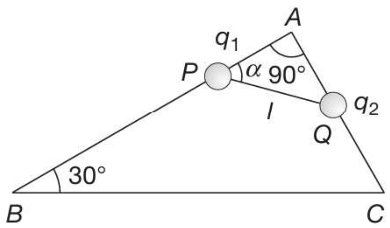
JEE Advanced - Electromagnetism

Chapter: Electromagnetism

Questions List

1 A rigid insulated wire frame in the form of a right-angled triangle ABC is set in a vertical plane as shown in the figure.



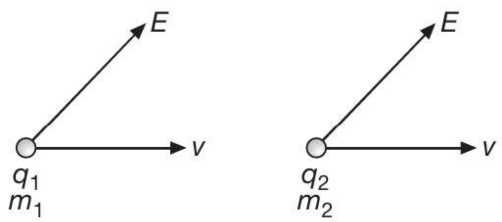
Two beads of equal masses m each and carrying the charges q_1 and q_2 are connected by a cord of length l and can slide without friction on the wires. Considering the case when the beads are stationary determine

(a) the angle α

(b) the tension in the cord and

(c)the normal reaction on the beads.

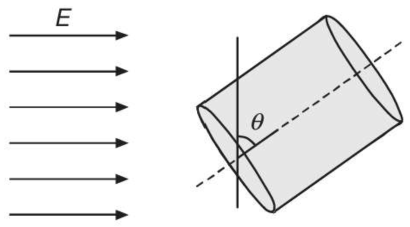
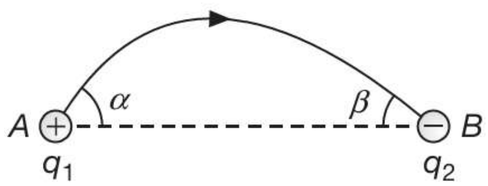
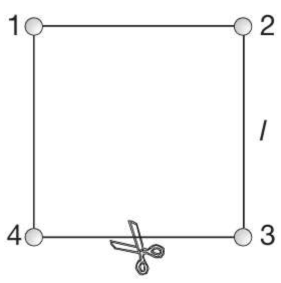
2 Two balls of charge q_1 and q_2 initially have velocities of equal magnitude and same direction. After a uniform electric field has been applied for a certain time interval, the direction of the first ball changes by 60°, and the velocity magnitude is reduced by half. The direction of velocity of the second ball changes thereby 90°.

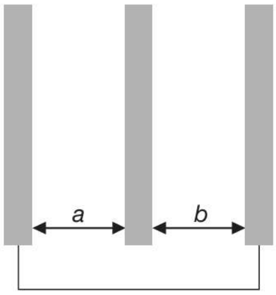


Initial condition

(a) Determine the magnitude of the charge to mass ratio of the second ball if it is equal to α_1 for the first ball.

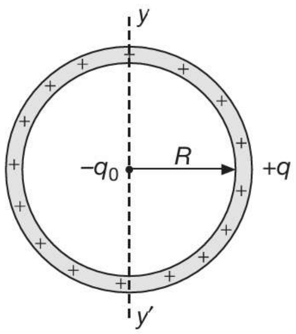
(b) In what ratio will the velocity of the second ball change? Ignore the electrostatic interaction between the balls.

<table>
<tr><td>3</td><td>A cylinder of height H and radius R is placed in a uniform electric field (as shown) such that the axis of the cylinder makes an angle θ with the vertical. Calculate the magnitude of flux through that part of the cylinder where electric field lines are entering the cylinder.

</td></tr>
<tr><td>4</td><td>Two charges $+q_1$ and $-q_2$ are placed at A and B respectively. A line of force emanates from q_1 at an angle α with the line AB. At what angle will it terminate at $-q_2$?

</td></tr>
<tr><td>5</td><td>Inside an infinitely long circular cylinder charged uniformly with volume density ρ there is a circular cylindrical cavity. The distance between the axis of the cylinder and the axis of cavity is equal to $\vec{a}$. Find the electric field strength $\vec{E}$ inside the cavity. The permittivity is assumed to be equal to unity.</td></tr>
<tr><td>6</td><td>For a system of charges aligned in such a way that each individual charge is in equilibrium, prove that the electrostatic potential energy of the system is zero.</td></tr>
<tr><td>7</td><td>Four balls, each with mass m, are connected by four non-conducting strings to form a square with side l, as shown in Figure. The assembly is placed on a horizontal nonconducting frictionless surface. Balls 3 and 4 each have charge q , and balls 1 and 2 are uncharged. Find the speed of balls 3 and 4 after the string connecting them is cut and all four balls become colinear.

</td></tr>
<tr><td>8</td><td>Three identical metallic plates are kept parallel to one another at a separation of a and b. The outer plates are connected by a thin conducting wire, and a charge Q is placed on the central plate.</td></tr>
</table>

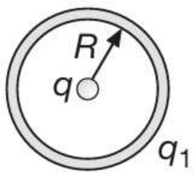


Find the final charges on all six faces of the plates shown.

9 Consider a thin spherical shell of radius R having charge q distributed uniformly on it. At the center of the shell, a negative point charge $-q_0$ is placed. The shell is cut in two identical hemispherical portions along a diametrical section yy' as shown. Due to mutual repulsion, the two hemispherical parts tend to move away from each other, but due to the attraction of $-q_0$, they may stay in equilibrium. Find the condition of equilibrium of the hemispherical shells. Calculate the minimum value of q_0 to attain this equilibrium.



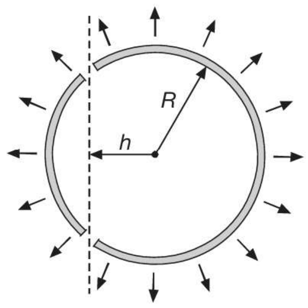
10 Figure shows a shell of radius R having charge q_1 uniformly distributed over it.



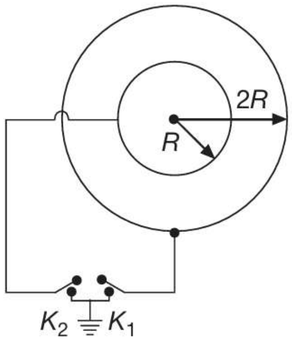
A point charge q is placed at the centre of the shell. Find the work required to increase the radius of the shell from R to R_1 ($R_1 > R$).

11 A point electric dipole with a moment $\vec{p}$ is placed in an external uniform electric field whose strength equals $\vec{E}_0$, with $\vec{p}$ parallel to $\vec{E}_0$. In this case, one of the equipotential surfaces enclosing the dipole forms a sphere. Find the radius of this sphere.

12 Consider a metal sphere of radius R that is cut in two along a plane whose minimum distance from the sphere's centre is h. The sphere is uniformly charged by a total electric charge Q. Find the force necessary to hold the two parts of the sphere together.



13 Two concentric shells of radii R and $2R$ are shown in figure. Initially a charge q is imparted to the inner shell. Now key K_1 is closed and opened and then key K_2 is closed and opened. After the keys K_1 and K_2 are alternately closed n times each, find the potential difference between the shells. Note that finally the key K_2 remains closed.



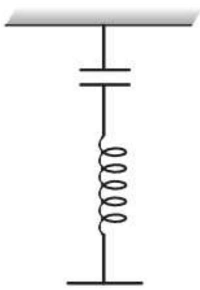
14 In case of two conducting spherical shells having radii a and $b (b > a)$ calculate the capacity of the system if

(a) shells are concentric and inner is given a charge while outer earthed

(b) shells are concentric and the outer is given a charge while the inner is earthed

(c) shells carry equal and opposite charges and are separated by a distance d. Can you try to solve this part, using the concept of self energy and interaction energy?

15 A parallel plate capacitor with air as a dielectric is arranged horizontally, such that its one plate is fixed and the other plate is connected with a perpendicular spring.



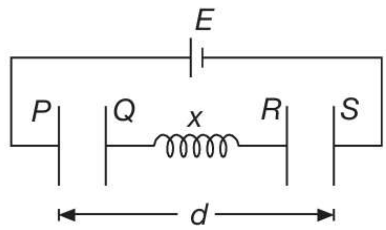
The area of each plate is A. In the steady position, the separation between the plates is d_0. When the capacitor is connected with an electric source with the voltage V, a new equilibrium appears, such that the new separation between the plates becomes d_1. Assuming mass of the upper plate to be m, find the

(a) spring constant K.

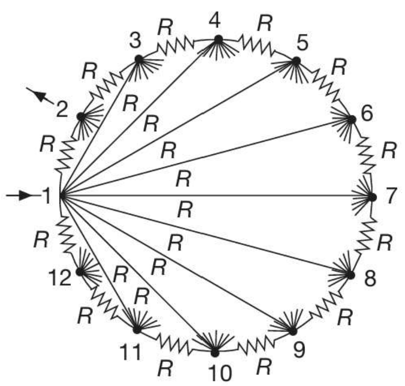
(b) maximum voltage for a given K in which an equilibrium is possible.

(c) angular frequency of the oscillating system around the equilibrium value d_1 (amplitude of the oscillation $<< d_1$).

16 Two parallel plate capacitors with area A are connected through a conducting spring of natural length l in series as shown. Plates P and S have fixed positions at separation d. Now the plates are connected by a battery of emf E as shown. If the extension in the spring in equilibrium is equal to the separation between the plates, find the spring constant k.



17 A network of 12 resistors each of value $R = 6\Omega$ are interconnected as shown in figure, being placed along the sides of a regular dodecagon. Each of the terminals 1, 2, 3, ..., 12 has been connected directly by insulated wires (other than nearest) each of the 9 terminals each of resistance R, there being 9 wires from each terminal making 108 wire connections totally. [Only one set of 9 wires, from terminal 1 have been shown].

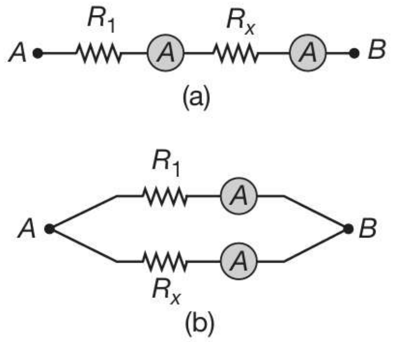


Find:

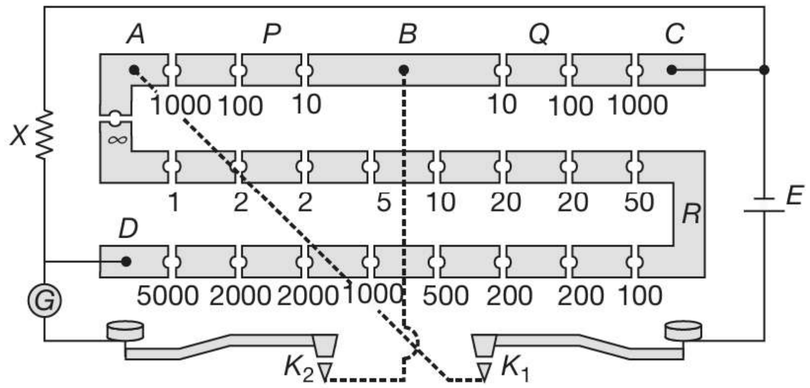
(a) the equivalent resistance of the network when the current enters at the terminal 1 and leaves at terminal 2.

(b) If the above network were generalised so that there are n (n = even) resistors each of resistance R placed along the sides of a regular n sided polygon and if each terminal point of a resistor were connected by $(n-2)$ insulated wires each of resistance R, directly to the $(n-2)$ terminals, other than its, nearest terminals, find the equivalent resistance across any two terminals of the network (i.e., current entering at one of the two terminals and leaving by the other).

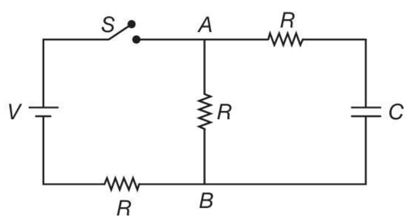
18 Consider two different ammeters in which the currents are proportional to the respective deflections of the needle. The first ammeter is connected to a resistor of resistance R_1 and the second to a resistor of unknown resistance R_x. Firstly, the ammeters are connected in series between points A and B (as shown in figure). In this case the readings of the ammeters are n_1 and n_2. Then the ammeters are connected in parallel between A and B (as shown in figure) and indicate N_1 and N_2. Determine the unknown resistance R_x of the second resistor.



19 Calculate the maximum and minimum values of unknown resistance X, which can be determined using the post office box shown in the figure?



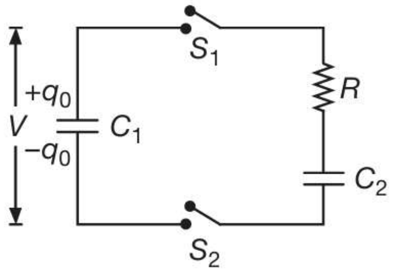
20 In the circuit shown in figure, the battery is an ideal one, with emf V. The capacitor is initially uncharged. The switch S is closed at time $t = 0$.



(a) Find the charge Q on the capacitor at time t.

(b) Find the current in AB at time t. What is its limiting value as $t \to \infty$?

21 The capacitor C_1 in figure initially carries a charge q_0. When the switches S_1 and S_2 are shut, capacitor C_1 is connected in series to a resistor R and a second capacitor C_2, which initially does not carry any charge.

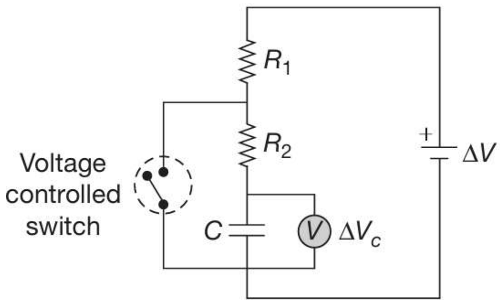


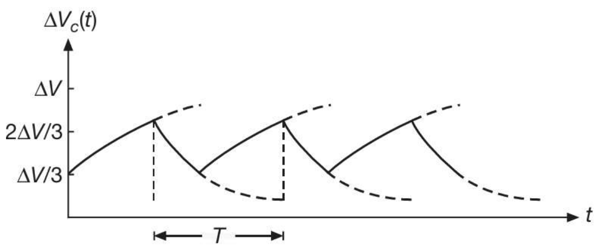
(a) Find the charges deposited on the capacitors and the current through R as a function of time.

(b) What is the heat lost in the resistor after a long time of closing the switch?

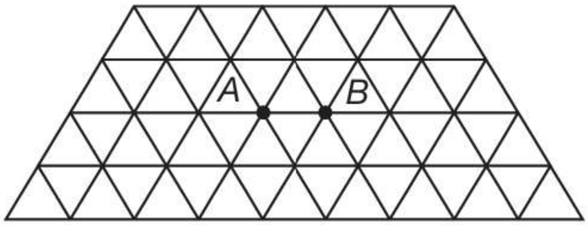
22 A conductor has a temperature independent resistance R and a total heat capacity C. At the moment $t = 0$ it is connected to a dc voltage V. Find the time dependence of the conductor's temperature T assuming the thermal power dissipated into surrounding space to vary as $q = k(T - T_0)$ where k is a constant, T_0 is the environmental temperature (equal to conductor's temperature at the initial moment).

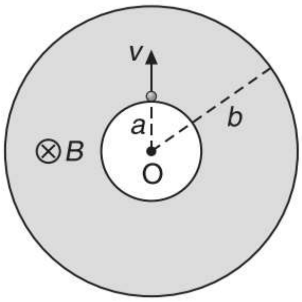
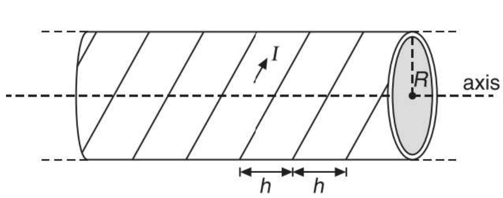
23 The switch in figure closes when $\Delta V_c > \dfrac{2\Delta V}{3}$ and opens when $\Delta V_c < \dfrac{\Delta V}{3}$. The voltmeter reads a voltage as plotted in figure. What is the period T of the waveform in terms of R_1, R_2 and C?

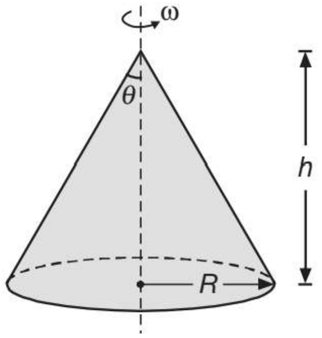
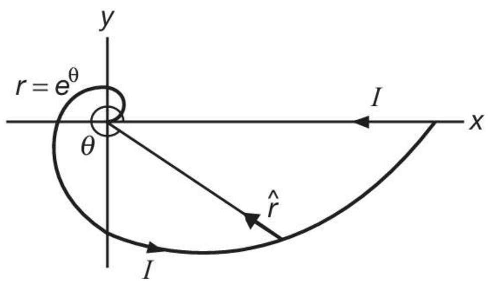
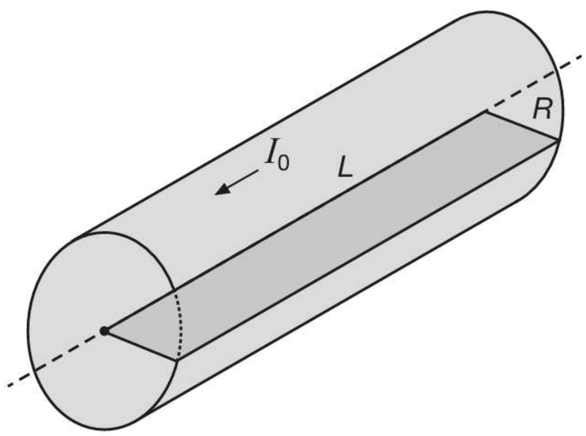




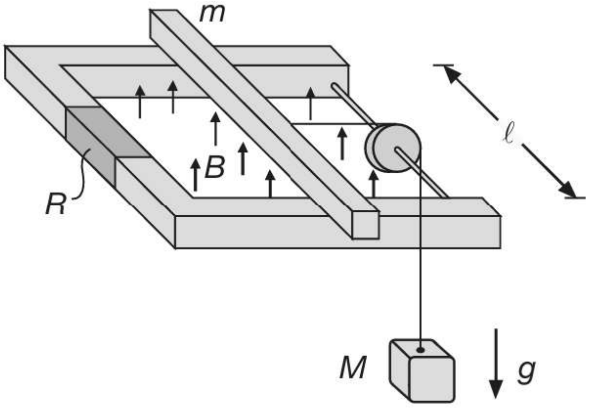
24 There is an infinite wire grid with cells in the form of equilateral triangles. The resistance of each wire between neighbouring joint connections is R_0. Fing the net resistance of the whole grid between the points A and B as shown



25	A uniform magnetic field B exists in the annular space enclosed between two cylindrical shells of the inner radius a and outer radius of b as shown in Figure.  An electron is projected from the surface of inner cylindrical shell perpendicular to it with some initial velocity. The magnetic field is directed along the common axis of the cylindrical shells. Calculate the maximum initial velocity with which this electron should be projected so that it will not hit the inner surface of the outer shell.
26	A particle of charge q and mass m is emitted at rest from the origin with zero initial velocity into a region of uniform electric and magnetic fields. The electric field $\vec{E} = E_0\hat{i}$ is acting along x-axis and magnetic field $\vec{B} = B_0\hat{j}$ along y-axis. Find: **(a)** the maximum x-coordinate of the particle. **(b)** the maximum speed of the particle. **(c)** the nature of the path followed by the particle.
27	Point charges Q_1 and Q_2 are constrained to move along the x and y-axes, respectively, with the same uniform speed v. At time $t = 0$, both the charges are at the origin. Calculate the magnetic force F acting on Q_2 due to the magnetic field of Q_1 at time t.
28	A thin conducting strip of width h is tightly wound in the shape of a very long cylindrical coil with cross-sectional radius R to make a single layer straight solenoid as shown in figure. 

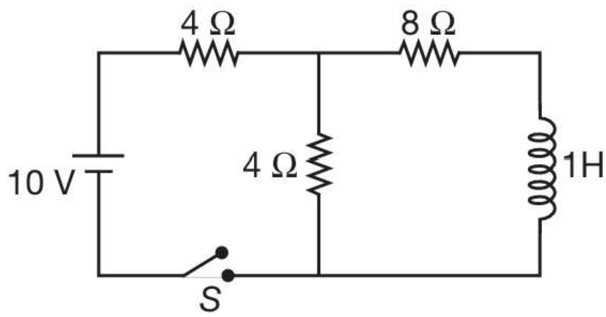
	A direct current I flows through the strip. Find the magnetic induction inside and outside the solenoid as a function of the distance r from its axis.
29	A charge Q is uniformly distributed over the slant surface of a thin walled right circular cone of semi-vertex angle θ and height h as shown in Figure. The cone is uniformly rotated about its axis at angular velocity ω. Calculate the magnetic dipole moment associated with the cone. 
30	A wire carrying a current I is bent into the shape of an exponential spiral, $r = e^{\theta}$ from $\theta = 0$ to $\theta = 2\pi$ as shown in figure. To complete a loop, the ends of the spiral are connected by a straight wire along the x-axis. Find the magnitude and direction of $\vec{B}$ at the origin. 
31	A very long, cylindrical wire of radius R carries a current I_0 uniformly distributed across the cross section of the wire. Calculate the magnetic flux through a rectangle that has one side of length L running down the centre of the wire and another side of length R, as shown in Figure. 

32 A bar of mass m is pulled horizontally across parallel rails by a massless string that passes over an ideal pulley and is attached to a suspended object of mass M as shown in Figure. The uniform magnetic field has a magnitude B and the distance between the rails is l. The rails are connected at one end by a load resistor R. Derive an expression that gives the horizontal speed of the bar as a function of time, assuming that the suspended object is released with the bar at rest at $t = 0$. Assume no friction between rails and bar. Also find the terminal velocity obtained by the bar.

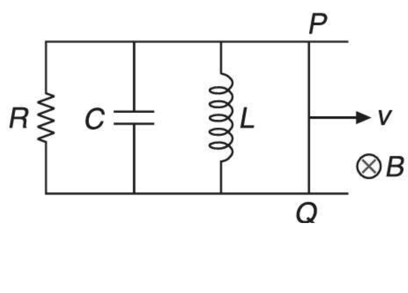


33 A long solenoid has n turns per unit length, radius R and carries a current I is kept in gravity free region. From its axis, at a distance twice that of its radius, a charge $+q$ and mass m is placed. If the current in solenoid is suddenly switched off, find the velocity attained by the charge.

34 The switch in Figure is open for $t < 0$ and then closed at time $t = 0$. Find the current in the inductor and the current in the switch as functions of time thereafter.



35 A conducting wire PQ of length $l = 1$ m is moved in a uniform magnetic field $B = 4$ T with constant velocity $v = 2\ \text{ms}^{-1}$ towards right as shown in Figure. If $R = 2\Omega$, $C = 1$ F and $L = 4$ H then, answer the following questions.



(a) If V_P and V_Q are potentials of the point P and Q respectively, then find $V_P - V_Q$.

(b) Find the potential difference across the inductor.

(c) What is the potential difference across the capacitor?

(d) Calculate the potential difference across the resistor.

(e) Determine the rate of change of current in the inductor.

(f) Find the current through capacitor at $t = 2$ s.

(g) Calculate the current through resistor at $t = 2$ s.

(h) What is the current through inductor at $t = 2$ s?

