# Analysis of Covariance

# Analysis of covariance

- ANOVA: explanatory variables categorical (divide data into groups)
- traditionally, analysis of covariance has categorical $x$'s plus one numerical $x$ ("covariate") to be adjusted for.
- `lm` handles this too.
- Simple example: two treatments (drugs) (`a` and `b`), with before and after scores.
- Does knowing before score and/or treatment help to predict after score?
- Is after score different by treatment/before score?

# Data

Treatment, before, after:

```
a 5 20
a 10 23
a 12 30
a 9 25
a 23 34
a 21 40
a 14 27
a 18 38
a 6 24
a 13 31
b 7 19
b 12 26
b 27 33
b 24 35
b 18 30
b 22 31
b 26 34
b 21 28
b 14 23
b 9 22
```

# Packages

```
library(tidyverse)
library(broom)
library(marginaleffects)
```

the last of these for predictions.

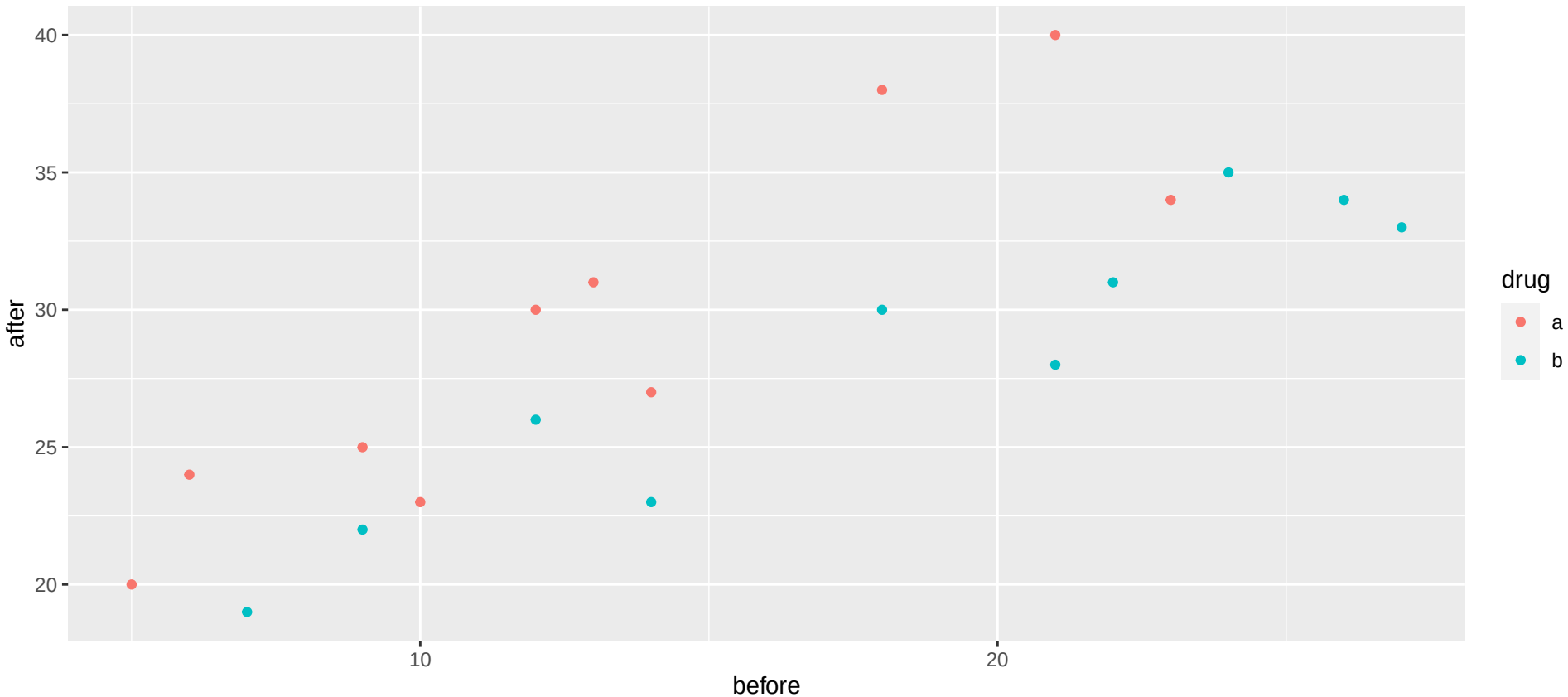
# Read in data

```
url <- "http://ritsokiguess.site/datafiles/ancova.txt"
prepost <- read_delim(url, " ")
prepost %>% sample_n(9) # randomly chosen rows
```

