18. ELECTROSTATICS OF CONDUCTORS

18.1. Properties of Conductors: Conductors contain mobile charge carriers. In metallic conductors, there charge carries are electrons. These electrons are free within the material but not free to leave the material. In an external electric field, they drift against the direction of the field. In electrolytic conductors, the charge carriers are both positive and negative ions.

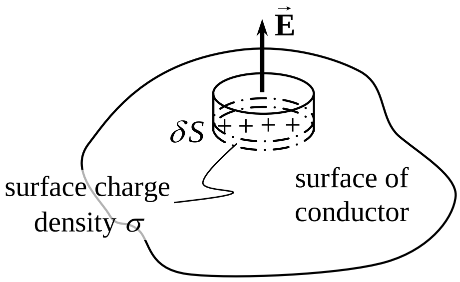
a. Inside a conductor, electrostatic field is zero: If a conductor is placed in an Electric Field, as long as electric field is not zero, the free charge carriers would experience force and drift. In the steady state situation, the free charges have so distributed themselves that the electric field is zero everywhere inside the material. This fact can be taken as the defining property of a conductor.

b. Electrostatic potential is constant throughout the volume of the conductor: This follows from the previous result. If a unit positive charge moves within material of the conductor, work done by electric field is zero and hence potential must not change. That is, there is no potential difference between any two points inside or on the surface of the conductor. If the conductor is charged, electric field normal to the surface exists; this means potential will be different for the surface and a point just outside the surface. In a system of conductors of arbitrary size, shape and charge configuration, each conductor is characterised by a constant value of potential, but this constant may differ from one conductor to the other.

c. The interior of a conductor can have no excess charge in the static situation: Consider any arbitrary volume element *V* inside a conductor. On the closed surface S bounding the volume element *V*, electrostatic field is zero. Thus the total electric flux through is zero. Hence, by Gauss's law, there is no net charge enclosed by *S*. But the surface *S* can be made as small as we want. A neutral conductor has equal amounts of positive and negative charges in every small volume or surface element. This means *there is* no *net charge at any point inside the conductor,* and *any excess charge must reside at the surface*

d. At the surface of a charged conductor, electrostatic field must be normal to the surface at every point: If Electric field were not normal to the surface, it would have some non-zero component along the surface. Free charges on the surface of the conductor would then experience force and move. In the static situation, therefore *E* should have no tangential component. Since Electric Field has no tangential component on the surface, no work is done in moving a small test charge on its surface, keeping its potential constant on its surface.

e. Electric field at the surface of a charged conductor $\vec{\mathbf{E}} = \frac{\sigma}{\varepsilon_o}\hat{\mathbf{n}}$: Where σ is the surface charge density

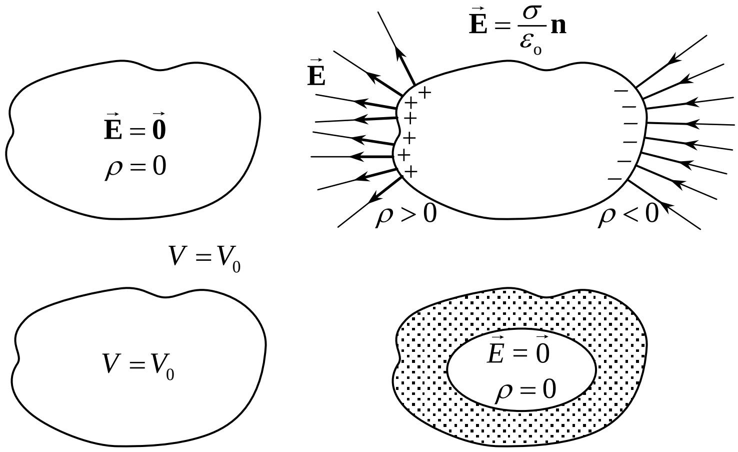


and $\hat{\mathbf{n}}$ is a unit vector normal to the surface in the outward direction. To derive the result, choose a pill box (a short cylinder) as the Gaussian surface about any point *P* on the surface, as shown. The pill box is partly inside and partly outside the surface of the conductor. It has a small area of cross section δS and negligible height. Just inside the surface, the electrostatic field is zero; just outside, the field is normal to the surface with magnitude *E*. Thus, the contribution to the total flux through the pill box comes only from the outside (circular) cross-section of the pill box. This equals $E\delta S$ since, over the small area δS, *E* may be considered constant and E and δS are parallel or anti-parallel. The charge enclosed by the pill box is $\sigma\delta S$. Using Gauss's law,

