

A Math Formula Extraction and Evaluation Framework for PDF Documents

Ayush Kumar Shah, Rochester Institute of Technology, USA

Abhisek Dey, Rochester Institute of Technology, USA

Richard Zanibbi, Rochester Institute of Technology, USA

{as1211, ad4529, rxzvcs}@rit.edu



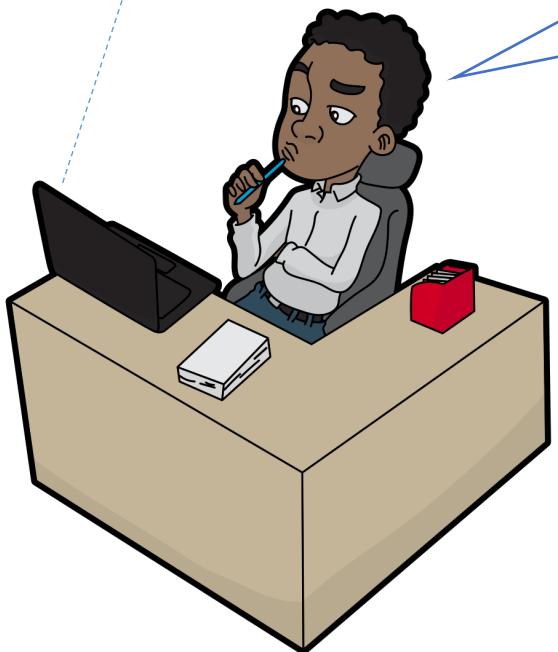
Document and Pattern Recognition Lab

RIT | Rochester Institute of Technology

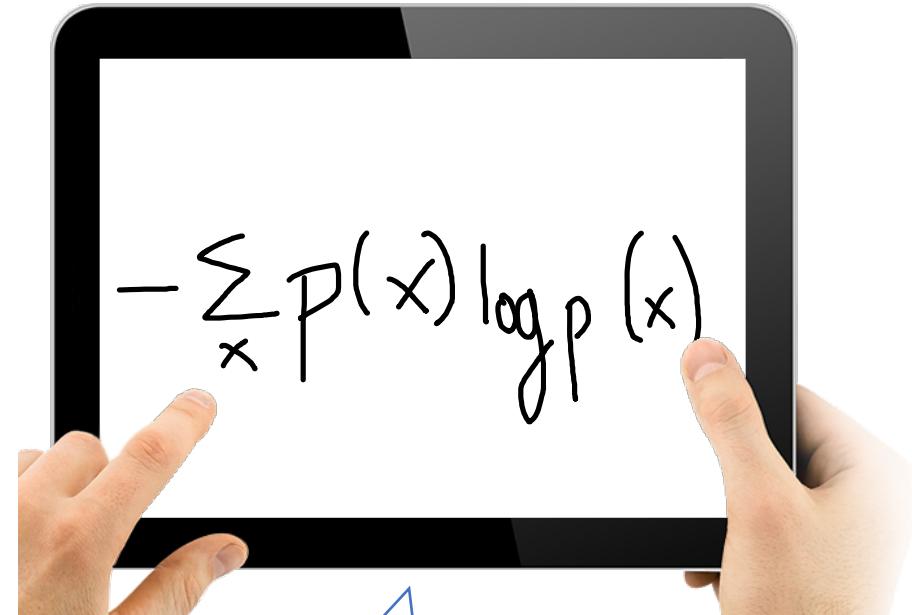


Motivation: Information Retrieval

$$-\sum_x p(x) \log p(x)$$

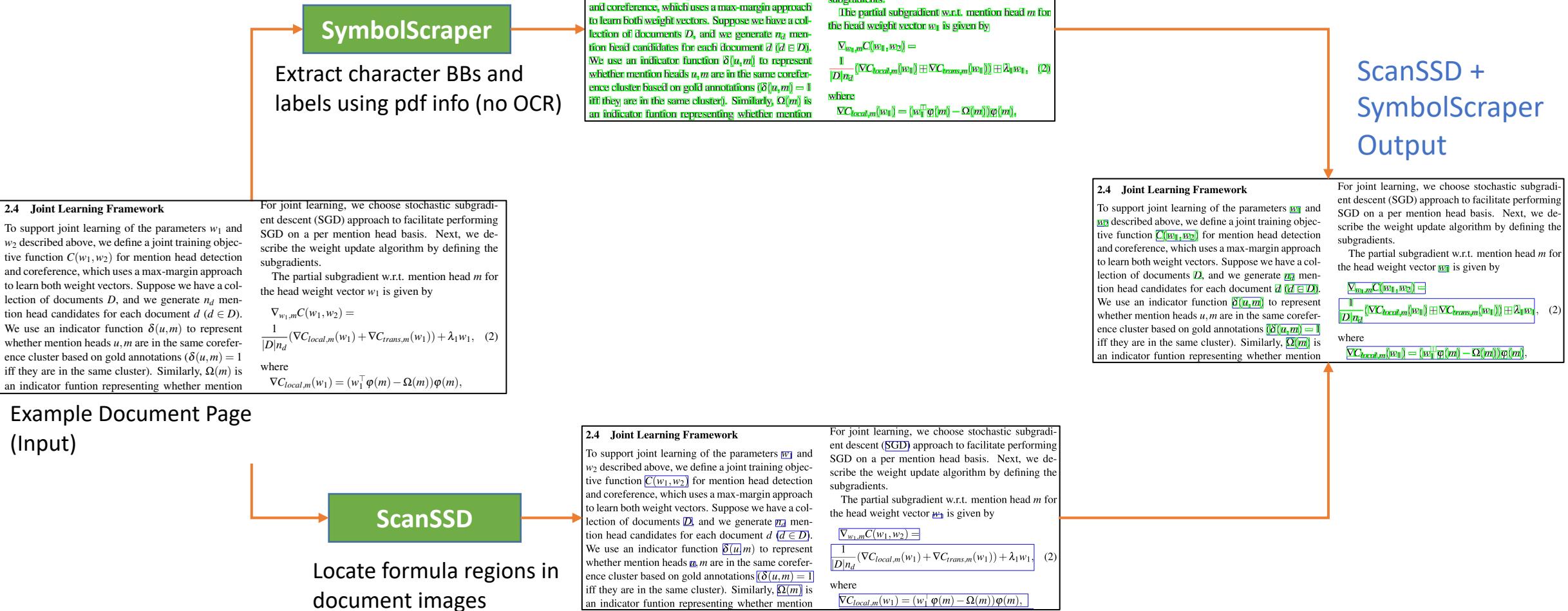


What is the name
of this formula?



Let's find documents
describing this formula.

