Cluster analysis

Cluster Analysis

- One side-effect of discriminant analysis: could draw picture of data (if 1st 2s `LD`s told most of story) and see which individuals "close" to each other.
- Discriminant analysis requires knowledge of groups.
- Without knowledge of groups, use *cluster analysis*: see which individuals close together, which groups suggested by data.
- Idea: see how individuals group into "clusters" of nearby individuals.
- Base on "dissimilarities" between individuals.
- Or base on standard deviations and correlations between variables (assesses dissimilarity behind scenes).

Packages

```
library(MASS) # for lda later
library(tidyverse)
library(spatstat) # for crossdist later
library(ggrepel)
library(conflicted)
conflict_prefer("select", "dplyr")
conflict_prefer("filter", "dplyr")
```

	English	Norwegian	Danish	Dutch	German

One to ten in 11 languages

	English	Norwegian	Danish	Dutch	German
1	one	en	en	een	eins
2	two	to	to	twee	zwei
3	three	tre	tre	drie	drei
4	four	fire	fire	vier	vier
5	five	fem	fem	vijf	funf
6	six	seks	seks	zes	sechs
7	seven	sju	syv	zeven	sieben
8	eight	atte	otte	acht	acht
9	nine	ni	ni	negen	neun
10	ten	ti	ti	tien	zehn

One to ten

	French	Spanish	Italian	Polish	Hungarian	Finnish
1	un	uno	uno	jeden	egy	yksi
2	deux	dos	due	dwa	ketto	kaksi
3	trois	tres	tre	trzy	harom	kolme
4	quatre	cuatro	quattro	cztery	negy	nelja
5	cinq	cinco	cinque	piec	ot	viisi
6	six	seis	sei	szesc	hat	kuusi
7	sept	siete	sette	siedem	het	seitseman
8	huit	ocho	otto	osiem	nyolc	kahdeksan
9	neuf	nueve	nove	dziewiec	kilenc	yhdeksan
10	dix	diez	dieci	dziesiec	tiz	kymmenen

Dissimilarities and languages example

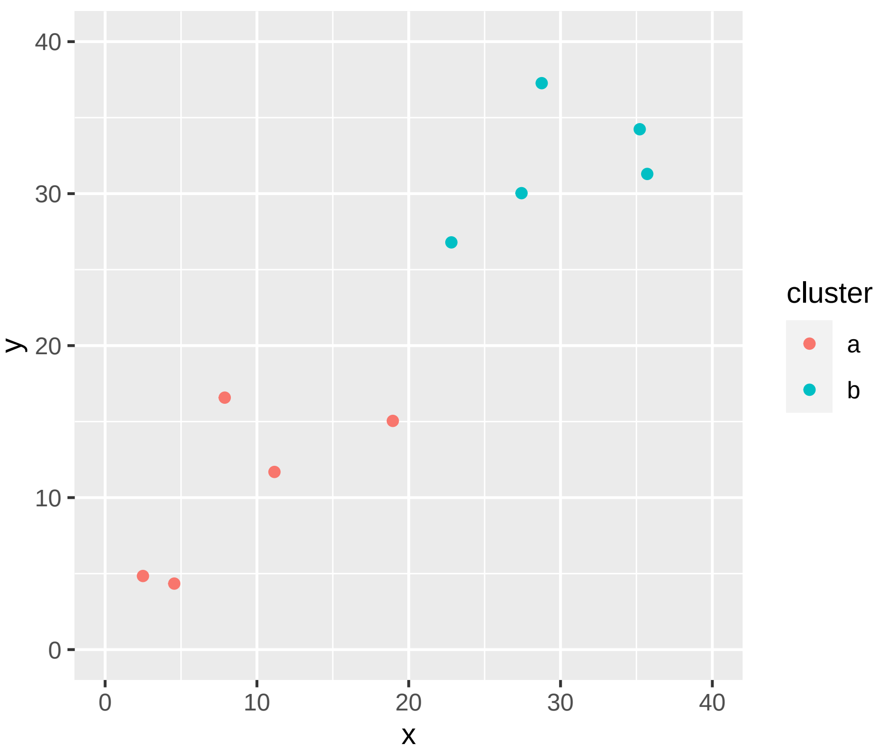
- Can define dissimilarities how you like (whatever makes sense in application).
- Sometimes defining "similarity" makes more sense; can turn this into dissimilarity by subtracting from some maximum.

- