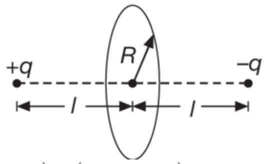
Questions based on Solid Angle

1. A point charge q is fixed at the origin. Calculate the electric flux through the infinite plane $y = a$.

S: $\phi_e = \dfrac{q}{2\varepsilon_0}$

2. Two point charges q and $-q$ separated by a distance 2ℓ. Find the flux of electric field strength vector across the circle of radius R placed with its centre coinciding with the midpoint of line joining the two charges in the perpendicular plane.



S: $\phi_{\text{total}} = \left(\phi_{\text{due to }+q}\right) + \left(\phi_{\text{due to }-q}\right)$

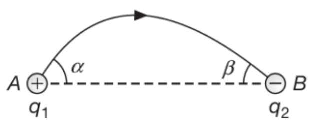
Since, we have seen already that

$$\phi = \frac{q}{2\varepsilon_0}(1-\cos\alpha) = \frac{q}{2\varepsilon_0}\left(1-\frac{\ell}{\sqrt{R^2+\ell^2}}\right)$$

$$\Rightarrow \quad \phi_{\text{total}} = 2\phi = \frac{q}{\varepsilon_0}\left(1-\frac{\ell}{\sqrt{R^2+\ell^2}}\right)$$

(Do not expect the answer to come to be zero as one charge is $+q$ and other $-q$. If both had been $+q$ or $-q$ then it would have been zero)

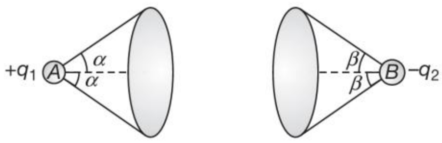
3. Two charges $+q_1$ and $-q_2$ are placed at A and B respectively. A line of force emanates from q_1 at angle α with the line AB. At what angle will it terminate at $-q_2$?



S: A line can leave $+q_1$ in a cone of apex angle α and then enter $-q_2$ in a cone of apex angle β.

So, flux due to the charge $+q_1$ is $\phi_1 = \dfrac{q_1}{2\varepsilon_0}(1-\cos\alpha)$

and that due to the charge $-q_2$ is $\phi_2 = \dfrac{q_2}{2\varepsilon_0}(1-\cos\beta)$.



Since, we know that only one line is leaving q_1 to enter $-q_2$. So, we can say

$$\frac{\phi_1}{\phi_2} = \frac{N_1}{N_2} = 1$$

$$\Rightarrow \quad \frac{q_1}{2\varepsilon_0}(1-\cos\alpha) = \frac{q_2}{2\varepsilon_0}(1-\cos\beta)$$

$$\Rightarrow \quad q_1\left[2\sin^2\left(\frac{\alpha}{2}\right)\right] = q_2\left[2\sin^2\left(\frac{\beta}{2}\right)\right]$$

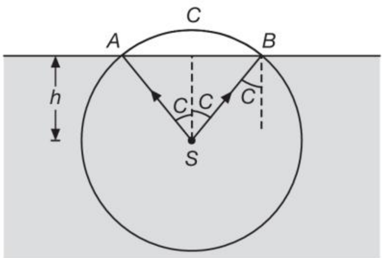
$$\Rightarrow \quad \sin\left(\frac{\beta}{2}\right) = \sqrt{\frac{q_1}{q_2}}\sin\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow \beta = 2\sin^{-1}\left[\sqrt{\frac{q_1}{q_2}}\sin\left(\frac{\alpha}{2}\right)\right]$$

4. A point source of light is placed at a distance h below the surface of a large and deep lake. Show that the fraction f of light that escapes directly from water surface is independent of h and is given by,

$$f = \frac{\left[1-\sqrt{1-\frac{1}{\mu^2}}\right]}{2}.$$

S: Due to TIR, light will be reflected back into the water for $i > C$. So only that portion of incident light will escape which passes through the cone of angle $\theta = 2C$.



So, the fraction of light escaping is given by

$$f = \frac{\text{Area of Surface } ACB}{\text{Total Area of Sphere}}$$

$$\Rightarrow \quad f = \frac{2\pi R^2(1-\cos C)}{4\pi R^2} = \frac{1-\cos C}{2}$$

Now, as f depends on C, which depends only on μ, hence f is independent of h.

Since, we know that

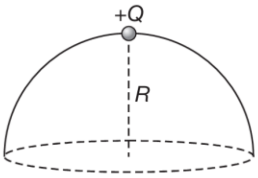
$$\sin C = \frac{1}{\mu}$$

$$\Rightarrow \quad \cos C = \frac{\sqrt{\mu^2-1}}{\mu} = \sqrt{1-\frac{1}{\mu^2}}$$

$$\Rightarrow \quad f = \frac{1}{2}\left(1-\sqrt{1-\frac{1}{\mu^2}}\right)$$

5. **[JEE (Advanced) 2017]**

A point charge $+Q$ is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct?



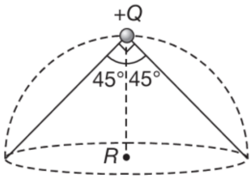
(A) The electric flux passing through the curved surface of the hemisphere is $-\dfrac{Q}{2\varepsilon_0}\left(1-\dfrac{1}{\sqrt{2}}\right)$.

