Factor Analysis

Principal components and factor analysis

- Principal components:
  - Purely mathematical.
  - Find eigenvalues, eigenvectors of correlation matrix.
  - No testing whether observed components reproducible, or even probability model behind it.
- Factor analysis:
  - some way towards fixing this (get test of appropriateness)
  - In factor analysis, each variable modelled as: "common factor" (eg. verbal ability) and "specific factor" (left over).
  - Choose the common factors to "best" reproduce pattern seen in correlation matrix.
  - Iterative procedure, different answer from principal components.

Packages

```
library(ggbiplot)
library(tidyverse)
library(conflicted)
conflict_prefer("mutate", "dplyr")
conflict_prefer("select", "dplyr")
conflict_prefer("filter", "dplyr")
conflict_prefer("arrange", "dplyr")
```

Example

- 145 children given 5 tests, called PARA, SENT, WORD, ADD and DOTS. 3 linguistic tasks (paragraph comprehension, sentence completion and word meaning), 2 mathematical ones (addition and counting dots).
- Correlation matrix of scores on the tests:

para	1	0.722	0.714	0.203	0.095
sent	0.722	1	0.685	0.246	0.181
word	0.714	0.685	1	0.170	0.113
add	0.203	0.246	0.170	1	0.585
dots	0.095	0.181	0.113	0.585	1

- Is there small number of underlying “constructs” (unobservable) that explains this pattern of correlations?

To start: principal components

Using correlation matrix. Read that first:

```
my_url <- "http://ritsokiguess.site/datafiles/rex2.txt"
kids <- read_delim(my_url, " ")
kids
```

test	para	sent	word	add	dots
para	1.000	0.722	0.714	0.203	0.095
sent	0.722	1.000	0.685	0.246	0.181
word	0.714	0.685	1.000	0.170	0.113
add	0.203	0.246	0.170	1.000	0.585
dots	0.095	0.181	0.113	0.585	1.000

Principal components on correlation matrix

Turn into R `matrix`, using column `test` as column names:

```
kids %>%
  column_to_rownames("test") %>%
  as.matrix() -> m
```

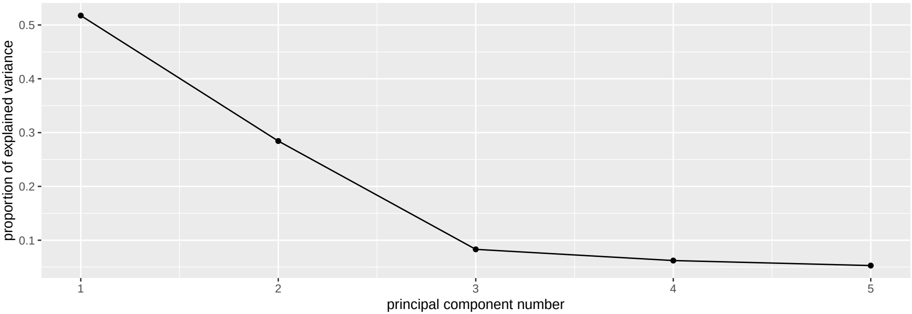
Principal components:

```
kids.0 <- princomp(covmat = m)
```

I used `kids.0` here since I want `kids.1` and `kids.2` later.

Scree plot

```
ggscreeplot(kids.0)
```



Principal component results

- Need 2 components. Loadings:

```
kids.0$loadings
```

```
##
## Loadings:
##      Comp.1 Comp.2 Comp.3 Comp.4 Comp.5
## para  0.534  0.245  0.114         0.795
## sent  0.542  0.164         0.660 -0.489
## word  0.523  0.247 -0.144 -0.738 -0.316
## add   0.297 -0.627  0.707
## dots  0.241 -0.678 -0.680         0.143
##
##                 Comp.1 Comp.2 Comp.3 Comp.4 Comp.5
## SS loadings        1.0    1.0    1.0    1.0    1.0
## Proportion Var     0.2    0.2    0.2    0.2    0.2
## Cumulative Var     0.2    0.4    0.6    0.8    1.0
```

Comments

- First component has a bit of everything, though especially the first three tests.
- Second component rather more clearly `add` and `dots`.
- No scores, plots since no actual data.
- See how factor analysis compares on these data.

Factor analysis

- Specify number of factors first, get solution with exactly that many factors.
- Includes hypothesis test, need to specify how many children wrote the tests.
- Works from correlation matrix via `covmat` or actual data, like `princomp`.
- Introduces extra feature, *rotation*, to make interpretation of loadings (factor-variable relation) easier.

Factor analysis for the kids data

- Create "covariance list" to include number of children who wrote the tests.
- Feed this into `factanal`, specifying how many factors (2).
- Start with the matrix we made before.

```
m
```

```
##        para  sent  word   add  dots
## para 1.000 0.722 0.714 0.203 0.095
## sent 0.722 1.000 0.685 0.246 0.181
## word 0.714 0.685 1.000 0.170 0.113
## add  0.203 0.246 0.170 1.000 0.585
## dots 0.095 0.181 0.113 0.585 1.000
```

```
ml <- list(cov = m, n.obs = 145)
kids.2 <- factanal(factors = 2, covmat = ml)
```

Uniquenesses

```
kids.2$uniquenesses
```

```
##      para      sent      word       add      dots
## 0.2424457 0.2997349 0.3272312 0.5743568 0.1554076
```

- Uniquenesses say how "unique" a variable is (size of specific factor). Small uniqueness means that the variable is summarized by a factor (good).
- Very large uniquenesses are bad; `add`'s uniqueness is largest but not large enough to be worried about.
- Also see "communality" for this idea, where *large* is good and *small* is bad.

Loadings

```
kids.2$loadings
```

```
##
## Loadings:
##      Factor1 Factor2
## para 0.867
## sent 0.820   0.166
## word 0.816
## add  0.167   0.631
## dots         0.918
##
##                Factor1 Factor2
## SS loadings      2.119   1.282
## Proportion Var   0.424   0.256
## Cumulative Var   0.424   0.680
```

- Loadings show how each factor depends on variables. Blanks indicate "small", less than 0.1.

