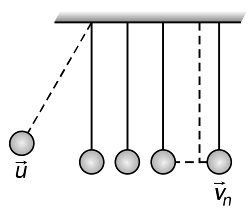
Calculate the velocities of the blocks when spring again comes to original length.

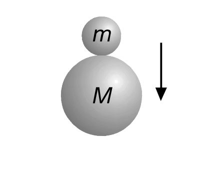
6. A small block of mass m moving with a speed v_0 collides elastically with an identical block kept at rest. Now the second block after travelling a distance d collides elastically with a heavy wall moving towards it with velocity u. The coefficient of friction between the blocks and the ground is μ. Find the maximum value of u for which no further collision takes place between the blocks.

7. n identical spheres of mass m are suspended with wires of equal length. The spheres are almost in contact with each other. Sphere 1 is pulled aside and released as shown in Figure.



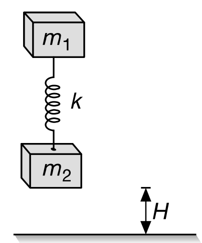
If sphere 1 strikes sphere 2 with velocity u, find an expression for velocity v_n of the nth sphere immediately after being struck by the one adjacent to it. Assume that the coefficient of restitution for each impact is e.

8. A small ball of mass m is placed on top of a super ball of mass M and the two balls are dropped to the floor from height h. How high does the small ball rise after the collision? Assume that the collisions with the superball are elastic and that $m \ll M$.



9. Three balls having masses m_1, m_2 and m_3 are lying in a straight line. The first ball moving with a certain velocity strikes the second ball directly and itself comes to rest. The second ball collides with the third and also comes to rest. If e is the coefficient of restitution for each ball, find the mass of ball m_2 in terms of m_1 and m_3.

10. A spring mass system is held at rest with the spring relaxed at a height H above the ground. Determine the minimum value of H so that the system has a tendency to rebound after hitting the ground. Given that the coefficient of restitution between m_2 and ground is zero.



11. Two particles of masses m and $2m$ moving in opposite directions collide elastically with velocity $2v$ and v, respectively. Find their velocities after collision.

12. A light spring of spring constant k is kept compressed between two blocks of masses m and M on a smooth horizontal surface. When released, the blocks acquire velocities in opposite directions. The spring loses contact with the block when it acquires natural length. If the spring was initially compressed through a distance x, find the final speeds of the two blocks.

OBLIQUE COLLISION

Physics of Oblique Collision

Before dealing with oblique collisions, we must keep in mind the following steps so that the problems can be solved with ease:

STEP-1: Draw a **common tangent line (*t*-line)** to the colliding surfaces at the point of impact. This line is called t-line.

STEP-2: Perpendicular to the common tangent line (t-line), draw the line of impact (also called n-line). The impact forces will be acting only along n-line (and have no component along t-line. **In other words, the impact forces are acting along the line joining the centres of the two bodies.**

STEP-3: Since impact forces have no components along t-line, hence the **individual momentum** or **individual velocity** of the particle must be conserved along t-line.

For A:

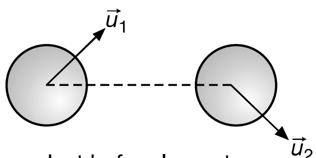
$$\left[(p_A)_i\right]_t = \left[(p_A)_f\right]_t$$

$$\Rightarrow \quad (u_1)_t = (v_1)_t$$

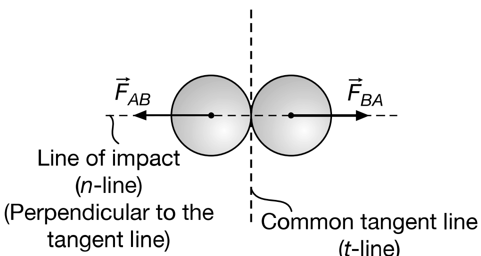
For B:

$$\left[(p_B)_i\right]_t = \left[(p_B)_f\right]_t$$

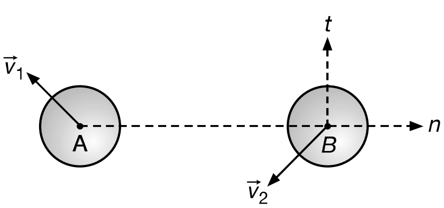
$$\Rightarrow \quad (u_2)_t = (v_2)_t$$



Just before Impact



During impact



Just after impact

STEP-4: The **total momentum** of the system $(A+B \text{ system})$ remains conserved along n-line.

