t-test Lab

The t-test is used to determine if there is a significant difference between the means of two groups.

It is commonly applied when the sample size is small ($n < 30$), and the population standard deviation is unknown.

The test statistic t is calculated as:

$$t = \frac{Difference\ in\ means}{Standard\ error\ of\ the\ difference}$$

Types of t-test:

1. One-Sample t-Test:

Compares the mean of a single sample to a known population mean.

One-Sample t-Test:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Where:

- $\bar{x}$: Sample mean
- μ: Population mean
- s: Sample standard deviation
- n: Sample size

Lab Question 1: A teacher claims that the average score of students in his class is 75. A sample of 10 students is taken, and their scores are as follows:

70, 80, 85, 90, 75, 60, 80, 85, 90, 95

We want to test whether the average score of this sample is significantly different from the claimed population mean of 75.

2. Independent (Two-Sample) t-Test or Unpaired t-Test:

Compares the means of two independent groups.

Independent Two-Sample t-Test:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Where:

- $\bar{x}_1, \bar{x}_2$: Sample means
- s_1^2, s_2^2: Sample variances
- n_1, n_2: Sample sizes

Lab Question 2: A researcher wants to compare the average heights of male and female students in a college. Two random samples are selected:

- Sample 1 (Male Students): 170, 165, 180, 175, 160, 172, 168, 177, 165, 180
- Sample 2 (Female Students): 160, 155, 150, 158, 165, 157, 162, 155, 160, 158

Test whether there is a significant difference in the average heights of male and female students at the 0.05 significance level.

3. Paired t-Test (Dependent t-Test):

Compares the means of the same group under different conditions.

Paired t-Test:

$$t = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}}$$

Where:

- $\bar{d}$: Mean of the differences between paired observations
- s_d: Standard deviation of the differences
- n: Number of pairs

Lab Question 3:

A company wants to assess whether a new training program improves employee productivity. The productivity of 12 employees is measured before and after attending the training program. The scores (measured in units of productivity) are as follows:

Before Training: 40, 50, 60, 45, 55, 48, 62, 49, 41, 53, 47, 59
After Training: 45, 55, 65, 50, 58, 52, 67, 54, 43, 58, 50, 62

Test whether the training program has a significant effect on productivity at a significance level of 0.05. Use a paired t-test for your analysis.

Z-test

One sample Z-test:

Lab Question 4:

A manufacturer claims that the average lifetime of a light bulb is 1,000 hours. You take a random sample of 50 light bulbs and find that the sample mean lifetime is 980 hours. The population standard deviation is known to be 80 hours. Test at the 0.05 significance level whether the manufacturer's claim is accurate.

Two sample Z-test:

Lab Question 5:

A researcher wants to compare the average scores of two groups of students who took different teaching methodologies.

- Group 1 (Method A): $\bar{X}1 = 75$, $n1 = 30$, $\sigma1=10$
- Group 2 (Method B): $\bar{X}2 = 70$, $n2 = 35$, $\sigma2=12$

The researcher wants to test if there is a significant difference between the two groups at a $\alpha=0.05$ significance level.

Steps for Conducting a t-Test:

1. **State the Hypotheses**:
   - Null Hypothesis (H_0): No significant difference ($\mu_1 = \mu_2$).
   - Alternative Hypothesis (H_a): Significant difference ($\mu_1 \neq \mu_2$, $\mu_1 > \mu_2$, or $\mu_1 < \mu_2$).
2. **Calculate the t-Statistic**:
   - Use the appropriate formula based on the type of t-test.
3. **Determine the Degrees of Freedom (df)**:
   - For a one-sample t-test: $df = n - 1$.
   - For an independent two-sample t-test: $df = n_1 + n_2 - 2$.
   - For a paired t-test: $df = n - 1$.
4. **Find the Critical t-Value or p-Value**:
   - Use a t-distribution table or statistical software.
5. **Make a Decision**:
   - If $|t| > t_{\text{critical}}$ or $p < \alpha$ (significance level, e.g., 0.05), reject the null hypothesis.

Note:

Unequal Variances (Welch's t-Test)

If the variances of the two groups are significantly different, we use Welch's t-test, which does not assume equal variances. The formula for df becomes:

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

Where:

- s_1^2: Variance of the first sample.
- s_2^2: Variance of the second sample.
- n_1, n_2: Sample sizes.