Multiway Frequency Tables

Multi-way frequency analysis

A study of gender and eyewear-wearing finds the following frequencies:

Gender	Contacts	Glasses	None
Female	121	32	129
Male	42	37	85

- Is there association between eyewear and gender?
- Normally answer this with chisquare test (based on observed and expected frequencies from null hypothesis of no association).
- Two categorical variables and a frequency.
- We assess in way that generalizes to more categorical variables.

The data file

```
gender contacts glasses none female 121 32 129 male 42 37 85
```

- This is not tidy!
- Two variables are gender and eyewear, and those numbers all frequencies.

```
my_url <- "http://ritsokiguess.site/datafiles/eyewear.txt"
(eyewear <- read_delim(my_url, " "))</pre>
```

```
## # A tibble: 2 x 4
## gender contacts glasses none
## <chr> <dbl> <dbl> <dbl> <dbl> <dbl> = 121
## 1 female 121 32 129
## 2 male 42 37 85
```

Tidying the data

```
## # A tibble: 6 \times 3
## gender eyewear frequency
  <chr> <chr>
                        <dbl>
##
## 1 female contacts
                          121
## 2 female glasses
                          32
## 3 female none
                          129
                        42
## 4 male contacts
## 5 male glasses
                          37
## 6 male none
                          85
```

Making tidy data back into a table

42

##

male

```
use pivot_wider
or this (we use it again later):

xt <- xtabs(frequency ~ gender + eyewear, data = eyes)

xt

## eyewear
## gender contacts glasses none
## female 121 32 129</pre>
```

37

85

Modelling

- Predict frequency from other factors and combos.
- glm with poisson family.

```
eyes.1 <- glm(frequency ~ gender * eyewear,
  data = eyes,
  family = "poisson"
)</pre>
```

Called log-linear model.

What can we get rid of?

nothing!

```
drop1(eyes.1, test = "Chisq")
## Single term deletions
##
## Model:
## frequency ~ gender * eyewear
##
                 Df Deviance AIC LRT Pr(>Chi)
## <none>
                       0.000 47.958
## gender:eyewear 2 17.829 61.787 17.829 0.0001345
##
## <none>
## gender:eyewear ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Conclusions

- ullet drop1 says what we can remove at this step. Significant = must stay.
- Cannot remove anything.
- Frequency depends on gender-wear combination, cannot be simplified further.
- Gender and eyewear are associated.
- Stop here.

prop.table

Original table:

```
## eyewear
## gender contacts glasses none
## female 121 32 129
## male 42 37 85
```

Calculate eg. row proportions like this:

```
prop.table(xt, margin = 1)
```

```
## eyewear

## gender contacts glasses none

## female 0.4290780 0.1134752 0.4574468

## male 0.2560976 0.2256098 0.5182927
```

Comments

- margin says what to make add to 1.
- More females wear contacts and more males wear glasses.

No association

##

Suppose table had been as shown below:

male 0.4901961 0.1045752 0.4052288

```
my_url <- "http://ritsokiguess.site/datafiles/eyewear2.txt"</pre>
eyewear2 <- read_table(my_url)</pre>
evewear2 %>%
   pivot longer(contacts:none, names to = "eyewear",
                values_to = "frequency") -> eyes2
xt2 <- xtabs(frequency ~ gender + eyewear, data = eyes2)
xt.2
##
           eyewear
## gender contacts glasses none
    female
                 150
                         30 120
##
##
    male 75 16 62
prop.table(xt2, margin = 1)
##
           eyewear
                        glasses
## gender contacts
                                     none
     female 0.5000000 0.1000000 0.4000000
##
```

Comments

- Females and males wear contacts and glasses in same proportions
 - though more females and more contact-wearers.
- No association between gender and eyewear.

Analysis for revised data

```
eyes.2 <- glm(frequency ~ gender * eyewear,
   data = eyes2,
   family = "poisson"
)
drop1(eyes.2, test = "Chisq")

