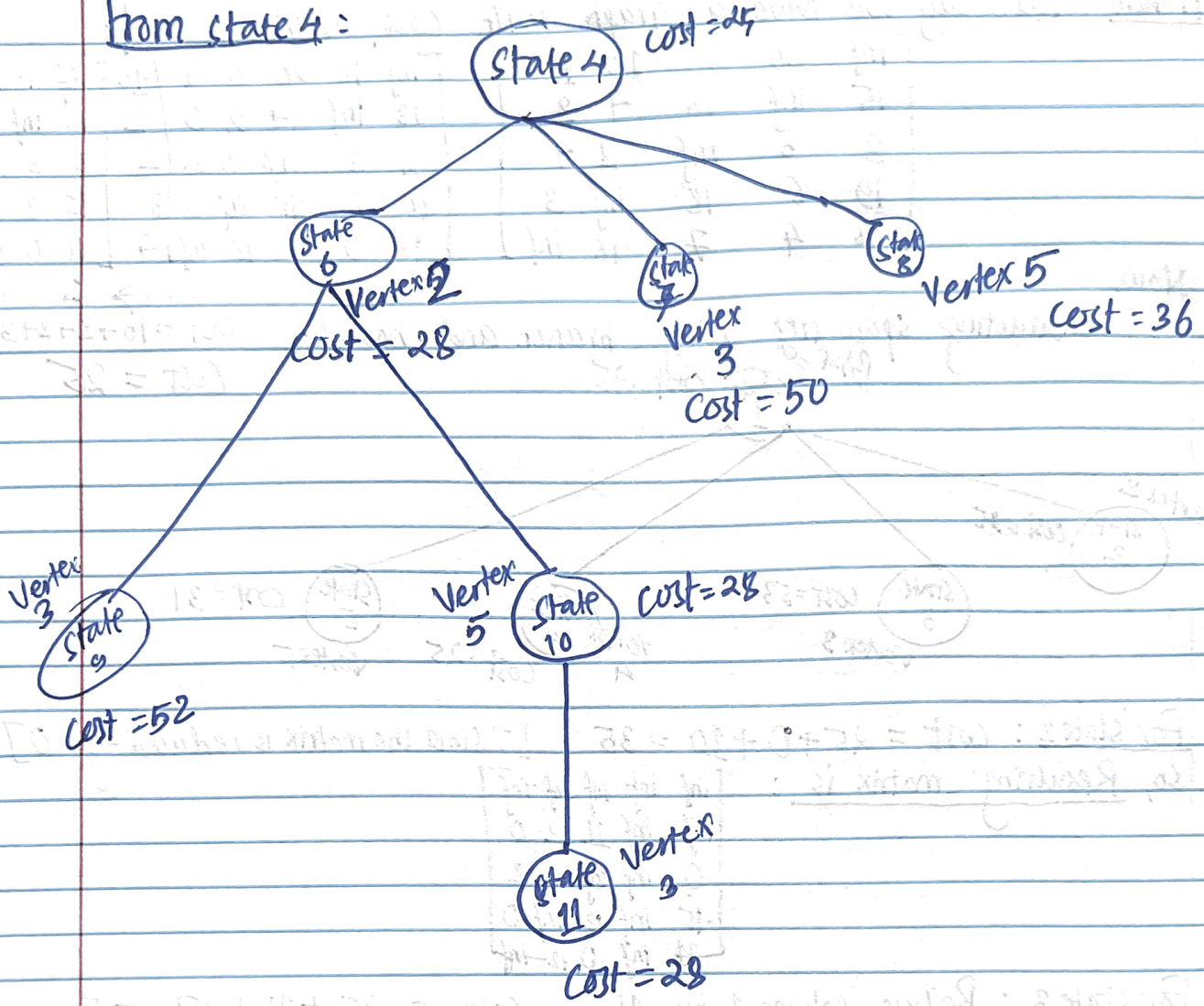
$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & \infty \end{bmatrix}$$

Similarly for state 4 : cost = 25 + 0 + 0 = 25

" " state 5 : cost = 25 + 5 + 1 = 31

Since, the smallest is state 4 with cost 25, we proceed in this node

From state 4 :



For state 6 : cost = 25 + 0 + 3 = 28

For state 7 : Cost = 25 + 13 + 12 = 50

For state 8 : cost = 25 + 11 + 0 = 36

so, The lowest cost is state 6 with 28.

so,

For state 9 : Cost = 28 + 13 + 11 = 52

For state 10 : cost = 28 + 0 + 0 = 28

so, lowest is state 10 with 28.

Similarly,

For state 11 :

$$\text{cost} = 28 + 0 + 0$$

The resulting matrix will be:

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

Thus, the path therefore is :

Solution path : $1 \rightarrow 4 \rightarrow 2 \rightarrow 5 \rightarrow 3 \rightarrow 1$

And min cost : 28.