1)

(a)(1) $\forall x,y \; add(x,y) \equiv x+y$

$\forall (x,y) \; add(x,y) \equiv add(y,x)$

(a)(2)

$\forall x,y \quad mult(x,y) \equiv x \times y$

$\forall (x,y) \quad mult(x,y) \equiv mult(y,x)$

(b) $\forall \; x,y \in Z \Leftrightarrow add(x,y) \in Z$

$\forall x,y \in Z \Leftrightarrow mult(x,y) \in Z$.

(c)

$\forall \; x,y,c \; mult(c, add(x,y)) \equiv add(mult(c,x), mult(c,y))$

(d). $\forall x,y,z \quad add(x, add(y,z)) \equiv add(add(x,y), z)$

$\forall \; x,y,z \quad mult(x, mult(y,z)) \equiv mult(mult(x,y), z)$

(e)

$\forall \; x \quad add(x,0) \equiv x$

$\forall \; x \quad mult(x,1) \equiv x$.

2) $\alpha = \forall x \left(P(x) \vee Q(x) \right)$

$\beta = \forall (x) P(x) \vee \forall (x) Q(x)$

let first $\forall x$ in α & β be for all eagles.

P be the function of all fast ~~eagles~~ birds.

Q be the function of all ~~sl~~ slow. ~~bad~~ birds.

let second $\forall x$ in β be the for all pegions.

$\Rightarrow$ $\forall$ eagles (P (eagles) $\vee$ Q (eagles)).

does not entail

($\forall$ eagles P(eagles)) $\vee$

($\forall$ Pegions Q(Pegions))

$\therefore$ Hence proved.

4) let x be an element.

let $P(x)$ be x is able.

$Q(x)$ be x is willing to prevent evil.

$R(x)$ be preventing evil.

$S(x)$ be x is malevolent.

$T(x)$ be T is impotent.

(a) $(P(x) \land Q(x)) \Rightarrow R(x)$

$\neg(P(x) \land Q(x)) \lor R(x)$

$\boxed{(\neg P(x) \lor \neg Q(x)) \lor R(x)}$.

(b) $(\neg P(x)) \Rightarrow T(x)$

$\boxed{P(x) \lor T(x)}$

(c) $\neg Q(x) \Rightarrow S(x)$

$\boxed{Q(x) \lor S(x)}$

(d) $\boxed{\neg R(x)}$

(e) $(\exists(x = zeus)) \Rightarrow (\neg T(x) \lor \neg S(x))$.

$\boxed{\neg(\exists(x = zeus)) \lor (\neg T(x) \lor \neg S(x))}$.

$\boxed{\neg P(x) \lor \neg Q(x)}$

$\boxed{\neg P(x) \lor S(x)}$

$\boxed{\neg(\exists x = z) \lor \neg P(x) \lor \neg T(x)}$

$\boxed{\neg \exists(x = zeus)}$

Therefore zeus doesn't exist.

5)

(a) $\exists x \; \neg (P(x) \Rightarrow P(x))$

$\in x \; \neg (\neg P(x) \lor P(x))$

$\exists x \; (P(x) \land \neg P(x))$.

$\Rightarrow$ ∴ Contradiction ∵ Hence proved.

(b) $\neg((\neg \exists x \, P(x)) \Rightarrow (\forall x \, \neg P(x)))$.

$\neg (\neg (\forall x (\neg P(x))) \lor (\forall x \, \neg P(x)))$.

$\neg (\exists x \, P(x) \lor (\forall x \, \neg P(x)))$.

$(\neg \exists x \, P(x)) \land (\neg \forall x \, \neg P(x))$

$(\forall x \, \neg P(x)) \land (\exists x \, P(x))$.

$\Rightarrow$ Contradiction ∴ Hence proved.