For A (along n-line)

$$\Delta \vec{p}_A = \int \vec{F}_{AB} dt$$

$$\Rightarrow \quad m_1\left[(\vec{v}_1)_n - (\vec{u}_1)_n\right] = \int \vec{F}_{AB} \qquad \ldots(1)$$

For B (along n-line)

$$\Delta \vec{p}_B = \int \vec{F}_{BA} dt$$

$$\Rightarrow \quad m_2\left[(\vec{v}_2)_n - (\vec{v}_2)_n\right] = \int \vec{F}_{BA} dt \qquad \ldots(2)$$

Since $\vec{F}_{AB} = -\vec{F}_{BA}$ {from Newton's Third Law}

$$\Rightarrow \quad m_1(\vec{v}_1)_n - m_1(\vec{v}_1)_n = -m_2(\vec{v}_2)_n + m_2(\vec{v}_2)_n$$

$$\Rightarrow \quad m_1(\vec{v}_1)_n + m_2(\vec{v}_2)_n = m_1(\vec{v}_1)_n + m_2(\vec{v}_2)_n$$

$$\begin{pmatrix}\text{Total initial}\\ \text{momentum of}\\ A+B \text{ system}\end{pmatrix}_{\substack{\text{along}\\ n\text{-line}}} = \begin{pmatrix}\text{Total final}\\ \text{momentum of}\\ A+B \text{ system}\end{pmatrix}_{\substack{\text{along}\\ n\text{-line}}}$$

STEP-5: For elastic oblique collisions,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

STEP-6: For inelastic oblique collision (instead of STEP-5), if e be the coefficient of restitution at the point of impact, then

$$e = -\left[\frac{(v_2)_n - (v_1)_n}{(u_2)_n - (u_1)_n}\right]$$

Conceptual Note(s)

To summarise, we observe that we select two axis very smartly, the t-axis and the n-axis. Along t-axis (or t-line) no component of force acts, so individual momentum of particles is constant along t-axis. Along n-axis (or n-line) the net force on collective system is zero, so total momentum of system is conserved along n-line.

ILLUSTRATION 74

A ball of mass m hits a floor with a speed v_0 making an angle of incidence θ with the normal. The coefficient of restitution is e. Find the speed of the reflected ball and the angle of reflection ϕ of the ball.

SOLUTION

Since impact force has got no component along t-line, because it always acts along n-line, so the component of velocity v_0 along common tangent direction $v_0 \sin\alpha$ will remain unchanged. Let v be the velocity of ball just after impact such that angle of reflection is ϕ with the vertical. So,

$$v\sin\phi = v_0 \sin\theta \qquad \ldots(1)$$

Also, we know that

$$e = -\left[\frac{(v_2)_n - (v_1)_n}{(u_2)_n - (u_1)_n}\right]$$

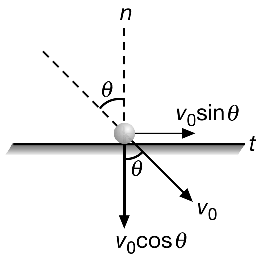
Now think of **1 as Ball** and **2 as Ground**