Mathematical Formula Extraction: Overview



Mathematical Formula Extraction: Overview

ScanSSD Output

2.4 Joint Learning Framework

To support joint learning of the parameters w_1 and w_2 described above, we define a joint training objective function $C(w_1, w_2)$ for mention head detection and coreference, which uses a max-margin approach to learn both weight vectors. Suppose we have a collection of documents D , and we generate n_d mention head candidates for each document $d \in D$. We use an indicator function $\delta(u, m)$ to represent whether mention heads u, m are in the same coreference cluster based on gold annotations ($\delta(u, m) = 1$ iff they are in the same cluster). Similarly, $\Omega(m)$ is an indicator function representing whether mention

For joint learning, we choose stochastic subgradient descent (SGD) approach to facilitate performing SGD on a per mention head basis. Next, we describe the weight update algorithm by defining the subgradients.

The partial subgradient w.r.t. mention head m for the head weight vector w_1 is given by

$$\nabla_{w_1, m} C(w_1, w_2) = \frac{1}{|D| n_d} (\nabla C_{local, m}(w_1) + \nabla C_{trans, m}(w_1)) + \lambda_1 w_1, \quad (2)$$

where

$$\nabla C_{local, m}(w_1) = (w_1^\top \varphi(m) - \Omega(m)) \varphi(m),$$

$$(\delta(u, m) = 1)$$

Formula Box

CC Extraction

CC Boxes

$$(\delta(u, m) = 1)$$

Combine

SS Boxes + labels

$$(\delta(u, m) = 1)$$

$$(\delta(u, m) =$$

$$(\delta(u, m) = 1)$$

$$(\delta(u, m) = -$$

QD-GGA

Parse formula structure

ScanSSD + SymbolScraper Output

2.4 Joint Learning Framework

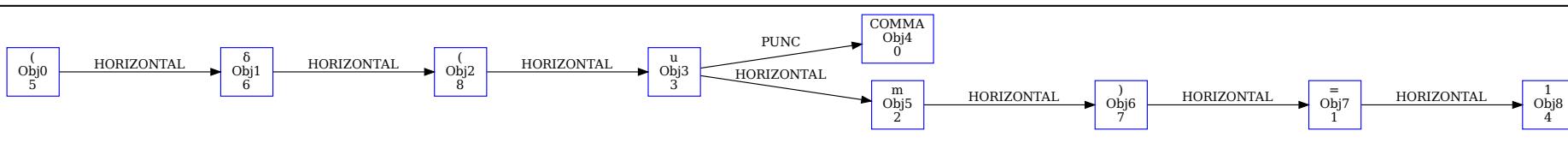
To support joint learning of the parameters w_1 and w_2 described above, we define a joint training objective function $C(w_1, w_2)$ for mention head detection and coreference, which uses a max-margin approach to learn both weight vectors. Suppose we have a collection of documents D , and we generate n_d mention head candidates for each document $d \in D$. We use an indicator function $\delta(u, m)$ to represent whether mention heads u, m are in the same coreference cluster based on gold annotations ($\delta(u, m) = 1$ iff they are in the same cluster). Similarly, $\Omega(m)$ is an indicator function representing whether mention

For joint learning, we choose stochastic subgradient descent (SGD) approach to facilitate performing SGD on a per mention head basis. Next, we describe the weight update algorithm by defining the subgradients.

The partial subgradient w.r.t. mention head m for the head weight vector w_1 is given by

$$\nabla_{w_1, m} C(w_1, w_2) = \frac{1}{|D| n_d} (\nabla C_{local, m}(w_1) + \nabla C_{trans, m}(w_1)) + \lambda_1 w_1, \quad (2)$$

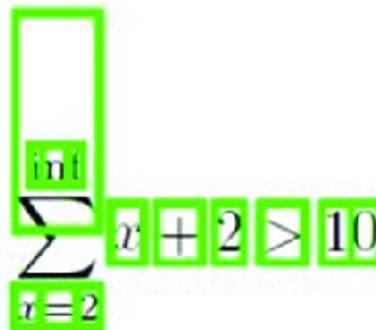
where

$$\nabla C_{local, m}(w_1) = [w_1^\top q[m] - \Omega(m)] q[m],$$


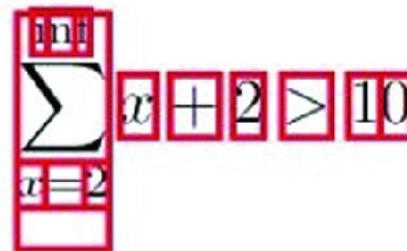
Symbol Layout Tree (Output)

SymbolScaper: Extracting Symbols in PDF

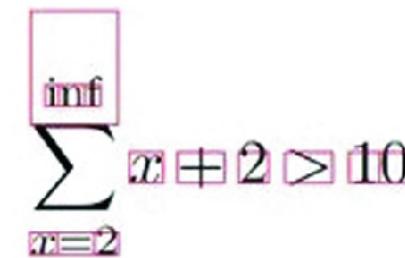
- Based on Apache PDFBox
- Avoids OCR in **born-digital** PDF documents and instead uses **vector drawing commands** in PDF
- Unicode, writing line position and attributes derived from PDF encoding
- ‘em box’ or underlying character outlines (**glyphs**) represent symbol outlines in a font as boxes