(i) Find the current through wire PQ at $t = 2$ s.

(j) Calculate the force required to move the wire with the given constant velocity of 2 ms^{-1} at t=2s.

(k) Find the energy supplied per second by the source at $t = 2$ s.

(l) Calculate the total power generated by the applied external force at $t = 2$ s.

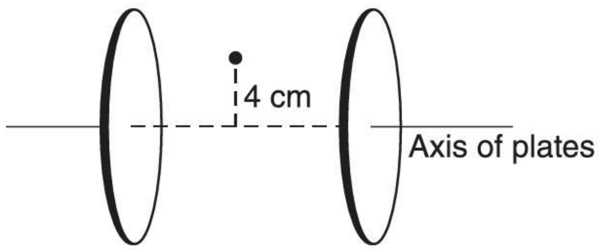
(m) What is the magnetic energy stored per second in inductor at $t = 2$ s?

(n) Find the energy dissipated per second in the resistor at $t = 2$ s.

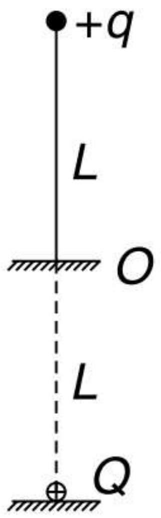
(o) Calculate the electrostatic energy stored per second in capacitor at $t = 2$ s.

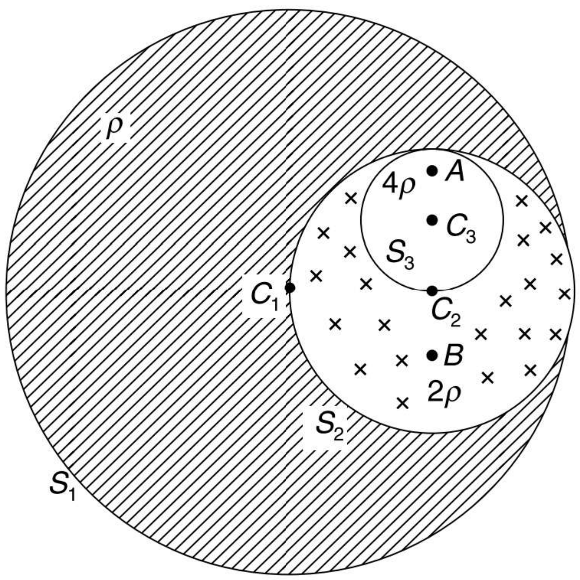
36 Prove that in a series LCR circuit, the frequencies at which the current amplitude falls to $\frac{1}{\sqrt{2}}$ of the current at resonance are separated by an interval equal to $\frac{R}{2\pi L}$.

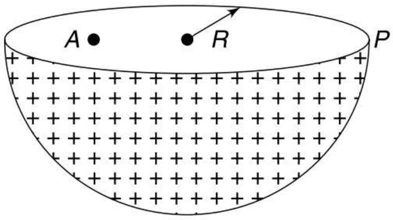
37 Two circular plates each of radius 0.1 m are used to form a parallel plate capacitor. If displacement current between the plates is 2π ampere, then calculate the magnetic field produced by displacement current 4 cm from the axis of the plates.

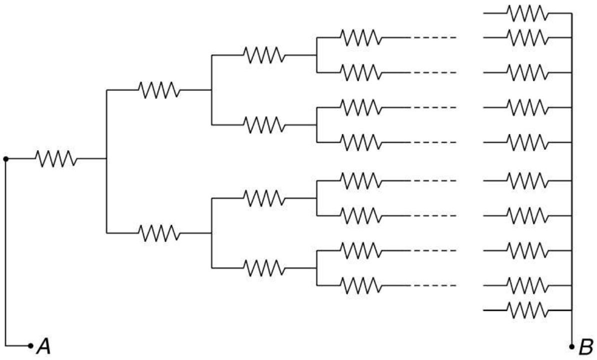
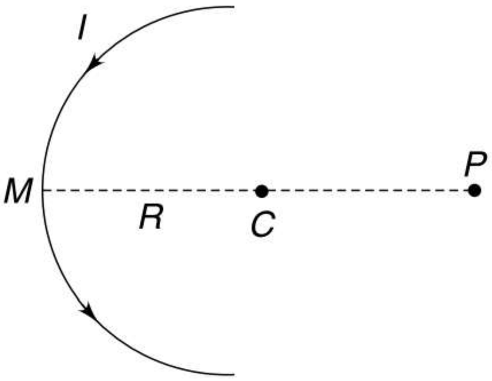


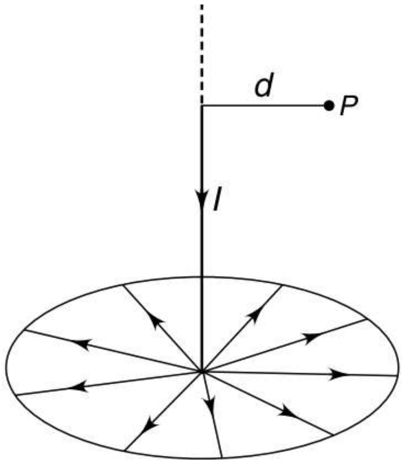
38 A particle of mass m and charge q is attached to a light insulating thread of length L. The other end of the thread is secured at point O. Exactly below point O, there is a small ball having charge Q fixed on an insulating horizontal surface. The particle remains in equilibrium vertically above the ball with the string taut. Distance of the ball from point O is L. Find the minimum value of Q for which the particle will be in a stable equilibrium for any gentle horizontal push given to it.



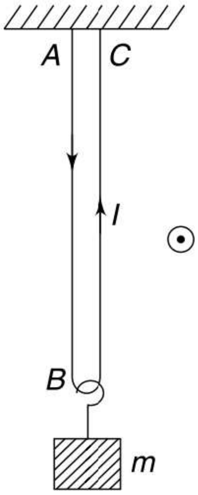
39	In the figure shown, spheres S_1, S_2 and S_3 have radii $4R$, $2R$ and R respectively. C_1, C_2 and C_3 are centers of the three spheres lying in a plane. Angle $\angle C_1C_2C_3$ is right angle. Sphere S_3 has a uniformly spread volume charge density of 4ρ. The remaining part of S_2 has uniform charge density of 2ρ and the left over part of S_1 has a uniform charge density of ρ.  **(a)** Find electric field at a point A at a distance $\frac{R}{8}$ from C_3 on the line C_2C_3 (see figure) **(b)** Find electric field at point B at a distance $\frac{R}{4}$ from C_2 on the line C_3C_2 (see figure)
40	A hemispherical bowl of radius R carries a uniform surface charge density of σ. Find potential at a point P located just outside the rim of the bowl (see figure). Also calculate the potential at a point A located at a distance $R/2$ from the centre on the equatorial plane.



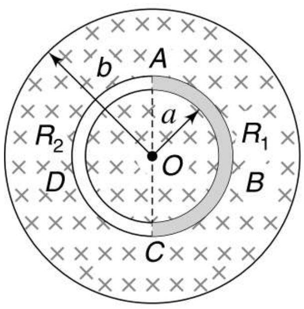
41	An infinite network of resistances has been made as shown in the Figure. Each resistance is R. Find the equivalent resistance between A and B. 
42	A current carrying wire is in the shape of a semicircle of radius R and has current I. M is midpoint of the arc and point P lies on extension of MC at a distance $2R$ from M. Find the magnetic field due to circular arc at point P. 
43	A straight current carrying wire has its one end attached to an infinity conducting sheet (shown as a circle in the figure). The other end of the wire goes to infinity and the wire is perpendicular to the sheet. The current spreads uniformly on the surface of the sheet. Calculate the magnitude of magnetic induction field at a point P at a distance d from the straight wire. Current in the wire is I.



44 A light freely deformable conducting wire with insulation has its two ends (A and C) fixed to the ceiling. The two vertical parts of the wire are close to each other. A load of negligible mass m is attached to the middle of the wire. The entire region has a uniform horizontal magnetic field B directed out of the plane of the figure. Prove that the two parts of the wire take the shape of circular arcs when a current I is passed through the wire. Neglect the magnetic interaction between the two parts of the wire.



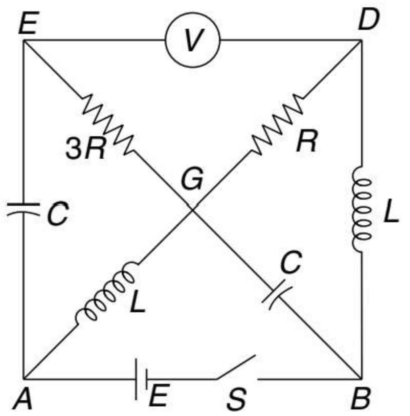
45 A uniform magnetic field B exists perpendicular to the plane of the Figure in a circular region of radius b with its centre at O. A circular conductor of radius $a < b$ and centre at O is made by joining two semicircular wires ABC and ADC. The two segments have same cross section but different resistances R_1 and R_2 respectively. The magnetic field is increased with time and there is an induced current in the conductor.



(a) Find the ratio of electric fields inside the conductors ABC and ADC.

(b) Explain why the electric field in two conductors is different despite the fact that the magnetic field is symmetrical.

46 In the circuit shown, switch S is kept closed and the circuit is in steady state.

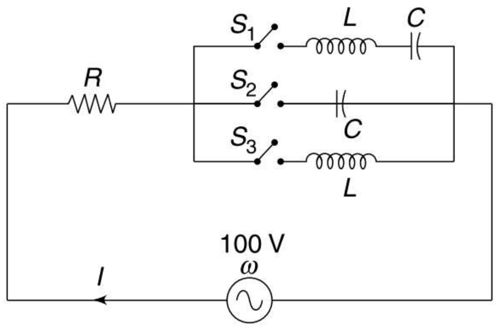


(a) Find reading of the ideal voltmeter.

(b) Now the switch is opened. Find the reading of the voltmeter immediately after the switch is opened.

(c) Find the heat dissipated in resistance R after the switch is opened.

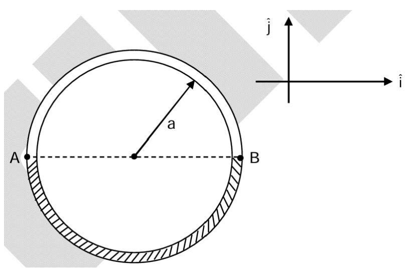
47 In the circuit shown in the figure, one of the three switches is kept closed and other two are open. The value of resistance is $R = 20\Omega$. When the angular frequency (ω) of the 100 V source is adjusted to 500 rad/s, 1000 rad/s and 2000 rad/s it was found that the current I was 4A, 5A and 4A respectively.



(a) Which switch is closed? (S_1, S_2 or S_3)

(b) Find the value of L and C.

48 Figure shows a conducting neutral ring of radius a made from two different wires of uniform cross-section as shown.



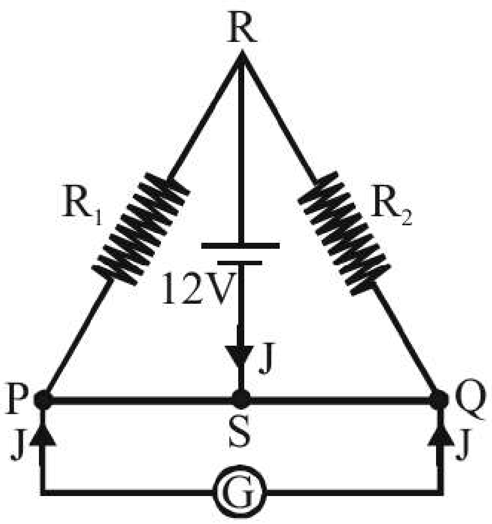
A magnetic field exists inside the ring which varies with time as $B = kt$ (k is a constant). Neglect any charge that appears on the surface of the ring. The upper half of the ring has resistance twice as compared to the lower half.

(a) For a uniform ring (same resistivity throughout), determine the potential difference between points A and B.

(b) For the given non-uniform ring (as in the diagram), find the conservative electric field along the tangent of the circular ring.

(c) For the given non-uniform ring (as in the diagram), calculate the potential difference between points B and A, V_{BA}.

49 A Wheatstone bridge is set up as shown. $R_1 = 10\Omega$ and $R_2 = 20\Omega$. The wire PQ has a length of 10 cm and a resistance of $1\Omega/\text{cm}$. A 12V battery is used as the source. The initial position of the jockey J is at the center of PQ (5 cm from P and 5 cm from Q). Analyze the following scenarios to achieve a balanced Wheatstone Bridge.

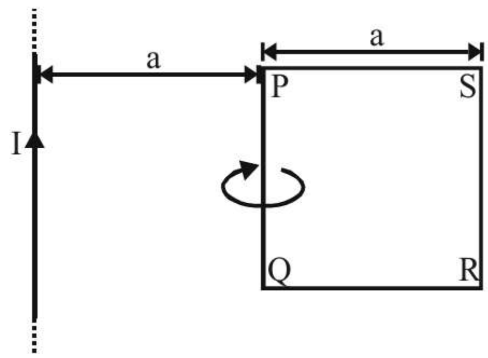


(a) Starting from the center, how should the jockey at S be moved along wire PQ to achieve a null deflection if jockeys at P and Q are fixed at ends P and Q respectively?

(b) Assuming the jockey at S is maintained fixed at the center S and end Q respectively. How should the jockey at P be moved along wire PQ to achieve a null deflection, such that a portion of the wire is in series with the resistor R_1? Specify the direction and distance of movement from end P.

(c) Assuming the jockey at S is maintained fixed at the center S and end P respectively. How should the jockey at Q be moved along wire PQ to achieve a null deflection, such that a portion of the wire is in series with the resistor R_2? Specify the direction and distance of movement from end Q.

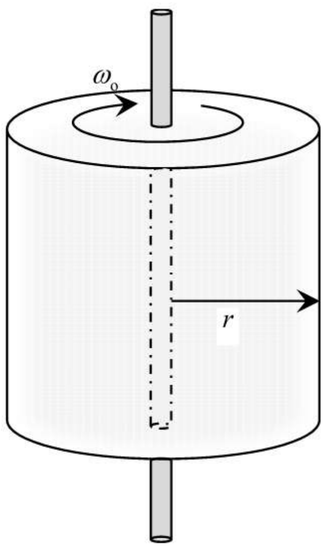
50 A square frame of resistance R and side $a = 20$ cm and a long straight wire carrying a current $I = 10$ amp are located in the same plane. The frame is rotated through an angle of 120° about the side PQ. The amount of charge flown through the loop during this time is $q = 4 \times 10^{-7}$ coulomb. Find the value of R.



51 A parallel plate capacitor is filled with a dielectric whose permittivity varies with the applied voltage according to the law $\epsilon = \alpha U$, where $\alpha = 1\ \mathrm{V}^{-1}$. The same capacitor (but containing no dielectric), initially charged to a voltage $U_0 = 156$ V, is connected in parallel to the first "nonlinear" uncharged capacitor. Determine the final voltage U across the capacitors.

52 The values of two resistors are $(5.0 \pm 0.2)\Omega$ and $(10.0 \pm 0.1)\Omega$. What is percentage error in the equivalent resistance when they are connected in parallel?

53 A long, uniformly charged line with positive linear charge density λ is surrounded by a large number of particles of negative charge having magnitude of specific charge α, each revolving around the line charge with the same angular velocity ω_0.

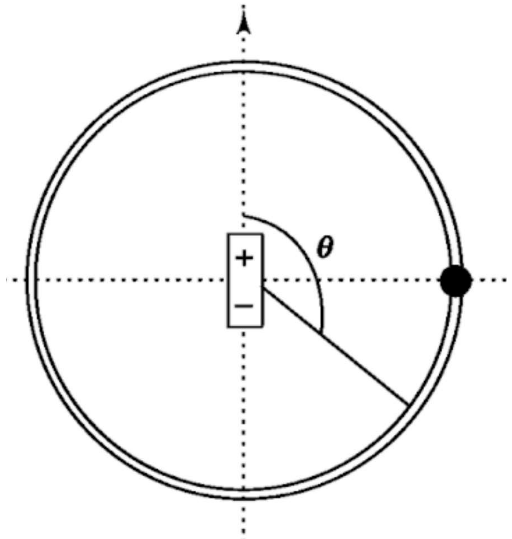


(a) For a charged particle revolving at a distance r from the line charge, the necessary centripetal acceleration is provided by only the electrostatic forces due to other charges around it. If the net charge per unit length (including the rod and carried by the revolving particles) present inside a coaxial cylindrical region of radius r is given as $\lambda(r)$, find a relationship between α, $\lambda(r)$, and ω_0.

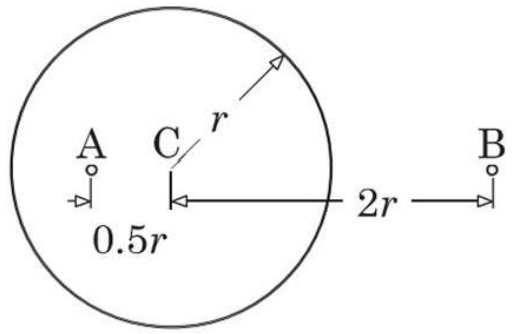
(b) The volume charge density present at a distance r from the line charge is $\rho(r)$. Derive an expression for $\rho(r)$ in terms of α, ω_0, and r.
(c) The net electric flux enclosed through a coaxial cylindrical surface of length l and radius r is $\Phi(r)$. How does $\Phi(r)$ vary with r? Explain your answer.

54 A solid sphere having uniform charge density $\rho = 1\ \mathrm{C/m^3}$ is traveling with a constant velocity $\vec{v} = (2\ \mathrm{m/s})\hat{i}$. Consider a point in space P which remains inside the sphere for a time interval of $\Delta t = 1$ s. During this time interval, the electric field at point P changes from $\vec{E}_1$ to $\vec{E}_2$. Find the value of $(9\epsilon_0) \times |\vec{E}_1 - \vec{E}_2|$ in SI units.

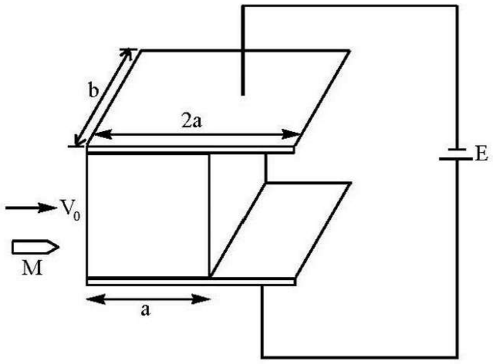
55 A small charged bead with charge Q can slide on a circular frictionless, insulating wire frame. A point-like dipole of dipole moment $\vec{p}$ is fixed at the center of the frame so that $\vec{p}$ is in the plane of the frame. Initially, the bead is on the plane of symmetry of the dipole. The bead is released from rest. Ignore the effect of gravity. Find the magnitude of the velocity of the bead as a function of its angular position θ, where $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$.



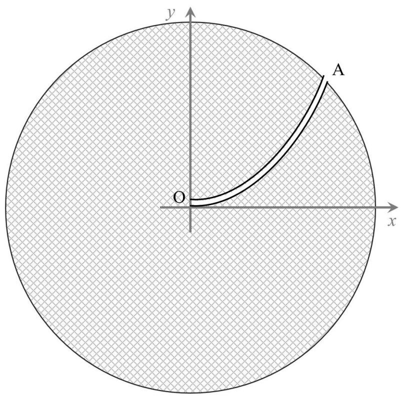
56 Consider a thin conducting shell of radius r carrying total charge q. Two point charges q and $2q$ are placed on points A and B, which are at distances $0.5r$ and $2r$ from the centre C of the shell respectively. If the shell is earthed, how much charge will flow to the earth?



57 A parallel plate capacitor is half filled with a solid dielectric of relative permittivity 'K' and mass 'M'. A cell of voltage E is connected across the capacitor. The plates of the capacitor are smooth and fixed. Distance between plates is d. A bullet of mass 'M' hits the dielectric and gets embedded in it. It is found that the dielectric just exits the capacitor completely. The speed of the bullet is

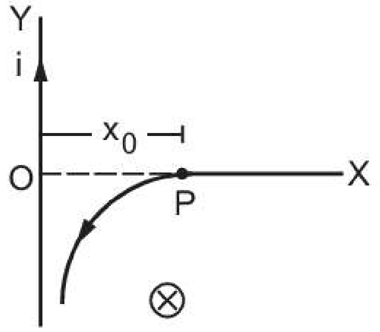


58 A magnetic field out of the screen is increasing in a cylindrical region of radius R at a rate of α with respect to time. The center of the circular region is at the origin O. A smooth tunnel is formed in this plane, defined by the equation $y = \dfrac{\sqrt{2}x^2}{R}$, as shown:



A charge particle q enters the tunnel at point A with a kinetic energy K_o. Determine the kinetic energy of the particle at the origin O.

59 A long, straight wire carries a current i. A particle having a positive charge q and mass m, kept at a distance x_0 from the wire, is projected towards it with a speed v. Find the minimum separation between the wire and the particle.