Comments

- Factor 1 clearly the “linguistic” tasks, factor 2 clearly the “mathematical” ones.
- Two factors together explain 68% of variability (like regression R-squared).
- Which variables belong to which factor is *much* clearer than with principal components.

Are 2 factors enough?

```r
kids.2$STATISTIC
```

```
## objective
## 0.5810578
```

```r
kids.2$dof
```

```
## [1] 1
```

```r
kids.2$PVAL
```

```
## objective
##  0.445898
```

P-value not small, so 2 factors OK.

1 factor

```
kids.1 <- factanal(factors = 1, covmat = ml)
kids.1$STATISTIC
```

```
## objective
##  58.16534
```

```
kids.1$dof
```

```
## [1] 5
```

```
kids.1$PVAL
```

```
##    objective
## 2.907856e-11
```

1 factor rejected (P-value small). Definitely need more than 1.

Places rated, again

- Read data, transform, rerun principal components, get biplot:

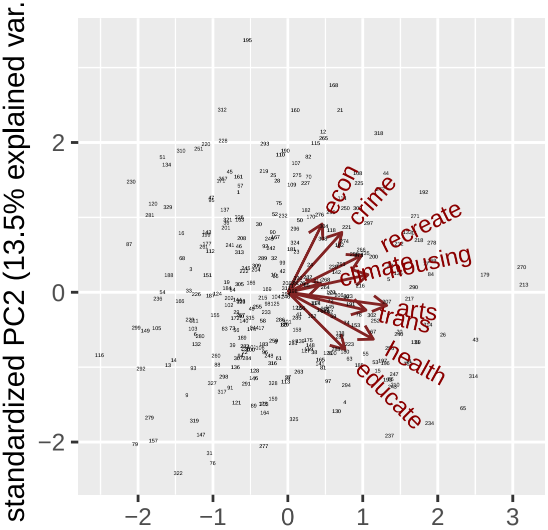
```
my_url <- "http://ritsokiguess.site/datafiles/places.txt"
places0 <- read_table2(my_url)
```

```
## Warning: `read_table2()` was deprecated in readr 2.0.0.
## i Please use `read_table()` instead.
```

```
places0 %>%
  mutate(across(-id, \(x) log(x))) -> places
places %>% select(-id) -> places_numeric
places.1 <- princomp(places_numeric, cor = TRUE)
g <- ggbiplot(places.1, labels = places$id,
         labels.size = 0.8)
```

- This is all exactly as for principal components (nothing new here).

```
g
```


standardized PC2 (13.5% explained var.
econ
crime
recreate
housing
climate
arts
trans
health
educate
2
0
−2
−2
−1
0
1
2
3

Comments

- Most of the criteria are part of components 1 *and* 2.
- If we can rotate the arrows counterclockwise:
    - economy and crime would point straight up
        - part of component 2 only
    - health and education would point to the right
        - part of component 1 only
- would be easier to see which variables belong to which component.
- Factor analysis includes a rotation to help with interpretation.

Factor analysis

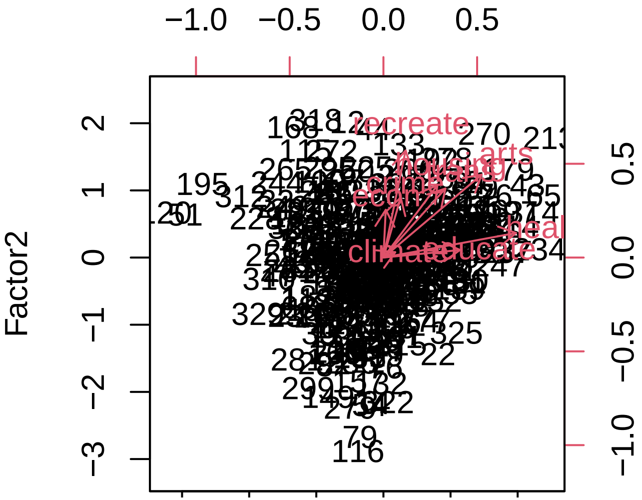
- Have to pick a number of factors *first*.
- Do this by running principal components and looking at scree plot.
- In this case, 3 factors seemed good (revisit later):

```
places.3 <- factanal(places_numeric, 3, scores = "r")
```

- There are different ways to get factor scores. These called "regression" scores.

A bad biplot

```
biplot(places.3$scores, places.3$loadings,
  xlabs = places$id)
```



Comments

- I have to find a way to make a better biplot!
- Some of the variables now point straight up and some straight across (if you look carefully for the red arrows among the black points).
- This should make the factors more interpretable than the components were.

Factor loadings

```
places.3$loadings
```

```
##
## Loadings:
##          Factor1 Factor2 Factor3
## climate                   0.994
## housing   0.360   0.482   0.229
## health    0.884   0.164
## crime     0.115   0.400   0.205
## trans     0.414   0.460
## educate   0.511
## arts      0.655   0.552   0.102
## recreate  0.148   0.714
## econ              0.318  -0.114
##
##                Factor1 Factor2 Factor3
## SS loadings      1.814   1.551   1.120
## Proportion Var   0.202   0.172   0.124
## Cumulative Var   0.202   0.374   0.498
```

Comments on loadings

- These are at least somewhat clearer than for the principal components:
  - Factor 1: health, education, arts: "well-being"
  - Factor 2: housing, transportation, arts (again), recreation: "places to be"
  - Factor 3: climate (only): "climate"
- In this analysis, economic factors don't seem to be important.

Factor scores

- Make a dataframe with the city IDs and factor scores:

```
cbind(id = places$id, places.3$scores) %>%
  as_tibble() -> places_scores
```

- Make percentile ranks again (for checking):

```
places %>%
  mutate(across(-id, \(x) percent_rank(x))) -> places_pr
```

Highest scores on factor 1, “well-being”:

- for the top 4 places:

```
places_scores %>%
  slice_max(Factor1, n = 4)
```

id	Factor1	Factor2	Factor3
213	2.469554	1.7788333	0.5060550
65	2.385729	0.9247505	-0.2870941
234	2.324346	0.1216503	0.5239580
314	2.221426	0.6714295	0.5208330

Check percentile ranks for factor 1

```
places_pr %>%
  select(id, health, educate, arts) %>%
  filter(id %in% c(213, 65, 234, 314))
```

id	health	educate	arts
65	0.9969512	0.9634146	0.9969512
213	1.0000000	0.7225610	1.0000000
234	0.9908537	1.0000000	0.9847561
314	0.9847561	0.9939024	0.9908537

- These are definitely high on the well-being variables.
- City #213 is not so high on education, but is highest of all on the others.