$$E\delta S = \frac{|\sigma|\delta S}{\varepsilon_0}$$

$$E = \frac{|\sigma|}{\varepsilon_0}$$

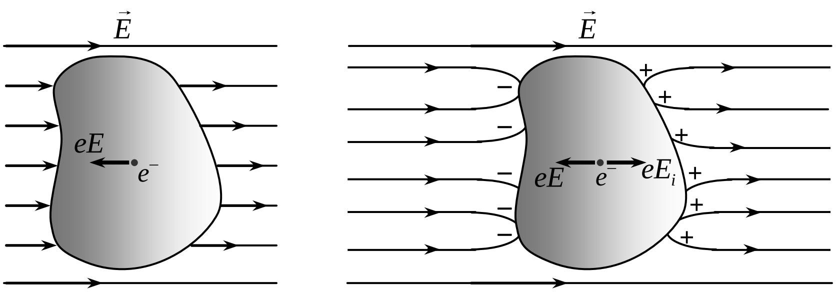
18.2. **Electrostatic shielding:** Consider a conductor with a cavity, with no charges inside the cavity. The electric field inside the cavity is zero, whatever be the size and shape of the cavity and whatever be the charge on the conductor and the external fields in which it might be placed. We have proved a simple case of this result already: the electric field inside a charged spherical shell is zero. But the vanishing of electric field in the (charge-free) cavity of a conductor is, as mentioned above, a very general result. A related result is that even if the conductor is charged or charges are induced on a neutral conductor by an external field, all charges reside only on the outer surface of a conductor with cavity. Whatever be the charge and field configuration outside, any cavity in a conductor remains shielded from outside electric influence: the *field inside the cavity* is *always* zero. This is known as *electrostatic shielding.* The effect can be made use of in protecting sensitive instruments from outside electrical influence. Figure gives a summary of the important electrostatic properties of a conductor.



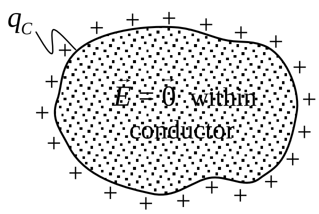
(Fig. 2.19 Important electrostatic properties of conductors)

18.3. **Charge induction on Conducting Surfaces:**

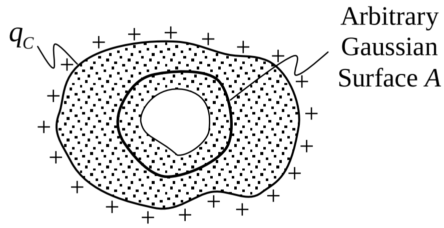
a. ***Charge induction on outer surface*:** Whenever a metal body is placed in an electric field $\vec{\mathbf{E}}_{ext}$, the free electrons in the body experience electrostatic force $-e\vec{\mathbf{E}}_{ext}$ (in opposite direction) as shown in figure. Due to this, electrons start to drift against the electric field, as shown in figure, causing development of negative and positive charges on the body surface (called induced charges) as shown. Due to the induced charges, inside the metal body an electric field $\vec{\mathbf{E}}_{ind}$ is developed in the direction opposite to external electric field. If $\left|\vec{\mathbf{E}}_{ind}\right| < \left|\vec{\mathbf{E}}_{ext}\right|$, free electrons in the body still experience a force and drift, due to which induced chares q_1 increases and hence the induced electric field $\vec{\mathbf{E}}_{ind}$ increases in magnitude. The process of drifting of charge will continue until the electric field inside the surface becomes zero. Thus if a metal body is placed in an electric field , charge induction on the body surface starts and after steady state is reached the net electric field inside the body becomes zero.



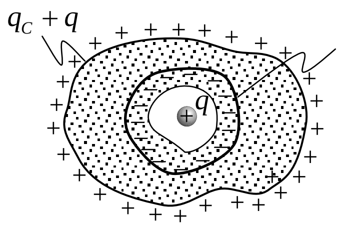
b. ***Charge induction on inner surface (on cavity surface) of solid conductor:*** Suppose we place a charge q inside a cavity within a conductor (see figure). The conductor is uncharged and is insulated from the charge q. Again Electric Field is zero everywhere on the shown Gaussian surface A. So according to Gauss's law the total charge inside this surface must be zero. Therefore there must be a charge $-q$ distributed on the surface of the cavity. The total charge on the conductor must remain zero, so a charge $+q$ must appear on the outer surface.