Since ground is at rest finally as well as initially, so $(v_2)_n = (u_2)_n = 0$

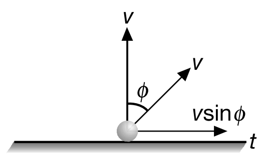
$$\Rightarrow \quad e = -\left(\frac{0 - v\cos\phi}{0 - (-v_0\cos\theta)}\right)$$

$$\Rightarrow \quad e = \frac{v\cos\phi}{v_0\cos\theta}$$

$$\Rightarrow \quad v\cos\phi = e(v_0\cos\theta) \qquad \ldots(2)$$



Just Before impact



Just After impact

To calculate the speed of ball after impact, squaring (1) and (2) and adding, to get

$$v^2 = v_0^2\sin^2\theta + e(v_0\cos\theta)^2$$

$$\Rightarrow \quad v = v_0\sqrt{\sin^2\theta + e^2\cos^2\theta}$$

To calculate the angle of reflection, divide (1) by (2), to get

$$\tan\phi = \frac{\tan\theta}{e}$$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{\tan\theta}{e}\right)$$

ILLUSTRATION 75

A sphere, of mass m, impinges obliquely on a sphere, of mass M, which is at rest. Show that, if $m = eM$, the directions of motion of the spheres after impact are at right angles.

SOLUTION

Let v_1 and v_2 be the components of velocity of m before collision, then we have

	Mass m		Mass M	
	***t*-line**	***n*-line**	***t*-line**	***n*-line**
Before collision	v_1	v_2	0	0
After collision	v_1	$v_3 = 0$	0	v_4

$$v_3 = \left(\frac{M - eM}{m + eM}\right)v_2 = 0 \qquad \{\because\ m = 2m\}$$

and $v_4 = \left(\dfrac{m + eM}{m + M}\right)v_2 \neq 0$

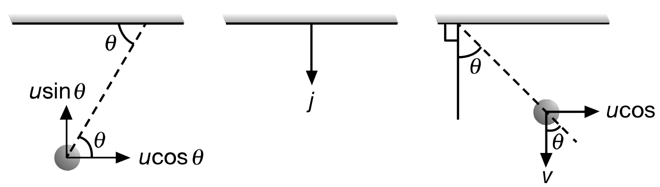
So, after collision m is along t-line while M along n-line and since t-line is perpendicular to n-line, so after impact both spheres move at right angles to each other.

ILLUSTRATION 76

The coefficient of restitution between a snooker ball and the side cushion is $\frac{1}{3}$. If the ball hits the cushion and then rebounds at right angles to its original direction, show that the angles made with the side cushion by the direction of motion before and after impact are 60° and 30° respectively.

SOLUTION

Let the original speed be u, in a direction making an angle θ with the side cushion.



Using the law of restitution

$$v = \frac{1}{3}(u\sin\theta)$$

After impact, $\tan\theta = \dfrac{u\cos\theta}{v} = \dfrac{3\cos\theta}{\sin\theta}$

$$\Rightarrow \quad \tan^2\theta = 3$$

$$\Rightarrow \quad \tan\theta = \sqrt{3}$$

$$\Rightarrow \quad \theta = 60^\circ$$

Therefore, the directions of motion before and after impact are at 60° and 30° to the cushion.

ILLUSTRATION 77

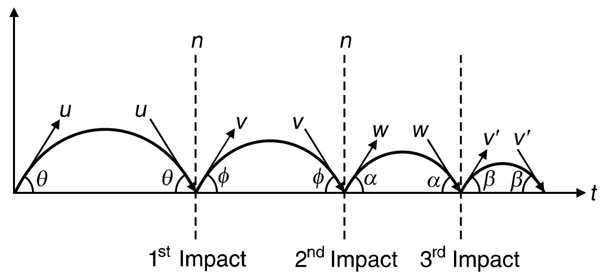
A projectile is launched with initial velocity u making angle θ with the horizontal. Assuming the coefficient of restitution between the projectile and the ground to be e, find the following:

(a) the total horizontal distance travelled before it comes to rest.

(b) the total time lapsed before it comes to rest.
(c) the average horizontal velocity during the tenure.
(d) the launch angle of the projectile just after n[th] impact.