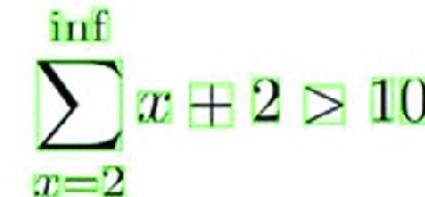
PDF Miner



PyMuPDF



PDFBox

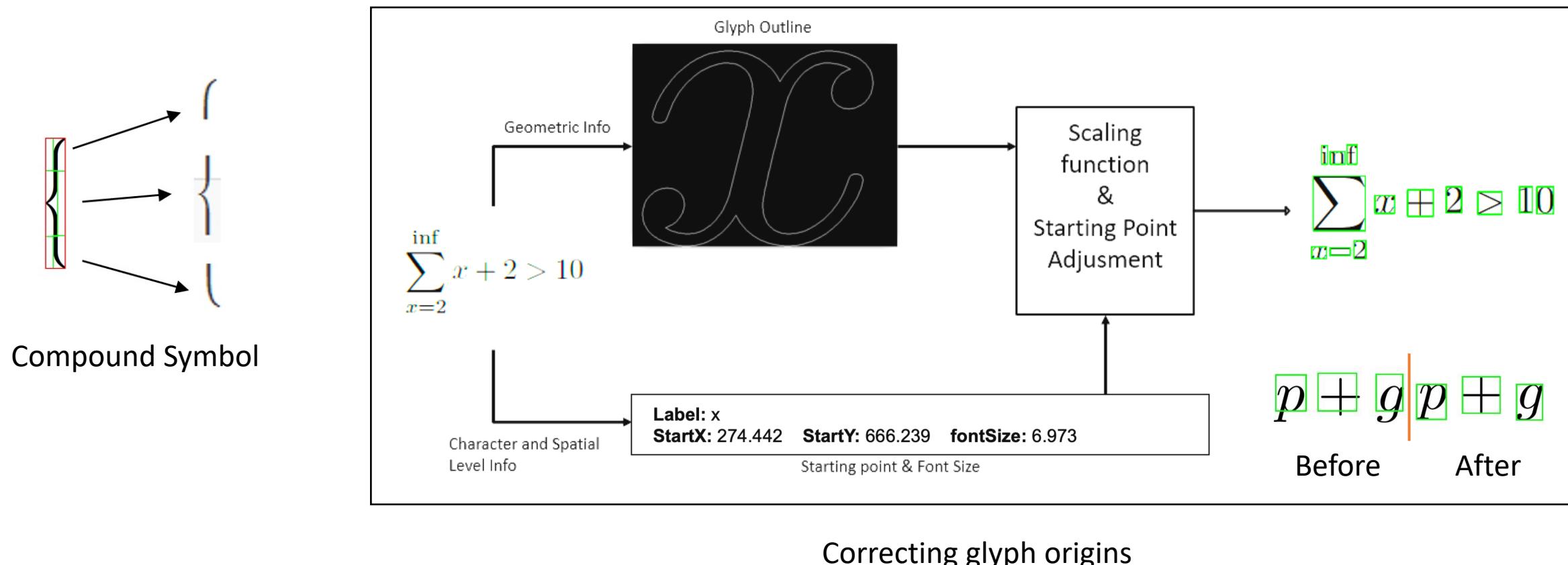


SymbolScaper

Unlike other methods, SymbolScaper uses glyphs to fine-tune bounding box locations

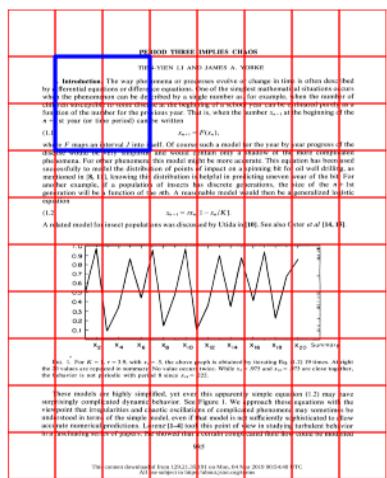
SymbolScraper: Extracting Symbols in PDF

- Glyphs and font scaling information used to obtain precise bounding box locations
- Compound characters (large braces, square roots, etc.) are formed of 2 or more characters

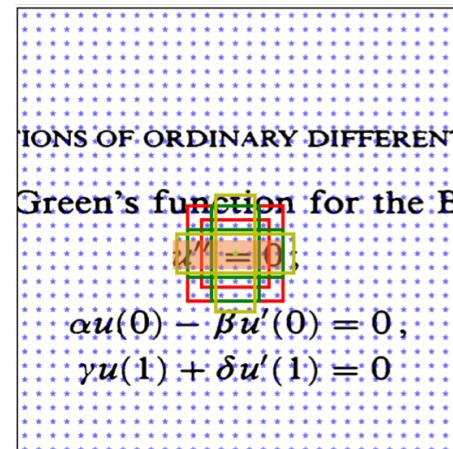


ScanSSD: Locating Formula Regions

- **Scanning Single-Shot Detector**, CNN which locates formula bounding boxes using a sliding window
- 600 dpi images broken into windows of 1200×1200 pixels, SSD applied in each window at 10% stride
- Non-Maximal Suppression selects the highest confidence regions from overlapping detections
- Wider default boxes sizes used with aspect ratios of 5, 7, and 10 -> increased recall



A window (in Blue) slides across the grid (in Red)
(50% stride)

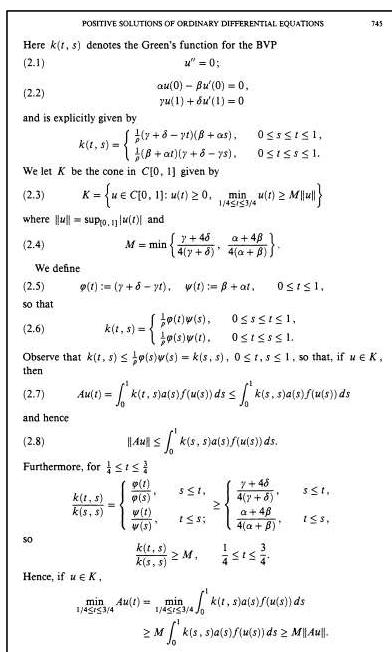


Default boxes around a grid point

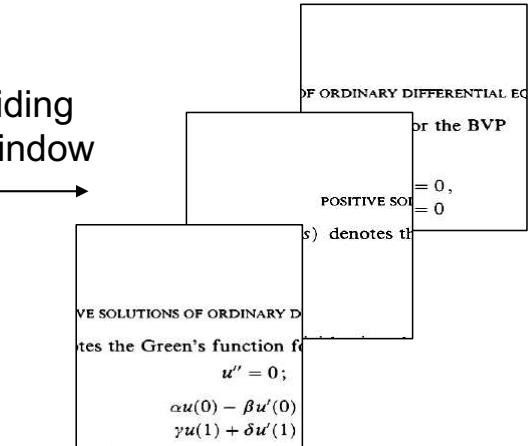
ScanSSD: Locating Formula Regions

- A sliding window divides the page into windows which are processed by ScanSSD
- The partial predictions at the window-level are pooled together and the final regions are identified using pixel-wise voting (stitching)