Highest scores on factor 2, "places to be":

```
places_scores %>%
  slice_max(Factor2, n = 4)
```

id	Factor1	Factor2	Factor3
318	-1.0124822	2.046099	-0.0957484
12	-0.5397677	2.019278	-3.8038802
168	-1.3485804	1.940127	0.2734326
44	-0.1492306	1.916696	-0.5558097

Check percentile ranks for factor 2

```
places_pr %>%
  select(id, housing, trans, arts, recreate) %>%
  filter(id %in% c(318, 12, 168, 44))
```

id	housing	trans	arts	recreate
12	0.9329268	0.7286585	0.6036585	0.8963415
44	0.9268293	0.9634146	0.7347561	0.9878049
168	0.8323171	0.8719512	0.4420732	0.9786585
318	0.8810976	0.7439024	0.6676829	0.9634146

- These are definitely high on housing and recreation.
- Some are (very) high on transportation, but not so much on arts.
- Could look at more cities to see if #168 being low on arts is a fluke.

Highest scores on factor 3, “climate”:

```
places_scores %>%
  slice_max(Factor3, n = 4)
```

id	Factor1	Factor2	Factor3
227	-0.1838076	0.3850312	2.039536
218	0.8809663	0.8966425	2.021177
269	0.9315705	1.1891810	1.977220
270	1.5031372	1.8418356	1.938905

Check percentile ranks for factor 3

```
places_pr %>%
  select(id, climate) %>%
  filter(id %in% c(227, 218, 269, 270))
```

id	climate
218	0.9969512
227	0.9908537
269	0.9939024
270	0.9969512

This is very clear.

Uniquenesses

- We said earlier that the economy was not part of any of our factors:

```
places.3$uniquenesses
```

```
##   climate   housing    health     crime     trans   educate
## 0.0050000 0.5859175 0.1854084 0.7842407 0.6165449 0.7351921
##      econ
## 0.8856382
```

- The higher the uniqueness, the less the variable concerned is part of any of our factors (and that maybe another factor is needed to accommodate it).
- This includes economy and maybe crime.

Test of significance

We can test whether the three factors that we have is enough, or whether we need more to describe our data:

```
places.3$PVAL
```

```
##    objective
## 1.453217e-14
```

- 3 factors are not enough.
- What would 5 factors look like?

Five factors

```
places.5 <- factanal(places_numeric, 5, scores = "r")
places.5$loadings
```

```
##
## Loadings:
##          Factor1 Factor2 Factor3 Factor4 Factor5
## climate                           0.131   0.559
## housing   0.286   0.505   0.289  -0.113   0.475
## health    0.847   0.214                   0.187
## crime             0.196   0.143   0.948   0.181
## trans     0.389   0.515           0.175
## educate   0.534
## arts      0.611   0.564           0.172   0.145
## recreate          0.705           0.115   0.136
## econ                      0.978   0.135
##
##                Factor1 Factor2 Factor3 Factor4 Factor5
## SS loadings      1.628   1.436   1.087   1.023   0.658
## Proportion Var   0.181   0.160   0.121   0.114   0.073
## Cumulative Var   0.181   0.340   0.461   0.575   0.648
```

Comments

- On (new) 5 factors:
  - Factor 1 is health, education, arts: same as factor 1 before.
  - Factor 2 is housing, transportation, arts, recreation: as factor 2 before.
  - Factor 3 is economy.
  - Factor 4 is crime.
  - Factor 5 is climate and housing: like factor 3 before.
- The two added factors include the two "missing" variables.
- Is this now enough?

```
places.5$PVAL
```

```
##    objective
## 0.0009741394
```

- No. My guess is that the authors of Places Rated chose their 9 criteria to capture different aspects of what makes a city good or bad to live in, and so it was too much to hope that a small number of factors would come out of these.

A bigger example: BEM sex role inventory

- 369 women asked to rate themselves on 60 traits, like “self-reliant” or “shy”.
- Rating 1 “never or almost never true of me” to 7 “always or almost always true of me’ ’.
- 60 personality traits is a lot. Can we find a smaller number of factors that capture aspects of personality?
- The whole BEM sex role inventory on next page.

The whole inventory

1. self reliant	21. reliable	41. warm
2. yielding	22. analytical	42. solemn
3. helpful	23. sympathetic	43. willing to take a stand
4. defends own beliefs	24. jealous	44. tender
5. cheerful	25. leadership ability	45. friendly
6. moody	26. sensitive to other's needs	46. aggressive
7. independent	27. truthful	47. gullible
8. shy	28. willing to take risks	48. inefficient
9. conscientious	29. understanding	49. acts as a leader
10. athletic	30. secretive	50. childlike
11. affectionate	31. makes decisions easily	51. adaptable
12. theatrical	32. compassionate	52. individualistic
13. assertive	33. sincere	53. does not use harsh language
14. flatterable	34. self-sufficient	54. unsystematic
15. happy	35. eager to soothe hurt feelings	55. competitive
16. strong personality	36. conceited	56. loves children
17. loyal	37. dominant	57. tactful
18. unpredictable	38. soft spoken	58. ambitious
19. forceful	39. likable	59. gentle
20. feminine	40. masculine	60. conventional