## Single term deletions
##
## Model:</pre>
```

No longer any association. Take out interaction.

No interaction

```
eyes.3 <- update(eyes.2, . ~ . - gender:eyewear)
drop1(eyes.3, test = "Chisq")
## Single term deletions
##
## Model:
## frequency ~ gender + eyewear
## Df Deviance AIC LRT Pr(>Chi)
## <none> 0.047 43.515
## gender 1 48.624 90.091 48.577 3.176e-12 ***
## eyewear 2 138.130 177.598 138.083 < 2.2e-16 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

    More females (gender effect)
```

more contact-wearers (eyewear effect)

Chest pain, being overweight and being a smoker

- In a hospital emergency department, 176 subjects who attended for acute chest pain took part in a study.
- Each subject had a normal or abnormal electrocardiogram reading (ECG), were overweight (as judged by BMI) or not, and were a smoker or not.
- How are these three variables related, or not?

The data

In modelling-friendly format:

ecg bmi smoke count abnormal overweight yes 47 abnormal overweight no 10 abnormal normalweight yes 8 abnormal normalweight no 6 normal overweight yes 25 normal overweight no 15 normal normalweight yes 35 normal normalweight no 30

First step

```
my_url <- "http://ritsokiguess.site/datafiles/ecg.txt"
chest <- read_delim(my_url, " ")
chest.1 <- glm(count ~ ecg * bmi * smoke,
    data = chest,
    family = "poisson"
)
drop1(chest.1, test = "Chisq")

## Single term deletions</pre>
```

That 3-way interaction comes out.

Removing the 3-way interaction

```
chest.2 <- update(chest.1, . ~ . - ecg:bmi:smoke)</pre>
drop1(chest.2, test = "Chisq")
## Single term deletions
##
## Model:
## count ~ ecg + bmi + smoke + ecg:bmi + ecg:smoke + bmi:smoke
   Df Deviance AIC LRT Pr(>Chi)
##
## <none> 1.3885 53.096
## ecg:bmi 1 29.0195 78.727 27.6310 1.468e-07 ***
## ecg:smoke 1 4.8935 54.601 3.5050 0.06119 .
## bmi:smoke 1 4.4689 54.176 3.0803 0.07924 .
## ---
## Signif. codes:
## 0 '*** 0.001 '** 0.01 '* 0.05 '. ' 0.1 ' ' 1
At \alpha = 0.05, bmi:smoke comes out.
```

Removing bmi:smoke

```
chest.3 <- update(chest.2, . ~ . - bmi:smoke)</pre>
drop1(chest.3, test = "Chisq")
## Single term deletions
##
## Model:
## count ~ ecg + bmi + smoke + ecg:bmi + ecg:smoke
    Df Deviance AIC LRT Pr(>Chi)
##
## <none> 4.469 54.176
## ecg:bmi 1 36.562 84.270 32.094 1.469e-08 ***
## ecg:smoke 1 12.436 60.144 7.968 0.004762 **
## ---
## Signif. codes:
```

ecg:smoke has become significant. So we have to stop.

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Understanding the final model

- Thinking of ecg as "response" that might depend on anything else.
- What is associated with ecg? Both bmi on its own and smoke on its own, but *not* the combination of both.
- ecg:bmi table:

```
xtabs(count ~ ecg + bmi, data = chest)
```

```
## bmi
## ecg normalweight overweight
## abnormal 14 57
## normal 65 40
```

 Most normal weight people have a normal ECG, but a majority of overweight people have an abnormal ECG. That is, knowing about BMI says something about likely ECG.

ecg:smoke

##

• ecg:smoke table:

```
xtabs(count ~ ecg + smoke, data = chest)
```

```
## ecg no yes
## abnormal 16 55
## normal 45 60
```

smoke

- Most nonsmokers have a normal ECG, but smokers are about 50–50 normal and abnormal ECG.
- Don't look at smoke: bmi table since not significant.

Simpson's paradox: the airlines example

	Alaska Airlines		America West	
Airport	On time	Delayed	On time	Delayed
Los Angeles	497	62	694	117
Phoenix	221	12	4840	415
San Diego	212	20	383	65
San Francisco	503	102	320	129
Seattle	1841	305	201	61
Total	3274	501	6438	787

Use status as variable name for "on time/delayed".

- Alaska: 13.3% flights delayed (501/(3274 + 501)).
- America West: 10.9% (787/(6438 + 787)).
- America West more punctual, right?

Arranging the data

 Can only have single thing in columns, so we have to construct column names like this:

