

Modular exponentiation :

Let's say we have  $16^{10} \bmod 3$   $\rightarrow$  unsolvable easily

$$((16^5 \bmod 3) (16^5 \bmod 3)) \bmod 3$$

$$((16 \bmod 3) (16^4 \bmod 3)) \bmod 3$$

$$((16 \bmod 3) (16^4 \bmod 3)) \bmod 3$$

$$(1 \cdot (16^4 \bmod 3)) \bmod 3$$

$$(1 \cdot (16^4 \bmod 3)) \bmod 3$$

$$(1 \cdot 1) \bmod 3$$

$$(1 \cdot 1) \bmod 3$$

11  
1

11  
1

1 Ans

## Miller Rabin Primality Test

Step 1: find  $n-1 = 2^k \times m$

Step 2: Choose 'a' such that  $1 < a < n-1$

Step 3: compute  $b_0 = a^m \pmod{n}$ ;  $\dots$   $b_i$

↓ Result of  $b_i$

$+1 \rightarrow$  Composite

$-1 \rightarrow$  Probabably Prime.

$$b_i = b_{i-1}^2 \pmod{n}$$

eg. 561

$$n = 561$$

1)  $560 = 2^4 \times 35$ ,  $k=4$ ,  $m=35$

2)  $a=2$ ,  $a > 1$  and  $a < 560$

3)  $b_0 = 2^{35} \pmod{561}$

↓  
This is modular  
exponentiation (use it)

is  $b_0 = \pm 1 \pmod{561} \Rightarrow \text{No}$

↓ so  
Calculate  $b_1 = b_0^2 \pmod{n} = 263^2 \pmod{561}$

$$b_3 = 1 \pmod{561} \rightarrow \text{composite.}$$