Some of the data

```r
my_url <- "http://ritsokiguess.site/datafiles/factor.txt"
bem <- read_tsv(my_url)
bem
```

subno	helpful	reliant	defbel	yielding	cheerful	indpt	athlet	shy	assert	strpers	forceful	affect	flatter	loyal	analyt	feminine	sympathy	moody	sensitiv	undstand	compass	leaderab	soothe	risk	decide	selfsuff	conscien	dominant	m-cu
1	7	7	5	5	7	7	7	1	7	7	2	7	4	7	1	2	6	3	7	7	7	6	7	2	7	7	7	1	1
2	5	6	6	6	2	3	3	3	4	1	3	5	4	5	6	5	5	4	5	5	4	4	4	3	3	5	5	4	1
3	7	6	4	4	5	5	2	3	4	4	3	5	2	7	5	6	5	2	4	6	6	4	7	3	5	6	6	2	1
4	6	6	7	4	6	6	3	4	4	3	3	5	3	7	6	6	5	2	6	6	6	2	6	5	5	6	6	4	2
5	6	6	7	4	7	7	7	2	7	7	5	7	4	7	7	4	7	3	7	6	7	7	7	7	7	5	7	6	4
7	5	6	7	4	6	6	2	4	4	2	2	5	3	7	7	7	6	4	5	5	5	3	5	1	3	7	7	2	1
8	6	4	6	6	6	3	1	3	3	4	1	7	7	6	4	4	7	3	6	6	6	1	6	1	4	4	4	1	1
9	7	6	7	5	6	7	5	2	5	6	6	7	3	7	7	6	6	4	6	6	6	6	5	4	4	6	7	4	1
10	7	6	6	4	4	5	2	2	5	7	6	6	6	6	7	6	4	4	6	6	5	5	5	4	4	6	5	5	9
11	7	4	7	4	7	5	2	1	5	6	4	7	7	7	7	5	5	1	5	6	6	4	6	2	1	5	5	4	2
13	7	7	7	4	6	7	1	3	6	6	4	6	5	7	7	5	7	6	6	7	7	5	6	5	4	4	6	3	2
14	7	7	5	5	7	1	3	5	6	7	1	7	7	7	3	5	7	5	7	7	7	2	7	3	1	1	5	2	1
15	7	7	7	7	7	7	7	1	7	7	7	7	5	7	7	7	7	1	7	7	7	7	7	7	7	7	7	7	1
23	5	6	7	5	6	6	1	1	6	6	7	7	7	7	7	6	6	4	6	6	5	4	4	2	4	4	5	7	1
25	6	6	7	4	4	7	7	1	5	5	7	6	1	7	4	4	7	5	4	7	5	4	5	7	6	7	7	5	1
26	6	6	5	5	7	6	2	2	5	5	4	4	5	7	7	4	6	2	6	6	5	6	6	3	3	6	6	4	1

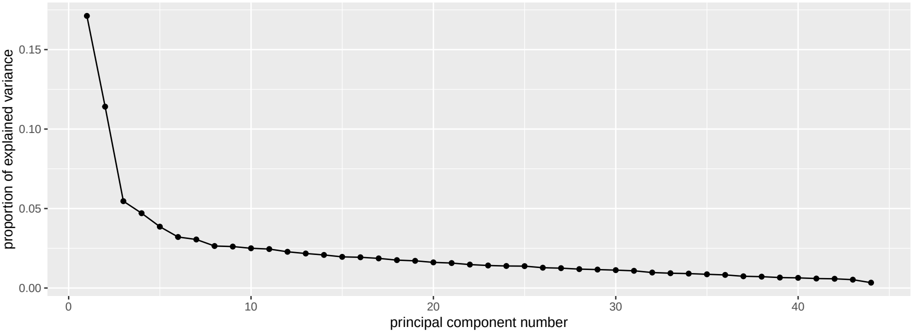
Principal components first

…to decide on number of factors:

```
bem.pc <- bem %>%
  select(-subno) %>%
  princomp(cor = T)
```

The scree plot

```
(g <- ggscreeplot(bem.pc))
```

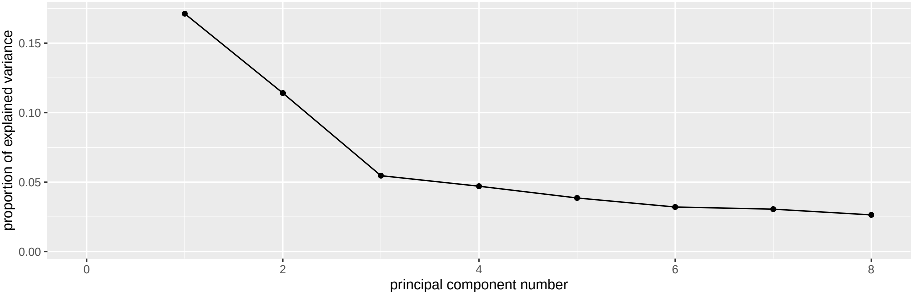


- No obvious elbow.

Zoom in to search for elbow

Possible elbows at 3 (2 factors) and 6 (5):

```
g + scale_x_continuous(limits = c(0, 8))
```