```
airport aa_ontime aa_delayed aw_ontime aw_delayed
LosAngeles 497
                                       117
                      62
                              694
Phoenix 221
                      12
                             4840
                                       415
SanDiego 212
                      20
                             383
                                        65
SanFrancisco 503
                     102 320
                                       129
Seattle 1841
                     305
                           201
                                        61
```

Read in:

```
my_url <- "http://ritsokiguess.site/datafiles/airlines.txt"
airlines <- read_table2(my_url)</pre>
```

```
## Warning: `read_table2()` was deprecated in readr 2.0.0.
## Please use `read table()` instead.
```

Tidying

 Some tidying gets us the right layout, with frequencies all in one column and the airline and delayed/on time status separated out. This uses one of the fancy versions of pivot_longer:

The data frame punctual

```
A tibble: 20 x 4
##
      airport
                    airline status
                                       freq
      <chr>>
                                      <dbl>
##
                    <chr>>
                             <chr>>
##
    1 LosAngeles
                             ontime
                                        497
                    aa
                                         62
##
    2 LosAngeles
                             delayed
                    aa
##
    3 LosAngeles
                             ontime
                                        694
                    ลพ
##
                                        117
    4 LosAngeles
                             delayed
                    aw
##
    5 Phoenix
                             ontime
                                        221
                    aa
##
    6 Phoenix
                             delayed
                                         12
                    aa
##
    7 Phoenix
                             ontime
                                       4840
                    aw
                                        415
##
    8 Phoenix
                             delayed
                    ลพ
##
    9 SanDiego
                             ontime
                                        212
                    aa
   10 SanDiego
                             delayed
                                         20
                    aa
  11 SanDiego
                             ontime
                                        383
                    ลพ
   12 SanDiego
                                         65
                    aw
                             delayed
   13 SanFrancisco aa
                                        503
                             ontime
## 14 SanFrancisco aa
                                        102
                             delayed
      SanFrancisco aw
                             ontime
                                        320
  16 SanFrancisco aw
                             delayed
                                        129
## 17 Seattle
                             ontime
                                       1841
                    aa
  18 Seattle
                    aa
                             delayed
                                        305
                                        201
## 19 Seattle
                             ontime
                    aw
   20 Seattle
                             delayed
                                         61
                    aw
```

Proportions delayed by airline

Two-step process: get appropriate subtable:

```
xt <- xtabs(freq ~ airline + status, data = punctual)
xt
## status</pre>
```

airline delayed ontime
aa 501 3274

aw 787 6438

• and then calculate appropriate proportions:

```
prop.table(xt, margin = 1)
```

```
## status

## airline delayed ontime

## aa 0.1327152 0.8672848

## aw 0.1089273 0.8910727
```

More of Alaska Airlines' flights delayed (13.3% vs. 10.9%).

Proportion delayed by airport, for each airline

```
xt <- xtabs(freq ~ airline + status + airport, data = punctual)
xp <- prop.table(xt, margin = c(1, 3))
ftable(xp,
    row.vars = c("airport", "airline"),
    col.vars = "status"
)</pre>
```

```
##
                                  delayed
                                           ontime
                        status
## airport
             airline
## LosAngeles
                               0.11091234 0.88908766
                aa
##
                               0.14426634 0.85573366
                ลพ
                               0.05150215 0.94849785
## Phoenix
                aa
##
                               0.07897241 0.92102759
                ลพ
## SanDiego
                               0.08620690 0.91379310
                aa
##
                               0.14508929 0.85491071
                aw
  SanFrancisco aa
                               0.16859504 0.83140496
                               0.28730512 0.71269488
##
                aw
                               0.14212488 0.85787512
## Seattle
                aa
##
                               0.23282443 0.76717557
                ลพ
```

Simpson's Paradox

Airport	Alaska	America West
Los Angeles	11.4	14.4
Phoenix	5.2	7.9
San Diego	8.6	14.5
San Francisco	16.9	28.7
Seattle	14.2	23.2
Total	13.3	10.9

- America West more punctual overall,
- but worse at every single airport!
- How is that possible?
- Log-linear analysis sheds some light.

Model 1 and output