| drug | before | after |
|---|---|---|
| b | 7 | 19 |
| b | 24 | 35 |
| a | 12 | 30 |
| a | 14 | 27 |
| a | 23 | 34 |
| b | 14 | 23 |
| b | 22 | 31 |
| a | 18 | 38 |
| b | 21 | 28 |

# Making a plot

```
ggplot(prepost, aes(x = before, y = after, colour = drug)) +
  geom_point()
```



# Comments

- As before score goes up, after score goes up.
- Red points (drug A) generally above blue points (drug B), for comparable before score.
- Suggests before score effect *and* drug effect.

# The means

```
prepost %>%
  group_by(drug) %>%
  summarize(
    before_mean = mean(before),
    after_mean = mean(after)
  )
```

| drug | before_mean | after_mean |
|---|---|---|
| a | 13.1 | 29.2 |
| b | 18.0 | 28.1 |

- Mean "after" score slightly higher for treatment A.
- Mean "before" score much higher for treatment B.
- Greater *improvement* on treatment A.

# Testing for interaction

```
prepost.1 <- lm(after ~ before * drug, data = prepost)
anova(prepost.1)
```

| | Df | Sum Sq | Mean Sq | F value | Pr(>F) |
|---|---|---|---|---|---|
| before | 1 | 430.92384 | 430.923838 | 62.68945 | 0.0000006 |
| drug | 1 | 115.30596 | 115.305957 | 16.77435 | 0.0008442 |
| before:drug | 1 | 12.33708 | 12.337080 | 1.79476 | 0.1990662 |
| Residuals | 16 | 109.98313 | 6.873945 | NA | NA |

- Interaction not significant. Will remove later.

# Predictions

Set up values to predict for:

```
summary(prepost)
```

```
##      drug                before          after
##  Length:20          Min.   : 5.00   Min.   :19.00
##  Class :character   1st Qu.: 9.75   1st Qu.:23.75
##  Mode  :character   Median :14.00   Median :29.00
##                     Mean   :15.55   Mean   :28.65
##                     3rd Qu.:21.25   3rd Qu.:33.25
##                     Max.   :27.00   Max.   :40.00
```

```
new <- datagrid(before = c(9.75, 14, 21.25),
                drug = c("a", "b"), model = prepost.1)
```