but is 2 really good?

```
summary(bem.pc)
```

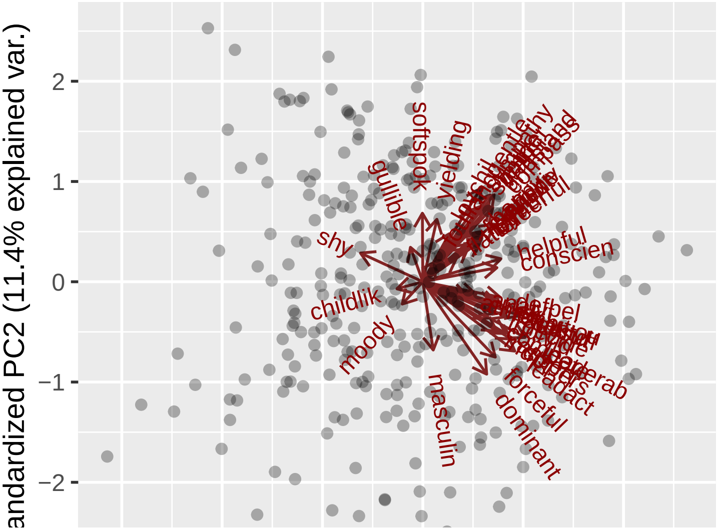
```
## Importance of components:
##                           Comp.1    Comp.2     Comp.3     Comp.4     Comp.5
## Standard deviation     2.7444993 2.2405789 1.55049106 1.43886350 1.30318840
## Proportion of Variance 0.1711881 0.1140953 0.05463688 0.04705291 0.03859773
## Cumulative Proportion  0.1711881 0.2852834 0.33992029 0.38697320 0.42557093
##                            Comp.6     Comp.7     Comp.8     Comp.9    Comp.10
## Standard deviation     1.18837867 1.15919129 1.07838912 1.07120568 1.04901318
## Proportion of Variance 0.03209645 0.03053919 0.02643007 0.02607913 0.02500974
## Cumulative Proportion  0.45766738 0.48820657 0.51463664 0.54071577 0.56572551
##                           Comp.11    Comp.12    Comp.13    Comp.14   Comp.15
## Standard deviation     1.03848656 1.00152287 0.97753974 0.95697572 0.9287543
## Proportion of Variance 0.02451033 0.02279655 0.02171782 0.02081369 0.0196042
## Cumulative Proportion  0.59023584 0.61303238 0.63475020 0.65556390 0.6751681
##                           Comp.16    Comp.17   Comp.18    Comp.19    Comp.20
## Standard deviation     0.92262649 0.90585705 0.8788668 0.86757525 0.84269120
## Proportion of Variance 0.01934636 0.01864948 0.0175547 0.01710652 0.01613928
## Cumulative Proportion  0.69451445 0.71316392 0.7307186 0.74782514 0.76396443
##                           Comp.21    Comp.22    Comp.23    Comp.24    Comp.25
## Standard deviation     0.83124925 0.80564654 0.78975423 0.78100835 0.77852606
## Proportion of Variance 0.01570398 0.01475151 0.01417527 0.01386305 0.01377506
## Cumulative Proportion  0.77966841 0.79441992 0.80859519 0.82245823 0.83623330
##                           Comp.26    Comp.27    Comp.28    Comp.29    Comp.30
```

Comments

- Want overall fraction of variance explained ("cumulative proportion'') to be reasonably high.
- 2 factors, 28.5%. Terrible!
- Even 56% (10 factors) not that good!
- Have to live with that.

Biplot

```r
ggbiplot(bem.pc, alpha = 0.3)
```



Comments

- Ignore individuals for now.
- Most variables point to 1 o'clock or 4 o'clock.
- Suggests factor analysis with rotation will get interpretable factors (rotate to 12 o'clock and 3 o'clock, for example).
- Try for 2-factor solution (rough interpretation, will be bad):

```
bem %>%
  select(-subno) %>%
  factanal(factors = 2) -> bem.2
```

- Show output in pieces (just print `bem.2` to see all of it).

Uniquenesses, sorted

```
sort(bem.2$uniquenesses)
```

```
##  leaderab   leadact      warm    tender  dominant    gentle
## 0.4091894 0.4166153 0.4764762 0.4928919 0.4942909 0.5064551
##  forceful   strpers   compass     stand  undstand    assert
## 0.5631857 0.5679398 0.5937073 0.6024001 0.6194392 0.6329347
##    soothe    affect    decide  selfsuff  sympathy     indpt
## 0.6596103 0.6616625 0.6938578 0.7210246 0.7231450 0.7282742
##   helpful    defbel      risk   reliant   individ   compete
## 0.7598223 0.7748448 0.7789761 0.7808058 0.7941998 0.7942910
##  conscien     happy  sensitiv     loyal  ambitiou       shy
## 0.7974820 0.8008966 0.8018851 0.8035264 0.8101599 0.8239496
##  softspok  cheerful  masculin  yielding  feminine  truthful
## 0.8339058 0.8394916 0.8453368 0.8688473 0.8829927 0.8889983
##   lovchil    analyt    athlet   flatter  gullible     moody
## 0.8924392 0.8968744 0.9229702 0.9409500 0.9583435 0.9730607
##  childlik  foullang
## 0.9800360 0.9821662
```

Comments

- Mostly high or very high (bad).
- Some smaller, eg.: Leadership ability (0.409), Acts like leader (0.417), Warm (0.476), Tender (0.493).
- Smaller uniquenesses captured by one of our two factors.
- Larger uniquenesses are not: need more factors to capture them.

Factor loadings, some

```
bem.2$loadings
```

```
##
## Loadings:
##          Factor1 Factor2
## helpful   0.314   0.376
## reliant   0.453   0.117
## defbel    0.434   0.193
## yielding -0.131   0.338
## cheerful  0.152   0.371
## indpt     0.521
## athlet    0.267
## shy      -0.414
## assert    0.605
## strpers   0.657
## forceful  0.649  -0.126
## affect    0.178   0.554
## flatter           0.223
## loyal     0.151   0.417
## analyt    0.295   0.127
## feminine  0.113   0.323
## sympathy          0.526
## moody            -0.162
## sensitiv  0.135   0.424
```

Making a data frame

There are too many to read easily, so make a data frame. A bit tricky:

```
bem.2$loadings %>%
  unclass() %>%
  as_tibble() %>%
  mutate(trait = rownames(bem.2$loadings)) -> loadings
loadings %>% slice(1:8)
```

Factor1	Factor2	trait
0.3137466	0.3764849	helpful
0.4532904	0.1171406	reliant
0.4336574	0.1926030	defbel
-0.1309965	0.3376293	yielding
0.1523718	0.3705305	cheerful
0.5212403	0.0058703	indpt
0.2670788	0.0755429	athlet
-0.4144579	-0.0653728	shy

Pick out the big ones on factor 1

Arbitrarily defining > 0.4 or < -0.4 as "big":

```
loadings %>% filter(abs(Factor1) > 0.4)
```

Factor1	Factor2	trait
0.4532904	0.1171406	reliant
0.4336574	0.1926030	defbel
0.5212403	0.0058703	indpt
-0.4144579	-0.0653728	shy
0.6049588	0.0330048	assert
0.6569855	0.0207776	strpers
0.6487190	-0.1264058	forceful
0.7654924	0.0695136	leaderab
0.4416176	0.1612384	risk
0.5416796	0.1128080	decide
0.5109964	0.1336268	selfsuff
0.6676490	-0.2448558	dominant
0.6066864	0.1718489	stand
0.7627129	-0.0406672	leadact
0.4448064	0.0891461	individ
0.4504188	0.0532073	compete
0.4136498	0.1368696	ambitiou