```
punctual.1 <- glm(freq ~ airport * airline * status,</pre>
  data = punctual, family = "poisson"
drop1(punctual.1, test = "Chisq")
## Single term deletions
##
## Model:
## freq ~ airport * airline * status
##
                          Df Deviance AIC
                                                I.R.T
## <none>
                               0.0000 183.44
## airport:airline:status 4 3.2166 178.65 3.2166
                          Pr(>Chi)
##
## <none>
## airport:airline:status 0.5223
```

Remove 3-way interaction

```
punctual.2 <- update(punctual.1, ~ . - airport:airline:status)</pre>
drop1(punctual.2, test = "Chisq")
## Single term deletions
##
## Model:
## freq ~ airport + airline + status + airport:airline + airport:status +
      airline:status
##
##
                  Df Deviance AIC LRT Pr(>Chi)
                          3.2 178.7
## <none>
## airport:airline 4 6432.5 6599.9 6429.2 < 2.2e-16
## airport:status 4 240.1 407.5 236.9 < 2.2e-16
## airline:status 1
                     45.5 218.9 42.2 8.038e-11
##
## <none>
## airport:airline ***
## airport:status ***
## airline:status
                  ***
## ---
## Signif. codes:
```

Understanding the significance

airline:status:

xt <- xtabs(freq ~ airline + status, data = punctual)
prop.table(xt, margin = 1)</pre>

```
## status

## airline delayed ontime

## aa 0.1327152 0.8672848

## aw 0.1089273 0.8910727
```

- More of Alaska Airlines' flights delayed overall.
- Saw this before.

Understanding the significance (2)

• airport:status:

xt <- xtabs(freq ~ airport + status, data = punctual)
prop.table(xt, margin = 1)</pre>

```
## status

## airport delayed ontime

## LosAngeles 0.13065693 0.86934307

## Phoenix 0.07780612 0.92219388

## SanDiego 0.12500000 0.87500000

## SanFrancisco 0.21916509 0.78083491

## Seattle 0.15199336 0.84800664
```

- Flights into San Francisco (and maybe Seattle) are often late, and flights into Phoenix are usually on time.
- Considerable variation among airports.

Understanding the significance (3)

• airport:airline:

```
xt <- xtabs(freq ~ airport + airline, data = punctual)
prop.table(xt, margin = 2)</pre>
```

```
##
                airline
  airport
                         aa
                                    aw
    LosAngeles 0.14807947 0.11224913
##
##
    Phoenix
            0.06172185 0.72733564
    SanDiego 0.06145695 0.06200692
##
##
    SanFrancisco 0.16026490 0.06214533
##
    Seattle
               0.56847682 0.03626298
```

- What fraction of each airline's flights are to each airport.
- Most of Alaska Airlines' flights to Seattle and San Francisco.
- Most of America West's flights to Phoenix.

The resolution

- Most of America West's flights to Phoenix, where it is easy to be on time.
- Most of Alaska Airlines' flights to San Francisco and Seattle, where it is difficult to be on time.
- Overall comparison looks bad for Alaska because of this.
- But, comparing like with like, if you compare each airline's performance to the same airport, Alaska does better.
- Aggregating over the very different airports was a (big) mistake: that was the cause of the Simpson's paradox.
- Alaska Airlines is more punctual when you do the proper comparison.

Ovarian cancer: a four-way table

- Retrospective study of ovarian cancer done in 1973.
- Information about 299 women operated on for ovarian cancer 10 years previously.
- Recorded:
 - stage of cancer (early or advanced)
 - type of operation (radical or limited)
 - X-ray treatment received (yes or no)
 - 10-year survival (yes or no)
- Survival looks like response (suggests logistic regression).
- Log-linear model finds any associations at all.

The data

after tidying:

