

# Vector and matrix algebra

## Packages for this section

- ▶ This is (almost) all base R! We only need this for one thing later:

```
library(tidyverse)
```

# Vector addition

Adds 2 to each element.

► Adding vectors:

```
u <- c(2, 3, 6, 5, 7)
v <- c(1, 8, 3, 2, 0)
u + v
```

```
[1] 3 11 9 7 7
```

► Elementwise addition. (Linear algebra: vector addition.)

## Adding a number to a vector

- ▶ Define a vector, then “add 2” to it:

```
u
```

```
[1] 2 3 6 5 7
```

```
k <- 2
```

```
u + k
```

```
[1] 4 5 8 7 9
```

- ▶ adds 2 to *each* element of u.

# Scalar multiplication

As per linear algebra:

$k$

$[1] \ 2$

$u$

$[1] \ 2 \ 3 \ 6 \ 5 \ 7$

$k * u$

$[1] \ 4 \ 6 \ 12 \ 10 \ 14$

► Each element of vector multiplied by 2.

# “Vector multiplication”

What about this?

```
u
```

```
[1] 2 3 6 5 7
```

```
v
```

```
[1] 1 8 3 2 0
```

```
u * v
```

```
[1] 2 24 18 10 0
```

Each element of  $u$  multiplied by *corresponding* element of  $v$ . Could be called elementwise multiplication.

(Don't confuse with “outer” or “vector” product from linear algebra, or indeed “inner” or “scalar” multiplication, for which the answer is a number.)

## Combining different-length vectors

- ▶ No error here (you get a warning). What happens?

```
u
```

```
[1] 2 3 6 5 7
```

```
w <- c(1, 2)
```

```
u + w
```

```
[1] 3 5 7 7 8
```

- ▶ Add 1 to first element of `u`, add 2 to second.
- ▶ Go back to beginning of `w` to find something to add: add 1 to 3rd element of `u`, 2 to 4th element, 1 to 5th.

## How R does this

- ▶ Keep re-using shorter vector until reach length of longer one.
- ▶ “Recycling”.
- ▶ If the longer vector's length not a multiple of the shorter vector's length, get a warning (probably not what you want).
- ▶ Same idea is used when multiplying a vector by a number: the number keeps getting recycled.



# Matrices

- ▶ Create matrix like this:

```
(A <- matrix(1:4, nrow = 2, ncol = 2))
```

	[,1]	[,2]
[1,]	1	3
[2,]	2	4

- ▶ First: stuff to make matrix from, then how many rows and columns.
- ▶ R goes down columns by default. To go along rows instead:

```
(B <- matrix(5:8, nrow = 2, ncol = 2, byrow = TRUE))
```

	[,1]	[,2]
[1,]	5	6
[2,]	7	8

- ▶ One of `nrow` and `ncol` enough, since R knows how many things in the matrix.

## Adding matrices

What happens if you add two matrices?

A

	[,1]	[,2]
[1,]	1	3
[2,]	2	4

B

	[,1]	[,2]
[1,]	5	6
[2,]	7	8

A + B

	[,1]	[,2]
[1,]	6	9
[2,]	9	12

# Adding matrices

- ▶ Nothing surprising here. This is matrix addition as we and linear algebra know it.

# Multiplying matrices

► Now, what happens here?

A

	[,1]	[,2]
[1,]	1	3
[2,]	2	4

B

	[,1]	[,2]
[1,]	5	6
[2,]	7	8

A \* B

	[,1]	[,2]
[1,]	5	18
[2,]	14	32

# Multiplying matrices?

- ▶ *Not* matrix multiplication (as per linear algebra).
- ▶ Elementwise multiplication. Also called *Hadamard product* of A and B.

# Legit matrix multiplication

Like this:

A

	[,1]	[,2]
[1,]	1	3
[2,]	2	4

B

	[,1]	[,2]
[1,]	5	6
[2,]	7	8

A %\*% B

	[,1]	[,2]
[1,]	26	30
[2,]	38	44

## Reading matrix from file

► The usual:

```
my_url <- "http://ritsokiguess.site/datafiles/m.txt"
M <- read_delim(my_url, " ", col_names = FALSE )
M
```

```
# A tibble: 3 x 2
```

	X1	X2
	<dbl>	<dbl>
1	10	9
2	8	7
3	6	5

```
class(M)
```

```
[1] "spec_tbl_df" "tbl_df"      "tbl"         "data.frame"
```

but...