SOLUTION

About t-line, individual velocity of particle is conserved, so we have



$$v\cos\phi = u\cos\theta \qquad \ldots(1)$$

$$w\cos\alpha = v\cos\phi = u\cos\theta \qquad \ldots(2)$$

About n-line

$$\begin{pmatrix}\text{Speed After}\\ \text{Collision}\end{pmatrix}_{\substack{\text{along}\\ n\text{-line}}} = e\begin{pmatrix}\text{Speed Before}\\ \text{Collision}\end{pmatrix}_{\substack{\text{along}\\ n\text{-line}}}$$

After 1[st] Impact

$$v\sin\phi = e(u\sin\theta) \qquad \ldots(3)$$

After 2[nd] Impact

$$w\sin\alpha = e(v\sin\phi) = e^2(u\sin\theta) \qquad \ldots(4)$$

After 3[rd] Impact

$$v'\sin\beta = e(w\sin\alpha) = e^2(v\sin\phi) = e^3(u\sin\theta) \ldots(5)$$

(a) Now, $R_1 = \dfrac{2}{g}(u\cos\theta)(u\sin\theta) = R$

and $R_2 = \dfrac{2}{g}(v\cos\phi)(v\sin\phi)$

$$\Rightarrow \quad R_2 = \frac{2}{g}(u\cos\theta)(eu\sin\theta) = eR$$

Similarly, $R_3 = e^2 R$

Total horizontal distance travelled by the projectile before it comes to rest is

$$x = R + eR + e^2R + e^3R + \ldots$$

$$\Rightarrow \quad x = R(1+e+e^2+\ldots)$$

$$\Rightarrow \quad x = \frac{R}{1-e}$$

$$\Rightarrow \quad x = \frac{2u^2\sin(2\theta)}{g(1-e)}$$

Total time lapsed is

$$T = \frac{2u\sin\theta}{g} + \frac{2v\sin\phi}{g} + \frac{2w\sin\alpha}{g} + \ldots$$

$$\Rightarrow \quad T = \frac{2u\sin\theta}{g}(1+e+e^2+\ldots)$$

$$\Rightarrow \quad T = \frac{2u\sin\theta}{g}\left(\frac{1}{1-e}\right)$$

(c) Average Horizontal Velocities

$$(v_{av})_x = \frac{\text{Total Horizontal Displacement}}{\text{Total Time Taken}}$$

$$\Rightarrow \quad (v_{av})_x = \frac{x}{t} = \frac{\left(\dfrac{2u^2\sin(2\theta)}{g(1-e)}\right)}{\left(\dfrac{2u\sin\theta}{g(1-e)}\right)}$$

Since $\sin(2\theta) = 2\sin\theta\cos\theta$

$$\Rightarrow \quad (v_{av})_x = u\cos\theta$$

(d) From (3) and (1), we get

$$\tan\phi = e\tan\theta$$

From (4) and (2), we get

$$\tan\alpha = e\tan\phi = e^2\tan\theta$$

So, after nth impact, we get

$$\tan(\theta_n) = e^n\tan\theta$$

$$\Rightarrow \quad \theta_n = \tan^{-1}(e^n\tan\theta)$$

Conceptual Note(s)

In an oblique collision between two bodies A and B if body B is at rest, then after impact, the body at rest i.e., B will move along the n-line (or the line of impact). Because, for the body B, at rest, we have, along t-line