```
stage operation xray survival freq
early radical no no 10
early radical no yes 41
early radical yes no 17
early radical yes yes 64
early limited no no 1
early limited no yes 13
early limited yes no 3
early limited yes yes 9
advanced radical no no 38
advanced radical no yes 6
advanced radical yes no 64
advanced radical yes yes 11
advanced limited no no 3
advanced limited no yes 1
advanced limited yes no 13
advanced limited yes yes 5
```

Reading in data

5 early limited

6 early limited

```
my url <- "http://ritsokiguess.site/datafiles/cancer.txt"
cancer <- read_delim(my_url, " ")</pre>
cancer %>% slice(1:6)
## # A tibble: 6 x 5
##
     stage operation xray survival
                                     freq
## <chr> <chr> <chr> <chr> <chr> <chr> <chr> <dbl>
## 1 early radical no no
                                      10
## 2 early radical no yes
                                      41
## 3 early radical yes
                                      17
                         no
## 4 early radical yes
                                      64
                         yes
```

no

yes

no

no

13

Model 1

hopefully looking familiar by now:

```
cancer.1 <- glm(freq ~ stage * operation * xray * survival,
   data = cancer, family = "poisson"
)</pre>
```

Output 1

See what we can remove:

Non-significant interaction can come out.

Model 2

```
cancer.2 <- update(cancer.1, . ~ . - stage:operation:xray:survival)</pre>
drop1(cancer.2, test = "Chisq")
## Single term deletions
##
## Model:
## freq ~ stage + operation + xray + survival + stage:operation +
##
      stage:xray + operation:xray + stage:survival + operation:survival +
##
      xray:survival + stage:operation:xray + stage:operation:survival +
##
       stage:xray:survival + operation:xray:survival
##
                            Df Deviance
                                          ATC
                                                   I.R.T
## <none>
                                0.60266 96.732
## stage:operation:xray 1 2.35759 96.487 1.75493
## stage:operation:survival 1 1.17730 95.307 0.57465
## stage:xray:survival
                        1 0.95577 95.085 0.35311
## operation:xrav:survival 1 1.23378 95.363 0.63113
##
                            Pr(>Chi)
## <none>
                             0.1853
## stage:operation:xray
## stage:operation:survival
                             0.4484
## stage:xray:survival
                             0.5524
## operation:xrav:survival
                             0.4269
```

Take out stage:xray:survival

```
cancer.3 <- update(cancer.2, . ~ . - stage:xray:survival)</pre>
drop1(cancer.3, test = "Chisq")
## Single term deletions
##
## Model:
## freq ~ stage + operation + xray + survival + stage:operation +
##
       stage:xray + operation:xray + stage:survival + operation:survival +
##
      xray:survival + stage:operation:xray + stage:operation:survival +
      operation:xrav:survival
##
##
                            Df Deviance
                                           AIC
                                                   LRT
## <none>
                                0.95577 95.085
## stage:operation:xray 1 3.08666 95.216 2.13089
## stage:operation:survival 1 1.56605 93.696 0.61029
## operation:xray:survival 1 1.55124 93.681 0.59547
##
                            Pr(>Chi)
## <none>
## stage:operation:xray
                             0.1444
## stage:operation:survival 0.4347
## operation:xray:survival
                             0.4403
```

operation:xray:survival comes out next.

Remove operation:xray:survival

```
cancer.4 <- update(cancer.3, . ~ . - operation:xray:survival)</pre>
drop1(cancer.4, test = "Chisq")
## Single term deletions
##
## Model:
## freq ~ stage + operation + xray + survival + stage:operation +
##
      stage:xray + operation:xray + stage:survival + operation:survival +
##
      xray:survival + stage:operation:xray + stage:operation:survival
                           Df Deviance ATC
                                                I.R.T
##
## <none>
                                1.5512 93.681
                    1 1.6977 91.827 0.1464
## xray:survival
## stage:operation:xray 1 6.8420 96.972 5.2907
## stage:operation:survival 1 1.9311 92.061 0.3799
##
                           Pr(>Chi)
## <none>
## xray:survival
                   0.70196
## stage:operation:xray 0.02144 *
## stage:operation:survival 0.53768
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Comments

- stage:operation:xray has now become significant, so won't remove that.
- Shows value of removing terms one at a time.
- There are no higher-order interactions containing both xray and survival, so now we get to test (and remove) xray:survival.