Factor 2, the big ones

```
loadings %>% filter(abs(Factor2) > 0.4)
```

Factor1	Factor2	trait
0.1778911	0.5537994	affect
0.1512127	0.4166622	loyal
0.0230146	0.5256654	sympathy
0.1347697	0.4242037	sensitiv
0.0911130	0.6101294	undstand
0.1135064	0.6272223	compass
0.0606175	0.5802714	soothe
0.1189301	0.4300698	happy
0.0795698	0.7191610	warm
0.0511381	0.7102763	tender
-0.0187322	0.7022768	gentle

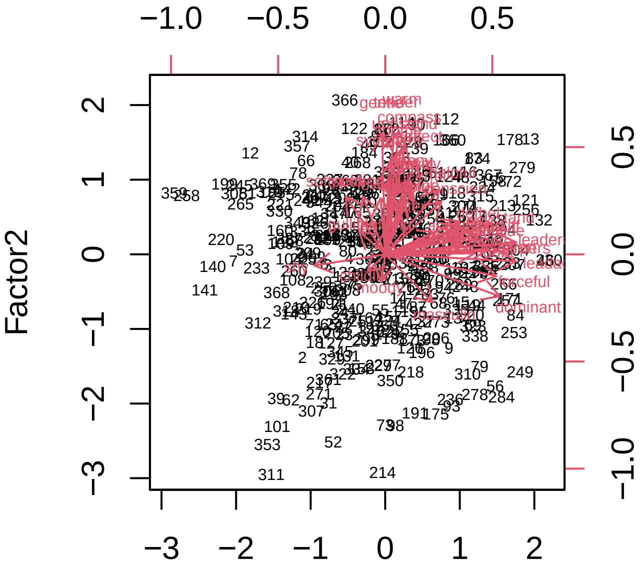
Plotting the two factors

- A bi-plot, this time with the variables reduced in size. Looking for unusual individuals.
- Have to run `factanal` again to get factor scores for plotting.

```
bem %>% select(-subno) %>%
  factanal(factors = 2, scores = "r") -> bem.2a
biplot(bem.2a$scores, bem.2a$loadings, cex = c(0.5, 0.5))
```

- Numbers on plot are row numbers of `bem` data frame.

The (awful) biplot



Comments

- Variables mostly up (“feminine”) and right (“masculine”), accomplished by rotation.
- Some unusual individuals: 311, 214 (low on factor 2), 366 (high on factor 2), 359, 258 (low on factor 1), 230 (high on factor 1).

Individual 366

```
bem %>% slice(366) %>% glimpse()
```

```
## Rows: 1
## Columns: 45
## $ subno    <dbl> 755
## $ helpful  <dbl> 7
## $ reliant  <dbl> 7
## $ defbel   <dbl> 5
## $ yielding <dbl> 7
## $ cheerful <dbl> 7
## $ indpt    <dbl> 7
## $ athlet   <dbl> 7
## $ shy      <dbl> 2
## $ assert   <dbl> 1
## $ strpers  <dbl> 3
## $ forceful <dbl> 1
## $ affect   <dbl> 7
## $ flatter  <dbl> 9
## $ loyal    <dbl> 7
## $ analyt   <dbl> 7
## $ feminine <dbl> 7
## $ sympathy <dbl> 7
## $ moody    <dbl> 1
## $ sensitiv <dbl> 7
## $ undstand <dbl> 7
## $ compass  <dbl> 6
## $ leaderab <dbl> 3
## $ soothe   <dbl> 7
## $ risk     <dbl> 7
## $ decide   <dbl> 7
## $ selfsuff <dbl> 7
## $ conscien <dbl> 7
```

Comments

- Individual 366 high on factor 2, but hard to see which traits should have high scores (unless we remember).
- Idea 1: use percentile ranks as before.
- Idea 2: Rating scale is easy to interpret. So *tidy* original data frame to make easier to look things up.

Tidying original data

```
bem %>%
  ungroup() %>%
  mutate(row = row_number()) %>%
  pivot_longer(c(-subno, -row), names_to="trait",
               values_to="score") -> bem_tidy
bem_tidy
```

subno	row	trait	score
1	1	helpful	7
1	1	reliant	7
1	1	defbel	5
1	1	yielding	5
1	1	cheerful	7
1	1	indpt	7
1	1	athlet	7
1	1	shy	1
1	1	assert	7
1	1	strpers	7
1	1	forceful	2
1	1	affect	7
1	1	flatter	4
1	1	loyal	7
1	1	analyt	1

Recall data frame of loadings

```
loadings %>% slice(1:10)
```

Factor1	Factor2	trait
0.3137466	0.3764849	helpful
0.4532904	0.1171406	reliant
0.4336574	0.1926030	defbel
-0.1309965	0.3376293	yielding
0.1523718	0.3705305	cheerful
0.5212403	0.0058703	indpt
0.2670788	0.0755429	athlet
-0.4144579	-0.0653728	shy
0.6049588	0.0330048	assert
0.6569855	0.0207776	strpers

Want to add the factor scores for each trait to our tidy data frame `bem_tidy`. This is a left-join (over), matching on the column `trait` that is in both data frames (thus, the default):

Looking up loadings

```
bem_tidy %>% left_join(loadings) -> bem_tidy
```

```
## Joining, by = "trait"
```

```
bem_tidy %>% sample_n(12)
```

subno	row	trait	score	Factor1	Factor2
212	121	leaderab	7	0.7654924	0.0695136
35	25	masculin	5	0.2758770	-0.2802764
501	288	moody	1	-0.0233054	-0.1624628
89	55	lovchil	7	-0.0270572	0.3268500
358	209	defbel	6	0.4336574	0.1926030
81	47	moody	2	-0.0233054	-0.1624628
30	20	strpers	6	0.6569855	0.0207776
592	348	masculin	2	0.2758770	-0.2802764
252	146	forceful	5	0.6487190	-0.1264058
83	49	compete	3	0.4504188	0.0532073
23	14	selfsuff	4	0.5109964	0.1336268
494	281	reliant	7	0.4532904	0.1171406