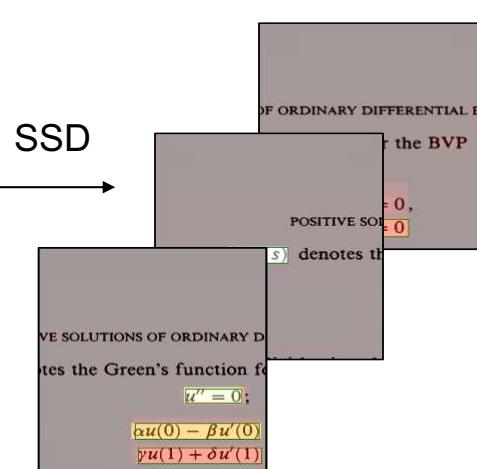
Input Page



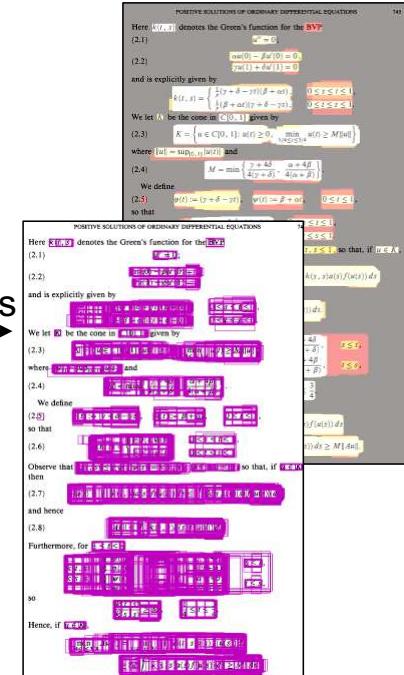
Sliding Window



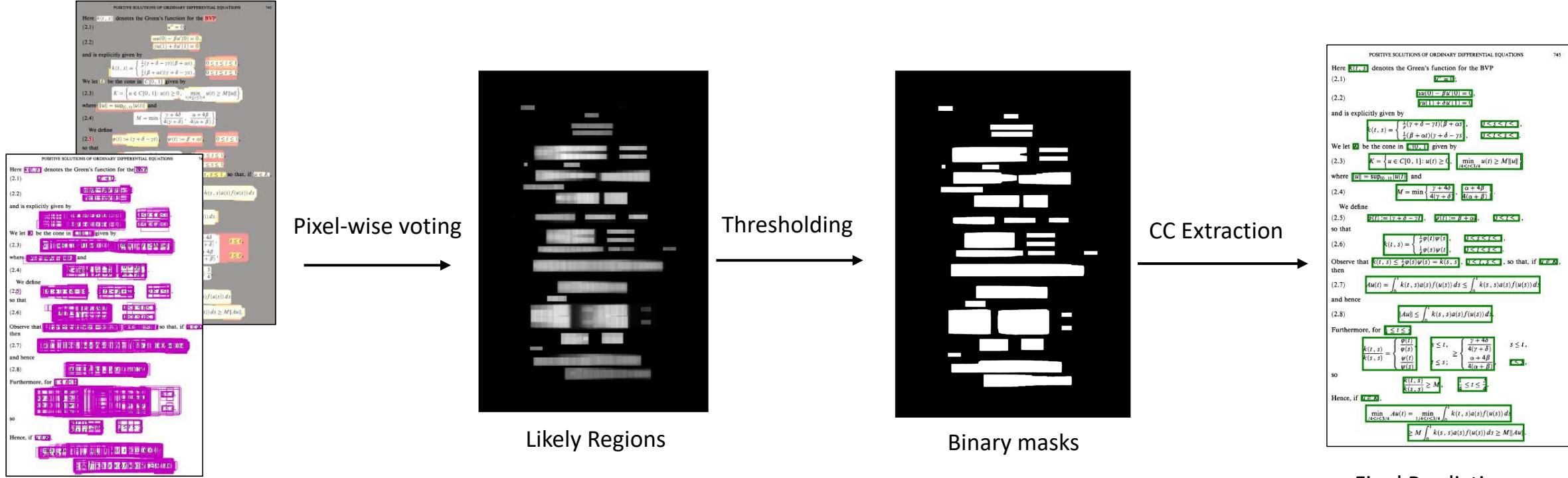
SSD



Stitch Patches



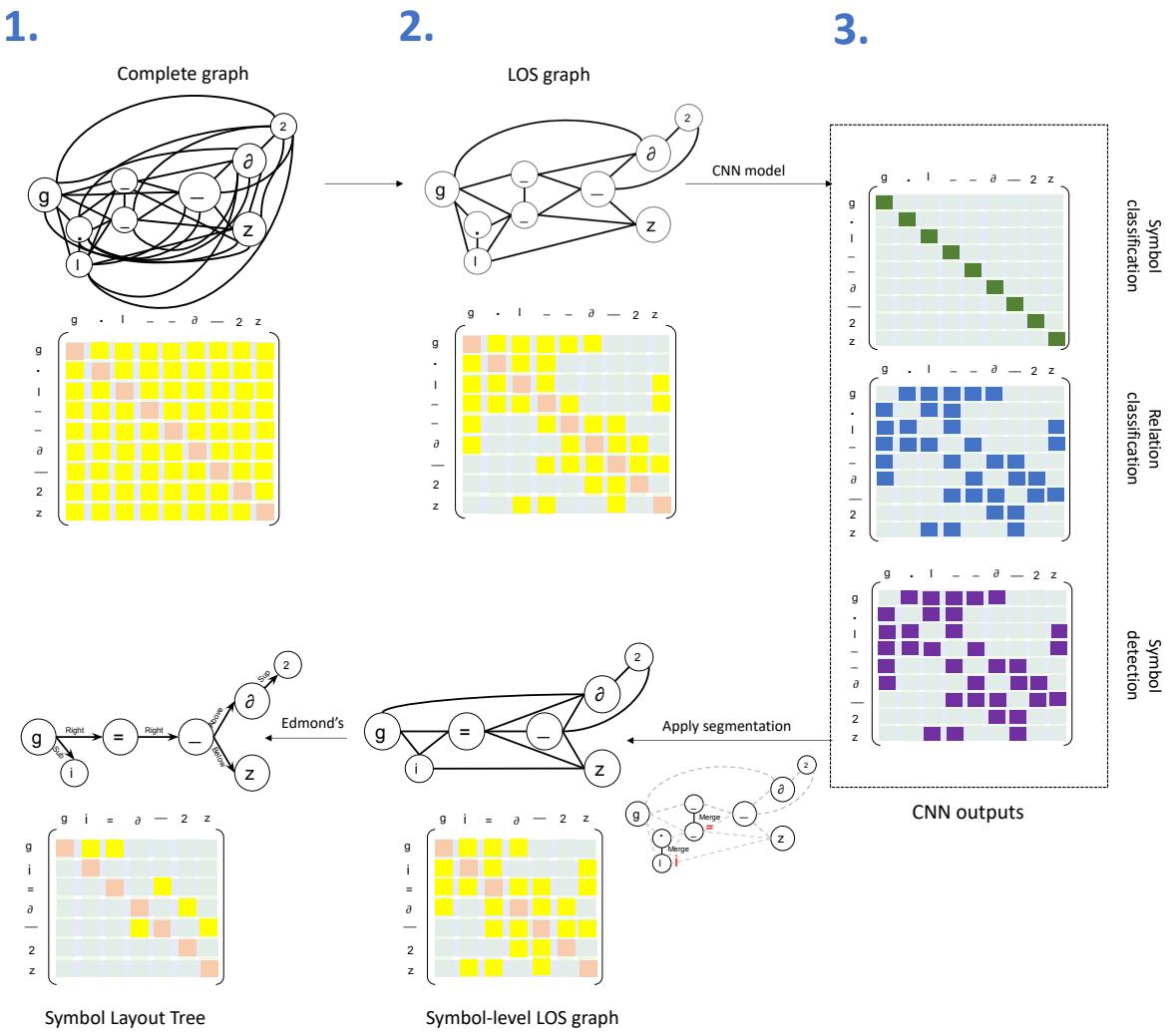
ScanSSD: Locating Formula Regions



- Liu, W., Anguelov, D., Erhan, D., Szegedy, C., Reed, S., Fu, C.Y., Berg, A.C.: **SSD: Single shot multibox detector.** In: European conference on computer vision. pp. 21–37. Springer (2016)
- Mali, P., Kukkudapu, P., Mahdavi, M., Zanibbi, R.: **ScanSSD: Scanning Single Shot Detector for Mathematical Formulas in PDF Document Images.** arXiv:2003.08005 [cs] (2020)