(B) The component of the electric field normal to the flat surface is constant over the surface

(C) Total flux through the curved and the flat surfaces is $\dfrac{Q}{\varepsilon_0}$

(D) The circumference of the flat surface is an equipotential

S: (a) $\Omega = 2\pi(1-\cos\theta)$, where $\theta = 45°$



$$\Rightarrow \quad \phi = -\frac{\Omega}{4\pi}\times\frac{Q}{\varepsilon_0}$$

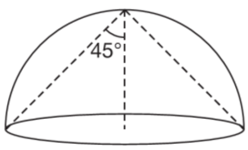
$$\Rightarrow \quad \phi = -\frac{2\pi(1-\cos\theta)}{4\pi}\frac{Q}{\varepsilon_0}$$

$$\Rightarrow \quad \phi = -\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$$

The negative sign signifies that the flux is due to field lines entering a surface.

(b) The component of the electric field perpendicular to the flat surface will decrease so we move away from the centre as the distance increases (magnitude of electric field decreases) as well as the angle between the normal and electric field will increase. Hence, the component of the electric field normal to the flat surface is not constant.

(c) Total flux ϕ due to charge Q is $\dfrac{Q}{\varepsilon_0}$.



So, ϕ through the curved and flat surface will be less than $\dfrac{Q}{\varepsilon_0}$.

(d) Since, the circumference is equidistant from Q it will be equipotential $V = \dfrac{Q}{4\pi\varepsilon_0\sqrt{2}R}$.

Hence, (A) and (D) are correct.

6. **[JEE (Advanced) 2019]**

A charged shell of radius R carries a total charge Q. Given ϕ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its centre same as that of the shell. Here, centre of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct?

(ε_0 is the permittivity of free space)

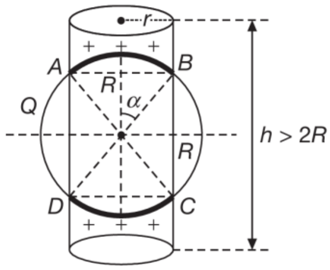
(A) If $h > 2R$ and $r = \dfrac{3R}{5}$ then $\phi = \dfrac{Q}{5\varepsilon_0}$

(B) If $h < \dfrac{8R}{5}$ and $r = \dfrac{3R}{5}$ then $\phi = 0$

(C) If $h > 2R$ and $r > R$ then $\phi = \dfrac{Q}{\varepsilon_0}$

(D) If $h > 2R$ and $r = \dfrac{4R}{5}$ then $\phi = \dfrac{Q}{5\varepsilon_0}$

S: **For OPTION (A)**



Since $h > 2R$ and $r = \dfrac{3R}{5}$, so we get

$$\sin\alpha = \frac{r}{R} = \frac{3}{5}$$

$$\Rightarrow \quad \cos\alpha = \frac{4}{5}$$

Since $\phi = 2\left[\dfrac{Q}{2\varepsilon_0}(1-\cos\alpha)\right]$ ...(1)

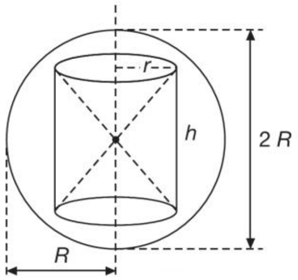
$$\Rightarrow \quad \phi = \frac{Q}{5\varepsilon_0}$$

Please note that a factor of 2 is used in equation (1), because we have two surfaces i.e., AB and CD of the shell that are enclosed by the cylinder.

For OPTION (B)

$$h < \frac{8R}{5} \text{ and } r = \frac{3R}{5}$$

$$h < 2R \text{ and } r < R$$

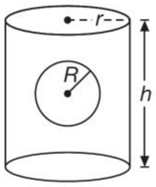


$\Rightarrow \quad Q_{\text{enclosed}} = 0$

$\Rightarrow \quad \phi = 0$

For OPTION (C)

$$h > 2R \text{ and } r > R$$

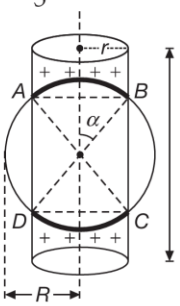


$\Rightarrow \quad Q_{\text{enclosed}} = Q$

$\Rightarrow \quad \phi = \frac{Q}{\varepsilon_0}$

For OPTION (D)

$$h > 2R \text{ and } r = \frac{4R}{5} \text{ i.e., } r < R$$



$$Q_{\text{enclosed}} = Q[1 - \cos\alpha]$$

Since, $\sin\alpha = \frac{r}{R} = \frac{3}{5}$

$\Rightarrow \quad \cos\alpha = \frac{3}{5}$

$\Rightarrow \quad \phi = \frac{2Q}{2\varepsilon_0}(1 - \cos\alpha)$

$\Rightarrow \quad \phi = \frac{Q}{\varepsilon_0}\left(1 - \frac{3}{5}\right)$

$\Rightarrow \quad \phi = \frac{2Q}{5\varepsilon_0}$

Hence, (A), (B) and (C) are correct.