Analysis of Covariance

Analysis of covariance

- ANOVA: explanatory variables categorical (divide data into groups)
- traditionally, analysis of covariance has categorical x's plus one numerical x ("covariate") to be adjusted for.
- `lm` handles this too.
- Simple example: two treatments (drugs) (`a` and `b`), with before and after scores.
- Does knowing before score and/or treatment help to predict after score?
- Is after score different by treatment/before score?

Data

Treatment, before, after:

```
a 5  20
a 10  23
a 12  30
a 9  25
a 23  34
a 21  40
a 14  27
a 18  38
a 6  24
a 13  31
b 7  19
b 12  26
b 27  33
b 24  35
b 18  30
b 22  31
b 26  34
b 21  28
b 14  23
b 9  22
```

Packages

`tidyverse` and `broom`:

```
library(tidyverse)
library(broom)
library(marginaleffects)
```

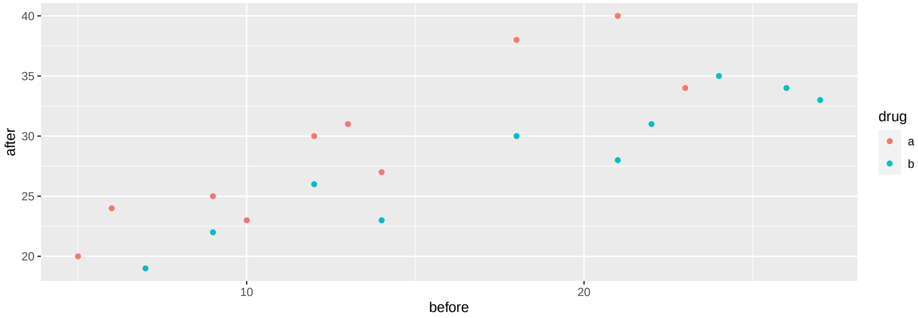
Read in data

```
url <- "http://ritsokiguess.site/datafiles/ancova.txt"
prepost <- read_delim(url, " ")
prepost %>% sample_n(9) # randomly chosen rows
```

drug	before	after
b	7	19
b	24	35
a	12	30
a	14	27
a	23	34
b	14	23
b	22	31
a	18	38
b	21	28

Making a plot

```
ggplot(prepost, aes(x = before, y = after, colour = drug)) +
  geom_point()
```



Comments

- As before score goes up, after score goes up.
- Red points (drug A) generally above blue points (drug B), for comparable before score.
- Suggests before score effect *and* drug effect.

The means

```
prepost %>%
  group_by(drug) %>%
  summarize(
    before_mean = mean(before),
    after_mean = mean(after)
  )
```

drug	before_mean	after_mean
a	13.1	29.2
b	18.0	28.1

- Mean "after" score slightly higher for treatment A.
- Mean "before" score much higher for treatment B.
- Greater *improvement* on treatment A.

Testing for interaction

```
prepost.1 <- lm(after ~ before * drug, data = prepost)
anova(prepost.1)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
before	1	430.92384	430.923838	62.68945	0.0000006
drug	1	115.30596	115.305957	16.77435	0.0008442
before:drug	1	12.33708	12.337080	1.79476	0.1990662
Residuals	16	109.98313	6.873945	NA	NA

- Interaction not significant. Will remove later.

Predictions

Set up values to predict for:

```
summary(prepost)
```

```
##      drug               before           after      
##  Length:20          Min.   : 5.00   Min.   :19.00  
##  Class :character   1st Qu.: 9.75   1st Qu.:23.75  
##  Mode  :character   Median :14.00   Median :29.00  
##                     Mean   :15.55   Mean   :28.65  
##                     3rd Qu.:21.25   3rd Qu.:33.25  
##                     Max.   :27.00   Max.   :40.00  
```

```
new <- datagrid(before = c(9.75, 14, 21.25), drug = c("a", "b'
```

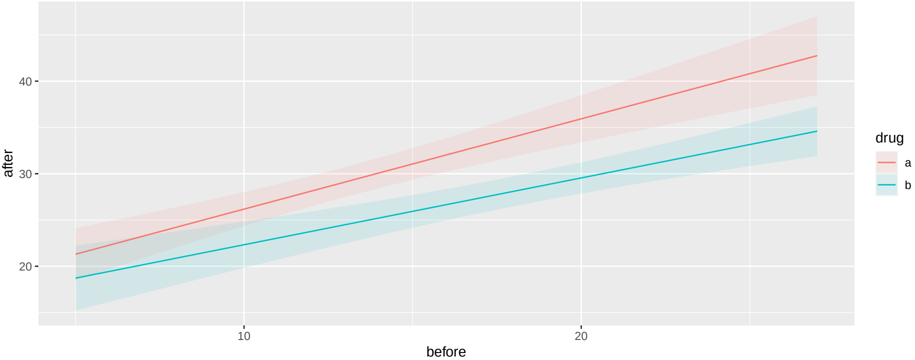
and then

```
cbind(predictions(prepost.1, newdata = new)) %>%
  select(drug, before, estimate)
```

drug	before	estimate
a	9.75	25.93250
b	9.75	22.14565
a	14.00	30.07784
b	14.00	25.21304
a	21.25	37.14929
b	21.25	30.44565

Predictions (with interaction included), plotted

```
plot_cap(model = prepost.1, condition = c("before", "drug"))
```



Lines almost parallel, but not quite.

Taking out interaction

```
prepost.2 <- update(prepost.1, . ~ . - before:drug)
anova(prepost.2)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
before	1	430.9238	430.923838	59.88958	0.0000006
drug	1	115.3060	115.305957	16.02516	0.0009209
Residuals	17	122.3202	7.195306	NA	NA

- Take out non-significant interaction.
- `before` and `drug` strongly significant.
- Do predictions again and plot them.