QD-GGA: Recognizing Formula Structure (Parsing)

1. **Construct** graph over CCs
2. **Prune**: Convert to LOS graph
3. **Classify** edges as merge/split and relationships, nodes as symbols
4. **New LOS graph**: detected symbols
5. **Extract MST** using Edmond's arborescence algorithm

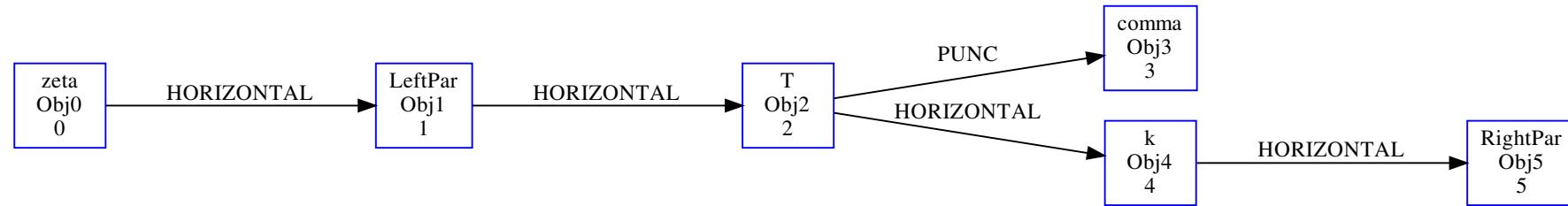


Inputs

$$\zeta(T, k)$$

Input Formula with
CCs or Symbols

Outputs



Symbol Layout Tree (SLT)

```
<math xmlns="http://www.w3.org/1998/Math/MathML">
<mrow>
  <mi xml:id="0:">\zeta</mi>
  <mrow>
    <mo xml:id="1:">(</mo>
    <mrow>
      <mi xml:id="2:">T</mi>
      <mo xml:id="3:">, </mo>
    </mrow>
    <mrow>
      <mi xml:id="4:">k</mi>
      <mo xml:id="5:">)</mo>
    </mrow>
  </mrow>
</mrow>
</math>
```

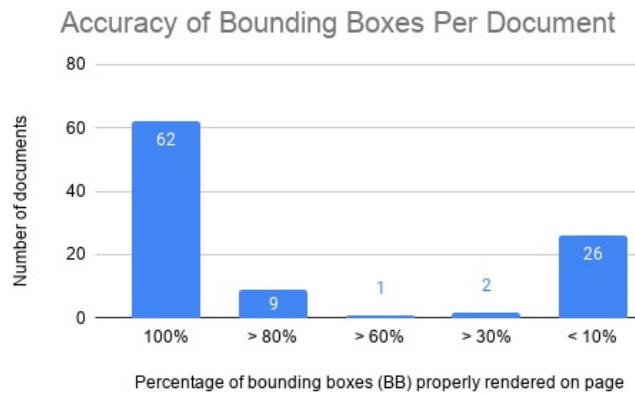
SLT in MathML

```
\(\zeta\left( T, k \right)\)
```

SLT in LATEX

SymbolScraper Results

Summary of SymbolScraper Accuracy



Hardware Specifications and Speed

Storage	HDD
Dataset Size	100 pages
Total time	28 mins, 19 secs
Average time	1.7 secs/page

ScanSSD Results

Formula Detection Results for TFD-ICDAR2019

	IOU ≥ 0.75			IOU ≥ 0.5		
	Precision	Recall	F-score	Precision	Recall	F-score
ScanSSD	0.774	0.690	0.730	0.851	0.759	0.802
RIT 2	0.753	0.625	0.683	0.831	0.670	0.754
RIT 1	0.632	0.582	0.606	0.744	0.685	0.713
Mitchiking	0.191	0.139	0.161	0.369	0.270	0.312
Samsung*	0.941	0.927	0.934	0.944	0.929	0.936

*Used character information

Hardware Specifications and Speed

Storage	HDD
RAM	32 GB
Graphics	Nvidia RTX 2080 Ti
Processor	AMD Ryzen 7 2700
Dataset Size	233 pages
Total time	4 hrs, 33 mins, 31 secs
Average time	70.4 secs/page

QD-GGA Results

Formula Recognition Results for InftyMCCDB-2^[1] Test set

Metrics	Value
Structure rate	92.56
Structure + Classification rate	85.94

Hardware Specifications and Speed

Storage	HDD
RAM	32 GB
Graphics	Nvidia GTX 1080
Processor	Intel(R) Core(TM) i7-9700KF
Dataset Size	6830 images
Total time	26 mins, 25 secs
Average time	232 ms/formula

[1] <https://zenodo.org/record/3483048#.XaCwmOdKiVo>

Mali,P.,Kukkadapu,P.,Mahdavi,M.,Zanibbi,R.: ScanSSD: Scanning Single Shot Detector for Mathematical Formulas in PDF Document Images. arXiv:2003.08005 [cs] (2020)

Mahdavi, M.; Sun, L.; Zanibbi, R.: Visual Parsing with Query-Driven Global Graph Attention (QD-GGA). In Conference on Computer Vision and Pattern Recognition Workshops (2020)

Recognition Results Visualization (HTML)

MathSeer Pipeline Results Visualization

Pdf name: K15-1002

Page: 4

[Previous page](#) [Home](#) [Next page](#)

Page image

2.3 Joint Inference Framework

We extend expression (1) to facilitate joint inference on mention heads and coreference as follows:

$$\begin{aligned} \arg \max_y \sum_{u \in \mathcal{U}, v \in \mathcal{M}} f_{u,v} y_{u,v} + \sum_{m \in \mathcal{M}} g_m y_m, \\ \text{s.t. } \sum_{u \in v} y_{u,v} \leq 1 \quad \forall v \in \mathcal{M}, \\ \sum_{u \in v} y_{u,v} \leq y_v \quad \forall v \in \mathcal{M}, \\ y_{u,v} \in \{0, 1\}, \quad y_m \in \{0, 1\} \quad \forall u, v, m \in \mathcal{M}. \end{aligned}$$

Here, \mathcal{M} is the set of all mention head candidates. y_m is the decision variable for mention head candidate m . $y_{u,v} = 1$ if and only if the mention head m is chosen. To consider coreference decisions and mention head decisions together, we add the constraint $\sum_{u \in v} y_{u,v} \leq y_v$, which ensures that if a candidate mention head u is not chosen, then it will not have coreference links with other mention heads.

