Modern Physics

Questions

1. De Broglie Wavelength in Center of Mass Frame

Two identical non-relativistic particles move at right angles to each other, possessing de Broglie wavelengths λ_1 and λ_2. Find the de Broglie wavelength of each particle in the frame of their center of mass.

2. Force on a Hemispherical Mirror due to Radiation Pressure

A point source of light O is placed at the center of curvature of a hemispherical perfectly reflecting surface of radius of curvature R. Find the force on the hemisphere due to the light falling on it if the source emits a power P.

3. Maximum Kinetic Energy of Photoelectrons

The surface of a metal of work function ϕ_0 is illuminated by light whose electric field component varies with time as $E = E_0[1 + \cos(\omega t)]\sin(\omega_0 t)$. Calculate the maximum kinetic energy of photoelectrons emitted from the surface.

4. Photoelectric Effect with a Metallic Sphere

In a photoelectric effect setup, a point source of light of power 3.2×10^{-3} W emits mono-energetic photons of energy 5 eV. The source is located at a distance of 0.8 m from the center of a stationary metallic sphere of work function 3 eV and of radius 8×10^{-3} m. The efficiency of photoelectron emission is one for every 10^6 incident photons. Assume that the sphere is isolated and electrons are instantly swept away after emission.

(a) Calculate the number of photoelectrons emitted per second.

(b) Find the ratio of the wavelength of incident light to the de Broglie wavelength of the fastest photoelectrons emitted.

(c) Explain why it is observed that the photoelectron emission stops at a certain time t after the light source is switched on.

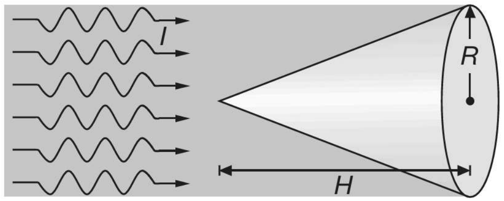
(d) Evaluate the time t.

5. Standing Waves of Electrons in an Atomic Array

Assume that the de Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one-dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance d between the atoms of the array is 2 Å. A similar standing wave is again formed if d is increased to 2.5 Å but not for any intermediate value of d. Find the energy of the electron in eV and the least value of d for which the standing wave of the type described above can form.

6. Force on a Reflecting Cone due to Light Beam

A cone of radius R and height H with a perfectly reflecting lateral surface is placed in the path of a light beam of intensity I as shown. Calculate the force exerted by the light beam on this cone.



7. Bohr's Correspondence Principle

Using Bohr's theory, show that when n is very large, the frequency of radiation emitted by a hydrogen atom due to the transition of an electron from n to $(n-1)$ is equal to the frequency of revolution of the electron in its orbit.

8. Bohr Model for a Mu-Mesonic Atom

A particle of charge equal to that of an electron, $-e$, and mass 208 times the mass of the electron (called a μ-meson) moves in a circular orbit around a nucleus of charge +3e (take the mass of the nucleus to be infinite). Assuming that the Bohr model of the atom is applicable to this system:

(i) Calculate the radius of the nth Bohr orbit.

(ii) Find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for the hydrogen atom.

(iii) Find the wavelength of radiation emitted when the μ-meson jumps from the third orbit to the first orbit (Rydberg's constant = $1.097 \times 10^7 m^{-1}$).

9. Bohr's Model for an Imaginary Atom

An imaginary particle has a charge equal to that of an electron and a mass 100 times the mass of the electron. It moves in a circular orbit around a nucleus of charge +4e. Take the mass of the nucleus to be infinite. Assuming that the Bohr's model is applicable to this system:

(a) Derive an expression for the radius of the n^{th} Bohr orbit.

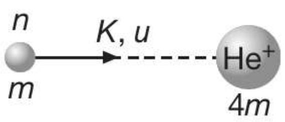
(b) Find the wavelength of the radiation emitted when the particle jumps from the fourth orbit to the second orbit.