Remove xray:survival

```
cancer.5 <- update(cancer.4, . ~ . - xray:survival)</pre>
drop1(cancer.5, test = "Chisq")
## Single term deletions
##
## Model:
## freq ~ stage + operation + xray + survival + stage:operation +
       stage:xray + operation:xray + stage:survival + operation:survival +
##
##
       stage:operation:xray + stage:operation:survival
                            Df Deviance ATC
##
                                                  I.R.T
## <none>
                                1.6977 91.827
## stage:operation:xray 1 6.9277 95.057 5.2300
## stage:operation:survival 1 2.0242 90.154 0.3265
##
                            Pr(>Chi)
## <none>
## stage:operation:xray
                        0.0222 *
## stage:operation:survival 0.5677
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Remove stage:operation:survival

```
cancer.6 <- update(cancer.5, . ~ . - stage:operation:survival)</pre>
drop1(cancer.6, test = "Chisq")
## Single term deletions
##
## Model:
## freq ~ stage + operation + xray + survival + stage:operation +
      stage:xray + operation:xray + stage:survival + operation:survival +
##
      stage:operation:xray
##
                       Df Deviance ATC
                                          I.R.T
##
## <none>
                            2.024 90.154
## stage:survival 1 135.198 221.327 133.173
## operation:survival 1 4.116 90.245 2.092
## stage:operation:xray 1 7.254 93.384 5.230
##
                       Pr(>Chi)
## <none>
                 <2e-16 ***
## stage:survival
## operation:survival 0.1481
## stage:operation:xray 0.0222 *
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Last step?

```
Remove operation:survival.
```

```
cancer.7 <- update(cancer.6, . ~ . - operation:survival)</pre>
drop1(cancer.7, test = "Chisq")
## Single term deletions
##
## Model:
## freq ~ stage + operation + xray + survival + stage:operation +
       stage:xray + operation:xray + stage:survival + stage:operation:xray
##
##
                       Df Deviance
                                       ATC
                                              I.R.T
## <none>
                             4.116 90.245
                     1 136.729 220.859 132.61
## stage:survival
## stage:operation:xray 1 9.346 93.475 5.23
                       Pr(>Chi)
##
## <none>
## stage:survival
                      <2e-16 ***
## stage:operation:xray 0.0222 *
## ---
## Signif. codes:
```

Multiway Frequency Tables

Conclusions

- What matters is things associated with survival (survival is "response").
- Only significant such term is stage:survival:

```
xt <- xtabs(freq ~ stage + survival, data = cancer)
prop.table(xt, margin = 1)</pre>
```

```
## survival

## stage no yes

## advanced 0.8368794 0.1631206

## early 0.1962025 0.8037975
```

- Most people in early stage of cancer survived, and most people in advanced stage did not survive.
- This true regardless of type of operation or whether or not X-ray treatment was received. These things have no impact on survival.

What about that other interaction?

```
xt <- xtabs(freq ~ operation + xray + stage, data = cancer)
ftable(prop.table(xt, margin = 3))</pre>
```

```
##
                         advanced
                  stage
                                         early
  operation xray
  limited
                        0.02836879 0.08860759
             nο
                        0.12765957 0.07594937
##
             yes
## radical
                        0.31205674 0.32278481
             nο
##
                        0.53191489 0.51265823
             yes
```

- Out of the people at each stage of cancer (since margin=3 and stage was listed 3rd).
- The association is between stage and xray only for those who had the limited operation.
- For those who had the radical operation, there was no association between stage and xray.
- This is of less interest than associations with survival.

General procedure

- Start with "complete model" including all possible interactions.
- drop1 gives highest-order interaction(s) remaining, remove least non-significant.
- Repeat as necessary until everything significant.
- Look at subtables of significant interactions.
- Main effects not usually very interesting.
- Interactions with "response" usually of most interest: show association with response.