Answer Keys

1	**(a)** $\alpha = 60^\circ$ **(b)** $T = mg + \dfrac{q_1 q_2}{4\pi\epsilon_0 l^2}$ **(c)** $\sqrt{3}mg; mg$
2	**(a)** $\dfrac{4\alpha_1}{3}$ **(b)** $\dfrac{1}{\sqrt{3}}$
3	$E(2RH\cos\theta + \pi R^2 \sin\theta)$
4	$2\sin^{-1}\left[\sqrt{\dfrac{q_1}{q_2}}\sin\dfrac{\alpha}{2}\right]$
5	$\dfrac{\rho\vec{a}}{2\epsilon_0}$
6	To prove
7	$\sqrt{\dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{3ml}}$
8	Counting sequencially from left, Face 1: $q_2 = \dfrac{Q}{2}$ Face 2: $-q_1 = -\dfrac{Qb}{a+b}$ Face 3: $q_1 = \dfrac{Qb}{a+b}$ Face 4: $Q - q_1 = \dfrac{Qa}{a+b}$ Face 5: $q_1 - Q = -\dfrac{Qa}{a+b}$ Face 6: $q_3 = \dfrac{Q}{2}$

9	$\frac{q}{2}$
10	$\frac{1}{4\pi\epsilon_0}\left(\frac{q_1^2}{2R}+\frac{q_1 q}{R}-\frac{q_1^2}{2R_1}-\frac{q_1 q}{R_1}\right)$
11	$\sqrt[3]{\frac{p}{4\pi\epsilon_0 E_0}}$
12	$\frac{Q^2(R^2-h^2)}{32\pi\epsilon_0 R^4}$
13	$\frac{-q}{4\pi\epsilon_0 2^{n+1}R}$
14	**(a)** $C=\frac{4\pi\epsilon_0 ab}{b-a}$ **(b)** $C_1=\frac{4\pi\epsilon_0 b^2}{b-a}$ **(c)** $C_2=\frac{4\pi\epsilon_0}{\left(\frac{1}{a}+\frac{1}{b}-\frac{2}{d}\right)}$
15	**(a)** $K=\frac{\epsilon_0 AV^2}{2d_1^2(d_0-d_1)}$ **(b)** $V_{max}=\frac{2d_0}{3}\sqrt{\frac{2Kd_0}{3A\epsilon_0}}$ **(c)** $\omega=\sqrt{\frac{K}{m}\left(\frac{3d_1-2d_0}{d_1}\right)}$
16	$k=\frac{27\epsilon_0 AE^2}{8(d-l)^3}$
17	**(a)** 1Ω **(b)** $\frac{2R}{n}$
18	$R_x=\frac{R_1 n_2 N_1}{n_1 N_2}$
19	$X_{max}=1111\text{ k}\Omega;\ X_{min}=0.01\Omega$
20	**(a)** $Q=\frac{CV}{2}(1-e^{-2t/3RC})$ **(b)** $I_2=\frac{V}{2R}-\frac{V}{6R}e^{-2t/3RC}$, Limiting value = $\frac{V}{2R}$
21	**(a)** $q(t)=q_0\left[\left(1-\frac{C}{C_2}\right)e^{-\frac{t}{RC}}+\frac{C}{C_2}\right]$ where $C=\frac{C_1C_2}{C_1+C_2}$, $I(t)=\frac{q_0}{RC_1}e^{-\frac{t}{RC}}$

	(b) Heat Loss = $\dfrac{q_0^2 C_2}{2C_1(C_1+C_2)}$
22	$T = T_0 + \dfrac{V^2}{kR}\left(1 - e^{-\dfrac{kt}{C}}\right)$
23	$T = (R_1 + 2R_2)C\log_e 2$
24	$\dfrac{R_0}{3}$
25	$v = \dfrac{eB(b^2 - a^2)}{2mb}$
26	**(a)** Maximum x-coordinate = $\dfrac{2E_0}{B_0\omega}$ where $\omega = \dfrac{qB_0}{m}$ **(b)** Maximum speed = $\dfrac{2E_0}{B_0}$ **(c)** Cycloidal path
27	$\dfrac{\mu_0 Q_1 Q_2}{8\sqrt{2}\pi t^2}\hat{i}$
28	Outside: $B_{out} = \dfrac{\mu_0 I}{2\pi r}$ Inside: $B_{inside} = \dfrac{\mu_0 I}{h}\sqrt{1 - \left(\dfrac{h}{2\pi R}\right)^2}$
29	$M = \dfrac{1}{4}Q\omega R^2 = \dfrac{1}{4}Q\omega h^2 \tan^2\theta$
30	$\dfrac{\mu_0 I}{4\pi}(1 - e^{-2\pi})$ out of the page
31	$\dfrac{\mu_0 I_0 L}{4\pi}$
32	$v(t) = \dfrac{MgR}{B^2 l^2}\left[1 - e^{-\dfrac{B^2 l^2 t}{R(M+m)}}\right]$ Terminal Velocity: $v_T = \dfrac{MgR}{B^2 l^2}$
33	$\dfrac{\mu_0 n I q R}{4m}$
34	Inductor Current: $I_1 = 0.5(1 - e^{-10t})$ A Switch Current: $I = (1.5 - 0.25e^{-10t})$ A
35	**(a)** 8 V

	(b) 8 V **(c)** 8 V **(d)** 8 V **(e)** 2 As^{-1} **(f)** 0 **(g)** 4 A **(h)** 4 A **(i)** 8 A **(j)** 32 N **(k)** 64 W **(l)** 64 W **(m)** 32 W **(n)** 32 W **(o)** 0 W
36	$f_1 - f_2 = \frac{R}{2\pi L}$
37	5×10^{-6} T
38	$\frac{32\pi\epsilon_0 L^2 mg}{q}$
39	**(a)** $E_A = \frac{\sqrt{41}}{6}\frac{\rho R}{\epsilon_0}$ **(b)** $E_B = \frac{\sqrt{5749}}{75}\frac{\rho R}{\epsilon_0}$
40	$V_P = V_A = \frac{\sigma R}{2\epsilon_0}$
41	$2R$
42	$\frac{\mu_0 I}{2\pi R}\ln(\sqrt{2}+1)$
43	$B = \frac{\mu_0 I}{2\pi d}$
44	Proved in the solution
45	**(a)** $\frac{E_1}{E_2} = \frac{R_1}{R_2}$ **(b)** Explained in the solution
46	**(a)** E **(b)** 2E

	(c) $\frac{1}{2}E^2\left(\frac{L}{R^2}+C\right)$
47	**(a)** S_1 **(b)** $L = 10\,\mathrm{mH}, C = 100\mu F$
48	**(a)** 0 **(b)** $E_c = \frac{ak}{6}$ **(c)** $V_{BA} = \frac{\pi a^2 k}{6}$
49	(a) Jockey S should be moved $\frac{5}{3}$ cm towards P from the center. (b) Jockey P should be moved 2 cm towards S from end P. (c) Achieving balance by moving jockey Q **alone** in the described configuration may not be directly practically possible to increase resistance.
50	$\ln(2)\Omega$
51	12 V
52	3%
53	**(a)** $\lambda(r) = \frac{2\pi\epsilon_0\omega_0^2 r^2}{\alpha}$ **(b)** $\rho = \frac{2\epsilon_0\omega_0^2}{\alpha}$ **(c)** $\Phi(r)$ is proportional to r^2 and thus increases with increase in r.
54	6
55	$v = \sqrt{\frac{-2Qp\cos\theta}{4\pi\epsilon_0 m r^2}}$
56	3q
57	$V_0 = \sqrt{\frac{2\epsilon_0 baE^2}{dM}(K-1)}$
58	$K_O = K_o + \frac{q\alpha R^2}{12}$
59	$x = x_0 e^{-\frac{2\pi mv}{\mu_0 qi}}$

Question Solving Steps

<table>
<tr><td>1</td><td>

1. **(a)** Draw free body diagrams for each bead. Apply equilibrium conditions in the x and y directions. These x and y directions to be chosen appropriatly
2. **(b)** Solve the equilibrium equations to find the tension in the cord.
3. **(c)** Analyze the forces after the cord is cut. Set the net force on each bead to zero to find the condition for equilibrium.

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<tr><td>2</td><td>

1. Write the impulse-momentum equations for both balls, resolving the velocities into x and y components: $\frac{q_i}{m_i}\vec{E}\Delta t = m_i(\vec{v}_i' - \vec{v}_i)$, where $i = 1, 2$, $\vec{v}_i$ is the initial velocity, and $\vec{v}_i'$ is the final velocity.
2. For the first ball, $\vec{v}_1 = v\hat{i}$ and $\vec{v}_1' = \frac{v}{2}(\cos 60^\circ\hat{i} + \sin 60^\circ\hat{j})$. For the second ball, $\vec{v}_2 = v\hat{i}$ and $\vec{v}_2' = v_2\hat{j}$.
3. Compare the x and y components of the impulse-momentum equation for the first ball to find $\frac{q_1}{m_1}E_x\Delta t$ and $\frac{q_1}{m_1}E_y\Delta t$.
4. Compare the x and y components for the second ball to find $\frac{q_2}{m_2}E_x\Delta t$ and $\frac{q_2}{m_2}E_y\Delta t$.
5. Divide the equations to find $\frac{q_2/m_2}{q_1/m_1}$ and v_2.

</td></tr>
<tr><td>3</td><td>

1. The flux is given by $\phi = \vec{E} \cdot \vec{A}$, where $\vec{A}$ is the area vector perpendicular to the surface.
2. When viewed from the -x direction, the projected area perpendicular to the electric field consists of two semicircular areas and a rectangular area.
3. The area of each semicircle is $\frac{1}{2}\pi R^2$, and their projected area perpendicular to E is $\frac{1}{2}\pi R^2 \sin\theta$. The total projected area due to two semicircles is then $\pi R^2 \sin\theta$.
4. The rectangular area is $2RH$, and its projected area perpendicular to E is $2RH\cos\theta$.
5. The total projected area $A_\perp$ is the sum of the projected semicircular and rectangular areas.
6. Calculate the flux using $\phi = EA_\perp$.

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</table>

4	1. The flux through a cone of apex angle θ due to a charge q is given by $\phi = \frac{q}{2\epsilon_0}(1 - \cos\theta)$. 2. The flux leaving $+q_1$ at angle α is equal to the flux entering $-q_2$ at angle β. Therefore, $\frac{q_1}{2\epsilon_0}(1 - \cos\alpha) = \frac{q_2}{2\epsilon_0}(1 - \cos\beta)$. 3. Use the trigonometric identity $1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$ and solve for β.
5	1. The electric field inside a uniformly charged infinite cylinder of charge density ρ at a distance r from the axis is $E = \frac{\rho r}{2\epsilon_0}$. The direction is radial. 2. Let $\vec{r}$ be the position vector of point P from the center of the larger cylinder, and $\vec{r_1}$ be the position vector of P from the center of the cavity. The vector $\vec{a}$ points from the center of the larger cylinder to the center of the cavity. 3. The electric field $\vec{E}$ at P due to the larger cylinder (as if there were no cavity) is $\vec{E} = \frac{\rho\vec{r}}{2\epsilon_0}$. 4. The electric field $\vec{E_1}$ at P due to a cylinder with charge density $-\rho$ occupying the cavity would be $\vec{E}_1 = \frac{-\rho\vec{r_1}}{2\epsilon_0}$. 5. By the principle of superposition, the actual field inside the cavity is $\vec{E}_{net} = \vec{E} - \vec{E}_1 = \frac{\rho}{2\epsilon_0}(\vec{r} - \vec{r_1}) = \frac{\rho\vec{a}}{2\epsilon_0}$.
6	1. Assume a system of charges where each charge is in equilibrium. 2. Consider scaling the system by increasing inter-particle distances by a factor k. 3. Argue that equilibrium is maintained after scaling due to the inverse square law. 4. Show that the work done by electrostatic forces during scaling is zero because the net force on each charge remains zero. 5. Relate zero work done to zero change in potential energy. 6. Consider scaling to infinity, where potential energy is zero. Conclude that the initial potential energy must also be zero.
7	1. The initial electrostatic potential energy is $U_i = \frac{1}{4\pi\epsilon_0}\frac{q^2}{l}$.

	2. When the string is cut, the charged balls repel each other, converting potential energy into kinetic energy. The center of mass remains stationary. When the speed is maximum, the four balls form a straight line. 3. The final potential energy is $U_f = \frac{1}{4\pi\epsilon_0}\frac{q^2}{3l}$. Each of the four balls acquires kinetic energy $\frac{1}{2}mv^2$. When the velocity becomes maximum, balls 3 and 4 are equidistant from balls 1, 2 and center of mass, respectively. 4. Apply conservation of energy: $U_i = U_f + 4(\frac{1}{2}mv^2)$. 5. Solve for v.
8	1. Let q_1, q_2, and q_3 be the charges on the faces as shown in the diagram. The net charge on plates A and C is zero. 2. Since plates A and C are connected by a wire, they are at the same potential. Use the relationship between electric field and potential difference for parallel plates: $V = Ed$, where E is the electric field and d is the separation between plates. 3. The electric field inside a conductor is zero. Apply this condition to plate C. 4. Solve the resulting equations to find q_1, q_2, and q_3.
9	1. The outward pressure P_1 due to the charge q on the shell is given by $P_1 = \frac{\sigma^2}{2\epsilon_0}$, where $\sigma = \frac{q}{4\pi R^2}$ is the surface charge density. 2. The electric field E at the surface of the shell due to the charge $-q_0$ at the center is $E = \frac{q_0}{4\pi\epsilon_0 R^2}$ (radially inwards). 3. The inward pressure P_2 due to this field is $P_2 = \sigma E$. 4. For equilibrium, $P_2 \geq P_1$. Substitute the expressions for P_1 and P_2 and solve for the minimum value of q_0.
10	1. The initial potential energy U_i is the sum of the shell's self-energy (SE) and the interaction energy (IE) between the shell and the point charge. The self energy of shell $(\text{SE})_{q_1}$ is given by $\frac{q_1^2}{8\pi\epsilon_0 R}$ and the interaction energy q and q_1 is $\frac{qq_1}{4\pi\epsilon_0 R}$. 2. The final potential energy U_f is calculated similarly, using R_1 instead of R .

	3. The work done is $W = U_i - U_f$.
11	1. The electric field due to the dipole is $E = \frac{1}{4\pi\epsilon_0}\frac{p}{R^3}\sqrt{1+3\cos^2\theta}$. Its direction makes an angle α with the radial direction, given by $\tan\alpha = \frac{1}{2}\tan\theta$. 2. The external field E_0 is parallel to the dipole moment p. Resolve E and E_0 into radial and tangential components. 3. On the equipotential surface, the tangential component of the net electric field is zero. Therefore, $E_0 \sin\theta = E \sin\alpha$. 4. Substitute the expressions for E and $\sin\alpha$ (from $\tan\alpha$) and solve for R.
12	1. The electric field at the surface of a uniformly charged sphere is $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}$. The surface charge density is $\sigma = \frac{Q}{4\pi R^2}$. 2. The electric field exerts a force $\Delta F = \frac{1}{2}\sigma E \Delta A$ on a small area ΔA. The factor of 1/2 comes because the field is zero inside the sphere and E outside, so the average is E/2. 3. The pressure p is $p = \frac{\Delta F}{\Delta A} = \frac{1}{2}\sigma E = \frac{Q^2}{32\pi\epsilon_0 R^4}$. 4. The force required to hold the sphere together is the product of this pressure and the cross-sectional area: $F = pa = p\pi(R^2 - h^2)$.
13	1. Calculate the charge induced on the outer shell when K_1 is closed. 2. Calculate the charge on the inner shell after K_2 is closed. 3. Determine the pattern of charge distribution after n cycles. 4. Calculate the potential on the outer shell and inner shell after n cycles. 5. Find the potential difference between the shells.
14	1. **(a)** Use the concept of earthing and potential due to spherical shells. 2. **(b)** Apply the same principles as in **(a)** with the inner shell earthed. 3. **(c)** Calculate the net electric field and integrate to find the potential difference, or use the energy method by calculating self and interaction energies.
15	1. **(a)** Apply equilibrium conditions to the plate.

	2. **(b)** Differentiate the expression for V^2 with respect to d1 and set it to zero to find the maximum voltage. 3. **(c)** Write the force equation for a small displacement from equilibrium and compare it with the standard equation of SHM.
16	1. Apply KVL to the circuit to determine the charge on the capacitors. 2. Calculate the electrostatic force on one of the movable plates. 3. Determine the force exerted by the spring. 4. Equate the electrostatic force and spring force at equilibrium. 5. Use the given condition that plate separation equals spring extension.
17	1. **(a)** Exploit the symmetry of the circuit to simplify it. 2. **(b)** Generalize the simplification process for n resistors.
18	1. Write the expressions for the currents in terms of the deflections and proportionality constants. 2. Apply the condition for series connection (equal currents). 3. Apply the condition for parallel connection (equal voltage drops). 4. Eliminate the proportionality constants to find Rx.
19	1. Recall the formula for the unknown resistance in a post office box: $X = \frac{QR}{P}$. 2. To maximize X, maximize Q and R, and minimize P. 3. To minimize X, minimize Q and R, and maximize P.
20	1. Apply Kirchhoff's second law to the two loops in the circuit. 2. Express the current through the capacitor in terms of dQ/dt. 3. Solve the resulting differential equation to find the charge Q as a function of time. 4. Differentiate Q(t) to find the current I1(t). 5. Use Kirchhoff's current law to find I2(t).
21	1. **(a)** Use KVL and the relationship between current and charge to set up a differential equation. Solve the equation to get q(t) and then differentiate

	to get I(t). 2. **(b)** Calculate initial and final energy stored in the capacitors and find the difference.
22	1. Write the energy balance equation. 2. Formulate the differential equation relating temperature and time. 3. Integrate the differential equation to find the time dependence of the temperature.
23	1. Determine the time constant for the discharging phase when the switch is closed. 2. Calculate the time taken for the voltage to drop from (2/3)ΔV to (1/3)ΔV. 3. Determine the time constant for the charging phase when the switch is open. 4. Calculate the time taken for the voltage to rise from (1/3)ΔV to (2/3)ΔV. 5. Add the two times to find the period T.
24	1. Apply symmetry to determine the current distribution in the grid. 2. Use superposition to find the total current flowing through the branch AB. 3. Apply Thevenin's theorem to calculate the equivalent resistance.
25	1. Determine the radius of the circular path of the electron in terms of its velocity and the magnetic field. 2. Relate this radius to the radii of the inner and outer shells. 3. Solve for the maximum velocity.
26	1. Write the expression for the Lorentz force acting on the particle. 2. Apply Newton's second law to determine the equations of motion. 3. Solve the differential equations to find the velocity components vx and vz as functions of time. 4. Integrate the velocity components to find the x and z coordinates as functions of time. 5. Analyze the equations to determine the nature of the path and the maximum displacement and speed.

27	1. Calculate the magnetic field B due to Q1 at the position of Q2. 2. Determine the Lorentz force F on Q2 due to the magnetic field B.
28	1. Calculate the current density j. 2. Resolve j into components jx and jy. 3. Apply Ampere's circuital law to find the magnetic field outside the solenoid (using jx). 4. Apply Ampere's circuital law to find the magnetic field inside the solenoid (using jy).
29	1. Express the magnetic moment dM of a small loop element in terms of its charge, radius, and angular velocity. 2. Relate the charge dq of the loop element to the total charge Q and the area of the cone. 3. Express the radius r of the loop element in terms of the height h and the semi-vertex angle θ. 4. Integrate dM over the height of the cone to find the total magnetic moment M.
30	1. Write down the Biot-Savart law for the magnetic field due to a current element. 2. Determine the angle between dl and r. 3. Express dl in terms of dr. 4. Integrate the Biot-Savart law over the spiral path. 5. Determine the direction of the magnetic field using the right-hand rule.
31	1. Apply Ampere's law to find the magnetic field B(r) inside the wire. 2. Consider an infinitesimal strip of width dr and length L within the rectangle. 3. Calculate the magnetic flux dΦ through this strip. 4. Integrate dΦ from r=0 to r=R to find the total magnetic flux.
32	1. Write the force balance equations for the hanging mass and the bar. 2. Express the current in the bar in terms of the induced EMF and the resistance R.

	3. Relate the induced EMF to the velocity of the bar and the magnetic field. 4. Combine the equations and solve the resulting differential equation for the velocity v(t). 5. To find the terminal velocity, set the acceleration to zero in the velocity equation.
33	1. Calculate the magnetic field inside the solenoid. 2. Apply Faraday's law to find the induced electric field outside the solenoid when the current is switched off. 3. Determine the impulse on the charge due to the induced electric field. 4. Relate the impulse to the change in momentum of the charge to find its final velocity.
34	1. Write down Kirchhoff's voltage law for the two loops in the circuit. 2. Solve the resulting system of equations to find the currents I and I1 as functions of time.
35	1. Use Fleming's right-hand rule to find the direction of induced current and the polarity of the induced emf. 2. Analyze the circuit to determine the potential differences across each element. 3. Use the appropriate formulas to calculate the currents, forces, and power/energy values.
36	1. Write the expression for the current amplitude in a series LCR circuit. 2. Set the current amplitude to $\frac{1}{\sqrt{2}}$ of the resonant current. 3. Solve for the frequencies f1 and f2. 4. Find the difference f1 - f2.
37	1. Find the displacement current density Jd. 2. Calculate the enclosed displacement current within a circle of radius 4 cm. 3. Apply Ampere-Maxwell's law to find the magnetic field at a distance of 4 cm from the axis.

38	1. Write the condition for equilibrium in the vertical direction. 2. Consider a small horizontal displacement and analyze the tangential forces. 3. For stable equilibrium, the tangential component of the electrostatic force should be greater than or equal to the tangential component of the weight.
39	1. Calculate the electric field due to each sphere at point A. 2. Add the electric fields vectorially to find the net electric field at A. 3. Repeat the process for point B.
40	1. Calculate the total charge Q on the hemispherical bowl. 2. Use the formula for the potential due to a uniformly charged sphere to find the potential at the center of the bowl. 3. Note that the potential at the rim is the same as the potential at the center due to symmetry.
41	1. Identify the repeating pattern in the infinite network. 2. Assume the equivalent resistance of the infinite network is R_0. 3. Express R_0 in terms of R and the equivalent resistance of the remaining part of the network. 4. Solve the resulting equation for R_0.
42	1. Apply the Biot-Savart law to a small element of the semicircular arc. 2. Express the magnetic field dB in terms of the current I, radius R, and the angle θ. 3. Integrate dB over the semicircle (from θ=0 to θ=π).
43	1. Construct a circular Amperian loop around the wire, passing through point P. 2. Use Ampere's law to relate the line integral of the magnetic field around the loop to the enclosed current. 3. Due to symmetry, the magnetic field has constant magnitude along the loop. Solve for B.

44	1. Consider a small segment of the wire and draw a free body diagram showing the forces acting on it. 2. The magnetic force on the segment is balanced by the components of tension along the wire. 3. Show that the radius of curvature is constant, implying a circular arc shape.
45	1. **(a)** Relate the electric field in each semicircular wire to the current density and resistance using Ohm's law. 2. **(b)** Explain how charge accumulation at junctions due to the induced EMF affects the electric field in each semicircular wire.
46	1. **(a)** Analyze the circuit in steady state, considering the capacitors as open circuits. 2. **(b)** Analyze the circuit immediately after the switch is opened considering that inductor current and capacitor voltage cannot change instantaneously. 3. **(c)** Calculate the total energy stored in the inductor and capacitor and equate it to the heat dissipated in the resistor.
47	1. **(a)** Analyze the impedance at ω = 1000 rad/s. Determine which switch configuration results in Z = R. 2. **(b)** Write the impedance equations at ω = 500 rad/s and ω = 1000 rad/s. Solve these equations simultaneously to find L and C.
48	1. **(a)** Apply Faraday's law to find the induced EMF in the ring. Since the ring is uniform, use this to find the potential difference. 2. **(b)** Use Ohm's law and the relationship between current density and electric field to find the electric field in each part of the non-uniform ring. 3. **(c)** Integrate the electric field along the path to find the potential difference between points A and B.
49	1. (a) For adjusting S, express R_3 and R_4 in terms of the position of S and solve the balanced bridge equation for the position of S.

	2. (b) For adjusting P, express the modified R'_1 in terms of added resistance due to moving P and solve the balanced bridge equation for the added resistance (and hence jockey position). 3. (c) For adjusting Q, express the modified R'_2 in terms of added resistance due to moving Q and solve the balanced bridge equation for the added resistance (and hence jockey position).
50	1. Calculate the change in magnetic flux through the loop as it rotates. Use Faraday's law to relate this change in flux to the charge flown through the loop.
51	1. Apply charge conservation. 2. Express the charge on each capacitor in terms of its capacitance and voltage. 3. Solve the resulting quadratic equation for the final voltage.
52	1. Calculate the equivalent resistance R. 2. Use partial derivatives to find the error in R. 3. Calculate the percentage error.
53	1. **(a)** Balance the electrostatic force and centripetal force to find the relation. 2. **(b)** Relate volume charge density to linear charge density using Gauss's Law and differential elements, then substitute to find rho(r). 3. **(c)** Use Gauss's law to find the flux as a function of radius. Analyze how this function depends on r.
54	1. Write the expressions for the electric fields at point P at the beginning and end of the time interval. 2. Find the displacement of the center of the sphere during this time interval. 3. Subtract the two electric field vectors to find the change in the electric field. 4. Multiply the magnitude of the change in the electric field by $9\epsilon_0$.
55	1. Calculate the initial potential energy of the bead (which is zero).