The charge q_C resides entirely on the surface of the conductor. The situation is electrostatic, so $\vec{E}=0$ within the conductor.



Because $\vec{E}=0$ at all points within the conductor, the electric field at all points n the Gaussian surface must be zero.

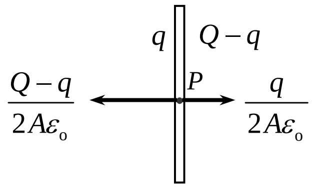


Foe $\vec{E}$ to be zero at all points on the Gaussian surface, the surface of the cavity must have a total charge $-q$.

c. ***Charge induction on large metal plates:***

i. If a metal plate is kept in uniform electric field the charge induced on it must be such that electric field inside it must be zero.

ii. If an isolated (isolated means no charges are near the plate) large conducting plate is given a charge then the charge distributes equally on its two surfaces. If total charge Q is given to the plate assuming charge q on left side of sheet, charge on right side of sheet must be $Q-q$ as shown. Since point P lies inside the conductor, electric field at this point must be zero. Hence,

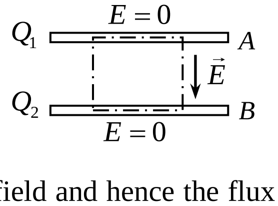


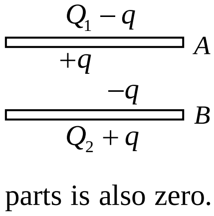
$$\frac{q}{2A\varepsilon_o}-\frac{Q-q}{2A\varepsilon_o}=0$$

$$\Rightarrow \qquad q={}^{Q}\!/_{2}$$

hence, charge is equally distributed on both sides

iii. If two conducting plates A and B are placed parallel to each other, the charges on the inner facing surfaces are of equal magnitude and opposite sign. Consider a Gaussian surface as shown in figure. Two faces of this closed surface lie completely inside the conductor where the electric field is zero. The flux through these faces is, therefore, zero. The other parts of the closed surface which are outside the conductor are parallel to the electric





field and hence the flux on these parts is also zero. The total flux of the electric field through the closed surface is, therefore zero. From Gauss’s law, the total charge inside this closed surface should be zero. Thus, the charge on the inner surface of A should be equal and opposite to that on the inner surface of B.

Example 24. A solid conductor with a cavity carries a total charge of +7 nC. Within the cavity, insulated from the conductor, is a point charge of –5 nC. How much charge is on each surface (inner and outer) of the conductor?

Solution: If the charge in the cavity is $q=-5$ nC, the charge on the inner cavity surface must be q' such that

$$q + q' = 0$$

so $$q' = 5 \text{ nC}$$

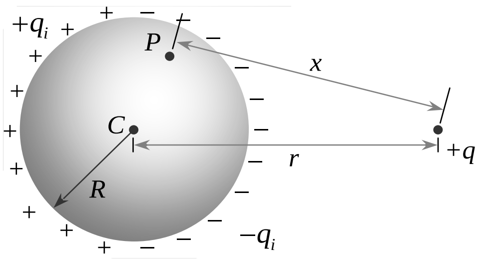
The conductor carries a total charge of +7 nC, none of which is in the interior of the material. If $+5$ nC is on the inner surface of the cavity, then there must be (+7 nC) – (+5 nC) = +2 nC on the outer surface of the conductor.

Example 15. A metal sphere of radius R is placed at a distance r from the point charge $+q$ $(r > R)$. There is a point P in the sphere at a distance x from $+q$. Find

(a) The net electric field at point P.

(b) The electric field at point P due to the induced charges on the surface of sphere.

(c) Electrostatic potential of point P



Solution: (a) We know that inside a metal body net electric field is always zero. Thus at point P electric field due to induced charges balances the electric field of the point charge $+q$, causing electric field at point P to be zero.