10. Hypothetical Hydrogen Atom with Modified Potential

Assume a hypothetical hydrogen atom in which the potential energy between the electron and proton at a separation r is given by $U = k \log_e \left(\frac{r}{2}\right)$, where k is a constant. For such a hypothetical hydrogen atom, calculate the radius of the nth Bohr orbit and the energy levels.

11. Inelastic Collision of Neutron with Helium Ion

A He^+ ion is at rest and in the ground state. A neutron with initial velocity u and kinetic energy K collides head-on with the He^+ ion. Calculate the minimum value of K so that there can be an inelastic collision between these two particles.



12. Moseley's Law: Determining Constants

In Moseley's equation, we have $\sqrt{f} = a(Z - b)$, where a and b are constants. Find their values with the help of the following data:

Element	Z	Wavelength of X-rays

Mo	42	0.71 Å
Co	27	1.785 Å

13. Black Body Radiation and Photoelectric Effect

A peak emission from a black body at a certain temperature occurs at a wavelength of $9000\mathring{A}$. On increasing the temperature, the total radiation emitted is increased 81 times. At the initial temperature, when the peak radiation from the black body is incident on a metal surface, it does not cause any photoemission from the surface. After the increase of temperature, the peak radiation from the black body caused photoemission. To bring these photoelectrons to rest, a potential equivalent to the excitation energy between the $n = 2$ and $n = 3$ Bohr levels of the hydrogen atom is required. Calculate the work function of the metal.

14. Radioactive Decay with Multiple Emission Modes

The mean lives of a radioactive substance are 1620 years and 405 years for α emission and β emission, respectively. Find out the time during which three-fourths of a sample will decay if it is decaying both by α emission and β emission simultaneously.

15. Radioactive Decay Time Calculation

A radioactive decay occurs as follows:

$$A \xrightarrow{T_{1/2}=8 \text{ yrs}} B$$

Initially, at time $t = 0$, only the substance A is present. Determine the time t at which, if a single atom is randomly selected from the sample, the probability of it being B is 15 times the probability of it being A.

16. Neutron Decay Fraction Calculation

A stream of neutrons has a kinetic energy of 0.0327 eV. Given that the half-life of the neutrons is 700 s and the mass of a neutron is 1.675×10^{-27} kg, determine the fraction of neutrons that will decay before they travel a distance of 10 m.

17. Estimation of Earth's Age Using Uranium Isotopes

Uranium ores on Earth today typically consist of 99.3% of the isotope ${}^{238}_{92}U$ and 0.7% of the isotope ${}^{235}_{92}U$. The half-lives of these isotopes are 4.47×10^9 years and 7.04×10^8 years, respectively. Assuming these isotopes were equally abundant when the Earth was formed, estimate the age of the Earth.

18. Proton Bombardment Energy for Nuclear Reaction

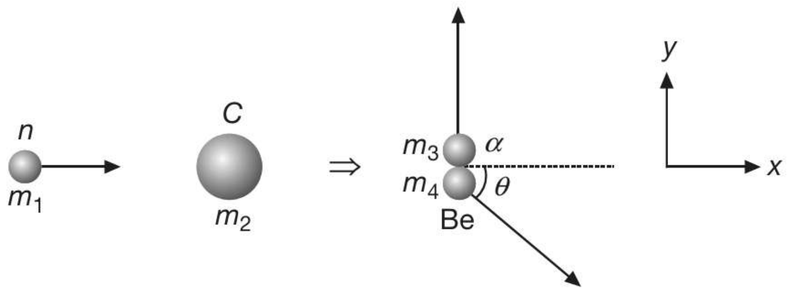
Determine the minimum kinetic energy that a bombarding proton must possess to cause the reaction

${}^7_3Li +^1_1 H \rightarrow^4_2 Be +^1_0 n$

The masses of the atoms are given as: Li atom = 7.01600 u, Be atom = 7.01693 u, H atom = 1.00783 u, and neutron = 1.00866 u.