2.4 Joint Learning Framework

To support joint learning of the parameters w_1 and w_2 described above, we define a joint training objective function $C(w_1, w_2)$ for mention head detection and coreference, which uses a max-margin approach to learn both weight vectors. Suppose we have a collection of documents \mathcal{D} , and we generate n_d mention head candidates for each document d ($d \in \mathcal{D}$). We use an indicator function $\delta(u, m)$ to represent whether mention heads u, m are in the same coreference cluster based on gold annotations ($\delta(u, m) = 1$ if they are in the same cluster). Similarly, $\Omega(m)$ is an indicator function representing whether mention head m is valid in the gold annotations.

For simplicity, we first define

$$\begin{aligned} u' &= \arg \max_{u \in \mathcal{M}} (w_2^\top \phi(u, m) - \delta(u, m)), \\ u'' &= \arg \max_{u \in \mathcal{M}, \delta(u, m)=1} w_2^\top \phi(u, m) \Omega(m). \end{aligned}$$

We then minimize the following joint training objective function $C(w_1, w_2)$.

$$C(w_1, w_2) = \frac{1}{|D|} \sum_{d \in D} \sum_m (C_{coref, m}(w_2) \\ + C_{local, m}(w_1) + C_{trans, m}(w_1)) + R(w_1, w_2).$$

$C(w_1, w_2)$ is composed of four parts. The first part is the loss function for coreference, where we have

$$\begin{aligned} C_{coref, m}(w_2) &= -w_2^\top \phi(u'', m) \Omega(m) \\ &\quad + (w_2^\top \phi(u', m) - \delta(u', m)) (\Omega(m) \vee \Omega(u')). \end{aligned}$$

It is similar to the loss function for a latent left-linking coreference model⁵. As the second component, we have the quadratic loss for the mention head detection model,

$$C_{local, m}(w_1) = \frac{1}{2} (w_1^\top \phi(m) - \Omega(m))^2.$$

Using the third component, we further maximize the margin between valid and invalid mention head candidates when they are selected as the best-left-link mention heads for any valid mention head. It can be represented as

$$C_{trans, m}(w_1) = \frac{1}{2} (w_1^\top \phi(u') - \Omega(u'))^2 \Omega(m).$$

The last part is the regularization term

$$R(w_1, w_2) = \frac{\lambda_1}{2} ||w_1||^2 + \frac{\lambda_2}{2} ||w_2||^2.$$

2.5 Stochastic Subgradient Descent for Joint Learning

For joint learning, we choose stochastic subgradient descent (SGD) approach to facilitate performing SGD on a per mention head basis. Next, we describe the weight update algorithm by defining the subgradients.

The partial subgradient w.r.t. mention head m for the head weight vector w_2 is given by

$$\nabla_{w_2, m} C(w_1, w_2) = \frac{1}{|D| n_d} (\nabla C_{local, m}(w_1) + \nabla C_{trans, m}(w_1)) + \lambda_1 w_1. \quad (2)$$

where

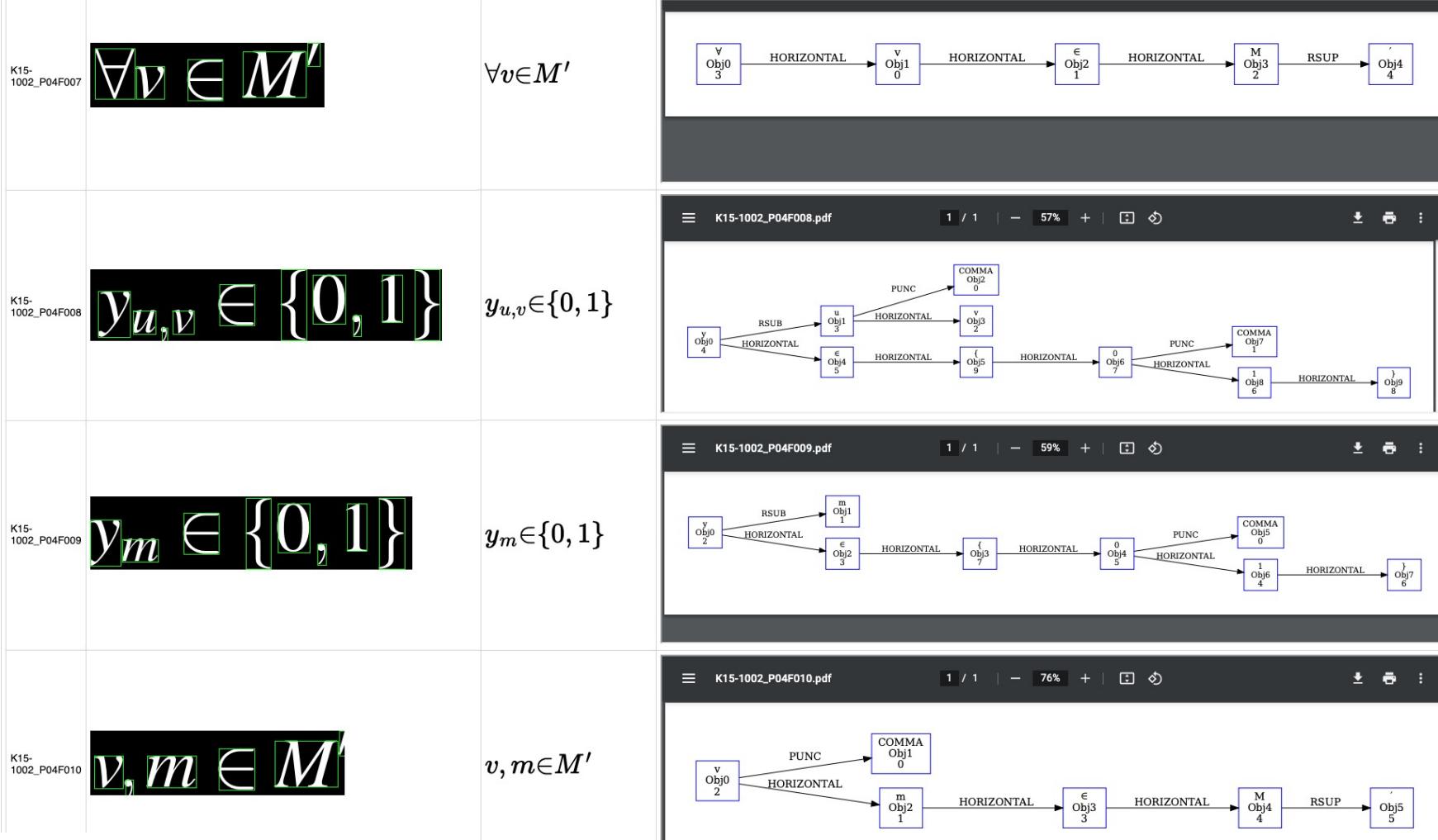
$$\begin{aligned} \nabla C_{local, m}(w_1) &= (w_1^\top \phi(m) - \Omega(m)) \phi(m), \\ \nabla C_{trans, m}(w_1) &= (w_1^\top \phi(u') - \Omega(u')) \phi(u') \Omega(m). \end{aligned}$$