(b) Electric field $\vec{E}_{ind}$ due to induced charges at P should be equal and opposite to electric field produced by charge $+q$ at P. So $\vec{E}_{ind}$ can be given as

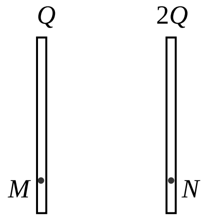
$$\vec{E}_{ind} = k\frac{q}{x^2} \qquad \text{(Directed toward point charge } +q\text{)}$$

(c) As inside the sphere at every point Electric field is zero, potential everywhere inside should be constant. At centre of sphere, potential is only due to the charge $+q$ as due to induced charges, potential at centre will be zero. Thus net potential at the centre of sphere can be given as

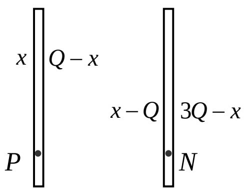
$$V_{centre} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

This potential will also be equal to Potential at point P.

Example 26. Two large parallel conducting sheets (placed at finite distance) are given charges Q and $2Q$ respectively. Find charges appearing on each surface.



Solution: Let there is x amount of charge on left side of first plate. So on its right side charge will be $Q - x$.



Now as proved in previous problem, charge on directly opposite face must be $x - Q$ as shown. Charge on the last face should be such that total charge on second plate becomes Q, hence $3Q - x$.

Now by property of conductor,

$$E_P = 0$$

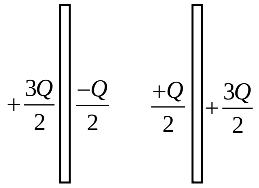
$$\Rightarrow \quad \frac{x}{2A\varepsilon_o} - \left\{ \frac{Q-x}{2A\varepsilon_o} + \frac{x-Q}{2A\varepsilon_o} + \frac{3Q-x}{2A\varepsilon_o} \right\} = 0$$

We can say, charge on left side of *P* is same as charge on right side of *P*

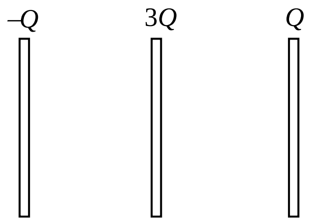
$$x = Q - x + x - Q + 3Q - x$$

$$\Rightarrow \quad x = \frac{3Q}{2},\ Q - x = \frac{-Q}{2}$$

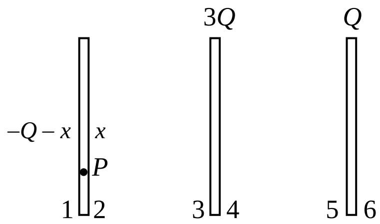
So, final charge distribution of plates is as shown



Example 27. Figure shows three large metallic plates with charges –*Q*, 3*Q* and *Q* respectively. Determine the final charges on all the surfaces.



Solution: We assume that charge on surface 2 is *x*. Following conservation of charge, we see that surfaces 1 has charge $(-Q-x)$. The electric field inside the metal plate is zero. So, field at *P* is zero.



Resultant field at *P*–

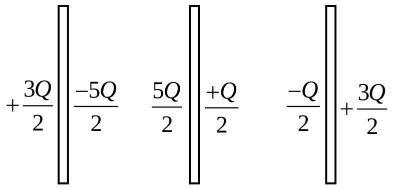
$$E_P = 0$$

$$\Rightarrow \quad \frac{-Q-x}{2A\varepsilon_o} = \frac{x+3Q+Q}{2A\varepsilon_o}$$

$$\Rightarrow \quad -Q - x = x + 4Q$$

$$\Rightarrow \quad x = \frac{-5Q}{2}$$

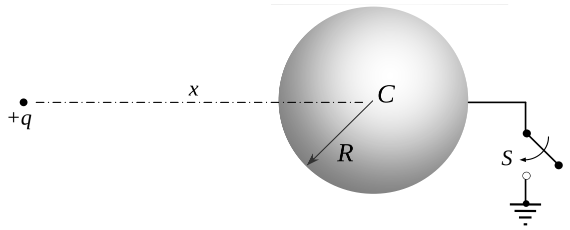
We see that charges on the facing surface of the plates are of equal magnitude and opposite sign. This can be in general proved by Gauss theorem also. Thus final charge distribution on all the surfaces is as shown in figure



19. Earthing of a conductor

Earth is assumed to be a large conducting sphere such that even if some charge (positive or negative) is given to earth, its potential remains negligible. Thus for bodies whose dimensions are negligible compared to earth we can assume that earth is at zero potential. If a conductor is connected to earth by a conducting wire (process is called earthing), then under steady state the Electric Field inside the conductor as well as inside the conducting connecting wire will become zero. So potential of both ends of connecting wire will achieve same value, causing the conductor to have zero potential.

