ELECTROMAGNETICS
Tutorial: Electric Field

Electric Field Intensity and Electric Flux Density

1. A point charge if 12nC is located at the origin. Four uniform line charges are located in the x=0 plane as follow: 80nc/m at y=-1 and -5m, -10 nC/m at y=-2 and -4m. Find the electric flux density D at P (0, -3, 2).
2. An infinite uniform line charge ρ_l = 2 nC/m lies along the x-axis in free space, while the point charge of 8nC each are located at (0, 0, 1). Find $\vec{E}$ at (2, 3, -4).
3. A uniform line charge density of 2-nC/m is located at y = 3 and z = 5. Find $\vec{E}$ at P (5, 6, 1).
4. Alone the z-axis there is a uniform line of charge $\rho_L = 4\pi$ Cm^{-1} and the x = 1 plane there is a surface charge with ρ_s = 20 cm^{-2}. Find the Electric Flux Density at (0.5, 0, 0).
5. Find the Electric flux density at point P (5, 4, 3) due to a uniform line charge of 2nC/m at x = 5, y=3, point charge 12 nC at Q(2, 0, 6) and uniform surface charge density of 0.2 nC/m^2 at x=2.
6. A uniform line charge density of 150μ C/m lies at x= 2, z = -4 and a uniform sheet of change equal to 25 nC/m^2 is placed at z = 5 plane. Find $\vec{D}$ at point (1, 2, 4) and convert it to the spherical coordinate system.
7. Find the electric field intensity in all three regions due to an infinite sheet parallel plate capacitor having surface charge density ρ_s c/m^2 and $-\rho_s$ c/m^2 and placed at y 0 and y = respectively.
8. Let a uniform line charge density, 3 nC/m, at y = 3; uniform surface charge density, 0.2 nC/m^2 at x = 2. Find $\vec{E}$ at the origin.
9. Find D at the point (-3, 4, 2) if the following charge distributions are present in free space point charge: +2 nC, at P (2, 0, 6); uniform line charge density, 3 nC/m, at x = -2, y = 3; uniform surface charge density, 0.2 nC/m^2 at x=2.
10. Surface charge densities of 200, -50 and ρ µC/m^2 are located at r=3, 5 and 7cm respectively. Find $\vec{D}$ at (i) r = 1cm (ii) r = 4.8cm (iii) r = 6.9cm. Find ρ if $\vec{D}$ = 0 at r = 9cm.
11. An infinite uniform line charge ρL = 2nC/m lies along the x-axis in free space, while point charges of 8nC each are located at (0, 0, 1) and (0, 0, -1). (a)Find $\vec{D}$ at (2, 3, 4).
12. Two uniform line charges, each 20 $_n$C/m, are located at y= 1, z= ±1m. Find the total electric flux leaving the surface of a sphere having a radius of 2 m, if it is centred at A (3, 1, 0).

Divergence and Divergence Theorem

13. State the physical significance of divergence. Derive the divergence theorem. Given the potential V =$\frac{10}{r^2}$ $sin\theta cos\emptyset$; find the electric density $\vec{D}$ at (2, $\frac{\pi}{2}$, 0).
14. Evaluate both sides of divergence theorem for the field **D** = 2xy $\mathbf{a_x}$ + x^2 $\mathbf{a_y}$ C/m2 and the rectangular parallelopiped formed by the planes x = 0 and 1, y = 0 and 2, and z = 0 and 3.
15. Given the field $\vec{D} = \frac{20}{\rho 2}(-sin^2\phi\ \vec{a}_r + \sin 2\emptyset\ \vec{a}_\phi)$, evaluate both sides of the divergence theorem for the region bounded 1 < ρ < 2, 0 < φ < 90°, 0<z<1.
16. Given the flux density $\vec{D}$ = ($2cos\ cos\ \theta$ /r^3) $\vec{a}_r$ + ($\sin\theta/r^3$) $\vec{a}_\theta$ C/m^2, evaluate both sides of the divergence theorem for the region defined by 1<r<2, 0<$\frac{\pi}{2}$, 0<$\emptyset < \frac{\pi}{2}$.

17. Given the field, $\vec{D} = \frac{5sinsin\,(\theta)\,cos\,(\emptyset}{r}$ a_r C/m2, find: (a) the volume charge density; (b) the total charge contained in the region r<2m; (c) the value of D at the surface r=2.
18. Verify the divergence theorem (evaluate both sides of the divergence theorem) for the function $\vec{A}$ =r^2 $\vec{a}_r$+rsin$\theta cos\emptyset\overrightarrow{a_\theta}$, over the surface of quarter of a hemisphere defined by: 0<r<3, 0<$\emptyset < \frac{\pi}{2}, 0 < \theta < \pi/2$.