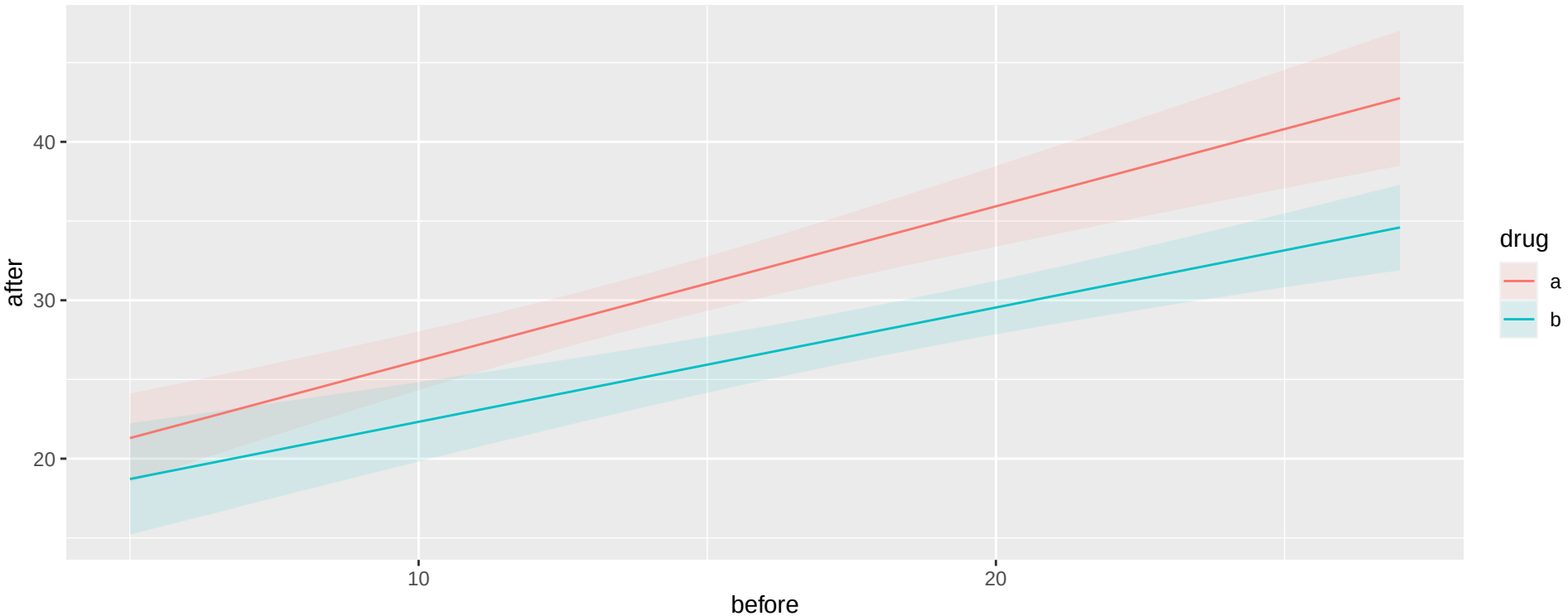
# and then

```
cbind(predictions(prepost.1, newdata = new)) %>%
  select(drug, before, estimate)
```

| drug | before | estimate |
|---|---|---|
| a | 9.75 | 25.93250 |
| b | 9.75 | 22.14565 |
| a | 14.00 | 30.07784 |
| b | 14.00 | 25.21304 |
| a | 21.25 | 37.14929 |
| b | 21.25 | 30.44565 |

# Predictions (with interaction included), plotted

```
plot_cap(model = prepost.1, condition = c("before", "drug"))
```




Lines almost parallel, but not quite.

# Taking out interaction

```
prepost.2 <- update(prepost.1, . ~ . - before:drug)
anova(prepost.2)
```

| | Df | Sum Sq | Mean Sq | F value | Pr(>F) |
|---|---|---|---|---|---|
| before | 1 | 430.9238 | 430.923838 | 59.88958 | 0.0000006 |
| drug | 1 | 115.3060 | 115.305957 | 16.02516 | 0.0009209 |
| Residuals | 17 | 122.3202 | 7.195306 | NA | NA |

- Take out non-significant interaction.
- `before` and `drug` strongly significant.
- Do predictions again and plot them.