	2. Determine the potential energy $U(\theta)$ of the bead at angle θ due to the electric dipole. 3. Apply the principle of conservation of energy: Initial kinetic energy + Initial potential energy = Final kinetic energy + Final potential energy. 4. Solve for the velocity v as a function of θ.
56	1. Calculate the potential at the surface of the conducting shell due to each external point charge. 2. Calculate the potential at the surface of the conducting shell due to the induced charge on the shell itself. 3. Set the sum of these potentials to zero (since the shell is earthed). 4. Solve for the induced charge, which is the charge that flows to the earth.
57	1. Calculate the capacitance of the partially filled capacitor as a function of the dielectric's position. 2. Find the force on the dielectric using the formula $F = \frac{E^2}{2}\frac{dC}{dx}$. 3. Calculate the work done to remove the dielectric completely. 4. Equate the work done to the initial kinetic energy of the bullet to find the speed of the bullet.
58	1. Determine the coordinates of A. 2. Find the equation of the line OA. 3. Calculate the area enclosed by the curve and the line OA. 4. Calculate the induced EMF using Faraday's law. 5. Apply the work-energy theorem to find the kinetic energy at O.
59	1. Calculate the magnetic field as a function of distance. 2. Use Newton's second law to find the equations of motion. 3. Consider the x-component of the magnetic force acting on the particle. 4. Relate the velocity components to the distance x. 5. Solve the differential equation.

Detailed Solutions

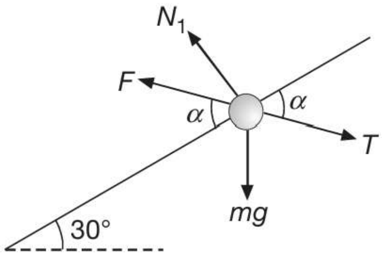
1 **(a)** The forces acting on bead at P having charge q_1 are

(i) Weight mg acting vertically downward

(ii) Tension T in the string along the length PQ

(iii) The electric force $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{l^2}$ between the beads along the length PQ

(iv) Normal reaction N_1 of wire on bead.



For bead P to be in equilibrium,

$$mg\cos 60^\circ = (T - F)\cos\alpha \quad \ldots(1)$$

and $N_1 = mg\cos 30^\circ + (T - F)\sin\alpha \quad \ldots(2)$

For bead Q to be in equilibrium,

$$mg\sin 60^\circ = (T - F)\sin\alpha \quad \ldots(3)$$

and $N_2 = mg\sin 60^\circ + (T - F)\cos\alpha \quad \ldots(4)$

Dividing (3) by (1)

$$\tan 60^\circ = \tan\alpha$$

Hence, $\alpha = 60^\circ$

(b) Hence, From (3),

$\Rightarrow T - F = mg$

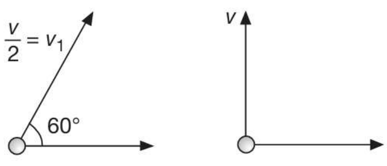
$\Rightarrow T = mg + F = mg + \dfrac{q_1 q_2}{4\pi\epsilon_0 l^2}$

(c) From (2) and (4), we have

$$N_1 = mg\cos 30^\circ + (T + F)\cos 60^\circ = \sqrt{3}mg$$

and $N_2 = mg\sin 30^\circ + mg\cos 60^\circ = mg$

2 **(a)** Let the electric field acting on each ball be given by $\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath}$.



Final condition

From Impulse Momentum equation, we have Impulse = Change in momentum.

Let the final velocities of the balls be $\vec{v}_1$ and $\vec{v}_2$. Noting that $v_1 = \dfrac{v}{2}$, we have

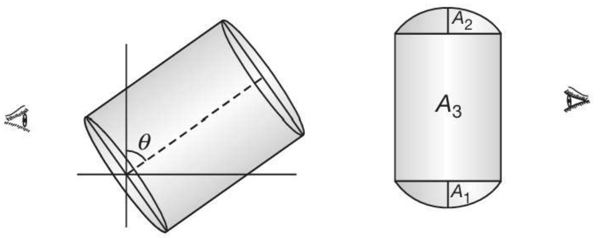
$$\frac{q_1}{m_1}(E_x\hat{\imath} + E_y\hat{\jmath})\Delta t = m_1\left[\frac{v}{2}(\cos 60^\circ\hat{\imath} + \sin 60^\circ\hat{\jmath}) - v\hat{\imath}\right] \ldots(1)$$

$$\frac{q_2}{m_2}(E_x\hat{\imath} + E_y\hat{\jmath})\Delta t = m_2[v_2(\cos 90^\circ\hat{\imath} + \sin 90^\circ\hat{\jmath}) - v\hat{\imath}] \ldots(2)$$

On comparing the x and y-components on both sides of equation (1), we get

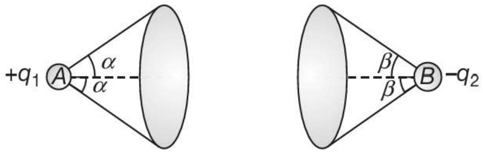
$$\frac{q_1}{m_1}E_x\Delta t = -\frac{3}{4}v \ ...(3)$$
$$\frac{q_1}{m_1}E_y\Delta t = \frac{\sqrt{3}}{4}v \ ...(4)$$
Similarly, from equation (2), we get
$$\frac{q_2}{m_2}E_x\Delta t = -v \ ...(5)$$
$$\frac{q_2}{m_2}E_y\Delta t = v_2 \ ...(6)$$
From equations (3) and (5), by dividing the equations expressing x-components, we get
$$\frac{\frac{q_1}{m_1}}{\frac{q_2}{m_2}} = \frac{3}{4} \Rightarrow \frac{q_2}{m_2} = \frac{4}{3}\frac{q_1}{m_1} = \frac{4}{3}\alpha_1$$
(b) Also $\dfrac{q_1}{m_1} = \dfrac{\sqrt{3}v}{4v_2} = \alpha_1$
$$\Rightarrow \frac{\sqrt{3}v}{4v_2} = \frac{3}{4}\frac{v}{v_2} \Rightarrow v_2 = \frac{v}{\sqrt{3}}$$

3 Looking at the cylinder from -x-axis, it appears as shown in the figure.



Perpendicular component of area $A_1 = A_1 + A_2 + A_3$.
Now $A_1 = A_2$
$$A_1 = \left(\frac{\pi R^2}{2}\right)\sin\theta$$
and $A_3 = 2HR\cos\theta$
Total perpendicular component of area is
$$A_\perp = 2\left(\frac{\pi R^2}{2}\right)\sin\theta + 2HR\cos\theta$$
$$A_\perp = 2RH\cos\theta + \pi R^2\sin\theta$$
Thus, flux passing through cylinder is
$$\phi = EA_\perp = E(2RH\cos\theta + \pi R^2\sin\theta)$$

4 A line can leave $+q_1$ in a cone of apex angle α and then enter $-q_2$ in a cone of apex angle β.
So, flux due to the charge $+q_1$ is
$\phi_1 = \dfrac{q_1}{2\epsilon_0}(1-\cos\alpha)$, and that due to the charge $-q_2$ is
$$\phi_2 = \frac{q_2}{2\epsilon_0}(1-\cos\beta).$$



Since, we know that only one line is leaving q_1 to enter $-q_2$. So, we can say

$$\frac{\phi_1}{\phi_2} = \frac{N_1}{N_2} = \frac{1}{1}$$

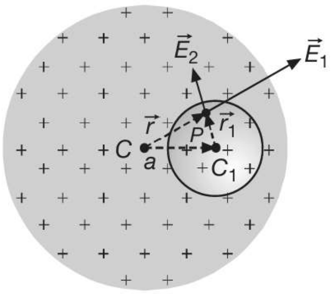
$$\Rightarrow \frac{q_1}{2\epsilon_0}(1 - \cos\alpha) = \frac{q_2}{2\epsilon_0}(1 - \cos\beta)$$

$$\Rightarrow q_1\left(2\sin^2\frac{\alpha}{2}\right) = q_2\left(2\sin^2\frac{\beta}{2}\right)$$

$$\Rightarrow \sin\frac{\beta}{2} = \sqrt{\frac{q_1}{q_2}}\sin\frac{\alpha}{2}$$

$$\Rightarrow \beta = 2\sin^{-1}\left[\sqrt{\frac{q_1}{q_2}}\sin\frac{\alpha}{2}\right]$$

5 Consider a point P in the cavity at a position vector $\vec{r}$ from the centre of cylinder and at a position vector $\vec{r}_1$ from the centre of cavity as shown.



If $\vec{E}$ be the electric field strength at P due to the complete charge of the cylinder (inside cavity also) then we know electric field inside a uniformly charged cylinder is given as

$$\vec{E} = \frac{\rho\vec{r}}{2\epsilon_0}$$

Similarly, if we assume that the charge is only there in the region of the cavity and hence this will also be a uniformly charged small cylinder. If $\vec{E}_1$ be the electric field only due to the cavity charge then

$$\vec{E}_1 = \frac{\rho\vec{r}_1}{2\epsilon_0}$$

Now the electric field due to the charged cylinder in the cavity at point P can be given as

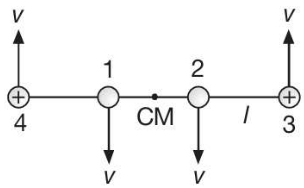
$$\vec{E}_{net} = \vec{E} - \vec{E}_1 \text{ (As now charge of cavity is removed)}$$

$$\Rightarrow \vec{E}_{net} = \frac{\rho}{2\epsilon_0}(\vec{r} - \vec{r}_1) = \frac{\rho\vec{a}}{2\epsilon_0}\ [\because \vec{r} - \vec{r}_1 = \vec{a}]$$

6 Consider a system of point charges where each charge is in equilibrium due to the net electrostatic force exerted by all other charges. We will analyze the electrostatic potential energy of this system.

Imagine scaling up the system, increasing the distances between every pair of charges by a common factor k ($k > 1$). Since electrostatic force is proportional to $1/r^2$, scaling all distances by k reduces each individual force by a factor of $1/k^2$. Because each charge was initially in equilibrium (net force of zero), and each force acting on a charge is scaled by the same factor, the net force on each charge remains zero after scaling.
Now, let's consider the work done on the system during this scaling process. This scaling is performed by an external agent, not the electrostatic forces. As each charge is moved outwards by the external agent, the net electrostatic force on it remains zero. Since work done on a charge is the integral of the net force on it over its displacement, and the net force is consistently zero, the work done by electrostatic forces on each charge during scaling is zero. Consequently, the total work done by electrostatic forces on the entire system, observed from any inertial frame, is also zero.
Because electrostatic forces are conservative, the work done by them is equal to the negative change in potential energy: $W = -\Delta U$. Since the work done is zero, the change in potential energy (ΔU) during scaling is also zero. This means the potential energy of the scaled system (U_{final}) is equal to the potential energy of the original system ($U_{initial}$).
If we scale the system to infinity ($k \to \infty$), the charges become infinitely separated, and their electrostatic interaction becomes zero. By definition, the electrostatic potential energy of an infinitely separated system of charges is zero ($U_{final} = 0$).
Since $U_{final} = U_{initial}$ and $U_{final} = 0$, we conclude that the initial potential energy $U_{initial}$ must also be zero.
Therefore, for a system of charges in equilibrium, the electrostatic potential energy of the system is zero.

7 Initially, since no external horizontal forces act on the set of four balls, the center of mass of the system stays fixed at the initial location of the center of the square. As the charged balls 4 and 3 swing out and away from each other, balls 1 and 2 move down with equal y-components of velocity.



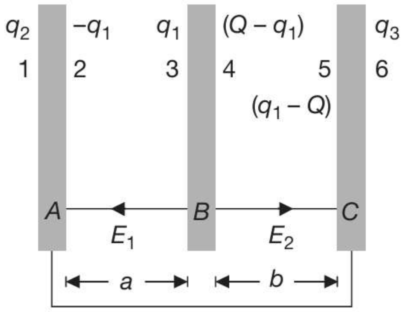
The maximum kinetic energy point is illustrated.
For system energy to be conserved,

$$\frac{1}{4\pi\epsilon_0}\frac{q^2}{l} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{3l} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0}\frac{2q^2}{3l} = 2mv^2$$

$$\Rightarrow v = \sqrt{\frac{1}{4\pi\epsilon_0}\frac{q^2}{3ml}}$$

8 Let the charge distribution in all the six faces be as shown in figure. While distributing the charge on different faces, we have used the fact that two opposite faces have equal and opposite charges on them.



Net charge on plates A and C is zero. Hence,

$$q_2 - q_1 + q_3 + (q_1 - Q) = 0$$

$\Rightarrow q_2 + q_3 = Q$ ...(1)

Further, A and C are at same potentials. If E_1 be the field between the plates A and B and that between B and C is E_2, then

$$V_A - V_B = E_2 b$$

$$V_B - V_C = E_1 a$$

Since $V_A = V_C$, so $V_A - V_B = V_C - V_B$, hence

$E_1 a = E_2 b \Rightarrow \dfrac{q_1}{A\epsilon_0}a = \dfrac{(Q - q_1)}{A\epsilon_0}b$

$\Rightarrow q_1 = \dfrac{Qb}{a+b}$ ...(2) [A = Area of plates]

Electric field inside any conducting plate (say inside C) is zero. Therefore,

$$\frac{q_2}{2A\epsilon_0} - \frac{q_1}{2A\epsilon_0} + \frac{q_1}{2A\epsilon_0} + \frac{(Q - q_1)}{2A\epsilon_0} - \frac{q_1}{2A\epsilon_0} + \frac{Q}{2A\epsilon_0} - \frac{q_3}{2A\epsilon_0} = 0$$

$\Rightarrow q_2 - q_3 = 0$ ...(3)

Solving equations (1), (2) and (3), we get

$$q_1 = \frac{Qb}{a+b}, \quad q_2 = \frac{Q}{2}, \quad q_3 = \frac{Q}{2}$$

Hence, the charges on the different faces are as follows:

Face 1: $q_2 = \dfrac{Q}{2}$

Face 2: $-q_1 = -\dfrac{Qb}{a+b}$

Face 3: $q_1 = \dfrac{Qb}{a+b}$

Face 4: $Q - q_1 = \dfrac{Qa}{a+b}$

Face 5: $q_1 - Q = -\dfrac{Qa}{a+b}$

Face 6: $q_3 = \dfrac{Q}{2}$

9 Let the outward electric pressure at every point of the spherical shell, due to its own charge q, be P_1. Then

$$P_1 = \frac{\sigma^2}{2\epsilon_0} = \frac{1}{2\epsilon_0}\left(\frac{q}{4\pi R^2}\right)^2 = \frac{q^2}{32\pi^2\epsilon_0 R^4} \ [\because \sigma = \frac{q}{4\pi R^2}]$$

Due to charge $-q_0$, the electric field on the surface of the shell is $E = \dfrac{q_0}{4\pi\epsilon_0 R^2}$ (radially inwards).

This electric field pulls every point of the shell in an inward direction. Let the inward pressure on the surface of the shell due to this negative point charge at the center be P_2. Then

$$P_2 = \sigma E = \left(\frac{q}{4\pi R^2}\right)\left(\frac{q_0}{4\pi\epsilon_0 R^2}\right) = \frac{q q_0}{16\pi^2\epsilon_0 R^4}$$

For equilibrium of the hemispherical shell, or for the shells not to separate,

$$P_2 \geq P_1$$

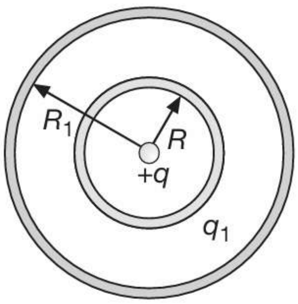
$$\Rightarrow \frac{q q_0}{16\pi^2\epsilon_0 R^4} \geq \frac{q^2}{32\pi^2\epsilon_0 R^4}$$

$$\Rightarrow q_0 \geq \frac{q}{2}$$

$$\Rightarrow q_{0_{min}} = \frac{q}{2}$$

10 Work done = Decrease in Potential Energy

$$\Rightarrow \text{Work} = U_i - U_f$$



Now $U_i = (\text{SE})_q + (\text{SE})_{q_1} + (\text{IE})$

$$\Rightarrow U_i = (\text{SE})_q + \frac{q_1^2}{8\pi\epsilon_0 R} + \frac{q_1 q}{4\pi\epsilon_0 R}$$

and $U_f = (\text{SE})_q + \dfrac{q_1^2}{8\pi\epsilon_0 R_1} + \dfrac{q_1 q}{4\pi\epsilon_0 R_1}$

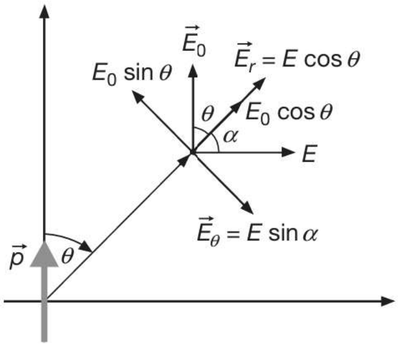
$$\Rightarrow \text{Work done} = U_i - U_f$$

$$W = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1^2}{2R} + \frac{q_1 q}{R} - \frac{q_1^2}{2R_1} - \frac{q_1 q}{R_1}\right)$$

11 The electric field due to a dipole is given by

$$E = \frac{1}{4\pi\epsilon_0}\frac{p}{R^3}\sqrt{1 + 3\cos^2\theta}$$

and its direction with the radius vector $\vec{r}$ is given by $\tan\alpha = \frac{1}{2}\tan\theta$, so that E_r and E_θ are the radial and transverse component of electric field due to the dipole and $E_0 \sin\theta$ and $E_0 \cos\theta$ are two rectangular components of external field E_0. Since, at equipotential surface, there is no tangential components of electric field. So $E_0 \sin\theta$ must be equal and opposite to $E_\theta = E\sin\alpha$.



Hence, $E_0 \sin\theta = E\sin\alpha$

$$\Rightarrow E_0 = \frac{E\sin\alpha}{\sin\theta} = \frac{1}{4\pi\epsilon_0}\frac{p}{R^3}\frac{\sqrt{1+3\cos^2\theta}}{\sin\theta} \times \frac{\sin\theta}{\sqrt{1+3\cos^2\theta}} \times \frac{1}{\sin\theta} \times \frac{x}{1}$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0 R^3}p = E_0 \Rightarrow 4\pi\epsilon_0 R^3 E_0 = p$$

$$R = \sqrt[3]{\frac{p}{4\pi\epsilon_0 E_0}}$$

12 At the surface of the charged sphere, whether it consists of a single piece or two pieces close together, the electric field strength from Gauss's law is

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}.$$

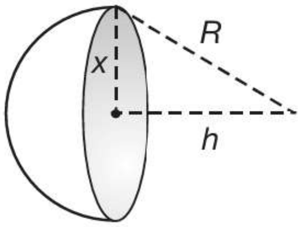
The electric charge per unit surface area is $\sigma = \frac{Q}{4\pi R^2}$.

This electric field exerts a force $\Delta F = \frac{1}{2}\sigma E\Delta Q$ on the charge $\Delta Q = \sigma\Delta A$ which resides on a surface of area ΔA, as illustrated in figure. The reason for the factor of $\frac{1}{2}$ is that the electric field strength is E at the outer surface of the sphere is zero inside and hence its average value comes out to be $\frac{E}{2}$.

The force per unit area exerted by the charges on the pieces of the sphere is therefore

$$\frac{\Delta F}{\Delta A} = \frac{Q^2}{32\pi\epsilon_0 R^4} = p$$

The required force can be considered to be analogous to the force with which two pieces of the sphere and this force is also the product of p and the cross-sectional area of the intersection plane and sphere.



So, $F = pa = p(\pi x^2)$

$\Rightarrow F = p\pi(R^2 - h^2) = \dfrac{Q^2}{32\pi\epsilon_0 R^4}(R^2 - h^2)$

13 When K_1 is closed first time, outer sphere is earthed and the potential on it becomes zero. Let the charge on it be q_1.

V_i = Potential due to charge on inner sphere and that due to charge on outer sphere

$$V_i = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{2R} + \frac{q_1}{2R}\right] = 0$$

$\Rightarrow q_1 = -q$

When K_2 is closed first time, the potential V_2 on inner sphere becomes zero as it is earthed. Let the new charge on inner sphere be q_2.

$$0 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{R} + \frac{1}{4\pi\epsilon_0}\frac{(-q)}{2R}$$

$\Rightarrow q_2 = \dfrac{q}{2}$

Now when K_1 will be closed second time, charge on outer sphere will be $-q_2$, i.e., $\dfrac{-q}{2}$.

After one event involving closure and opening of K_1 and K_2, charge is reduced to half its initial value.

Similarly, when K_1 will be closed n^{th} time, charge on outer sphere will be $\dfrac{-q}{2^{n-1}}$

After closing of K_2 n^{th} time, charge on inner shell will be reduced to half the previous value.

Be negative of half the charge on outer shell, i.e., $\left(+\dfrac{q}{2^n}\right)$ and potential on it will be zero.

For potential of outer shell,

$$V_0 = \frac{1}{4\pi\epsilon_0}\left(\frac{\frac{q}{2^n}}{2R}\right) + \frac{1}{4\pi\epsilon_0}\left(\frac{\frac{-q}{2^{n-1}}}{2R}\right)$$

$$\Rightarrow V_0 = \frac{-q(-1+2)}{4\pi\epsilon_0 \cdot 2^{n+1}R} = \frac{-q}{4\pi\epsilon_0 \cdot 2^{n+1}R}$$

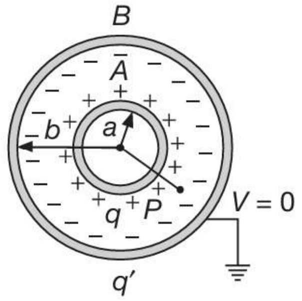
Potential difference $= V_0 - V_i = \dfrac{-q}{4\pi\epsilon_0 2^{n+1}R} - 0$

$$\Rightarrow V_0 - V_i = \frac{-q}{4\pi\epsilon_0 2^{n+1}R}$$

14 **(a)** Since the outer sphere is earthed hence, its potential will be zero, i.e., $V_B = 0$. If q' is the charge induced on shell B, then

$$\frac{q+q'}{4\pi\epsilon_0 b} = 0$$

$$\Rightarrow q' = -q$$



So electric field at a point P between the shells is

$$E = E_A + E_B = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \text{ [as } E_B = E_{in} = 0]$$

$$\Rightarrow \frac{-dV}{dr} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \text{ [as } E = -\frac{dV}{dr}]$$

$$\Rightarrow \int_V^0 dV = \frac{q}{4\pi\epsilon_0}\int_a^b \frac{dr}{r^2}$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Since, $C = \frac{q}{V}$

$$\Rightarrow C = \frac{4\pi\epsilon_0 ab}{b-a} \quad \ldots(1)$$

From this it is clear that

(i) As $b \to \infty$ equation (1) reduces to $C \to 4\pi\epsilon_0 a$.

So, we can say that, a spherical conductor is a spherical capacitor with its other plate of infinite radius.

(ii) As a and b both become very large such that $b-a$ has a finite and small value, say d, then $4\pi ab = 4\pi a^2 = 4\pi\frac{A}{d} = A$ and so equation (1) reduces to $C = \frac{\epsilon_0 A}{d}$ and hence we observe that a spherical capacitor behaves as a parallel plate capacitor when its spherical surfaces have large radii and are close to each other.