$$(v_B)_t = (u_B)_t = 0$$

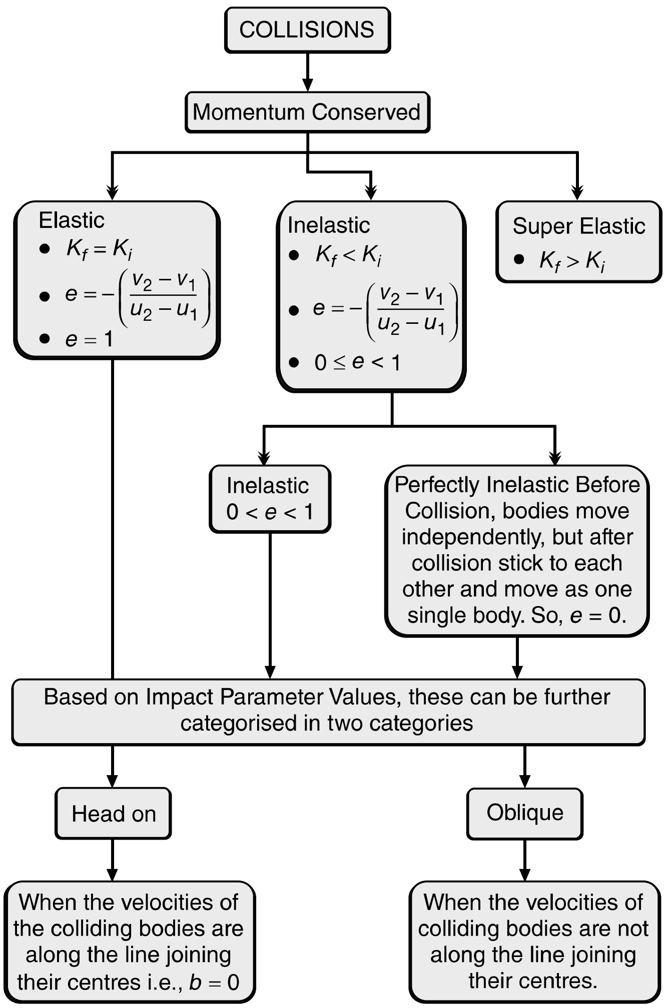
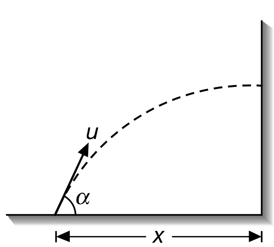


ILLUSTRATION 78

A ball is projected from a given point with velocity u at some angle with the horizontal and after hitting a vertical wall returns to the same point. If the distance of the point from the wall is x, then prove that $x < \frac{eu^2}{(1+e)g}$, where e is the coefficient of restitution.

SOLUTION

The situation discussed in the problem is shown in Figure.



Since, $\frac{x}{u\cos\alpha} + \frac{x}{eu\cos\alpha} = T = \frac{2u\sin\alpha}{g}$

$$\Rightarrow \quad x = \frac{eu^2\sin 2\alpha}{(1+e)g}$$

At $\alpha = 45°$, $\sin(2\alpha) = 1$, so the value of x is maximum and is given by

$$x_{\max} = \frac{eu^2}{(1+e)g}$$

Hence, all other values of α will yield a value of x that happens to be less than $x_{\max} = \frac{eu^2}{(1+e)g}$.

$$\Rightarrow \quad x < x_{\max}$$

$$\Rightarrow \quad x < \frac{eu^2}{(1+e)g}$$

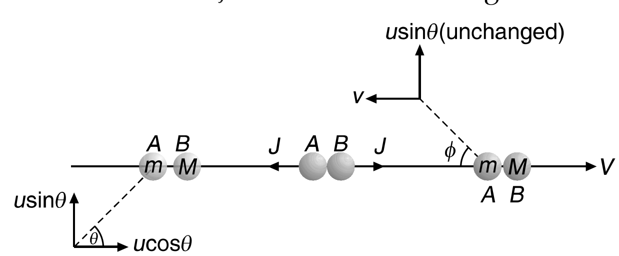
$$\Rightarrow \quad x_{\max} = \frac{eu^2}{(1+e)g} \text{ at } 2\alpha = 90°$$

ILLUSTRATION 79

Two smooth spheres A and B, of equal radius but masses m and M, are free to move on a horizontal table. A is projected with speed u towards B which is at rest. On impact, the line joining their centres is inclined at an angle θ to the velocity of A before impact. If e is the coefficient of restitution between the spheres, find the speed with which B begins to move. If A's path after impact is perpendicular to its path before impact, show that $\tan^2\theta = \frac{eM - m}{M + m}$.