Individual 366, high on Factor 2

So now pick out the rows of the tidy data frame that belong to individual 366 (`row=366`) and for which the `Factor2` score exceeds 0.4 in absolute value (our "big" from before):

```
bem_tidy %>% filter(row == 366, abs(Factor2) > 0.4)
```

subno	row	trait	score	Factor1	Factor2
755	366	affect	7	0.1778911	0.5537994
755	366	loyal	7	0.1512127	0.4166622
755	366	sympathy	7	0.0230146	0.5256654
755	366	sensitiv	7	0.1347697	0.4242037
755	366	undstand	7	0.0911130	0.6101294
755	366	compass	6	0.1135064	0.6272223
755	366	soothe	7	0.0606175	0.5802714
755	366	happy	7	0.1189301	0.4300698
755	366	warm	7	0.0795698	0.7191610
755	366	tender	7	0.0511381	0.7102763
755	366	gentle	7	-0.0187322	0.7022768

As expected, high scorer on these.

Several individuals

Rows 311 and 214 were *low* on Factor 2, so their scores should be low. Can we do them all at once?

```
bem_tidy %>% filter(
  row %in% c(366, 311, 214),
  abs(Factor2) > 0.4
)
```

subno	row	trait	score	Factor1	Factor2
369	214	affect	1	0.1778911	0.5537994
369	214	loyal	7	0.1512127	0.4166622
369	214	sympathy	4	0.0230146	0.5256654
369	214	sensitiv	7	0.1347697	0.4242037
369	214	undstand	5	0.0911130	0.6101294
369	214	compass	5	0.1135064	0.6272223
369	214	soothe	3	0.0606175	0.5802714
369	214	happy	4	0.1189301	0.4300698
369	214	warm	1	0.0795698	0.7191610
369	214	tender	3	0.0511381	0.7102763
369	214	gentle	2	-0.0187322	0.7022768
534	311	affect	5	0.1778911	0.5537994
534	311	loyal	4	0.1512127	0.4166622

Individual by column

Un-tidy, that is, `pivot_wider`:

```
bem_tidy %>%
  filter(
    row %in% c(366, 311, 214),
    abs(Factor2) > 0.4
  ) %>%
  select(-subno, -Factor1, -Factor2) %>%
  pivot_wider(names_from=row, values_from=score)
```

trait	214	311	366
affect	1	5	7
loyal	7	4	7
sympathy	4	4	7
sensitiv	7	4	7
undstand	5	3	7
compass	5	4	6
soothe	3	4	7
happy	4	3	7
warm	1	3	7
tender	3	4	7
gentle	2	3	7

366 high, 311 middling, 214 (sometimes) low.

Individuals 230, 258, 359

These were high, low, low on factor 1. Adapt code:

```
bem_tidy %>%
  filter(row %in% c(359, 258, 230), abs(Factor1) > 0.4) %>%
  select(-subno, -Factor1, -Factor2) %>%
  pivot_wider(names_from=row, values_from=score)
```

trait	230	258	359
reliant	7	4	1
defbel	7	1	1
indpt	7	7	1
shy	2	7	5
assert	7	3	1
strpers	7	1	3
forceful	7	1	1
leaderab	7	1	1
risk	7	5	7
decide	7	1	2
selfsuff	7	4	1
dominant	7	1	1
stand	7	1	6
leadact	7	1	1
individ	7	3	3
compete	6	2	1
ambitiou	7	2	4

Is 2 factors enough?

Suspect not:

```
bem.2$PVAL
```

```
##     objective
## 1.458183e-150
```

2 factors resoundingly rejected. Need more. Have to go all the way to 15 factors to not reject:

```
bem %>%
  select(-subno) %>%
  factanal(factors = 15) -> bem.15
bem.15$PVAL
```

```
## objective
##  0.132617
```

Even then, only just over 50% of variability explained.

What's important in 15 factors?

- Let's take a look at the important things in those 15 factors.
- Get 15-factor loadings into a data frame, as before:

```
bem.15$loadings %>%
  unclass() %>%
  as_tibble() %>%
  mutate(trait = rownames(bem.15$loadings)) -> loadings
```

- then show the highest few loadings on each factor.

Factor 1 (of 15)

```
loadings %>%
  arrange(desc(abs(Factor1))) %>%
  select(Factor1, trait) %>%
  slice(1:10)
```

Factor1	trait
0.8127595	compass
0.6756043	undstand
0.6611293	sympathy
0.6408327	sensitiv
0.5971006	soothe
0.3481290	warm
0.2797159	gentle
0.2788627	tender
0.2501505	helpful
0.2340594	conscien

Compassionate, understanding, sympathetic, soothing: thoughtful of others.

Factor 2

```
loadings %>%
  arrange(desc(abs(Factor2))) %>%
  select(Factor2, trait) %>%
  slice(1:10)
```

Factor2	trait
0.7615492	strpers
0.7160312	forceful
0.6981500	assert
0.5041921	dominant
0.3929344	leaderab
0.3669560	stand
0.3507080	leadact
-0.3131682	softspok
-0.2866862	shy
0.2602525	analyt

Strong personality, forceful, assertive, dominant: getting ahead.

Factor 3

```
loadings %>%
  arrange(desc(abs(Factor3))) %>%
  select(Factor3, trait) %>%
  slice(1:10)
```

Factor3	trait
0.6697542	reliant
0.6475496	selfsuff
0.6204018	indpt
0.3899607	helpful
-0.3393605	gullible
0.3333813	individ
0.3319003	decide
0.3294806	conscien
0.2877396	leaderab
0.2804170	defbel

Self-reliant, self-sufficient, independent: going it alone.