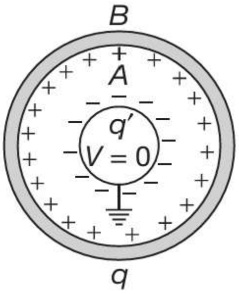
(b) In this situation $V_A = 0$ so if q' is the charge induced on shell A, then

$$\frac{1}{4\pi\epsilon_0}\left(\frac{q'}{a} + \frac{q}{b}\right) = 0$$

$$\Rightarrow q' = -\frac{a}{b}q$$

Since no field will exist at P due to B as P lies inside the shells, so the field at a point P, between the conductor is

$$E = E_A + E_B = \frac{1}{4\pi\epsilon_0}\frac{q'}{r^2}$$



$$\Rightarrow \frac{dV}{dr} = \frac{1}{4\pi\epsilon_0}\left(-\frac{a}{b}\frac{q}{r^2}\right)$$

$$\Rightarrow C_1 = \frac{b}{a}C(< C) \text{ [as } b > a] \text{ ...(2)}$$

Also, we note that in this situation q_B is not equal to $-q_A$ and hence the system is not a capacitor. However, we observe that

$$C_1 = \frac{4\pi\epsilon_0 b^2}{b-a} = \frac{4\pi\epsilon_0 ab}{b-a} + 4\pi\epsilon_0 b = C_{AB} + C_B$$

This system is equivalent to a spherical capacitor (of inner radius a and outer radius b and a spherical conductor of radius b connected in parallel. This is because the charge q given to the outer shell distributes in such a way that $\left(\frac{a}{b}\right)q$ remains on its inner side while the remaining $q\left[1-\left(\frac{a}{b}\right)\right]$ lies on its outer side. Hence,

$$\left|\frac{q_{in}}{q_{out}}\right|_B = \frac{\frac{a}{b}q}{q - \left(\frac{a}{b}\right)q} = \frac{a}{b-a}$$

and the system becomes equivalent to two capacitors in parallel having a common potential V.

(c) To find the capacitance in this situation, let us consider a point P at a distance r from centre of A on the line joining the centres of two spheres. If E be the net field at this point P, then

$$E = \frac{q}{4\pi\epsilon_0 r^2} + \frac{q}{4\pi\epsilon_0 (d-r)^2}$$

$$\Rightarrow \frac{dV}{dr} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r^2} + \frac{1}{(d-r)^2}\right] \text{ [as } E = \frac{dV}{dr}]$$

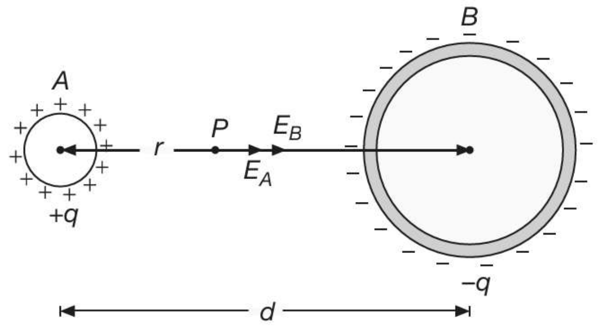
$$\Rightarrow \int_{V_A}^{V_B} dV = \frac{q}{4\pi\epsilon_0}\int_a^b \left[\frac{1}{r^2} + \frac{1}{(d-r)^2}\right]dr$$

$$\Rightarrow V_A - V_B = \frac{q}{4\pi\epsilon_0}\left[-\frac{1}{r} + \frac{1}{(d-r)}\right]_a^b$$

$$\Rightarrow V_A - V_B = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{a} + \frac{1}{b} - \frac{1}{(d-a)} - \frac{1}{(d-b)}\right]$$

For $d \gg a$ and $d \gg b$, we get

$$\Rightarrow V_A - V_B = V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right]$$



Since $C = \dfrac{q}{|\Delta V|}$

$$\Rightarrow C_2 = \frac{4\pi\epsilon_0}{\left(\dfrac{1}{a} + \dfrac{1}{b} - \dfrac{2}{d}\right)} \quad \ldots(3)$$

Using Concept of Self Energy and Interaction Energy

The self energy of a spherical conductor is

$$U_s = \frac{1}{2}qV = \frac{1}{2}\left[\frac{q^2}{4\pi\epsilon_0 R}\right] \text{ [as } V = \frac{q}{4\pi\epsilon_0 R}\text{]}$$

while interaction energy of two point charges separated by a distance r

$$U_i = qV = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

So total potential energy of the system is

$$U = \begin{bmatrix} Self \\ Energy \\ of A \end{bmatrix} + \begin{bmatrix} Self \\ Energy \\ of B \end{bmatrix} + \begin{bmatrix} Interaction \\ Energy \\ of AB \end{bmatrix}$$

$$\Rightarrow U = U_A + U_B + U_{AB}$$

$$\Rightarrow U = \frac{q^2}{8\pi\epsilon_0 a} + \frac{(-q)^2}{8\pi\epsilon_0 b} + \frac{q(-q)}{4\pi\epsilon_0 d}$$

$$\Rightarrow U = \frac{1}{2} \cdot \frac{q^2}{4\pi\epsilon_0}\left(\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right)$$

Since, we have $U = \dfrac{(q^2)}{2C}$

$$\Rightarrow C_2 = \frac{4\pi\epsilon_0}{\left(\dfrac{1}{a} + \dfrac{1}{b} - \dfrac{2}{d}\right)}$$

This result is identical to the result obtained earlier, using the method of electric field.

Now, if $d \to \infty$ then

$$C_2' = \frac{4\pi\epsilon_0}{\left(\dfrac{1}{a} + \dfrac{1}{b}\right)}$$

$$\Rightarrow C_2' < C_2$$

Also, $\dfrac{1}{C_2'} = \dfrac{1}{4\pi\epsilon_0 a} + \dfrac{1}{4\pi\epsilon_0 b} = \dfrac{1}{C_A} + \dfrac{1}{C_B}$

and hence the given system becomes equivalent to two capacitors $C_A(= 4\pi\epsilon_0 a)$ and $C_B(= 4\pi\epsilon_0 b)$ in series.

Also, we observe that in this ILLUSTRATION, **(a)** and **(c)** represent capacitors (as here the two conductors have equal and opposite charges) while arrangement given in **(b)** is not a capacitor (because q_B is not equal to $-q_A$).

15 **(a)** Let the initial extension in the spring, in equilibrium situation be x_0. Then, for equilibrium of lower plate 2, we must have Kx_0 (acting upwards) balancing mg (acting downwards).

$$Kx_0 = mg$$

When the voltage source is connected, then the plate separation changes from d_0 to d_1 as a result of which the extension of the spring becomes $(d_0 - d_1) + x_0$ so that net extension in the spring becomes $x_0 + (d_0 - d_1)$.

When equilibrium is attained again, we have

$$K[x_0 + (d_0 - d_1)] = mg + \frac{1}{2}\epsilon_0 E^2 A$$

Since, $Kx_0 = mg$

$$K(d_0 - d_1) = \frac{1}{2}\epsilon_0 \left(\frac{V}{d_1}\right)^2 A \quad \ldots(1)$$

$$\Rightarrow K = \frac{\epsilon_0 A V^2}{2d_1^2(d_0 - d_1)}$$

(b) From equation (1), we have

$$V^2 = \frac{2Kd_1^2(d_0 - d_1)}{\epsilon_0 A} = \frac{2K}{\epsilon_0 A}(d_0 d_1^2 - d_1^3) \quad \ldots(2)$$

Differentiating w.r.t. d_1, we get

$$2V\left(\frac{dV}{dd_1}\right) = \frac{2K}{A\epsilon_0}(2d_0 d_1 - 3d_1^2)$$

For V to be maximum, we have $\left(\frac{dV}{dd_1}\right) = 0$

$$\Rightarrow 2d_0 d_1 - 3d_1^2 = 0$$

$$\Rightarrow d_1 = \frac{2}{3}d_0$$

Substituting $d_1 = \frac{2d_0}{3}$ in (2), we get

$$V_{max}^2 = \frac{2K}{\epsilon_0 A}\left(\frac{2}{3}d_0\right)^2 \left(d_0 - \frac{2}{3}d_0\right)$$

$$\Rightarrow V_{max} = \frac{2d_0}{3}\sqrt{\frac{2Kd_0}{3A\epsilon_0}} \quad \ldots(3)$$

(c) Let a small displacement x be given to the upper plate in the downward direction from the equilibrium position. Then the net force on the plate is

$$F = -K[x_0 + (d_0 - d_1) + x] + mg + \frac{1}{2}\epsilon_0 A\left[\frac{V^2}{(d_1 - x)^2}\right]$$

$$\{\because E = \frac{V}{d_1 - x}\}$$

$$\Rightarrow F = -K(d_0 - d_1) - Kx + \frac{1}{2}\epsilon_0 A\frac{V^2}{d_1^2}\left(1 - \frac{x}{d_1}\right)^{-2}$$

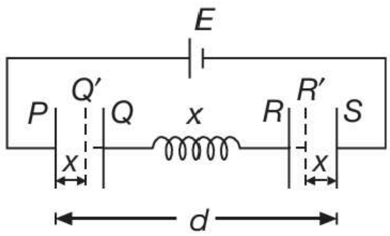
$$\Rightarrow F = -Kx\left(\frac{3d_1 - 2d_0}{d_1}\right)$$
$$\Rightarrow m\ddot{x} = -Kx\left(\frac{3d_1 - 2d_0}{d_1}\right)$$
$$\Rightarrow \ddot{x} + \frac{K}{m}\left(\frac{3d_1 - 2d_0}{d_1}\right)x = 0$$

Compare with the standard equation of SHM, $\ddot{x} + \omega^2 x = 0$, we get

$$\omega = \sqrt{\frac{K}{m}\left(\frac{3d_1 - 2d_0}{d_1}\right)}$$

16 Let charge on capacitors be q and separation between plates P and Q and R and S be x at any time.

Distance between plates P and Q and R and S is same because the force acting on them is same.



Capacitance of capacitor PQ is given by

$$C_1 = \frac{\epsilon_0 A}{x}$$

Capacitance of capacitor RS is given by

$$C_2 = \frac{\epsilon_0 A}{x}$$

From Kirchhoff's Voltage Law (KVL), we have

$$\frac{q}{C_1} + \frac{q}{C_2} = E$$
$$\Rightarrow q = \frac{\epsilon_0 AE}{2x}$$

At this moment extension in spring, $y = d - 2x - l$ and force on plate Q towards P is given by

$$F_1 = \frac{q^2}{2A\epsilon_0} = \frac{\epsilon_0^2 A^2 E^2}{8Ax^2\epsilon_0} = \frac{A\epsilon_0 E^2}{8x^2} \quad \ldots(1)$$

Spring force on plate Q due to extension in spring is given by

$$F_2 = ky$$

At equilibrium,

Separation between the plates = Extension in the spring

Thus $x = y = d - 2x - l$

$$\Rightarrow x = \frac{d - l}{3} \quad \ldots(2)$$
$$\Rightarrow F_1 = F_2 = kx = ky \quad \ldots(3)$$

From equations (1) and (3), we get

$$\frac{A\epsilon_0 E^2}{8x^2} = ky = kx$$

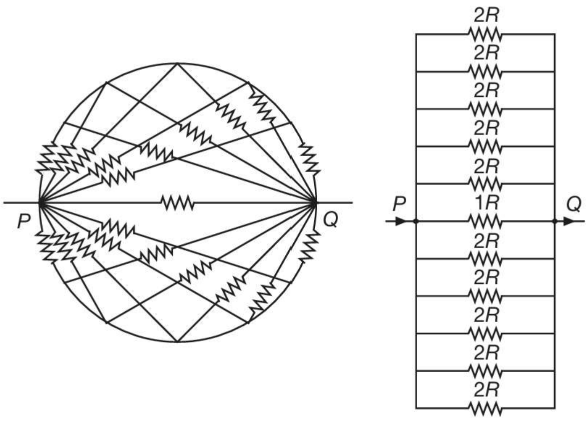
$$\Rightarrow x = \left(\frac{A\epsilon_0 E^2}{8k}\right)^{\frac{1}{3}} \quad \ldots (4)$$

From equations (2) and (4), we get

$$\left(\frac{d-l}{3}\right)^3 = \frac{A\epsilon_0 E^2}{8k}$$

$$\Rightarrow k = \frac{27\epsilon_0 AE^2}{8(d-l)^3}$$

17 **(a)** Since each of the terminals is connected by an insulated wire to each of the remaining 11 terminals by a resistor R, the symmetry shows all the twelve terminals to be symmetrically equivalent before any voltage is applied. However, asymmetry is introduced just at the point where current enters and also at the point where the current leaves the circuit. All other ten points are symmetry points, all at the same potential. Hence the given network reduces to the following network



Thus if X be the equivalent resistance between P and Q

$$\frac{1}{X} = \frac{1}{2R} + \frac{1}{2R} + \ldots \text{ to 10 terms } + \frac{1}{R}$$

$$\frac{1}{X} = \frac{10}{2R} + \frac{1}{R} = \frac{6}{R}$$

Hence $X = \dfrac{R}{6}$

The equivalent resistance $= \dfrac{R}{6} = \dfrac{6}{6} = 1\Omega$

(b) Proceeding in the same manner as in **(a)** we find that the equivalent resistor is given by X where

$$\frac{1}{X} = \frac{1}{2R} + \frac{1}{2R} + \ldots \text{ to } (n-2) \text{ terms } + \frac{1}{R}$$

$$\frac{1}{X} = \frac{n-2}{2R} + \frac{1}{R} = \frac{n}{2R}$$

Hence equivalent resistance $= \dfrac{2R}{n}$

The above symmetry simplification can be done only if n is even.

18 The currents in the ammeters are proportional to deflections produced, let α_1 and α_2 be the proportionality constants.

$$I_1 = \alpha_1 n_1$$
$$I_2 = \alpha_2 n_2$$

The ammeters are in series, hence

$$I_1 = I_2$$
$$\Rightarrow \alpha_1 n_1 = \alpha_2 n_2 \quad \ldots(1)$$

In the second arrangement the potential drops across resistors are equal as they are in parallel arrangement. Resistances of ammeters have been ignored assuming them to be ideal.

Thus we have $I_1' R_1 = I_2' R_x$

Also $I_1' = \alpha_1 N_1$ and $I_2' = \alpha_2 N_2$

$$\Rightarrow R_1 \alpha_1 N_1 = R_x \alpha_2 N_2 \quad \ldots(2)$$

From equations (1) and (2), eliminating α_1 and α_2, we get

$$R_1 N_1 = R_x N_2$$
$$\Rightarrow R_x = \frac{R_1 n_2 N_1}{n_1 N_2}$$

19

$$X = \frac{QR}{P}$$

So, maximum value of x is given by

$$X_{max} = \frac{Q_{max} R_{max}}{P_{min}}$$
$$\Rightarrow X_{max} = \frac{1000(11110)}{10}$$

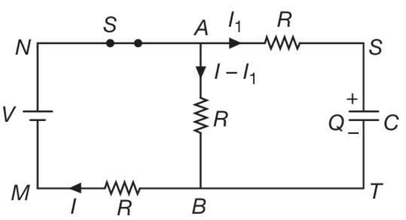
Note, as per the standard method of performing the Post Office Box experiment, we set the the ratio $\frac{Q}{P}$ to a multiple of 10. Therefore, Q_max is considered 1000, and not 1110.

$$\Rightarrow X_{max} = 1111\ \mathrm{k\Omega}$$
$$X_{min} = \frac{Q_{min} R_{min}}{P_{max}}$$
$$\Rightarrow X_{min} = \frac{(10)(1)}{1000}$$
$$\Rightarrow X_{min} = 0.01\Omega$$

20

Let at any time t charge on capacitor C be Q and currents are as shown. Since, charge Q will increase with time t. Therefore,

$$I_1 = \frac{dQ}{dt}$$



(a) Applying Kirchhoff's Second Law in Loop MNABM

$$V = (I - I_1)R + IR$$

$\Rightarrow V = 2IR - I_1R \quad \ldots(1)$

Similarly, applying Kirchhoff's Second Law in Loop MNSTM, we have

$$V = I_1R + \frac{Q}{C} + IR \quad \ldots(2)$$

Eliminating I from equations (1) and (2), we get

$$V = 3I_1R + \frac{2Q}{C}$$

$$\Rightarrow 3I_1R = V - \frac{2Q}{C}$$

$$\Rightarrow I_1 = \frac{1}{3R}\left(V - \frac{2Q}{C}\right)$$

$$\Rightarrow \frac{dQ}{dt} = \frac{1}{3R}\left(V - \frac{2Q}{C}\right)$$

$$\Rightarrow \frac{dQ}{V - \frac{2Q}{C}} = \frac{dt}{3R}$$

$$\Rightarrow \int_0^Q \frac{dQ}{V - \frac{2Q}{C}} = \int_0^t \frac{dt}{3R}$$

This equation gives

$$Q = \frac{CV}{2}(1 - e^{-2t/3RC})$$

(b) $I_1 = \dfrac{dQ}{dt} = \dfrac{V}{3R}e^{-2t/3RC}$

From equation (1), we get

$$I = \frac{V + I_1R}{2R} = \frac{V + \frac{V}{3}e^{-2t/3RC}}{2R}$$

So, current through AB is given by

$$I_2 = I - I_1 = \frac{V + \frac{V}{3}e^{-2t/3RC}}{2R} - \frac{V}{3R}e^{-2t/3RC}$$

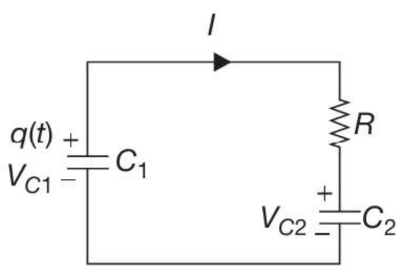
$$I_2 = \frac{V}{2R} - \frac{V}{6R}e^{-2t/3RC}$$

$$I_2 = \frac{V}{2R} \text{ as } t \to \infty$$

21 **(a)** Suppose at a moment t the charge deposited on C_1 is $q(t)$. Then,

$$V_{C_1} = \frac{q(t)}{C_1} \text{ and } V_{C_2} = \frac{q_0 - q(t)}{C_2}$$

$$V_R = IR \text{ where } I = -\frac{dq}{dt}$$



Applying KVL, we get

$$\frac{q}{C_1} - (\frac{q_0 - q}{C_2}) = IR = -R\frac{dq}{dt}$$

$$\Rightarrow R\frac{dq}{dt} + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)q = \frac{q_0}{C_2}$$

Put $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$, we get

$$R\frac{dq}{dt} + \frac{q}{C} = \frac{q_0}{C_2}$$

$$\Rightarrow \frac{dq}{dt} + \frac{q}{RC} = \frac{q_0}{C_2}\frac{q}{C}$$

$$\Rightarrow R\frac{dq}{dt} = \frac{1}{C_2}(\frac{C}{C_2}q_0 - q)$$

$$\Rightarrow \int \frac{dq}{\frac{q_0 C}{C_2} - q} = \int_0^t \frac{dt}{RC}$$

$$\Rightarrow \log_e \left(\frac{q - \frac{C}{C_2}q_0}{q_0 - \frac{C}{C_2}q_0}\right) = -\frac{t}{RC}$$

$$\Rightarrow q - \frac{C}{C_2}q_0 = q_0\left(1 - \frac{C}{C_2}\right)e^{-\frac{t}{RC}}$$

$$\Rightarrow q = q(t) = q_0\left[\left(1 - \frac{C}{C_2}\right)e^{-\frac{t}{RC}} + \frac{C}{C_2}\right] \text{ where } C = \frac{C_1 C_2}{C_1 + C_2}$$

$$\Rightarrow I(t) = -\frac{dq}{dt} = \frac{q_0}{RC_1}e^{-\frac{t}{RC}}$$

Charge on C_2, $q_2 = q_0 - q(t) = q_0 C_1 (1 - e^{-\frac{t}{RC}})$

So, we observe that the charge on q_1 decays and that on q_2 grows exponentially.

(b) Electrostatic energy at $t = 0$ is

$$U(0) = \frac{q_0^2}{2C_1}$$

Electrostatic energy at $t \to \infty$ is final energy given by

$$U_f = U(\infty) = \frac{q_0^2}{2(C_1 + C_2)}$$

Since, Heat Loss $= -\Delta U = U_i - U_f$

$$\Rightarrow -\Delta U = U(0) - U(\infty) = \frac{q_0^2 C_2}{2C_1(C_1 + C_2)}$$

22 Here energy is being generated in the resistance at a rate of $\frac{V^2}{R}$. Of which part of energy is being lost in the environment and the rest is utilized in raising the temperature of conductor. So, applying the Law of Conservation of Energy, we get

$$\begin{bmatrix} Energy \\ supplied \\ bythedc \\ sourceper \\ unittime \end{bmatrix} = \begin{bmatrix} Energylost \\ inthe \\ environment \\ per \\ unittime \end{bmatrix} + \begin{bmatrix} Energyused \\ inraisingthe \\ temperatureof \\ theconductor \\ perunittime \end{bmatrix}$$

Hence, $\frac{V^2}{R} = k(T - T_0) + C(\frac{dT}{dt})$

$\Rightarrow C\left(\frac{dT}{dt}\right) = \frac{V^2}{R} - k(T - T_0)$

$\Rightarrow \frac{dT}{\frac{V^2}{R} - k(T - T_0)} = \frac{dt}{C}$

Integrating the above expression, we get

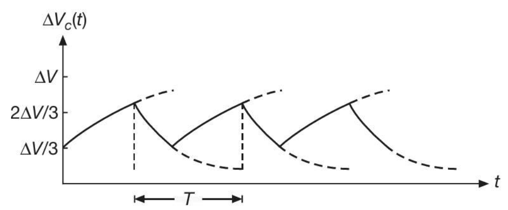
$$\int_{T_0}^{T} \frac{dT}{\frac{V^2}{R} - k(T - T_0)} = \int_0^t \frac{dt}{C}$$

Solving this equation, we get

$$T = T_0 + \frac{V^2}{kR}\left(1 - e^{-\frac{kt}{C}}\right)$$

23 Start at the point when the voltage has just reached $\frac{2\Delta V}{3}$ and the switch has just closed. The voltage is $\frac{2}{3}\Delta V$ and is decaying towards 0 V with a time constant R_2C.

$$\Delta V_c(t) = \left[\frac{2}{3}\Delta V\right] e^{-\frac{t_1}{R_2C}}$$



Let, $\Delta V_c(t) = \frac{1}{3}\Delta V$, at time t_1, then

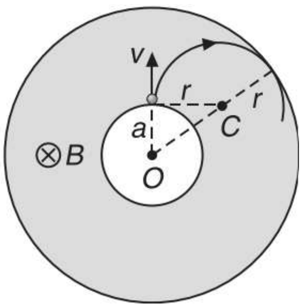
$$\frac{1}{3}\Delta V = \left[\frac{2}{3}\Delta V\right] e^{-\frac{t_1}{R_2C}}$$

$\Rightarrow e^{-\frac{t_1}{R_2C}} = \frac{1}{2} \Rightarrow t_1 = R_2C \log_e 2$

After the switch opens, the voltage is $\frac{1}{3}\Delta V$, increasing toward ΔV with time constant $(R_1 + R_2)C$, so

$$\Delta V_c(t) = \Delta V - \left[\frac{2}{3}\Delta V\right] e^{-\frac{t}{(R_1 + R_2)C}}$$