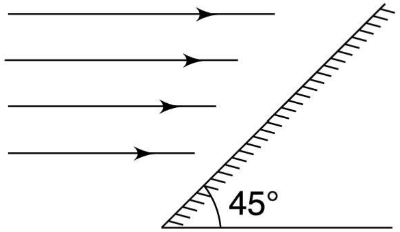
19. Kinetic Energy of Alpha Particles in Nuclear Reaction

A neutron with kinetic energy $K = 10$ MeV activates a nuclear reaction $n +^{12} C \rightarrow^9 Be + \alpha$. Find the kinetic energy of the alpha particles outgoing at a right angle to the direction of incoming neutrons. Take $u = 931.5$ MeV and the threshold energy of the reaction $(E_{th}) = 6.17$ MeV.



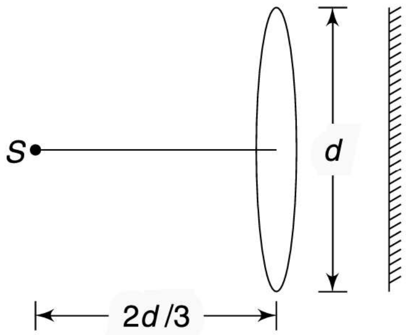
20. Direction of Force on a Mirror due to Reflected Light

A horizontal beam of light is incident on a plane mirror inclined at 45° to the horizontal. The percentage of light energy reflected from the mirror is 80%. Find the direction in which the mirror will experience force due to the incident light.



21. Force on Reflecting Surface due to a Light Source

A point source (S) of light having power 500 W is kept at the focus of a lens of aperture diameter d. The focal length of the lens is $\frac{2d}{3}$. Assume that 40% of the incident light energy is transmitted through the lens and the complete transmitted light is incident normally on a perfectly reflecting surface placed behind the lens. Calculate the force on the reflecting surface.



22. Pair Production Near Nucleus

It is possible for a photon to materialize into an electron and a positron. The process is called pair production. (a) Using conservation of energy and momentum prove that pair production cannot occur in empty space. (b) Argue qualitatively that such pair production is possible near a nucleus.

Answer Keys

1. The de Broglie wavelength of each particle in the center of mass frame is $\frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$.

2. The force on the hemisphere due to the light is $\frac{P}{2c}$.

3. The maximum kinetic energy of the photoelectrons is $K_{max} = \frac{h(\omega+\omega_0)}{2\pi} - \phi_0$.

4. (a) The number of photoelectrons emitted per second is 10^5 electrons/s.

(b) The ratio of the wavelength of incident light to the de Broglie wavelength is approximately 286.

(c) Emission stops because sphere reaches stopping potential.

(d) The time is approximately 111 s.

5. The energy of the electron is approximately 150.8 eV, and the least value of d is 0.5 Å.

6. The force exerted by the light beam on the cone is $\frac{2\pi IR^4}{c(R^2+H^2)}$.

7. For large n, $f_{\text{revolution}} = f_{\text{transition}}$, demonstrating Bohr's Correspondence Principle.

8. (i) $r_n = \dfrac{n^2h^2\epsilon_0}{624\pi m_e e^2}$

(ii) $n \approx 25$

(iii) $\lambda = 0.546\AA$

9. (a) $r_n = \dfrac{n^2h^2\epsilon_0}{400\pi m_e e^2}$

(b) $\lambda \approx 3\AA$

10. Radius of nth Bohr orbit $r_n = \dfrac{nh}{2\pi\sqrt{mk}}$ and Energy levels $E = k\log_e\left(\dfrac{nh}{4\pi\sqrt{mk}}\right) + \dfrac{k}{2}$.

11. The minimum value of K is greater than 51 eV.

12. The values of the constants are approximately $a = 5 \times 10^7\ (\mathrm{Hz})^{1/2}$ and $b = 1.37$.

13. The work function of the metal is 2.235 eV.

14. The time during which three-fourths of the sample will decay is 449 years.

15. 32 years

16. $3.96 \times 10^{-}6$

17. 5.97×10^{9} years

18. 1.89 MeV

19. 2.21 MeV

20. $\tan^{-1}(0.8)$ with the horizontal

21. $1.33 \times 10^{-7}\,\mathrm{N}$

22. (a) Cannot occur in empty space

(b) Possible near a nucleus