Electric Potential and Energy

19. Two uniform line charges, 8 nC/m each, are located at x = 1, z = 2, and at x = -1, y = 2 in free space. If the potential at the origin is 100V, find V at P (4, 1, 3).
20. A uniform sheet of charge ρ_s=40 C/m^2 is located in the plane x = 0 in free space. A uniform line charge ρ_L = 0.6 nC/m lies along the line x = 9, y = 4 in free space. Find the potential at point P (6, 8, -3) if V = 10V at A (2, 9, 3).
21. Find the energy stored in free space for the region 2mm$< r < 3mm$, 0$< \theta < 90°, 0 < \emptyset < 90°$, find the potential field if V=:

    a) $\frac{200}{r} V\ and\ \ b)\ \frac{300}{r^2}\ cos\theta V.$

22. Derive an expression to calculate the potential due to a dipole in terms of the dipole moment ($\vec{P}$). A dipole for which $\vec{P} = 3\overrightarrow{a_x}$ - $5\overrightarrow{a_y}$+$10\overrightarrow{a_z}$ nCm is located at the point (1, 2, -4). Find $\vec{E}$ at P.
23. A line charge of 8nC/m is located at x = -1, y = 2, a point charge of 6mC at y = -4 and a surface charge of 30 ρC/m^2 at z = 0. If the potential at origin is 100V, find the potential at P (4, 1, 3).
24. Given the potential function V = $\frac{20cos\theta}{r^2}$V in free space and point P is located at r = 3m, $= 60°, \emptyset = 30°$. Find a) $\vec{E}_P$ b) $\frac{dV}{dN}$ at P c) unit normal vector at p d) ρ_v at P.
25. The conducting planes 2x+3y= 12 and 2x+3y= 18 are at potentials of 100V and 0, respectively. Let ϵ= ϵ_0 and find: a) V at p (5, 2, 6); b) E at p (5, 2, 6).
26. Given the potential field V= 100xz/(x^2+4) volts in free space:

    a) Find $\vec{D}$ at the surface, z=0

    b) Show that the z=- surface is an equipotential surface

    c) Assume that the z=0 surface is a conductor and find the total charge on that portion of the conductor defined by 0<x<2, -3<y<0.

Current and Current Density

27. Derive the integral and point forms of continuity equation. In certain region, $\vec{J}$ = 3r^2 cos$\theta\overrightarrow{a_r}$ – r^2 sin$\theta\overrightarrow{a_\theta}$ A/m^2. Find the current crossing the surface define by $\theta = 30°$, 0<$\emptyset < 2\pi, 0 < r < 2$.
28. Using the continuity equation elaborate the concept of Relaxation Time Constant (RTC) with the necessary derivations. Let $\vec{J} = \frac{e^{-10^4 t}}{\rho^2}\hat{a}_\rho$A/m^2 be the current density in a

given region. At t=10 ms, calculate the amount of current passing through surface ρ = 2m, 0$\leq z \leq 3m, 0 \leq \emptyset \leq 2\pi$.

29. Define Relaxation Time Constant (RTC). Derive an expression for RTC. Given the vector current density $\vec{J} = 10\rho^2 z\overrightarrow{a_\rho} - 4\rho\cos^2\emptyset\overrightarrow{a_\emptyset}$ mA/m^2. Find the current flowing outward through the circular band ρ=3, 0<$\emptyset < 2\pi$, 2<z<2.8.
30. Explain the continuity equation. The current density in certain region is approximated by $\vec{J} = (\frac{0.1}{r})e^{-10^6 t}\overrightarrow{a_r}$ A/m^2 in spherical coordinates. (a) How much current is crossing the surface R = 50cm at t = 1μS? (b) Find $\rho_v(r, t)$ assuming that $\rho_v \to 0$ as t→∞.

Boundary Condition

31. Let the region z< 0 be composed of a uniform dielectric material for which ε_{R1} =3.2, while the region z>0 is characterized by ε_{R2}=2. Let D_1= $-30a_x+50a_y+70a_z$ nC/m^2 and find:

    a) D_{t1} (Tangential component of E in Region 1);

    b) Polarization (p_1);

    c) E_{n2} (Normal component of E in Region 2)

    d) E_{12} (Tangential component of E in Region 2)

32. The region z<0 contains a dielectric material for which ε_{r1} = 2.5 while the region z > 0 is characterized by ε_{r2} 4. Let $\overrightarrow{E_1}$ = $-30\hat{a}_x + 50\hat{a}_y + 70\hat{a}_z$ V/m. Find: a) $\vec{E}_2$ (b) $\vec{D}_2$ (c) Polarization in region 2 ($\vec{P}_2$).
33. Consider the region y<0 be composed of a uniform dielectric material for which the relative permittivity (ε_1) is 3.2 while the region y>0 is characterized by ε_1 = 2. Let the flux density in region 1 be $\vec{D}_1 = -30\vec{a}_x + 50\vec{a}_y + 70\vec{a}_z$ nC/m^2. Find:

    a) Magnitude of Flux density and Electric fields intensity at region 2.

    b) Polarization ($\vec{P}$) in region 1 and 2.

34. Use boundary condition to find $\vec{E}_2$ in the medium 2 with boundary located at plane z=0. Medium 1 is perfect dielectric characterized by ε_{r1}=2.5, medium 2 is perfect dielectric characterized by ε_{r2}=5, electric field in medium is $\vec{E}_1 = \vec{a}_x + 3\vec{a}_y + 3\vec{a}_z$ v/m.
35. The region X<0 is composed of a uniform dielectric material for which ε_{r1}=3.2, while the region X>0 is characterized by ε_{r2}=2. The electric flux density at region X<0 is $\vec{D}_1$ $= -30\vec{a}_x + 50\vec{a}_y + 70\vec{a}_z$ nC/m^2 then find polarization ($\vec{P}$) and electric field intensity ($\vec{E}$) in both regions.