Let $\Delta V_c(t) = \frac{2}{3}\Delta V$, in further time t_2, then

	$\Rightarrow \frac{2}{3}\Delta V = \Delta V - \frac{2}{3}\Delta V e^{-\frac{t_2}{(R_1+R_2)C}}$ $\Rightarrow e^{-\frac{t_2}{(R_1+R_2)C}} = \frac{1}{2}$ $\Rightarrow t_2 = (R_1+R_2)C\log_e 2$ $\Rightarrow T = t_1 + t_2 = (R_1+2R_2)C\log_e 2$
24	Since net resistance is to be found between A and B. So let a current I enter at A and then exit at B. When I enters at A then by symmetry a current $\frac{I}{6}$ must flow in the branch AB from A to B. For current I to exit from B, a current $\frac{I}{6}$ must flow in the branch AB from A to B. Super-imposing the two, we conclude that a current $\left(\frac{I}{6}+\frac{I}{6}\right)$ must flow in the branch AB from A to B. According to Thevenin's Theorem we have $I_{total}R_{eq} = V_{AB} = \left(\frac{I}{6}+\frac{I}{6}\right)R_0 = \frac{IR_0}{3}$ $\Rightarrow IR_{eq} = \frac{IR_0}{3}$ $\Rightarrow R_{eq} = \frac{R_0}{3}$
25	The electron follows a circular path with centre C such that it just escapes the outer shell tangentially and does not collide with it as shown in Figure.  The radius of the circular path followed by electron is given by $r = \frac{mv}{eB} \quad \ldots(1)$ Also, we observe that the length OC is given by $b - r = \sqrt{a^2 + r^2}$ $\Rightarrow b^2 + r^2 - 2br = a^2 + r^2$ $\Rightarrow r = \frac{b^2 - a^2}{2b} \quad \ldots(2)$ Equating (1) and (2), we get $\frac{mv}{eB} = \frac{b^2 - a^2}{2b}$

$$\Rightarrow v = \frac{eB(b^2 - a^2)}{2mb}$$

26 Electric field will provide the particle an acceleration (and therefore a velocity component) in x-direction and the magnetic field will rotate the particle in xz-plane (perpendicular to B). Hence, at any instant of time its velocity (and hence, position) will have only x and z components. Let at time t its velocity be,

$$\vec{v} = v_x\hat{i} + v_z\hat{k}$$

Net force on it at this instant is

$$\vec{F} = \vec{F_e} + \vec{F_m} = q\vec{E} + q(\vec{v} \times \vec{B})$$

$$\Rightarrow \vec{F} = q[E_0\hat{i} + (v_x\hat{i} + v_z\hat{k}) \times (B_0\hat{j})]$$

$$\Rightarrow \vec{F} = q(E_0 - v_zB_0)\hat{i} + qv_xB_0\hat{k}$$

From Newton's Second Law, we have $\vec{F} = m\vec{a}$

$$\Rightarrow \vec{a} = \frac{\vec{F}}{m} = a_x\hat{i} + a_z\hat{k}$$

where, $a_x = \dfrac{q}{m}(E_0 - v_zB_0) \quad \ldots(1)$ and $a_z = \dfrac{q}{m}v_xB_0 \quad \ldots(2)$

Differentiating Equation (1) w.r.t. time, we have,

$$\frac{da_x}{dt} = \frac{d^2v_x}{dt^2} = -\frac{qB_0}{m}\left(\frac{dv_z}{dt}\right)$$

Since, $\dfrac{dv_z}{dt} = a_z = \dfrac{qB_0}{m}v_x$

$$\Rightarrow \frac{d^2v_x}{dt^2} = -\left(\frac{qB_0}{m}\right)^2 v_x \quad \ldots(3)$$

Comparing this equation with the differential equation of SHM $\left[\dfrac{d^2y}{dt^2} = -\omega^2 y\right]$, we get

$$\omega = \frac{qB_0}{m}$$

and the general solution of Equation (3) is,

$$v_x = A\sin(\omega t + \phi) \quad \ldots(4)$$

At time $t = 0$, $v_x = 0$, hence, $\phi = 0$

Again, $\dfrac{dv_x}{dt} = A\omega\cos(\omega t) \quad [\text{as } \phi = 0]$

From equation (1),

$$a_x = \frac{qE_0}{m} \text{ at } t = 0, \text{ as } v_z = 0 \text{ at } t = 0$$

$$\Rightarrow A\omega = \frac{qE_0}{m}$$

$$\Rightarrow A = \frac{qE_0}{m\omega}$$

Substituting $\omega = \dfrac{qB_0}{m}$, we get $A = \dfrac{qE_0}{m\left[\dfrac{qB_0}{m}\right]} = \dfrac{E_0}{B_0}$

Therefore, equation (4) becomes,

$$v_x = \frac{E_0}{B_0}\sin(\omega t) \text{ where } \omega = \frac{qB_0}{m} \quad \ldots(5)$$

Now substituting value of v_x in equation (2), we get

$$a_z = \frac{dv_z}{dt} = \frac{qE_0}{m}\sin(\omega t)$$

$$\Rightarrow \int_0^{v_z} dv_z = \frac{qE_0}{m}\int_0^t \sin(\omega t)dt$$

$$\Rightarrow v_z = \frac{qE_0}{m\omega}(1-\cos(\omega t))$$

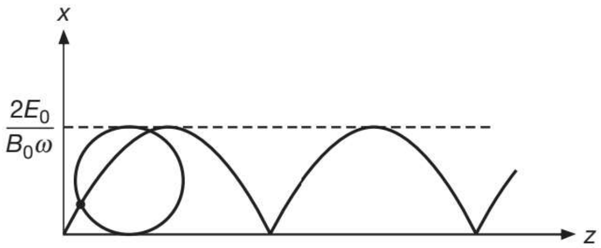
Again substituting $\omega = \dfrac{qB_0}{m}$, we get

$$v_z = \frac{E_0}{B_0}(1-\cos(\omega t)) \quad \ldots(6)$$

On integrating equations for v_x and v_z from (5) and (6), and knowing that at $t=0$, $x=0$ and $z=0$, we get

$$x = \frac{E_0}{B_0\omega}(1-\cos\omega t) \text{ and } z = \frac{E_0}{B_0}(\omega t - \sin\omega t)$$

These equations are the equations for a cycloid which is defined as the path generated by the point on the circumference of a wheel rolling on a ground.



In the present case, radius of the rolling wheel is $\dfrac{E_0}{B_0\omega}$. The maximum displacement along x-direction is $\dfrac{2E_0}{B_0\omega}$. The x-displacement becomes zero at $t = 0, \dfrac{2\pi}{\omega}, \dfrac{4\pi}{\omega}$, etc.

27 The expression for the magnetic field $\vec{B}$ at a point P at position vector $\vec{r}$ from the charge Q moving with a velocity $\vec{v}$ is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{Q(\vec{v}\times\vec{r})}{r^3}$$

Since both charges start from the origin at $t=0$, so the position vector of the charge Q_1 is $\vec{r_1} = \vec{v_1}t$ and the position vector of the charge Q_2 is $\vec{r_2} = \vec{v_2}t$. The vector displacement $\vec{r}$ of Q_2 w.r.t. Q_1 is

$$\vec{r_{12}} = (\vec{v_2} - \vec{v_1})t$$

The magnetic field at the position of Q_2 due to the charge Q_1 is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{Q_1(\vec{v_1}\times\vec{r_{12}})}{r_{12}^3} = \frac{\mu_0}{4\pi}\frac{Q_1[\vec{v_1}\times(\vec{v_2}-\vec{v_1})t]}{r_{12}^3} = \frac{\mu_0}{4\pi}\frac{Q_1(\vec{v_1}\times\vec{v_2})t}{r^3}$$

The force on a charge Q_2 moving with at velocity $\vec{v_2}$ in a magnetic field $\vec{B}$ is

$$\vec{F_{21}} = Q_2(\vec{v_2}\times\vec{B})$$

$$\Rightarrow \vec{F_{21}} = \left[\frac{\mu_0 Q_2 Q_1 t}{4\pi r^3}\right][\vec{v_2}\times(\vec{v_1}\times\vec{v_2})] \quad \ldots(1)$$

Taking $\vec{v_1} = v\hat{i}$ and $\vec{v_2} = v\hat{j}$, we get

$\vec{v}_1 \times \vec{v}_2 = v^2\hat{k}$

$\vec{v}_2 \times (\vec{v}_1 \times \vec{v}_2) = v^3\hat{\imath}$ and

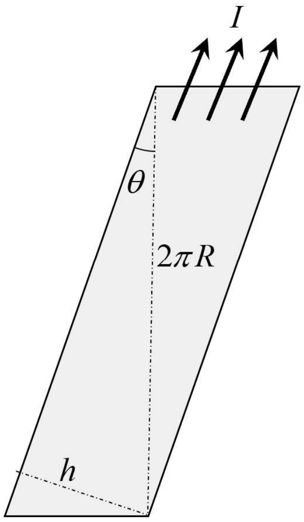
$|\vec{v}_1 - \vec{v}_2|^3 t^3 = (\sqrt{2}v)^3 t^3$

From equation (1), the magnetic force is given by

$$\vec{F}_{21} = \frac{\mu_0}{4\pi}\frac{Q_1Q_2t}{2\sqrt{2}v^3t^3}v^3\hat{\imath}$$

$\Rightarrow \vec{F}_{21} = \dfrac{\mu_0 Q_1 Q_2}{8\sqrt{2}\pi t^2}\hat{\imath}$, parallel to the x-axis.

28 Let us consider one loop, then open it and draw it as shown in Figure.



Since the current is flowing in a strip of width h so, the current per unit strip width is given by

$$j = \frac{I}{h}$$

This current per unit length (also called linear current density) can be resolved in two components j_x and j_y as shown in Figure.

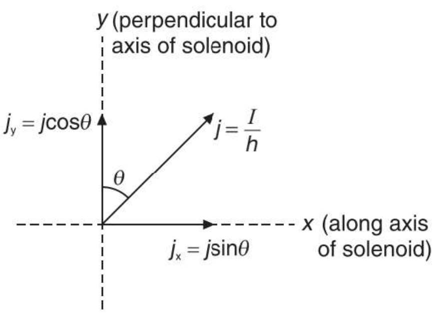


Figure (b)

Outside this type of solenoid, the magnetic field will be due to the component of current along the axis of solenoid as shown in Figure **(c)**.

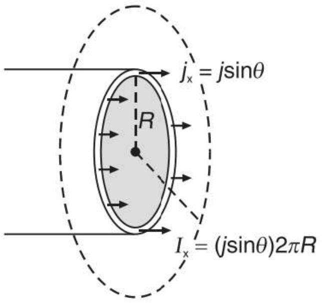


Figure (c)

For current density $j_x = j\sin\theta$ along the length of solenoid, by applying Amperes Circuital Law, we get

$$B_{out}(2\pi r) = \mu_0 I_x$$

where $I_x = j_x(2\pi R)$

$$\Rightarrow I_x = j\sin\theta(2\pi R)$$

From Figure **(a)**, we have

$$\sin\theta = \frac{h}{2\pi R}$$

$$\Rightarrow I_x = j\left(\frac{2\pi R}{h}\right)(2\pi R) = jh$$

$$\Rightarrow B_{out} = \frac{\mu_0 I_x}{2\pi r} = \frac{\mu_0 jh}{2\pi r} = \frac{\mu_0 I}{2\pi r}$$

Inside this type of solenoid, the magnetic field will be due to the component of current perpendicular to the axis of the solenoid. Due to component of current density perpendicular to length of solenoid, we have

$B = \mu_0 n I_y$, where

$$n = \frac{N}{h} = \frac{1}{h} \text{ and } I_y = (j\cos\theta)h = I\cos\theta$$

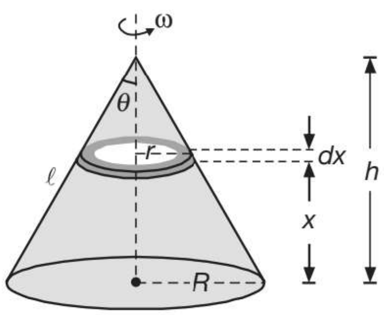
$$\Rightarrow B_{inside} = \mu_0\left(\frac{1}{h}\right)(I\cos\theta)$$

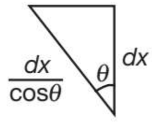
$$\Rightarrow B_{inside} = \frac{\mu_0 I}{h}\sqrt{1-\left(\frac{h}{2\pi R}\right)^2}$$

29

$$M = IA = \left(\frac{\text{Charge Circulating}}{\text{Period of Revolution}}\right)(\text{Area of loop})$$

Let us consider a small, current carrying loop element, having thickness dx at a distance x from the base of the cone as shown in Figure.





Magnified view of element

If dM is the magnetic moment of this small loop having charge dq, radius r and rotating with angular velocity ω then we have

$$dM = iA = \left[\frac{dq}{\left(\frac{2\pi}{\omega}\right)}\right] A = \frac{\omega}{2\pi}(dq)A$$

where A is area of the loop or the area circulated by the charge dq in the loop.
Also, we know that

$$dq = \sigma dA, \text{ where } \sigma = \frac{Q}{\pi R l}, \; dA = (2\pi r)\frac{dx}{\cos\theta}$$

and l is the slant height of the cone given by $l = \dfrac{h}{\cos\theta}$

$$\Rightarrow dM = \left(\frac{\omega}{2\pi}\right)\left(\frac{Q}{\pi R l}\right)(2\pi r)\left(\frac{dx}{\cos\theta}\right)(\pi r^2) \quad \ldots (1)$$

Also, from figure we observe that

$$\tan\theta = \frac{R}{h} = \frac{r}{h-x}$$

$$\Rightarrow r = \left(\frac{h-x}{h}\right)R$$

So, from (1), we get

$$dM = \left(\frac{\omega}{2\pi}\right)\left(\frac{Q\cos\theta}{\pi R h}\right)\left(\frac{2\pi r^2 R}{\cos\theta}\right)\left(\frac{h-x}{h}\right)^3 dx$$

$$\Rightarrow M = \int dM = \frac{\omega Q R^2}{h^4}\int_0^h (h-x)^3 dx$$

$$\Rightarrow M = \frac{\omega Q R^2}{h^4}\left[\frac{(h-x)^4}{-4}\right]_0^h$$

$$\Rightarrow M = \frac{\omega Q R^2}{h^4}\left(0 - \frac{h^4}{-4}\right)$$

$$\Rightarrow M = \frac{1}{4}Q\omega R^2 = \frac{1}{4}Q\omega h^2\tan^2\theta$$

Since we know that the Gyromagnetic ratio for such an arrangement is given by

$$\gamma = \frac{M}{L} = \frac{Q}{2m}$$

where M is to be calculated and $L = I\omega$. For a thin walled cone, $I = \dfrac{1}{2}mR^2$. So, we get

$$M = I\omega\frac{Q}{2m} = \left(\frac{1}{2}mR^2\right)\frac{\omega Q}{2m} = \frac{1}{4}Q\omega R^2$$

$$\Rightarrow M = \frac{1}{4}Q\omega h^2\tan^2\theta$$

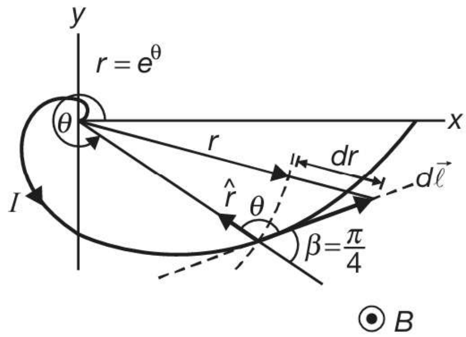
{we could have bypassed the process of integration}

30	Before we start with the problem, we must know and keep in mind that the angle β between a radial line and its tangent line at any point on the curve $r = f(\theta)$ are related to the function as

$$\tan\beta = \frac{r}{\frac{dr}{d\theta}}$$

From Biot-Savart's Law, we know that there is no contribution from the straight portion of the wire since $d\vec{l} \times \vec{r} = 0$. For the field of the spiral, we have

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{(d\vec{l} \times \hat{r})}{r^2}$$



$$\Rightarrow B = \frac{\mu_0 I}{4\pi}\int_0^{2\pi}\frac{|d\vec{l}|\sin\theta|\hat{r}|}{r^2}$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi}\int_0^{2\pi}\sqrt{2}dr\left[\sin\left(\frac{3\pi}{4}\right)\right]\frac{1}{r^2}$$

Thus in this case, we have $r = e^{\theta}$ and so we get $\tan\beta = 1$ and $\beta = \frac{\pi}{4}$. Therefore, the angle between $d\vec{l}$ and $\hat{r}$ is $(\pi - \beta) = \frac{3\pi}{4}$. Also

$$d\vec{l} = \frac{dr}{\sin\left(\frac{\pi}{4}\right)} = \sqrt{2}dr$$

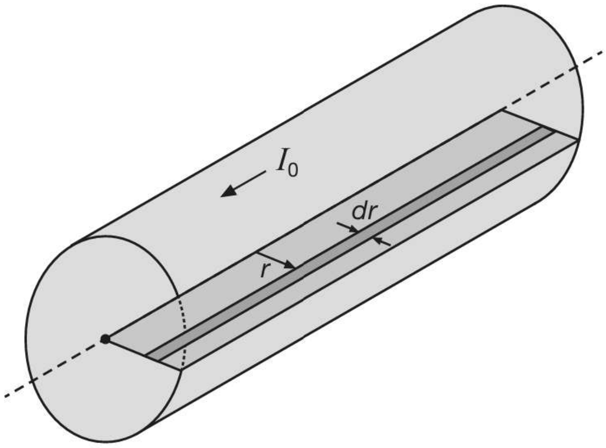
$$\Rightarrow B = \frac{\mu_0 I}{4\pi}\int_0^{2\pi} r^{-2}dr = \frac{-\mu_0 I}{4\pi}(r^{-1})|_0^{2\pi}$$

Substitute $r = e^{\theta}$, we get

$$B = \frac{-\mu_0 I}{4\pi}(e^{-\theta})|_0^{2\pi} = \frac{-\mu_0 I}{4\pi}(e^{-2\pi} - e^0)$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi}(1 - e^{-2\pi}) \text{ out of the page.}$$

31	Let us find the magnetic field at a distance r from the centre of the wire and divide the rectangle into narrow infinitesimal strips of width dr. The magnetic flux through each infinitesimal strip will then be integrated to get the total flux.



From Ampere's Law, the field inside the wire is

$$B = \left(\frac{\mu_0 I}{2\pi R^2}\right) r$$

The field B is normal to the area of the strip Ldr, so angle between B and dA is 0°. Hence,

$$d\phi_B = B.\,dA = \left(\frac{\mu_0 I_0 r}{2\pi R^2}\right)(Ldr)\cos(0^\circ)$$

$$\phi_B = \int d\phi = \frac{\mu_0 I_0 L}{2\pi R^2}\int_0^R rdr$$

$$\Rightarrow \phi_B = \frac{\mu_0 I_0 L}{4\pi}$$

32 For the suspended mass M, we have

$$Mg - T = Ma \quad \ldots(1)$$

For the bar connected to the suspended mass M,

$$T - BIl = ma \quad \ldots(2)$$

where $I = \dfrac{\varepsilon}{R} = \dfrac{Blv}{R} \quad \ldots(3)$

$$\Rightarrow Mg - BIl = (M+m)a$$

$$\Rightarrow Mg - \frac{B^2 l^2 v}{R} = (M+m)a$$

$$\Rightarrow a = \frac{dv}{dt} = \frac{Mg}{M+m} - \frac{B^2 l^2}{(R(M+m))} v$$

$$\Rightarrow a = \frac{dv}{dt} = \alpha - \beta v \text{ where } \alpha = \frac{Mg}{M+m}, \beta = \frac{B^2 l^2}{R(M+m)}$$

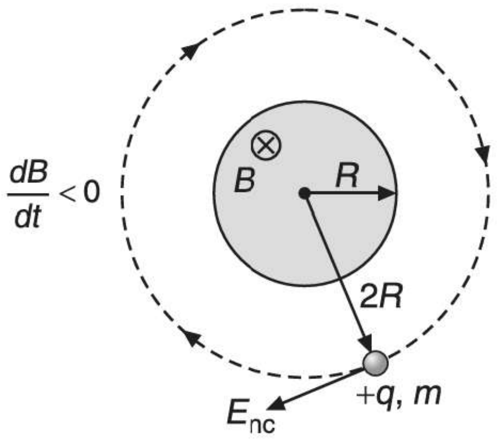
$$\Rightarrow \frac{dv}{dt} = \alpha - \beta v$$

$$\Rightarrow \int_0^v \frac{dv}{\alpha - \beta v} = \int_0^t dt$$

$$\Rightarrow -\frac{1}{\beta}\log_e(\alpha - \beta v)|_0^v = t$$

$$\Rightarrow \log_e\left(\frac{\alpha - \beta v}{\alpha}\right) = -\beta t$$

$$\Rightarrow v = \frac{\alpha}{\beta}(1 - e^{-\beta t})$$

	$\Rightarrow v = \dfrac{MgR}{B^2l^2}\left[1 - e^{-\dfrac{B^2l^2t}{R(M+m)}}\right]$ At the terminal velocity of the bar, say v_T, we have $\Rightarrow Mg - \dfrac{B^2l^2v_T}{R} = 0$ $\Rightarrow Mg = \dfrac{B^2l^2}{R}v_T$ $\Rightarrow v_T = \dfrac{MgR}{B^2l^2}$
33	The magnetic field inside the solenoid is along the axis of the solenoid and is given by $B_{inside} = \mu_0 nI$  Let the duration in which field drops to zero after switching off the current in the solenoid be Δt. Due to the switching off the current to the solenoid, an electric field E_{nc} is induced at a distance $x = 2R$ i.e., outside the solenoid in this time interval Δt. So, we have $E_{nc}(2\pi x) = -A\left(\dfrac{\Delta B}{\Delta t}\right) = -\pi R^2\left(\dfrac{0-\mu_0 nI}{\Delta t}\right)$ $\Rightarrow E_{nc}(2\pi x) = \pi R^2\left(\dfrac{\mu_0 nI}{\Delta t}\right)$ $\Rightarrow E_{nc} = R\dfrac{R}{2x}\dfrac{\mu_0 nI}{\Delta t} = \dfrac{R}{2(2R)}\dfrac{\mu_0 nI}{\Delta t} = \dfrac{R}{4}\left(\dfrac{\mu_0 nI}{\Delta t}\right) \quad \ldots(1)$ Due to this induced electric field, the charge $+q$ experiences an impulse in this duration which is given by $F\Delta t = mv - 0$ $\Rightarrow (qE_{nc})\Delta t = mv$ Using equation (1), we get $q\left[\dfrac{R}{4}\left(\dfrac{\mu_0 nI}{\Delta t}\right)\right]\Delta t = mv$ $\Rightarrow v = \dfrac{\mu_0 nIqR}{4m}$