The partial subgradient w.r.t. mention head m for the coreference weight vector w_2 is given by

$$\begin{aligned} \nabla_{w_2, m} C(w_1, w_2) &= \lambda_2 w_2 + \\ &\begin{cases} (\phi(u', m) - \phi(u'', m)) & \text{if } \Omega(m) = 1, \\ \phi(u', m) & \text{if } \Omega(m) = 0 \text{ and } \Omega(u') = 1, \\ 0 & \text{if } \Omega(m) = 0 \text{ and } \Omega(u') = 0. \end{cases} \quad (3) \end{aligned}$$

Here λ_1 and λ_2 are regularization coefficients which are tuned on the development set. To learn the mention head detection model, we consider two different parts of the gradient in expression (2). $\nabla C_{local, m}(w_1)$ is exactly the local gradient of mention head m while we add $\nabla C_{trans, m}(w_1)$ to represent

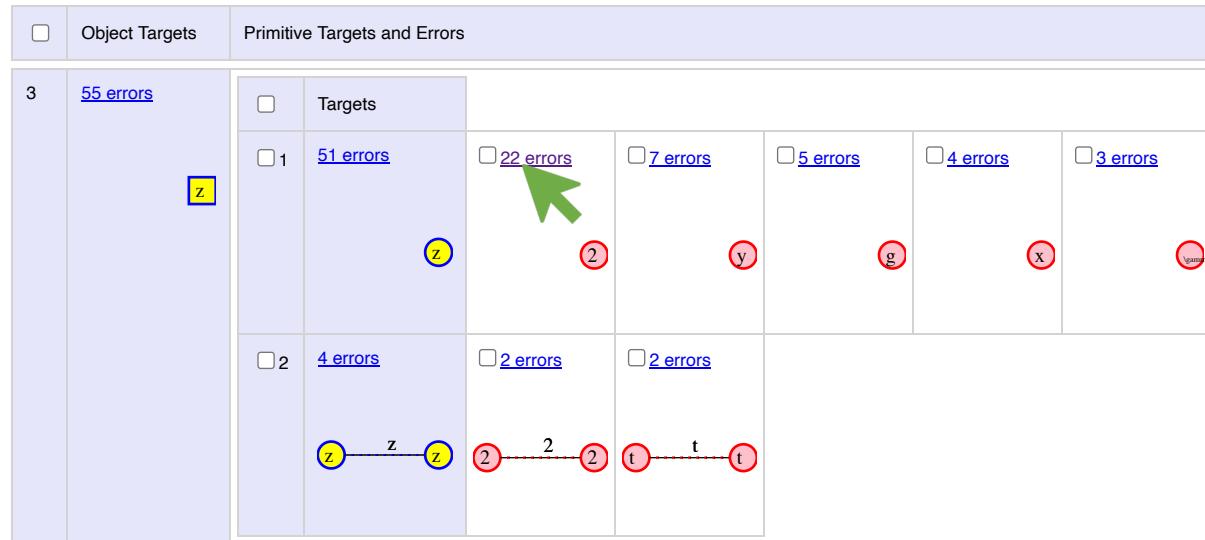
⁵More details can be found in Chang et al. (2013). The difference here is that we also consider the validity of mention heads using $\Omega(u) \vee \Omega(m)$.



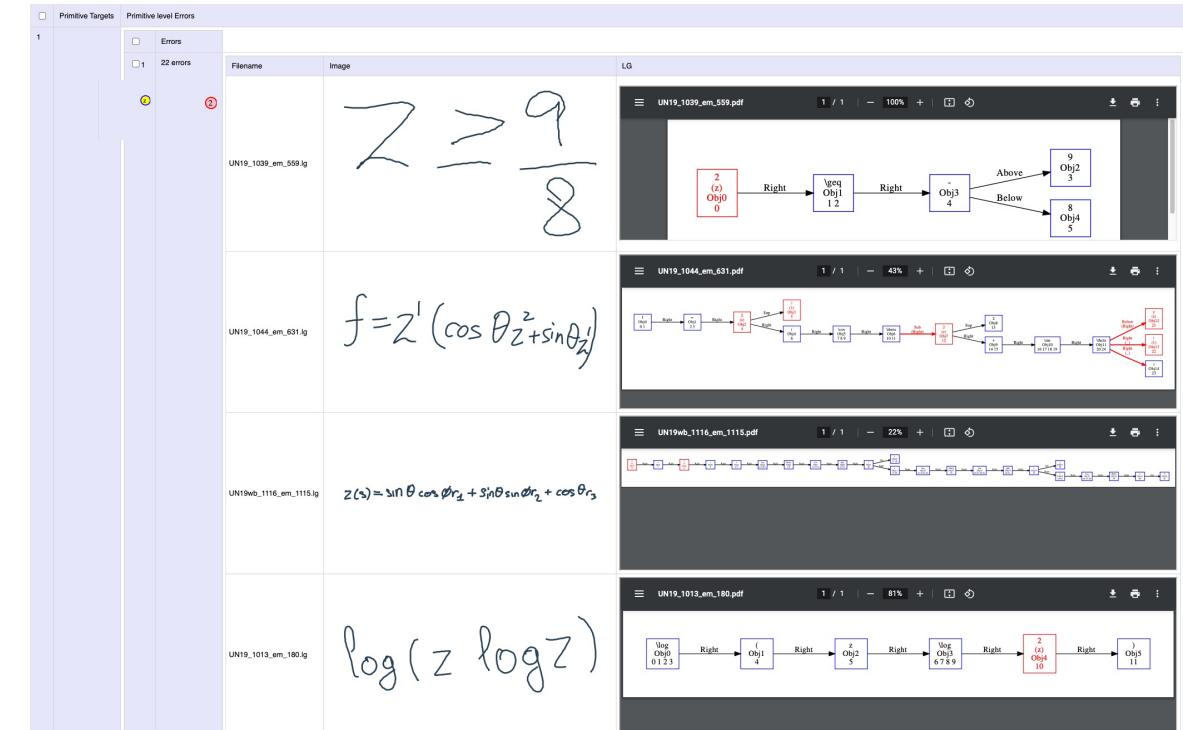
LgEval Extension: Error Visualization

Object Confusion Histograms

Object structures recognized incorrectly are shown at left, sorted by decreasing frequency. 95 incorrect targets, 1418 errors.