▶ except that M is not an R matrix, and thus this doesn't work:

```
v <- c(1, 3)
M %*% v
```

Error in M %\*% v: requires numeric/complex matrix/vector and



## Making a genuine matrix

Do this first:

```
M <- as.matrix(M)
```

```
M
```

```
      X1 X2
[1,] 10  9
[2,]  8  7
[3,]  6  5
```

```
v
```

```
[1] 1 3
```

and then all is good:

```
M %*% v
```

```
      [,1]
[1,]    37
[2,]    29
[3,]    21
```

# Linear algebra stuff

► To solve system of equations  $Ax = w$  for  $x$ :

A

	[,1]	[,2]
[1,]	1	3
[2,]	2	4

w

[1]	1	2
-----	---	---

`solve(A, w)`

[1]	1	0
-----	---	---

# Matrix inverse

- ▶ To find the inverse of A:

A

	[,1]	[,2]
[1,]	1	3
[2,]	2	4

`solve(A)`

	[,1]	[,2]
[1,]	-2	1.5
[2,]	1	-0.5

- ▶ You can check that the matrix inverse and equation solution are correct.

# Inner product

- ▶ Vectors in R are column vectors, so just do the matrix multiplication (`t()` is transpose):

```
a <- c(1, 2, 3)
b <- c(4, 5, 6)
t(a) %*% b
```

```
      [,1]
[1,]    32
```

- ▶ Note that the answer is actually a  $1 \times 1$  matrix.
- ▶ Or as the sum of the elementwise multiplication:

```
sum(a * b)
```

```
[1] 32
```

## Accessing parts of vector

- ▶ use square brackets and a number to get elements of a vector

```
b
```

```
[1] 4 5 6
```

```
b[2]
```

```
[1] 5
```

## Accessing parts of matrix

- ▶ use a row and column index to get an element of a matrix

A

	[,1]	[,2]
[1,]	1	3
[2,]	2	4

A[2,1]

[1] 2

- ▶ leave the row or column index empty to get whole row or column, eg.

A[1,]

[1] 1 3

# Eigenvalues and eigenvectors

- For a matrix  $A$ , these are scalars  $\lambda$  and vectors  $v$  that solve

$$Av = \lambda v$$

- In R, `eigen` gets these:

A

	[,1]	[,2]
[1,]	1	3
[2,]	2	4

```
e <- eigen(A)
```

# Eigenvalues and eigenvectors

```
e
```

```
eigen() decomposition
```

```
$values
```

```
[1]  5.3722813 -0.3722813
```

```
$vectors
```

```
      [,1]      [,2]
```

```
[1,] -0.5657675 -0.9093767
```

```
[2,] -0.8245648  0.4159736
```



## To check that the eigenvalues/vectors are correct

- ▶  $\lambda_1 v_1$ : (scalar) multiply first eigenvalue by first eigenvector (in column)

```
e$values[1] * e$vectors[,1]
```

```
[1] -3.039462 -4.429794
```

- ▶  $Av_1$ : (matrix) multiply matrix by first eigenvector (in column)

```
A %*% e$vectors[,1]
```

```
      [,1]
```

```
[1,] -3.039462
```

```
[2,] -4.429794
```

- ▶ These are (correctly) equal.
- ▶ The second one goes the same way.

# A statistical application of eigenvalues

- ▶ A negative correlation:

```
d <- tribble(
  ~x, ~y,
  10, 20,
  11, 18,
  12, 17,
  13, 14,
  14, 13
)
v <- cor(d)
v
```

	x	y
x	1.0000000	-0.9878783
y	-0.9878783	1.0000000

- ▶ `cor` gives the correlation matrix between each pair of variables (correlation between `x` and `y` is  $-0.988$ )

# Eigenanalysis of correlation matrix

```
eigen(v)
```

```
eigen() decomposition
```

```
$values
```

```
[1] 1.98787834 0.01212166
```

```
$vectors
```

```
      [,1]      [,2]
```

```
[1,] -0.7071068 -0.7071068
```

```
[2,]  0.7071068 -0.7071068
```

- ▶ first eigenvalue much bigger than second (second one near zero)
- ▶ two variables, but data nearly *one*-dimensional
- ▶ opposite signs in first eigenvector indicate that the one dimension is:
  - ▶ x small and y large at one end,
  - ▶ x large and y small at the other.