SOLUTION

When B is struck by the impulse J, it begins to move in the direction of J as shown in the diagram.



Along the line of centres, we apply

(a) Conservation of Linear Momentum,
i.e., $mu\cos\theta = MV - mv$ ...(1)

(b) Law of restitution,
i.e., $eu\cos\theta = V + v$ ...(2)

Solving equations (1) and (2), we get

$$v=\frac{(eM-m)u\cos\theta}{(M+m)}$$

and $V=\dfrac{(1+e)mu\cos\theta}{M+m}$

Hence, $\tan\phi=\dfrac{u\sin\theta}{v}=\dfrac{(M+m)\tan\theta}{(eM-m)}$

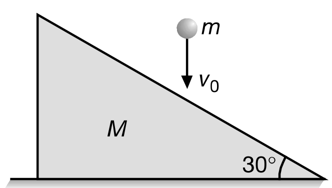
But the paths of A before and after impact are at right angles, therefore, $\cot\phi=\tan\theta$

$$\Rightarrow\quad \frac{(eM-m)}{(M+m)\tan\theta}=\tan\theta$$

$$\Rightarrow\quad \tan^2\theta=\frac{eM-m}{(M+m)}$$

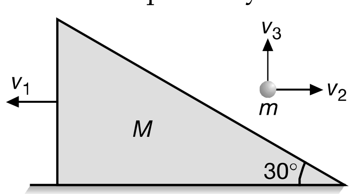
ILLUSTRATION 80

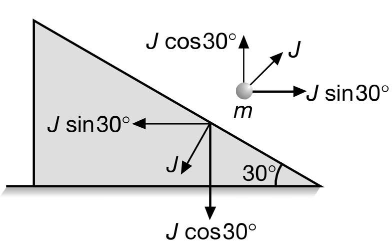
A ball of mass $m=1$ kg falling vertically with a velocity $v_0=2\ \text{ms}^{-1}$ strikes a wedge of mass $M=2$ kg kept on a smooth, horizontal surface as shown in figure. The coefficient of restitution between the ball and the wedge is $e=\dfrac{1}{2}$. Find the velocity of the wedge and the ball immediately after collision.



SOLUTION

Let, J be the impulse between ball and wedge during collision and v_1, v_2 and v_3 be the **components of velocity** of the wedge and the ball in horizontal and vertical directions respectively as shown in Figure.





Since, Impulse = Change in Momentum

$$\Rightarrow\quad J\sin(30^\circ)=Mv_1=mv_2,\ \text{along horizontal}$$

$$\Rightarrow\quad \frac{J}{2}=2v_1=v_2 \qquad \ldots(1)$$

Also, $J\cos(30^\circ)=m(v_3+v_0)$, along vertical

$$\Rightarrow\quad \frac{\sqrt{3}}{2}J=(v_3+2) \qquad \ldots(2)$$

Since $e=-\dfrac{(v_2)_n-(v_1)_n}{(u_2)_n-(u_1)_n}$

$$\begin{pmatrix}\text{Relative Velocity}\\ \text{of Separation just}\\ \text{after Impact}\end{pmatrix}_{n\ \text{line}}=e\begin{pmatrix}\text{Relative Velocity}\\ \text{of Approach just}\\ \text{before Impact}\end{pmatrix}_{n\ \text{line}}$$

$$\Rightarrow\quad (v_1+v_2)\sin(30^\circ)+v_3\cos(30^\circ)=\frac{1}{2}(v_0\cos(30^\circ))$$

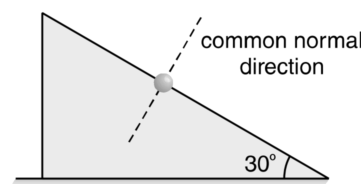
$$\Rightarrow\quad v_1+v_2+\sqrt{3}v_3=\sqrt{3} \qquad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$v_1=\frac{1}{\sqrt{3}}\ \text{ms}^{-1},\ v_2=\frac{2}{\sqrt{3}}\ \text{ms}^{-1}\ \text{and}\ v_3=0$$

Thus, velocities of wedge and ball are

$v_1=\dfrac{1}{\sqrt{3}}\ \text{ms}^{-1}$ and $v_2=\dfrac{2}{\sqrt{3}}\ \text{ms}^{-1}$ in horizontal direction as shown in Figure.