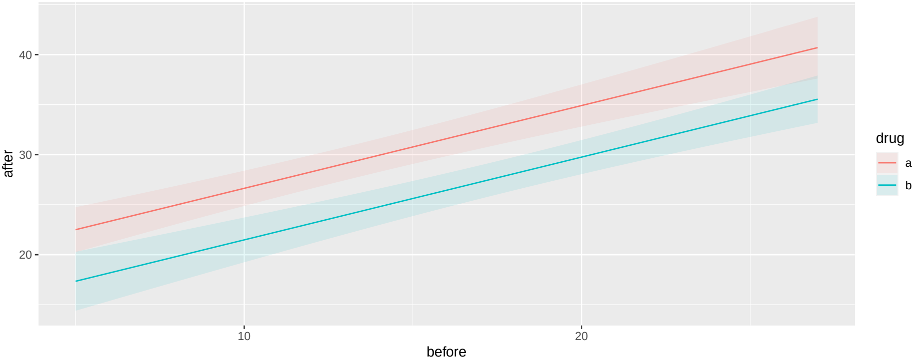
Predictions

```
predictions(prepost.2, variables = c("before", "drug"))
```

rowid	rowidcf	type	estimate	std.error	statistic	p.value	conf.low	conf.high	after	before	drug
1	1	response	22.49740	1.1480151	19.59678	0	20.24733	24.74747	20	5.0	a
2	2	response	22.49740	1.1480151	19.59678	0	20.24733	24.74747	23	5.0	a
3	3	response	22.49740	1.1480151	19.59678	0	20.24733	24.74747	30	5.0	a
4	4	response	22.49740	1.1480151	19.59678	0	20.24733	24.74747	25	5.0	a
5	5	response	22.49740	1.1480151	19.59678	0	20.24733	24.74747	34	5.0	a
6	6	response	22.49740	1.1480151	19.59678	0	20.24733	24.74747	40	5.0	a
7	7	response	22.49740	1.1480151	19.59678	0	20.24733	24.74747	27	5.0	a
8	8	response	22.49740	1.1480151	19.59678	0	20.24733	24.74747	38	5.0	a
9	9	response	22.49740	1.1480151	19.59678	0	20.24733	24.74747	24	5.0	a
10	10	response	22.49740	1.1480151	19.59678	0	20.24733	24.74747	31	5.0	a
11	11	response	22.49740	1.1480151	19.59678	0	20.24733	24.74747	19	5.0	a
12	12	response	22.49740	1.1480151	19.59678	0	20.24733	24.74747	26	5.0	a
13	13	response	22.49740	1.1480151	19.59678	0	20.24733	24.74747	33	5.0	a
14	14	re-	22.49740	1.1480151	19.59678	0	20.24733	24.74747	35	5.0	a

Plot of predicted values

```
plot_cap(prepost.2, condition = c("before", "drug"))
```



This time the lines are *exactly* parallel. No-interaction model forces them to have the same slope.

Different look at model output

- `anova(prepost.2)` tests for significant effect of before score and of drug, but doesn't help with interpretation.
- `summary(prepost.2)` views as regression with slopes:

```
summary(prepost.2)
```

```
##
## Call:
## lm(formula = after ~ before + drug, data = prepost)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.6348 -2.5099 -0.2038  1.8871  4.7453
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  18.3600     1.5115  12.147 8.35e-10
## before        0.8275     0.0955   8.665 1.21e-07
## drugb        -5.1547     1.2876  -4.003 0.000921
##
## (Intercept) ***
## before      ***
```

Understanding those slopes

```
tidy(prepost.2)
```

term	estimate	std.error	statistic	p.value
(Intercept)	18.3599949	1.5115326	12.146608	0.0000000
before	0.8274813	0.0955023	8.664520	0.0000001
drugb	-5.1546584	1.2876524	-4.003144	0.0009209

- `before` ordinary numerical variable; `drug` categorical.
- `lm` uses first category `druga` as baseline.
- Intercept is prediction of after score for before score 0 and *drug A*.
- `before` slope is predicted change in after score when before score increases by 1 (usual slope)
- Slope for `drugb` is *change* in predicted after score for being on drug B rather than drug A. Same for *any* before score (no interaction).

Summary

- ANCOVA model: fits different regression line for each group, predicting response from covariate.
- ANCOVA model with interaction between factor and covariate allows different slopes for each line.
- Sometimes those lines can cross over!
- If interaction not significant, take out. Lines then parallel.
- With parallel lines, groups have consistent effect regardless of value of covariate.