Factor 4

```
loadings %>%
  arrange(desc(abs(Factor4))) %>%
  select(Factor4, trait) %>%
  slice(1:10)
```

Factor4	trait
0.6956206	gentle
0.6920303	tender
0.5992467	warm
0.4465546	affect
0.3942568	softspok
0.2779793	lovchil
0.2444249	undstand
0.2442119	happy
0.2125905	loyal
0.2022861	soothe

Gentle, tender, warm (affectionate): caring for others.

Factor 5

```
loadings %>%
  arrange(desc(abs(Factor5))) %>%
  select(Factor5, trait) %>%
  slice(1:10)
```

Factor5	trait
0.6956846	compete
0.6743459	ambitiou
0.3453425	risk
0.3423456	individ
0.2808623	athlet
0.2695570	leaderab
0.2449656	decide
0.2064415	dominant
0.1928159	leadact
0.1854989	strpers

Ambitious, competitive (with a bit of risk-taking and individualism): Being the best.

Factor 6

```
loadings %>%
  arrange(desc(abs(Factor6))) %>%
  select(Factor6, trait) %>%
  slice(1:10)
```

Factor6	trait
0.8675651	leadact
0.6078869	leaderab
0.3378645	dominant
0.2014835	forceful
-0.1915632	shy
0.1789256	risk
0.1703440	masculin
0.1639190	decide
0.1594585	compete
0.1466037	athlet

Acts like a leader, leadership ability (with a bit of Dominant): Taking charge.

Factor 7

```
loadings %>%
  arrange(desc(abs(Factor7))) %>%
  select(Factor7, trait) %>%
  slice(1:10)
```

Factor7	trait
0.6698996	happy
0.6667105	cheerful
-0.5219125	moody
0.2191425	athlet
0.2126626	warm
0.1719953	gentle
-0.1640302	masculin
0.1601472	reliant
0.1472926	yielding
0.1410481	lovchil

Happy and cheerful.

Factor 8

```
loadings %>%
  arrange(desc(abs(Factor8))) %>%
  select(Factor8, trait) %>%
  slice(1:10)
```

Factor8	trait
0.6296764	affect
0.5158355	flatter
-0.2512066	softspok
0.2214623	warm
0.1878549	tender
0.1846225	strpers
-0.1804838	shy
0.1801992	compete
0.1658105	loyal
0.1548617	helpful

Affectionate, flattering: Making others feel good.

Factor 9

```
loadings %>%
  arrange(desc(abs(Factor9))) %>%
  select(Factor9, trait) %>%
  slice(1:10)
```

Factor9	trait
0.8633171	stand
0.3403294	defbel
0.2446971	individ
0.1941110	risk
-0.1715481	shy
0.1710978	decide
0.1197126	assert
0.1157729	conscien
0.1120308	analyt
-0.1115140	gullible

Taking a stand.

Factor 10

```
loadings %>%
  arrange(desc(abs(Factor10))) %>%
  select(Factor10, trait) %>%
  slice(1:10)
```

Factor10	trait
0.8075127	feminine
-0.2637851	masculin
0.2450718	softspok
0.2317560	conscien
0.2019203	selfsuff
0.1758423	yielding
0.1412707	gentle
0.1128203	flatter
0.1093453	decide
-0.0940798	lovchil

Feminine. (A little bit of not-masculine!)

Factor 11

```
loadings %>%
  arrange(desc(abs(Factor11))) %>%
  select(Factor11, trait) %>%
  slice(1:10)
```

Factor11	trait
0.9162259	loyal
0.1894908	affect
0.1588386	truthful
0.1246453	helpful
0.1044066	analyt
0.1007679	tender
0.0972046	lovchil
0.0963522	gullible
0.0935062	cheerful
0.0820760	conscien

Loyal.

Factor 12

```r
loadings %>%
  arrange(desc(abs(Factor12))) %>%
  select(Factor12, trait) %>%
  slice(1:10)
```

Factor12	trait
0.6106933	childlik
-0.2845004	selfsuff
-0.2786751	conscien
0.2588843	moody
0.2013245	shy
-0.1669301	decide
0.1542031	masculin
0.1455526	dominant
0.1379163	compass
-0.1297408	leaderab

Childlike. (With a bit of moody, shy, not-self-sufficient, not-conscientious.)

Factor 13

```
loadings %>%
  arrange(desc(abs(Factor13))) %>%
  select(Factor13, trait) %>%
  slice(1:10)
```

Factor13	trait
0.5729242	truthful
-0.2776490	gullible
0.2631046	happy
0.1885152	warm
-0.1671924	shy
0.1646031	loyal
-0.1438127	yielding
-0.1302900	assert
0.1137074	defbel
-0.1105583	lovchil

Truthful. (With a bit of happy and not-gullible.)

Factor 14

```
loadings %>%
  arrange(desc(abs(Factor14))) %>%
  select(Factor14, trait) %>%
  slice(1:10)
```

Factor14	trait
0.4429926	decide
0.2369714	selfsuff
0.1945034	forceful
-0.1862756	softspok
0.1604175	risk
-0.1484606	strpers
0.1461972	dominant
0.1279456	happy
0.1154479	compass
0.1054078	masculin

Decisive. (With a bit of self-sufficient and not-soft-spoken.)

Factor 15

```
loadings %>%
  arrange(desc(abs(Factor15))) %>%
  select(Factor15, trait) %>%
  slice(1:10)
```

Factor15	trait
-0.3244092	compass
0.2471884	athlet
0.2292980	sensitiv
0.1986878	risk
-0.1638296	affect
0.1632164	moody
-0.1118135	individ
0.1100678	warm
0.1047347	cheerful
0.1012342	reliant

Not-compassionate, athletic, sensitive: A mixed bag. ("Cares about self"?)

Anything left out? Uniquenesses

```
enframe(bem.15$uniquenesses, name="quality", value="uniq") %>%
  arrange(desc(uniq)) %>%
  slice(1:10)
```

quality	uniq
foullang	0.9136126
lovchil	0.8242992
analyt	0.8120934
yielding	0.7911748
masculin	0.7228739
athlet	0.7217327
shy	0.7033071
gullible	0.7000779
flatter	0.6625008
helpful	0.6516863

Uses foul language especially, also loves children and analytical. So could use even more factors.