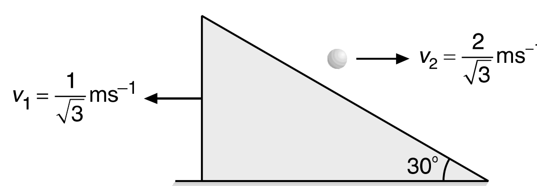
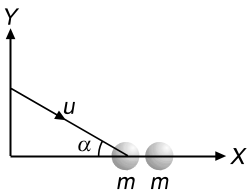


ILLUSTRATION 81

A ball moving translationally collides elastically with another stationary ball of the same mass. At the moment of impact the angle between the straight line passing through the centres of the balls and the direction of the initial motion of the striking ball is equal to $\alpha=45^\circ$. Assuming the balls to be smooth, find the fraction η of the kinetic energy of the striking ball that turned into potential energy at the moment of the maximum deformation.

SOLUTION

Let a ball of mass m with velocity u collides with stationary ball as shown in figure.



Applying the Law of Conservation of Momentum along x-axis, we have

$$mu\cos\alpha = mv_{1x} + mv_{2x}$$

At the moment of maximum deformation

$$v_{1x} = v_{2x}$$

$$\Rightarrow \quad u\cos\alpha = v_{1x} + v_{1x} = 2v_{1x}$$

$$\text{or} \quad v_{1x} = \frac{u\cos\alpha}{2}$$

Initial kinetic energy, $K_{\text{initial}} = \frac{1}{2}mu^2$

Final kinetic energy, $K_{\text{final}} = \frac{1}{2}mv_{1x}^2 + \frac{1}{2}mv_{2x}^2 + \frac{1}{2}mv_{1y}^2$

$$\Rightarrow \quad K_{\text{final}} = 2\left(\frac{1}{2}mv_{1x}^2\right) + \frac{1}{2}mv_{1y}^2$$

$$\Rightarrow \quad K_{\text{final}} = \frac{mu^2\cos^2\alpha}{4} + \frac{mu^2\sin^2\alpha}{2}$$

$$\Rightarrow \quad K_{\text{final}} = \frac{mu^2}{4}\cos^2 45° + \frac{mu^2}{2}\sin^2 45°$$

$$\Rightarrow \quad K_{\text{final}} = \frac{mu^2}{8} + \frac{mu^2}{4} = \frac{3mu^2}{8}$$

Therefore, potential energy at the moment of maximum deformation is

$$U = \frac{mu^2}{2} - \frac{3mu^2}{8} = \frac{mu^2}{8}$$

So, the desired fraction η is given by

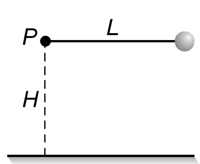
$$\eta = \frac{mu^2/8}{mu^2/2} = \frac{1}{4} = 0.25$$

Test Your Concepts-VII

Based on Oblique Collisions

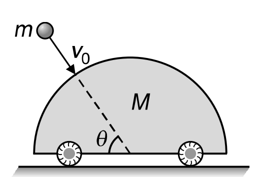
(Solutions on page H.88)

1. A point P is fixed at a height H above a perfectly inelastic smooth horizontal plane. A light inextensible string of length $L(>H)$ has one end attached to P and other is attached to a heavy particle. The particle is held at the level of P with string just taut and released from rest. Find the height of the particle above the plane when it is next instantaneously at rest.