34	For loop abefa, we have

For loop abefa, we have

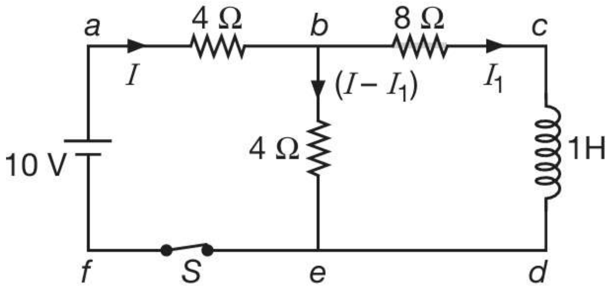
$$-4I - 4(I - I_1) + 10 = 0$$

$$\Rightarrow -4I_1 + 8I = 10 \quad \ldots(1)$$

For loop bcdeb, we have

$$-8I_1 - (1)\frac{dI_1}{dt} + 4(I - I_1) = 0$$

$$\Rightarrow -12I_1 + 4I = \frac{dI_1}{dt} \quad \ldots(2)$$



Multiplying (2) by 2 and then subtracting from (1), we get

$$-4I_1 + 24I_1 = 10 - 2\frac{dI_1}{dt}$$

$$\Rightarrow 20I_1 = 10 - 2\frac{dI_1}{dt}$$

$$\Rightarrow \frac{dI_1}{dt} = 5 - 10I_1$$

$$\Rightarrow \int_0^{I_1} \frac{dI_1}{5 - 10I_1} = \int_0^t dt$$

$$\Rightarrow -\frac{1}{10}\log_e(5 - 10I_1)\Big|_0^{I_1} = t$$

$$\Rightarrow \log_e\left(\frac{5 - 10I_1}{5}\right) = -10t$$

$$\Rightarrow 5 - 10I_1 = 5e^{-10t}$$

$$\Rightarrow I_1 = 0.5(1 - e^{-10t})\text{ A} \quad \{\text{current through the inductor}\}$$

The current through the switch is I, calculated from either of the equations to be

$$I = \frac{1}{8}(10 + 4I_1)$$

$$\Rightarrow I = 1.25 + 0.5I_1$$

$$\Rightarrow I = 1.25 + (0.5)(0.5)(1 - e^{-10t})$$

$$\Rightarrow I = 1.25 + 0.25 - 0.25e^{-10t}$$

$$\Rightarrow I = (1.5 - 0.25e^{-10t})\text{ A} \quad \{\text{current through the switch}\}$$

35 **(a)** Using Fleming's Right Hand Rule, we see that the induced current in the wire PQ is from Q to P. So, we have

$$V_P - V_Q = \varepsilon = Blv = (4)(1)(2) = 8\text{ V}$$

(b) Inductor being connected in parallel to the emf source PQ has same potential difference developed across the emf source. So

$$V_L = \varepsilon = Blv = 8\text{ V}$$

(c) Capacitor being connected in parallel to the emf source PQ has same potential difference developed across the emf source. So

$$V_c = \varepsilon = Blv = 8 \text{ V}$$

(d) Resistor being connected in parallel to the emf source PQ has same potential difference developed across the emf source. So

$$V_R = \varepsilon = Blv = 8 \text{ V}$$

(e) Since $V_L = 8 \text{ V}$

$$\Rightarrow L\frac{di}{dt} = 8$$

$$\Rightarrow 4\frac{di}{dt} = 8$$

$$\Rightarrow \frac{di}{dt} = 2 \text{ As}^{-1}$$

(f) Charge on the capacitor is

$$Q = C\varepsilon = (1)(8) = 8 \text{ coulomb}$$

$$\Rightarrow i_2 = \frac{dQ}{dt} = \frac{d}{dt}(8) = 0$$

(g) Current through resistor is

$$i_3 = \frac{\varepsilon}{R} = \frac{8}{2} = 4 \text{ A}$$

(h) Since rate of change of current in inductor is $\frac{di_1}{dt} = 2 \text{ As}^{-1}$

So, current in the inductor at $t = 2$ s is

$$i_1 = \left(\frac{di_1}{dt}\right)\Delta t = (2)(2) = 4 \text{ A}$$

(i) Since wire PQ is acting as source of emf and supplies current i_1 to inductor, i_2 to capacitor and i_3 to resistor, so current through wire PQ is

$$i = i_1 + i_2 + i_3 = 4 + 0 + 4 = 8 \text{ A}$$

(j) To keep the wire PQ moving with constant velocity, an external force that equals the magnetic force on the current carrying conductor should be applied on it.

$$\Rightarrow F_{external} = F_{magnetic} = Bil = (4)(8)(1) = 32 \text{ N}$$

(k) Energy supplied per second by the source is

$$P_{supplied} = \varepsilon i = (8)(8) = 64 \text{ W}$$

(l) Power generated by the applied external force is

$$P_{external\ force} = F_{ext}v = (32)(2) = 64 \text{ W}$$

(m) Magnetic energy stored per second in inductor is

$$\frac{dU_m}{dt} = \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = \frac{1}{2}L\frac{d}{dt}(i^2) = \frac{L}{2}2i\frac{di}{dt} = Li\frac{di}{dt}$$

Since $\frac{di1}{dt} = 2 \text{ As}^{-1}$ and $i_1 = 4 \text{ A}$ at $t = 2$ s

$$\Rightarrow \frac{dU_m}{dt} = Li_1\frac{di_1}{dt} = (4)(4)(2) = 32 \text{ W}$$

(n) Energy dissipated per second in the resistor at $t = 2$ s is

$$P_{Resistor} = i_3^2 R = (4^2)(2) = 32 \text{ W}$$

(o) Electrostatic stored per second in capacitor at $t = 2$ s is

$$\frac{dU_e}{dt} = \frac{d}{dt}\left(\frac{Q^2}{2C}\right) = \frac{1}{2C}\frac{d}{dt}(Q^2) = \frac{Q}{C}\frac{dQ}{dt} = \frac{Q}{C}i_2$$

Since $i_2 = 0$

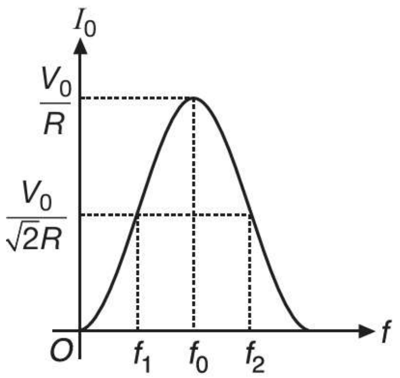
$$\Rightarrow \frac{dU_e}{dt} = 0$$

36 At resonance, we have

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow I_0 = \frac{V_0}{R} \quad \{\text{as Z} = \text{R}\}$$

According to the problem, we have $I = \dfrac{I_0}{\sqrt{2}}$



$$\Rightarrow \frac{1}{\sqrt{2}}\frac{V_0}{R} = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\Rightarrow 2R^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$\Rightarrow \omega L - \frac{1}{\omega C} = \pm R$$

If f_1 and f_2 are two corresponding frequencies, then

$$2\pi f_1 L - \frac{1}{2\pi f_1 C} = R \quad \ldots(1)$$

$$2\pi f_2 L - \frac{1}{2\pi f_2 C} = -R \quad \ldots(2)$$

Dividing (1) by f_2, (2) by f_1 and solving, we get

$$\frac{2\pi f_1 L}{f_2} - \frac{1}{2\pi f_1 f_2 C} = \frac{R}{f_2}$$

$$\frac{2\pi f_2 L}{f_1} - \frac{1}{2\pi f_1 f_2 C} = \frac{-R}{f_1}$$

Subtracting the above two equations:

$$2\pi L\left(\frac{f_1}{f_2} - \frac{f_2}{f_1}\right) = R\left(\frac{1}{f_2} + \frac{1}{f_1}\right)$$

$$2\pi L\left(\frac{f_1^2 - f_2^2}{f_1 f_2}\right) = R\left(\frac{f_1 + f_2}{f_1 f_2}\right)$$

$$2\pi L(f_1 - f_2)\left(\frac{f_1 + f_2}{f_1 f_2}\right) = R\left(\frac{f_1 + f_2}{f_1 f_2}\right)$$

$$2\pi L(f_1 - f_2) = R$$
$$f_1 - f_2 = \frac{R}{2\pi L}$$

37 Current density of the displacement current is

$$J_d = \frac{2\pi}{\pi(0.1)^2} = \frac{2}{0.01} = 200 \text{ Am}^{-2}$$

Since, $J_d = \dfrac{I_d}{A}$

$\Rightarrow i_d = J_d \times \pi(0.04)^2$

$\Rightarrow i_d = 200 \times \pi \times 16 \times 10^{-4}$

$\Rightarrow i_d = 32\pi \times 10^{-2}$ A

$\Rightarrow i_d = 1$ A

Since, $\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{enclosed}$

$\Rightarrow B \times 2\pi(0.04) = 4\pi \times 10^{-7} \times 1$

$\Rightarrow B = \dfrac{2 \times 10^{-7}}{4 \times 10^{-2}} = 0.5 \times 10^{-5}$

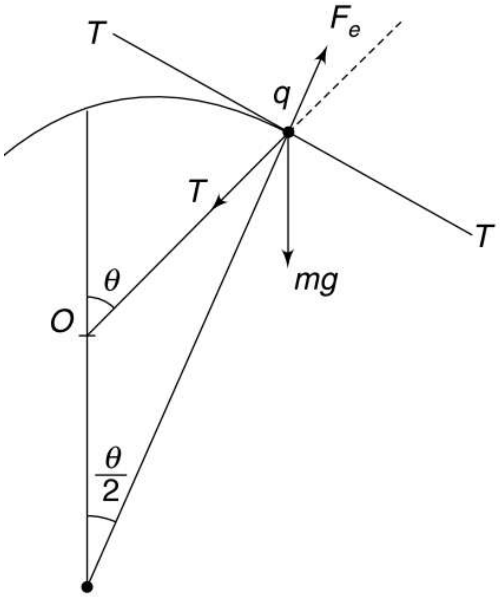
$\Rightarrow B = 5 \times 10^{-6}$ T

38 In equilibrium

$$\frac{KQq}{(2L)^2} = mg \Rightarrow Q \geq \frac{4L^2 mg}{Kq}$$

Now consider the particle in a slightly displaced position

Equilibrium will be stable if tangential component (along TT) of electrostatic repulsion is greater than the tangential component of mg.



$$\Rightarrow F_e \sin\left(\frac{\theta}{2}\right) \geq mg \sin\theta$$

For small displacement,

$$\sin\theta \approx \theta$$
$$\sin\frac{\theta}{2} \approx \frac{\theta}{2}$$

$$\therefore K\frac{qQ}{(2L)^2}\frac{\theta}{2} \geq mg\theta$$

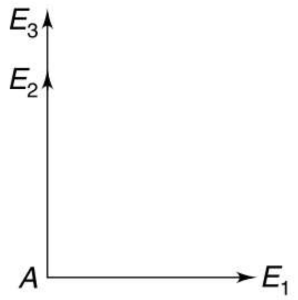
$$\therefore Q \geq \frac{8L^2mg}{Kq} = \frac{32\pi\epsilon_0 L^2 mg}{q}$$

39 **(a)** Field at A:

Due to ρ is $E_1 = \frac{\rho}{3\epsilon_0}(2R)$ parallel to C_1C_2

Due to 2ρ is $E_2 = \frac{2\rho}{3\epsilon_0}(R)$ parallel to C_2C_3

Due to 4ρ is $\frac{4\rho}{3\epsilon_0}\frac{R}{8}$ parallel to C_3A



$$\therefore E_A = \sqrt{E_1^2 + (E_2+E_3)^2} = \frac{\sqrt{41}}{6}\frac{\rho R}{\epsilon_0}$$

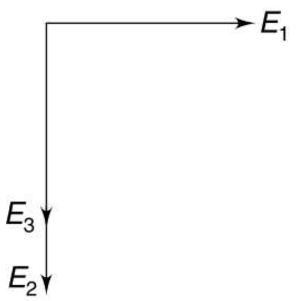
(b) Field at B

Due to ρ is $E_1 = \frac{\rho}{3\epsilon_0}2R$ parallel to C_1C_2

Due to 2ρ (including the smallest sphere's 2ρ) in S_2 is $E_2 = \frac{2\rho R}{3\epsilon_0 \cdot 4}$ in direction C_2B

Due to 2ρ in S_3 is

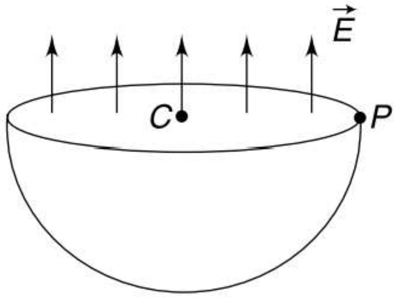
$$E_3 = \frac{1}{4\pi\epsilon_0}\frac{\frac{4}{3}\pi(R)^3 \cdot 2\rho}{\left(\frac{5R}{4}\right)^2} = \frac{32\rho R}{75\epsilon_0} \text{ in direction } C_3B$$



$$E_B = \sqrt{E_1^2 + (E_2+E_3)^2} = \frac{\sqrt{5749}}{75}\frac{\rho R}{\epsilon_0}$$

40 Potential at centre

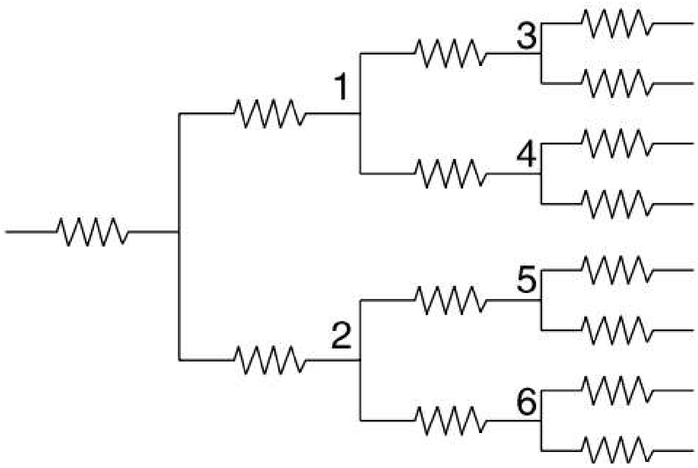
$$V_C = \frac{KQ}{R} = \frac{1}{4\pi\epsilon_0}\frac{2\pi R^2\sigma}{R} = \frac{\sigma R}{2\epsilon_0}$$

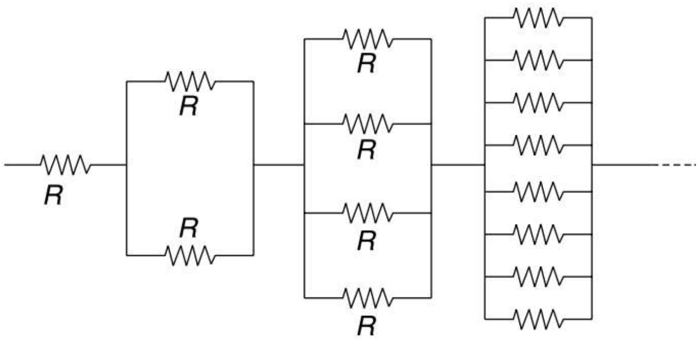


Potential at point P and A will also be same because the electric field at all points of the circular base is perpendicular to the circular surface. When one moves from C to P, he is travelling on an equipotential surface.
Therefore, potential at P and A is same and is equal to V_C.

$$V_P = V_A = \frac{\sigma R}{2\epsilon_0}$$

41 Points 1 and 2 are equipotential. They can be connected together. Points 3, 4, 5 and 6 are equipotential. Thee together. And so on.





$\therefore$ Equivalent is

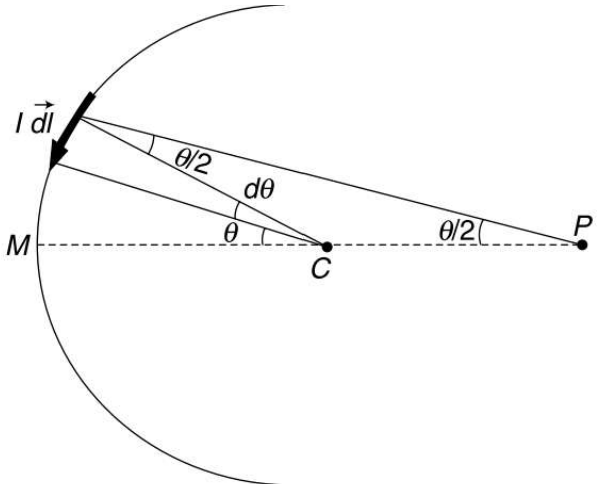
$$R_0 = R + \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots \infty\right]$$

$$= R\left[\frac{1}{1-\frac{1}{2}}\right] = 2R$$

42	Point P lies on the circumference of the circle. Consider an element subtending angle $d\theta$ at the centre of the circle. Field at P due to this element is given by Biot – Savart law.

$$dB = \frac{\mu_0 I}{4\pi}\frac{dl\sin(90^\circ - \frac{\theta}{2})}{l^2}$$

Where $dl = Rd\theta$ and $r = 2R\cos\left(\frac{\theta}{2}\right)$



$$\Rightarrow dB = \frac{\mu_0 I}{4\pi}\frac{Rd\theta\cos\left(\frac{\theta}{2}\right)}{[2R\cos\left(\frac{\theta}{2}\right)]^2} = \frac{\mu_0 I}{16\pi R}\sec\left(\frac{\theta}{2}\right)d\theta$$

Direction of dB is $\odot$

Note: If you consider an identical element in the other quadrant, field at P due to the element is -

$$dB = \frac{\mu_0 I}{4\pi}\frac{Rd\theta\sin(90^\circ - \frac{\theta}{2})}{[2R\cos(\frac{\theta}{2})]^2} = -\frac{\mu_0 I}{16\pi R}\sec\left(\frac{\theta}{2}\right)d\theta$$

All elements contribute in same direction and resultant field is

$$B = 2\times\frac{\mu_0 I}{16\pi R}\int_0^{\pi/2}\sec\left(\frac{\theta}{2}\right)d\theta$$
$$= \frac{\mu_0 I}{4\pi R}[2\ln(\sec\frac{\theta}{2} + \tan\frac{\theta}{2})]_0^{\pi/2}$$
$$= \frac{\mu_0 I}{4\pi R}[2\ln(\sqrt{2}+1) - 2\ln(1+0)]$$
$$= \frac{\mu_0 I}{4\pi R}2\ln(\sqrt{2}+1) = \frac{\mu_0 I}{2\pi R}\ln(\sqrt{2}+1)$$

43	Consider an Amperian loop in shape of a circle passing through P. Due to symmetry field at all points on the circle will have same magnitude and tangential direction.

$$\Rightarrow \oint \vec{B}\cdot\vec{dl} = \mu_0 I$$
$$\Rightarrow B(2\pi d) = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi d}$$

44 Magnetic force on any small element of the wire is perpendicular to its length. It means that the tension will be constant along the wire.

Consider a small segment of the wire subtending a small angle $\Delta\theta$ at the centre of curvature. Magnetic force is

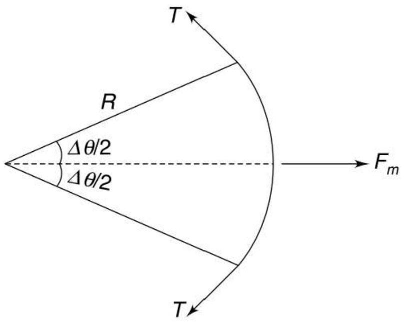
$$F_m = I(R\Delta\theta)B$$

This is balanced by components of tension

$$= 2T\sin\left(\frac{\Delta\theta}{2}\right) = T\Delta\theta$$

$$\Rightarrow T\Delta\theta = I(R\Delta\theta)B \Rightarrow T = IRB$$

$$\text{Or, } R = \frac{T}{IB} = \text{constant}$$

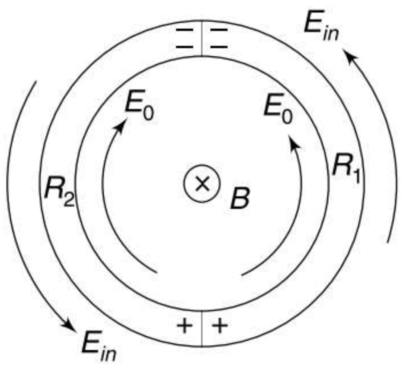


Radius of curvature remains constant means that the two segments will take the shape of circular arcs.

45 **(a)** Current (and hence current density) in the entire loop must be same.

From microscopic form of Ohm's law we can write

$$\sigma_1 E_1 = \sigma_2 E_2 = I \Rightarrow \frac{E_1}{E_2} = \frac{R_1}{R_2}$$

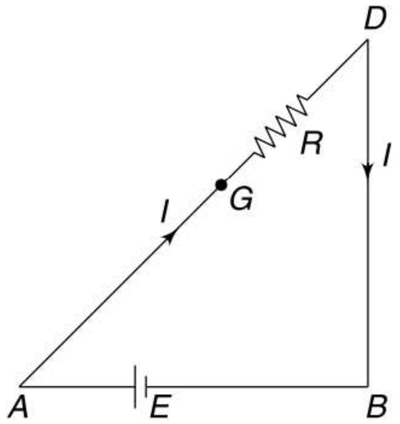


(b) The induced electric field must be uniform everywhere in the circular conductor. It is given by

$$2\pi a E_m = \pi a^2 \frac{dB}{dt} \Rightarrow E_m = \frac{a}{2}\frac{dB}{dt}$$

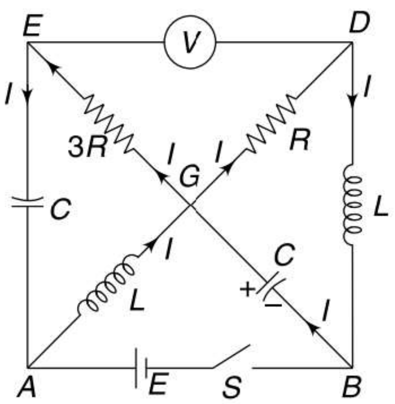
There is accumulation of charge at the junctions which produce additional electric field in the conductor. If $R_1 > R_2$, then $E_1 > E_2$. In this case the charge at the upper junction is negative and charge at the lower junction will be positive. In figure E_0' is electric field due to charge.

46 **(a)** There is no current through capacitors, voltmeters and inductors are zero resistance, when the circuit is in steady state. The effective circuit is as shown.



Reading of voltmeter $= E = V_E = V_G$ and $V_G - V_D = E$

(b) The current through inductors and the voltages across the capacitors cannot change immediately. Current, before the switch is opened, is $I = \dfrac{E}{R}$ through both inductors.



After opening the switch, the current in R and the inductor B and D must be same (since voltmeter does not conduct). It implies that current through R is still I and $V_B - V_D = RI = E$. The current I in GD will loop through GDBG. The current I in the inductor between A and G must loop through AGEA.

$\therefore V_D - V_G = -E$

$V_E - V_G = -3E$

So, reading of the voltmeter is

(c) When switch was opened the capacitor between A and E was uncharged. The circuit is effectively two disjoint loops -BGD and AGE.

Energy stored in L and C gets dissipated in R.

$$U_R = \frac{1}{2}LI^2 + \frac{1}{2}CE^2$$

$$= \frac{1}{2}L\left(\frac{E}{R}\right)^2 + \frac{1}{2}CE^2 = \frac{1}{2}E^2\left(\frac{L}{R^2} + C\right)$$

47 When current is 5A, the impedance is

$$Z = \frac{V}{I} = \frac{100}{5} = 20\Omega$$

This is equal to R.