Errors organized by decreasing frequency



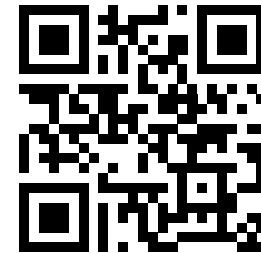
Specific instances where 'z' is misclassified as '2,'
seen after clicking on the '22 errors' link

LgEval Extension: Error Visualization

	Primitive Targets	Primitive level Errors									
1	<ul style="list-style-type: none"><input type="checkbox"/> Errors<input type="checkbox"/> 22 errors	<table border="1"><thead><tr><th>Filename</th><th>Image</th><th>LG</th></tr></thead><tbody><tr><td>UN19_1039_em_559.lg</td><td> A row of four handwritten digits: a '2', a '9', a '2', and an '8'.</td><td> An LG (Left-to-Right) diagram showing the sequence of tokens. It starts with a red box labeled '2 (z) Obj0 0'. An arrow labeled 'Right' points to a blue box labeled '\geq Obj1 1 2'. Another arrow labeled 'Right' points to a blue box labeled '- Obj3 4'. From '- Obj3 4', two arrows branch out: one labeled 'Above' points to a blue box labeled '9 Obj2 3', and one labeled 'Below' points to a blue box labeled '8 Obj4 5'. The background shows a PDF viewer interface with the file 'UN19_1039_em_559.pdf' open.</td></tr><tr><td>UN19_1044_em_631.lg</td><td>$f = z^i (\cos \theta_z^2 + \sin \theta_z^i)$</td><td> An LG diagram showing the sequence of tokens. It starts with a blue box labeled 'f Obj0 0 1'. An arrow labeled 'Right' points to a blue box labeled '= Obj1 2 3'. Another arrow labeled 'Right' points to a red box labeled '2 (z) Obj2 4'. From '2 (z) Obj2 4', two arrows branch out: one labeled 'Sup' points to a red box labeled '1 (1) Obj3 5', and one labeled 'Right' points to a blue box labeled 'Obj4 6'. An arrow labeled 'Right' points to a blue box labeled '\cos Obj5 7 8 9'. Finally, an arrow labeled 'Right' points to a red box labeled 'Sub (Right)'.</td></tr></tbody></table>	Filename	Image	LG	UN19_1039_em_559.lg	 A row of four handwritten digits: a '2', a '9', a '2', and an '8'.	 An LG (Left-to-Right) diagram showing the sequence of tokens. It starts with a red box labeled '2 (z) Obj0 0'. An arrow labeled 'Right' points to a blue box labeled '\geq Obj1 1 2'. Another arrow labeled 'Right' points to a blue box labeled '- Obj3 4'. From '- Obj3 4', two arrows branch out: one labeled 'Above' points to a blue box labeled '9 Obj2 3', and one labeled 'Below' points to a blue box labeled '8 Obj4 5'. The background shows a PDF viewer interface with the file 'UN19_1039_em_559.pdf' open.	UN19_1044_em_631.lg	$f = z^i (\cos \theta_z^2 + \sin \theta_z^i)$	 An LG diagram showing the sequence of tokens. It starts with a blue box labeled 'f Obj0 0 1'. An arrow labeled 'Right' points to a blue box labeled '= Obj1 2 3'. Another arrow labeled 'Right' points to a red box labeled '2 (z) Obj2 4'. From '2 (z) Obj2 4', two arrows branch out: one labeled 'Sup' points to a red box labeled '1 (1) Obj3 5', and one labeled 'Right' points to a blue box labeled 'Obj4 6'. An arrow labeled 'Right' points to a blue box labeled '\cos Obj5 7 8 9'. Finally, an arrow labeled 'Right' points to a red box labeled 'Sub (Right)'.
Filename	Image	LG									
UN19_1039_em_559.lg	 A row of four handwritten digits: a '2', a '9', a '2', and an '8'.	 An LG (Left-to-Right) diagram showing the sequence of tokens. It starts with a red box labeled '2 (z) Obj0 0'. An arrow labeled 'Right' points to a blue box labeled '\geq Obj1 1 2'. Another arrow labeled 'Right' points to a blue box labeled '- Obj3 4'. From '- Obj3 4', two arrows branch out: one labeled 'Above' points to a blue box labeled '9 Obj2 3', and one labeled 'Below' points to a blue box labeled '8 Obj4 5'. The background shows a PDF viewer interface with the file 'UN19_1039_em_559.pdf' open.									
UN19_1044_em_631.lg	$f = z^i (\cos \theta_z^2 + \sin \theta_z^i)$	 An LG diagram showing the sequence of tokens. It starts with a blue box labeled 'f Obj0 0 1'. An arrow labeled 'Right' points to a blue box labeled '= Obj1 2 3'. Another arrow labeled 'Right' points to a red box labeled '2 (z) Obj2 4'. From '2 (z) Obj2 4', two arrows branch out: one labeled 'Sup' points to a red box labeled '1 (1) Obj3 5', and one labeled 'Right' points to a blue box labeled 'Obj4 6'. An arrow labeled 'Right' points to a blue box labeled '\cos Obj5 7 8 9'. Finally, an arrow labeled 'Right' points to a red box labeled 'Sub (Right)'.									

Zoomed in: Specific instances where 'z' is misclassified as '2,' seen after clicking on the '22 errors' link

Conclusion and Future Work



- Open-source formula extraction pipeline for PDF documents
 - <https://www.cs.rit.edu/~dprl/software.html>
- PDF symbol extractor that identifies precise bounding box locations in born-digital PDFs
- A simple and effective algorithm for detection of math expressions using visual features alone
- Extended tools for visualizing recognition results and formula parsing errors
- **ScanSSD-XYc:** Unified page and window level merging using recursive XY Cuts avoiding NMS speeding up detection by 300 times approximately (included in the repository)

Future work

- **SymbolScraper:** Handle Type 3 Fonts and faster system for symbol extraction, better handling of compound characters
- **Pipeline:** End-to-End trainable system for detection and parsing

Thank You

This material is based on upon work supported by the Alfred P. Sloan Foundation under Grant No. G-2017-9827 and the National Science Foundation (USA) under Grant Nos. IIS-1717997 (MathSeer project) and 2019897 (MMLI project)

Thanks to R. Joshi, P. Mali, P. Kukkadapu, A. Keller, M. Mahdavi, and J. Diehl for their contribution to the formula extraction pipeline. Jian Wu provided the document collected used to evaluate SymbolScrapers



Alfred P. Sloan
FOUNDATION