2. A smooth sphere of mass m is moving on a horizontal plane with a velocity $3\hat{i} + \hat{j}$ when it collides with a vertical wall which is parallel to the vector $\hat{j}$. If the coefficient of restitution between the sphere and the wall is $\frac{1}{2}$, find the velocity of the sphere after impact, the loss in kinetic energy caused by the impact and the impulse $\vec{J}$ that acts on the sphere.

3. A putty ball of mass m strikes a smooth stationary wedge of mass M with a velocity v_0 making at an angle θ with the horizontal as shown in Figure.

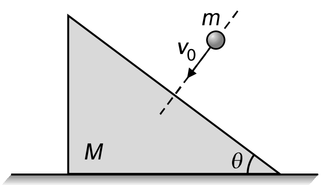


If the collision is perfectly inelastic, calculate the velocity of the wedge just after collision and the loss in kinetic energy of the system.

4. A sphere of mass m is moving with a velocity $4\hat{i} - \hat{j}$ when it hits a wall and rebounds with velocity $\hat{i} + 3\hat{j}$. Calculate the impulse it receives. Also calculate the coefficient of restitution between the sphere and the wall.

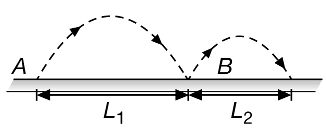
5. A ball of mass m, moving with a velocity u along x-axis, strikes another ball of mass $2m$ kept at rest. The first ball comes to rest after collision and the other breaks into two equal pieces. One of the pieces starts moving along y-axis with a speed v. What will be the speed of the other piece?

6. A ball of mass m normally hits a wedge of mass M lying at rest on a smooth horizontal surface as shown in Figure.

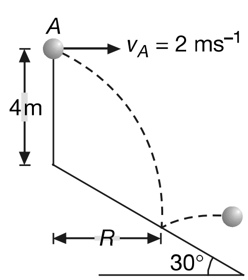


If v_0 be the velocity with which the ball hits the wedge, e be the coefficient of restitution between ball and the wedge, then calculate the velocity of wedge and ball just after collision.

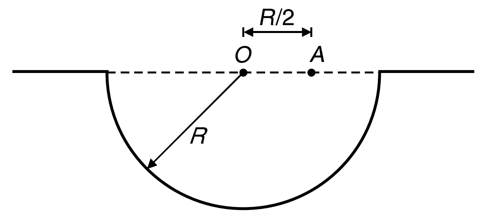
7. A projectile is launched from point A has a horizontal range L_1 as shown. If the coefficient of restitution at B is e, determine the distance L_2.



8. A 0.5 kg ball is thrown horizontally from point A with velocity $v_A = 2\ \text{ms}^{-1}$. Determine the horizontal distance R where the ball strikes the smooth inclined plane. If the coefficient of restitution is $e = 0.6$, determine the speed at which it bounces from the plane. Take $g = 9.8\ \text{ms}^{-2}$.



9. A semi cylindrical frictionless track of radius R is centred at the point O. A particle is released from point A such that $OA = R/2$ as shown in Figure.



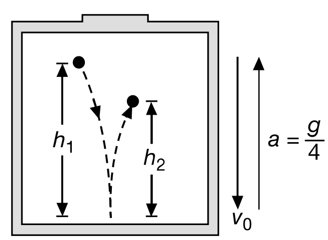
Calculate the coefficient of restitution e, if after collision with the track, the particle moves along the track. Also calculate e if the velocity of particle becomes horizontal just after collision with the track.

10. A ball is released from rest relative to the elevator at a distance h_1 above the floor. The speed of the elevator at the instant ball is released is v_0. Determine the bounce height h_2 of the ball

(a) if v_0 is constant and

(b) if an upward elevator acceleration $a = \dfrac{g}{4}$ begins at the instant the ball is released.

The coefficient of restitution for the impact is e.



11. A ball is projected from the ground with speed u at an angle α with horizontal. It collides with a wall at a distance a from the point of projection and returns to its original position. Find the coefficient of restitution between the ball and the wall.