Example 28. Consider a solid uncharged conducting sphere as shown in figure. A point charge q is placed in front of the sphere at a distance x from centre of sphere as shown. Find charge induced on the sphere after switch S is closed.



Solution: Some charge (not necessarily equal positive and negative) will be induced on the surface of conductor after switch S is closed. Potential of the centre of the sphere,

i. Due to charge $+q$ is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{x}$$

ii. Due to induced charges q_{ind} is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_{ind}}{R}$$

But potential of the conductor must be zero. Hence,

$$V = V_1 + V_2$$

$$\Rightarrow \quad 0 = \frac{1}{4\pi\varepsilon_0}\frac{q}{x} + \frac{1}{4\pi\varepsilon_0}\frac{q_{ind}}{R}$$

$$\Rightarrow \quad q_{ind} = -\frac{qR}{x}$$

20. Force on a charged conducting surface element:

In the presence of an electric field, a surface charge will experience a force. The force per unit area f experienced by a conducting surface element having charge density σ is:

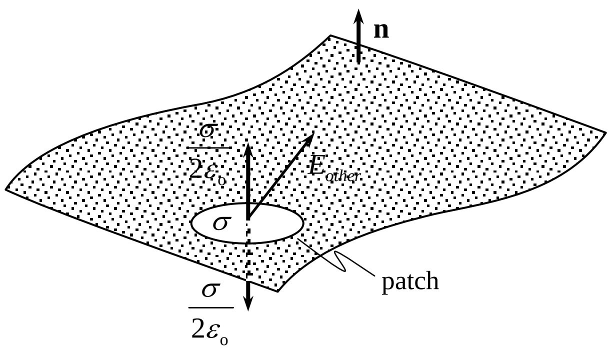
$$\vec{f} = \sigma\vec{E}$$

But the value of electric field $\vec{E}$ is *discontinuous* at the surface charge element, so which value are we supposed to use: $\vec{E}_{above}$ or $\vec{E}_{below}$?

The answer is that we should use the *average* of the two:

$$\vec{f} = \sigma\vec{E} = \frac{1}{2}\sigma\left(\vec{E}_{above} + \vec{E}_{below}\right)$$

The reason is very simple. To find force experienced by a given charge, the electric field we use is due to every other charge except the given charge.



Let's focus our attention on a small surface element surrounding the point in question (as shown in figure). Make it tiny enough so it is essentially flat and the surface charge on it is essentially constant. The *total* field consists of two parts:

(i) due to the element itself, and

(ii) due to everything else (other regions of the surface, as well as any external sources that may be present):

$$\vec{E}_{Total} = \vec{E}_{element} + \vec{E}_{other}$$

The force on the patch, then, is due exclusively to $\vec{E}_{other}$, and *this* suffers *no* discontinuity (if we removed the patch, the field in the "hole" would be perfectly smooth). The discontinuity is due entirely to the charge on the patch, which puts out a field $\left(\dfrac{\sigma}{2\varepsilon_0}\right)$ on either side, pointing away from the surface

Thus, $$\vec{E}_{above} = \vec{E}_{other} + \frac{\sigma}{2\varepsilon_0}\eta$$

$$\vec{E}_{below} = \vec{E}_{other} - \frac{\sigma}{2\varepsilon_0}\eta$$

And hence, $\vec{E}_{other} = \dfrac{1}{2}\left(\vec{E}_{other} + \vec{E}_{below}\right) = \vec{E}_{average}$

Averaging is really just a device for removing the contribution of the patch itself.

This argument applies to *any* surface charge; in the particular case of a conductor, the field is zero inside and $\left(\dfrac{\sigma}{\varepsilon_0}\eta\right)$ outside, so the average is and $\left(\dfrac{\sigma}{2\varepsilon_0}\eta\right)$, and the force per unit area is

$$P = \frac{\sigma^2}{2\varepsilon_0}.$$

This amounts to an outward **electrostatic pressure** on the surface, tending to push the conductor in the direction of the field, regardless of the sign of σ. Expressing the pressure in terms of the field just outside the surface,

$$P = \frac{1}{2}\varepsilon_0 \left|\vec{E}_{Total}\right|^2$$