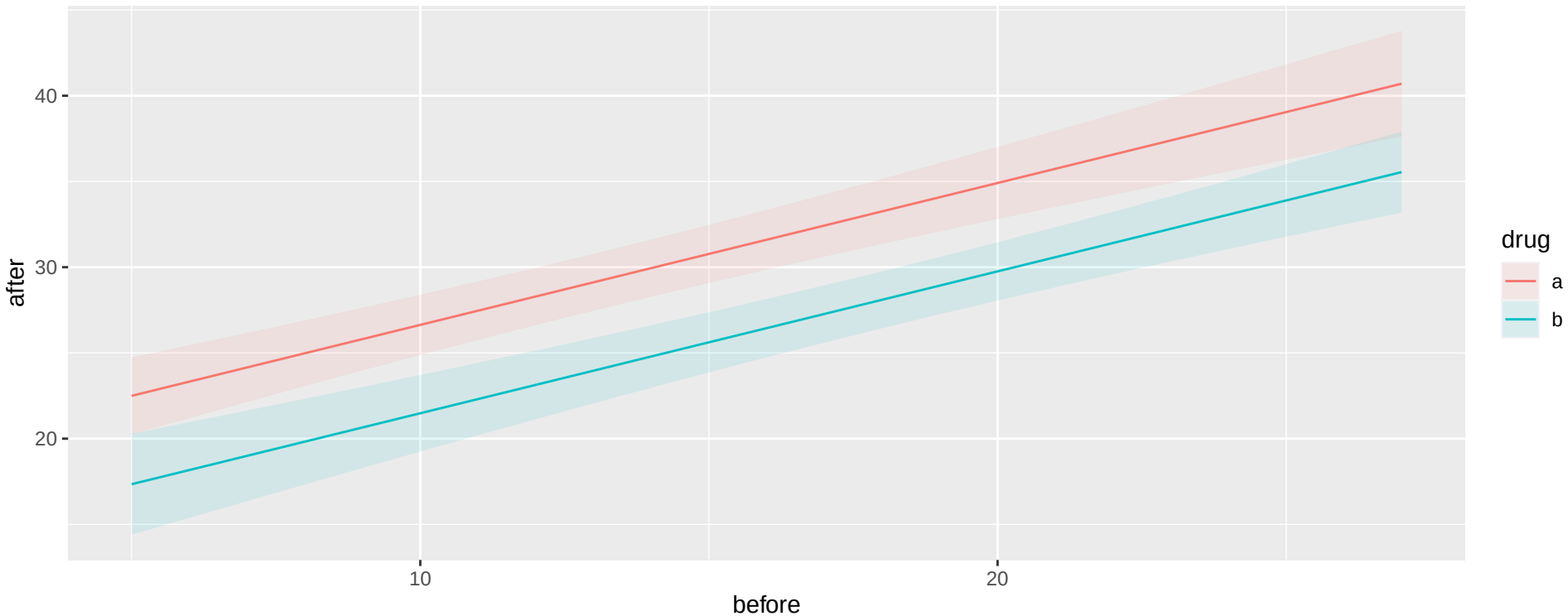
# Predictions

```
cbind(predictions(prepost.2, newdata = new)) %>%
  select(drug, before, estimate)
```

| drug | before | estimate |
|---|---|---|
| a | 9.75 | 26.42794 |
| b | 9.75 | 21.27328 |
| a | 14.00 | 29.94473 |
| b | 14.00 | 24.79007 |
| a | 21.25 | 35.94397 |
| b | 21.25 | 30.78931 |

# Plot of predicted values

```
plot_cap(prepost.2, condition = c("before", "drug"))
```




This time the lines are *exactly* parallel. No-interaction model forces them to have the same slope.

# Different look at model output

- `anova(prepost.2)` tests for significant effect of before score and of drug, but doesn't help with interpretation.
- `summary(prepost.2)` views as regression with slopes:

```
summary(prepost.2)
```

```
##
## Call:
## lm(formula = after ~ before + drug, data = prepost)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.6348 -2.5099 -0.2038  1.8871  4.7453
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  18.3600     1.5115  12.147 8.35e-10 ***
## before        0.8275     0.0955   8.665 1.21e-07 ***
## drugb        -5.1547     1.2876  -4.003 0.000921 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

# Understanding those slopes

```
tidy(prepost.2)
```

| term | estimate | std.error | statistic | p.value |
|---|---|---|---|---|
| (Intercept) | 18.3599949 | 1.5115326 | 12.146608 | 0.0000000 |
| before | 0.8274813 | 0.0955023 | 8.664520 | 0.0000001 |
| drugb | -5.1546584 | 1.2876524 | -4.003144 | 0.0009209 |

- `before` ordinary numerical variable; `drug` categorical.
- `lm` uses first category `druga` as baseline.
- Intercept is prediction of after score for before score 0 and *drug A*.
- `before` slope is predicted change in after score when before score increases by 1 (usual slope)
- Slope for `drugb` is *change* in predicted after score for being on drug B rather than drug A. Same for *any* before score (no interaction).

# Summary

- ANCOVA model: fits different regression line for each group, predicting response from covariate.
- ANCOVA model with interaction between factor and covariate allows different slopes for each line.
- Sometimes those lines can cross over!
- If interaction not significant, take out. Lines then parallel.
- With parallel lines, groups have consistent effect regardless of value of covariate.