Hence for $\omega = 1000$ rad/s

$$Z = R = 20\Omega$$

This is possible only if S_1 is closed and

$$\omega_r L = \frac{1}{\omega_r C}$$

$$\Rightarrow LC = \frac{1}{\omega_r^2} = \frac{1}{10^6} \quad \dots (i)$$

When $\omega = 500$ rad/s, $I = 4$ A

$$Z = \frac{100}{4} = 25\Omega$$

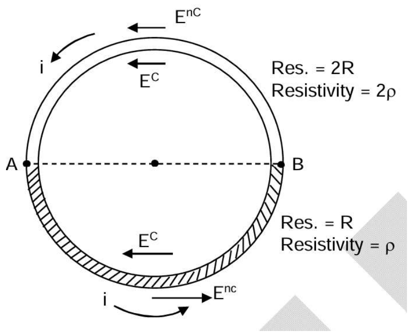
$$\sqrt{R^2 + X^2} = 25 \Rightarrow X = 15\Omega$$

$$\frac{1}{\omega C} - \omega L = 15 \quad \dots (ii)$$

[Note that X is not $\omega L - \frac{1}{\omega C}$, since on increasing the frequency from 500 rad/s to 1000 rad/s the impedance is decreasing]

Solving (i) and (ii) $L = 10$ mH, $C = 100 \mu F$

48



(a) For a uniform ring, the induced electric field is given by:

$$E_{nc} \cdot 2\pi a = \pi a^2 \frac{d(kt)}{dt} \Rightarrow E_{nc} = \frac{ak}{2}$$

The induced electric field is uniform and non-conservative. Due to absence of any electrostatic field, there is no potential difference across the ring; therefore, $V_{AB} = 0$.

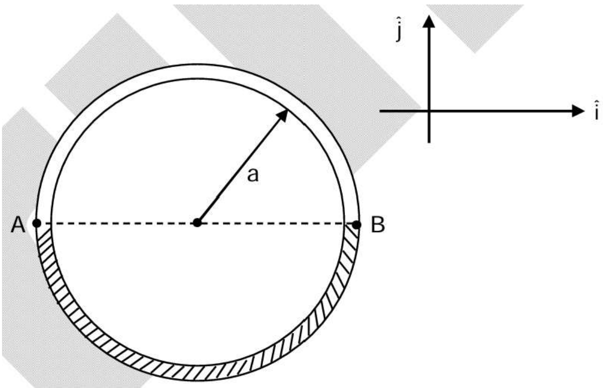
(b) For the non-uniform ring, let the current density be J. Since the current is the same throughout, and the upper half has twice the resistivity of the lower half, the electric fields will differ. The induced electric field (E_{nc}) is still given by $E_{nc} = \frac{ak}{2}$ along the circumference of the ring. The difference in the resultant electric field in upper and lower part of the ring is provided by an electrostatic field.

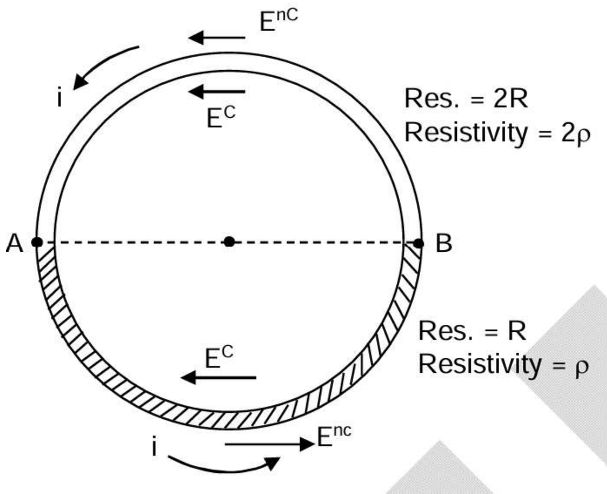
The electrostatic field ΔE in the ring is generated by some positive and some negative charges accumulated at junctions A and B. In the upper semi-ring, the resultant electric field now becomes $E_{\text{induced}} + \Delta E$ and for lower semiring electric field becomes $E_{\text{induced}} - \Delta E$

Let R and 2R be the resistance of the lower and upper halves, respectively. The current is the same throughout the ring, and the resistivity is ρ in the lower half and 2ρ in the upper half. The current density is given by J = E/ρ. Since the current is constant, the electric field and resistivity are inversely related. Therefore the electric field in the lower half is twice the electric field in the upper half. The electric field along the tangent is given by $E_c = \dfrac{ak}{6}$.

(c) For the given case, the potential difference is given by:

$$V_{BA} = \int_A^B \vec{E} \cdot d\vec{l} = \frac{ak}{6} \times \pi a = \frac{\pi a^2 k}{6}$$





49 For a Wheatstone bridge to be balanced, the ratio of resistances in adjacent arms must be equal. Let R_3 be the resistance of wire segment PS and R_4 be the resistance of wire segment SQ. The balanced condition is:

$$\frac{R_1}{R_3} = \frac{R_2}{R_4}$$

Given $R_1 = 10\Omega$ and $R_2 = 20\Omega$, the condition simplifies to $R_4 = 2R_3$. Also, the total resistance of wire PQ is 10 Ω, so $R_3 + R_4 = 10\Omega$.

(a) Adjusting Jockey S with Fixed P and Q:

To achieve balance by moving jockey S, we need to find lengths PS and SQ that satisfy the Wheatstone bridge condition. Let PS have length x cm. Then $R_3 = x\Omega$ and $R_4 = (10 - x)\Omega$. Substituting these into the balanced condition:

$$\frac{10}{x} = \frac{20}{10 - x}$$
$\Rightarrow 10(10 - x) = 20x$

$\Rightarrow 100 - 10x = 20x$

$\Rightarrow 30x = 100$

$\Rightarrow x = \frac{10}{3}\ \mathrm{cm} \approx 3.33\ \mathrm{cm}$

Since the jockey S is initially at the center (5 cm from P), it must be moved a distance of $5\ \mathrm{cm} - \frac{10}{3}\ \mathrm{cm} = \frac{5}{3}\ \mathrm{cm}$ towards P.

(b) Moving Jockey at P with Fixed S and Q:

To achieve balance by moving jockey P, let the displacement of jockey be x towards S. The resistance of wire PS becomes $(5 - x)\Omega$ and the resistance R_1 now contains an added resistance (in series) of x. The balance condition becomes:

$$\frac{R_1 + x}{5 - x} = \frac{R_2}{5}$$
$$\Rightarrow \frac{10 + x}{5 - x} = \frac{20}{5}$$
$\Rightarrow 10 + x = 20 - 4x$

$\Rightarrow x = 2\Omega$

Since the wire has resistance 1 Ω/cm, this corresponds to a length of 2 cm. Jockey P must be moved 2 cm towards S from end P.

(c) Moving Jockey at Q with Fixed S and P:

To achieve balance by moving jockey Q by length z cm towards S, we add a resistance $z\Omega$ in series with R_2. Here, S is fixed at the center, so $R_3 = 5\Omega$ and the resistance of wire SQ becomes $(5 - z)\Omega$. The balance condition becomes:

$$\frac{R_1}{5} = \frac{R_2 + z}{5 - z}$$
$$\Rightarrow \frac{10}{5} = \frac{20 + z}{5 - z}$$
$\Rightarrow 10 - 2z = 20 + z$

$\Rightarrow z = -10/3\Omega$

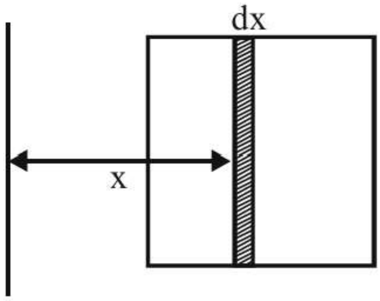
The negative sign indicates that we need to shift the jockey away from point S. However, in this setup, moving jockey Q along **away** from S will make the jockey outside the wire PQ. Therefore, no balanceing point would be achieved.

50 After moving the loop about side PQ through 120°, the normal on the side PS from the wire will be passing through midpoint C and the net magnetic flux through the loop in the new position will be zero.

The change is flux $\Delta\phi =$ Flux through the loop in the initial position

Flux through area element dA

$$d\phi = BdA\frac{\mu_0}{2\pi x}adx$$



$$\phi=\int_a^{2a}\frac{\mu_0 Ia}{2\pi x}dx=\frac{\mu_0 Ia}{2\pi}\ln(2)$$

Now the flux associated with the square loop after it is rotated by an angle 120° is zero. This is because no net magnetic field line penetrates through the square in this situation.

Charge flown through the square loop

$$=\frac{\Delta\phi}{R}=\frac{\frac{\mu_0 Ia}{2\pi}\ln(2)}{R}=4\times10^{-7}$$

$\Rightarrow R=\ln(2)\Omega$

51	The total charge remains constant. Let Q_0 be the initial charge on the linear capacitor, and Q_1 and Q_2 be the final charges on the nonlinear and linear capacitors, respectively. Then $Q_0=Q_1+Q_2$. The capacitance of the linear capacitor is C. The capacitance of the nonlinear capacitor is voltage-dependent: $C(U)=\frac{\epsilon A}{d}=\frac{\alpha UA}{d}=\alpha CU$ (where $C=\frac{\epsilon_0 A}{d}$). 3. $Q=CU$. Therefore, $Q_0=CU_0$, $Q_1=\alpha CU^2$, $Q_2=CU$. Substituting into the charge conservation equation gives $CU_0=\alpha CU^2+CU$. Simplifying and solving the quadratic equation $\alpha U^2+U-U_0=0$ using the quadratic formula yields $U=\frac{-1\pm\sqrt{1+4\alpha U_0}}{2\alpha}$. Since U must be positive, we take the positive root. Substituting $\alpha=1\ \mathrm{V}^{-1}$ and $U_0=156$ V, we get $U=\frac{\sqrt{1+4(1)(156)}-1}{2(1)}=12$ V.
52	Let $R_1=(5.0\pm0.2)\Omega$ and $R_2=(10.0\pm0.1)\Omega$. When connected in parallel, the equivalent resistance R is given by: $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$ Differentiating, we get: $-\frac{\delta R}{R^2}=-\frac{\delta R_1}{R_1^2}-\frac{\delta R_2}{R_2^2}$ $\frac{\delta R}{R}=R\left[\frac{\delta R_1}{R_1^2}+\frac{\delta R_2}{R_2^2}\right]$ The equivalent resistance is: $R=\frac{R_1R_2}{R_1+R_2}=\frac{5\times10}{5+10}=\frac{50}{15}=\frac{10}{3}\Omega\approx3.33\Omega$

	Percentage error is: $$\frac{\delta R}{R}\times 100 = 100R\left[\frac{\delta R_1}{R_1^2}+\frac{\delta R_2}{R_2^2}\right] = 100\times\frac{10}{3}\left[\frac{0.2}{5^2}+\frac{0.1}{10^2}\right]\approx 3$$
53	**(a)** For a charge q at distance r revolving with angular velocity ω_0: $$E=\frac{m\omega_0^2 r}{q}=\frac{\omega_0^2 r}{\alpha}$$ Also, to find electric field at distance r from axis, the entire cylinder of radius r can be assumed concentrated along the axis. Thus, if $\lambda(r)$ represents total linear charge density due to the revolving particles present inside coaxial cylindrical region of radius r: $$E=\frac{\lambda(r)}{2\pi\epsilon_0 r}$$ Equating, we get $$\frac{\omega_0^2 r}{\alpha}=\frac{\lambda(r)}{2\pi\epsilon_0 r}$$ $$\Rightarrow \lambda(r)=\frac{2\pi\epsilon_0\omega_0^2 r^2}{\alpha}$$ **(b)** If ρ is the charge density present at a distance r from axis, then total charge enclosed within a cylindrical region of inner radius r and outer radius $r+dr$, having a length l is: $$dq=\rho(2\pi r dr)l$$ $$\Rightarrow \frac{dq}{l}=\rho(2\pi r)dr$$ $$\Rightarrow d[\lambda(r)]=\rho(2\pi r)dr$$ $$\Rightarrow \frac{d[\lambda(r)]}{dr}=\rho(2\pi r)$$ Using the previously obtained expression of $\lambda(r)=\frac{2\pi\epsilon_0\omega_0^2 r^2}{\alpha}$: $$\frac{4\pi\epsilon_0\omega_0^2 r}{\alpha}=\rho(2\pi r)$$ $$\Rightarrow \rho=\frac{2\epsilon_0\omega_0^2}{\alpha}$$ **(c)** To find the total flux passing through a cylindrical region of radius r, we can find the net charge within the cylinder. Since net charge continuously decreases with increase in r (due to the r^2 dependence of $\lambda(r)$), the total flux also decreases with increasing r. Specifically, $\Phi(r)=\frac{q_{enc}}{\epsilon_0}$ $$\Rightarrow \Phi(r)=\frac{l\lambda(r)}{\epsilon_0}$$ $$\Rightarrow \Phi(r)=\frac{2\pi l\omega_0^2 r^2}{\alpha}.$$ So, $\Phi(r)$ is proportional to r^2 and thus increases with increase in r.
54	If the displacement of the center of the sphere is $\vec{OO'}=\Delta\vec{s}$ during this time, the initial and the final electric fields at the considered point P are:

	$\vec{E}_1 = \dfrac{\rho}{3\epsilon_0}\vec{OP}$ and $\vec{E}_2 = \dfrac{\rho}{3\epsilon_0}\vec{O'P}$ $\Rightarrow \vec{E}_1 - \vec{E}_2 = \dfrac{\rho}{3\epsilon_0}(\vec{OP} - \vec{O'P})$ $\Rightarrow \vec{E}_1 - \vec{E}_2 = \dfrac{\rho}{3\epsilon_0}(\Delta\vec{s})$ Given $\rho = 1\ \mathrm{C/m^3}$, $\Delta t = 1$ s, $v = 2$ m/s Displacement of the center of the sphere in $\Delta t = 1$ s: $\Delta\vec{s} = \vec{v}\Delta t = 2\hat{i}$ $\therefore \vec{E}_1 - \vec{E}_2 = \dfrac{\rho}{3\epsilon_0}(2\hat{i}) = \dfrac{2\hat{i}}{3\epsilon_0}$ $\therefore \lvert\vec{E}_1 - \vec{E}_2\rvert = \dfrac{2}{3\epsilon_0}$ $\therefore (9\epsilon_0) \times \lvert\vec{E}_1 - \vec{E}_2\rvert = (9\epsilon_0) \times \dfrac{2}{3\epsilon_0} = 6$
55	Let r be the radius of the circular wire frame. The initial potential energy of the bead when it is on the plane of symmetry of the dipole ($\theta = \pi/2$) is zero since the electric potential at any point on the bisector of an electric dipole is zero. The electric potential due to a dipole at a point (r,θ) is given by: $V = \dfrac{kp\cos\theta}{r^2} = \dfrac{p\cos\theta}{4\pi\epsilon_0 r^2}$ As the bead moves along the wire, its angular position changes to θ. The potential energy of the bead at this position is given by: $U = QV = \dfrac{Qp\cos\theta}{4\pi\epsilon_0 r^2}$ Since the bead starts from rest, its initial kinetic energy is zero. Let v be the speed of the bead at angular position θ. By conservation of energy: $\dfrac{1}{2}mv^2 + \dfrac{Qp\cos\theta}{4\pi\epsilon_0 r^2} = 0$ $\dfrac{1}{2}mv^2 = -\dfrac{Qp\cos\theta}{4\pi\epsilon_0 r^2}$ Since $\dfrac{\pi}{2} \le \theta \le \dfrac{3\pi}{2}$, $\cos\theta$ is negative, ensuring the kinetic energy remains positive. Thus, the magnitude of the velocity is: $v = \sqrt{-\dfrac{2Qp\cos\theta}{4\pi\epsilon_0 mr^2}} = \sqrt{\dfrac{-2Qp\cos\theta}{4\pi\epsilon_0 mr^2}}$
56	Let the initial charge on the conducting shell be q. Let the point charges be $q_A = q$ and $q_B = 2q$. When the conducting shell is earthed, its potential becomes zero. The potential of the shell is due to its own charge and the point charges q_A and q_B. Let q_{ind} be the charge induced on the conducting shell after earthing. The initial charge q is irrelevant to the charge flow because we are interested in the

charge that **flows** to earth due to earthing in the presence of external charges. We consider the initial charge on the shell to be zero for simplicity and find the charge induced upon earthing in presence of q_A and q_B.
The potential of the conducting shell (radius r) must be zero after earthing. The potential at the surface of the shell is the sum of potentials due to the point charges q_A, q_B and the induced charge q_{ind} on the shell itself.
Potential due to q_A at the shell: $V_A = \frac{Kq}{r}$
Potential due to q_B at the shell: $V_B = \frac{K(2q)}{2r} = \frac{Kq}{r}$
Potential due to induced charge q_{ind} on the shell: $V_{ind} = \frac{Kq_{ind}}{r}$
The total potential of the shell must be zero:

$$V_{total} = V_A + V_B + V_{ind} = 0$$
$$\frac{Kq}{r} + \frac{Kq}{r} + \frac{Kq_{ind}}{r} = 0$$
$$2\frac{Kq}{r} + \frac{Kq_{ind}}{r} = 0$$
$$2Kq + Kq_{ind} = 0$$
$$q_{ind} = -2q$$

Therefore, the charge induced on the shell is $-2q$.
If the shell initially had a charge q, then the charge that flows to earth is the difference between the initial charge and the induced charge required to make potential zero due to external charges.
Hence, amount of charge flow to the earth is $|q - (-2q)| = 3q$.

57 The force pulling the dielectric into the capacitor is given by $F = \frac{E^2}{2}\frac{dC}{dx}$, where $\frac{dC}{dx}$ is the change in capacitance per unit displacement of the dielectric.
The capacitance of the partially filled capacitor is $C(x) = \frac{\epsilon_0 b}{d}[(K-1)x + 2a]$, where x is the length of the dielectric inside the capacitor. Therefore, $\frac{dC}{dx} = \frac{\epsilon_0 b}{d}(K-1)$.
The magnitude of the force is $F = \frac{E^2}{2}\frac{\epsilon_0 b}{d}(K-1)$.
The work done by this force to pull the dielectric completely out of the capacitor (over a distance a) is $W = F \times a = \frac{E^2}{2}\frac{\epsilon_0 ba}{d}(K-1)$.
This work is done against the initial kinetic energy of the bullet and embedded dielectric. Applying momentum conservation during the collision, taking final velocity of bullet-dielectric combined system as V_1

$$MV_0 = 2MV_1$$
$$\Rightarrow V_1 = \frac{V_0}{2}$$

Now, initial kinetic energy is,

$$\frac{1}{2}(2M)V_1^2 = \frac{1}{4}MV_0^2$$

This kinetic energy was changed to zero, while the dielectric was exiting the capacitor plates. Hence,

$$\frac{1}{4}MV_0^2 = \frac{E^2}{2}\frac{\epsilon_0 ba}{d}(K-1)$$

Solving we get,

$$V_0^2 = \sqrt{\frac{2\epsilon_0 baE^2}{dM}(K-1)}$$

58 1. **Coordinates of Point A:** Point A lies on both the circle $x^2 + y^2 = R^2$ and the parabola $y = \frac{\sqrt{2}x^2}{R}$. Substituting the parabola equation into the circle equation gives:

$$x^2 + \frac{2x^4}{R^2} = R^2 \text{ Multiplying by } R^2\text{:}$$

$$R^2x^2 + 2x^4 = R^4 \text{ Let } u = x^2\text{:}$$

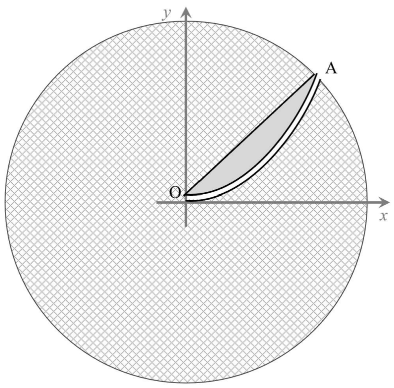
$$2u^2 + R^2u - R^4 = 0$$

Solving this quadratic equation and taking the positive root: $u = x^2 = \frac{R^2}{2}$.

Thus, $x_A = \frac{R}{\sqrt{2}}$ and $y_A = \frac{\sqrt{2}}{R}\left(\frac{R^2}{2}\right) = \frac{R}{\sqrt{2}}$

2. **Equation of Line OA:** The line OA is given by $y = x$.

3. **Calculating Area ΔA:**



$$\Delta A = \int_0^{R/\sqrt{2}} (x - \frac{\sqrt{2}x^2}{R})dx = \left[\frac{x^2}{2} - \frac{\sqrt{2}x^3}{3R}\right]_0^{R/\sqrt{2}} = \frac{R^2}{4} - \frac{R^3}{6R} = \frac{R^2}{12}$$

4. **Induced EMF and Work Done:**The induced emf in the area ΔA will be caused due to the induced electric field in the region. This induced electric field remains perpendicular to the straight line OA. So, along line OA there is no emf. So the complete emf is only along the curve of $y = \frac{\sqrt{2}x^2}{R}$

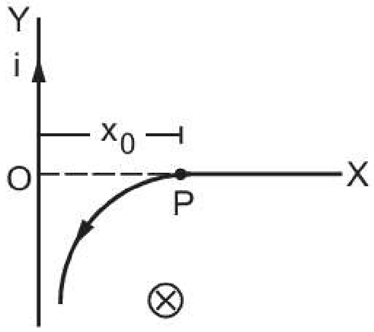
$$\text{EMF} = -\frac{d\Phi_B}{dt} = -\alpha\Delta A = -\frac{\alpha R^2}{12}$$

The work done is $W = q \times |EMF| = \frac{q\alpha R^2}{12}$.

5. **Kinetic Energy at O:**

$$K_O = K_A + W = K_o + \frac{q\alpha R^2}{12}$$

59 Let the particle be initially at P as shown in diagram below:



Take the wire as the y-axis and the foot of perpendicular from P to the wire as the origin. Take the line OP as the x-axis. The magnetic field $\vec{B}$ at any point to the right of the wire is along the negative z-axis. The magnetic force on the particle is, therefore, in the x-y plane. As there is no initial velocity along the z-axis, the motion will be in the x-y plane. Also, as the magnetic field is not uniform, the particle does not go along a circle. The force at time t is

$\vec{F} = q\vec{v} \times \vec{B}$

$$\vec{F} = q(v_x\hat{i} + v_y\hat{j}) \times \left(-\frac{\mu_0 i}{2\pi x}\hat{k}\right) = jqv_x\frac{\mu_0 i}{2\pi x} - iqv_y\frac{\mu_0 i}{2\pi x}$$

Thus

$$a_x = \frac{F_x}{m} = -\frac{\mu_0 qi}{2\pi m}\frac{v_y}{x} = -\lambda\frac{v_y}{x} \quad \ldots(i)$$

where $\lambda = \dfrac{\mu_0 qi}{2\pi m}$

Also,

$$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\frac{dx}{dt} = v_x\frac{dv_x}{dx} \quad \ldots(ii)$$

Since the force acting on the charged particle is only the magnetic force, the speed of the particle must not change. Thus,

$$v_x^2 + v_y^2 = v^2$$

Giving: $\quad 2v_x dv_x + 2v_y dv_y = 0$

$\Rightarrow v_x dv_x = -v_y dv_y \quad \ldots(iii)$

From (i), (ii) and (iii),

$$v_x\frac{dv_x}{dx} = -\frac{\lambda v_y}{x}$$

$$\Rightarrow \frac{dx}{x} = \frac{dv_y}{\lambda}$$

Initially, $x = x_0$ and $v_y = 0$.

At minimum separation from the wire, $v_x = 0$ so that $v_y = -v$.

Thus

$$\int_{x_0}^{x_{\min}} \frac{dx}{x} = \int_0^{-v} \frac{dv_y}{\lambda}$$

$$\ln \frac{x_{\min}}{x_0} = -\frac{v}{\lambda}$$

$$\Rightarrow x = x_0 e^{-v/\lambda}$$

$$\Rightarrow x = x_0 e^{-\frac{2\pi m v}{\mu_0 q i}}$$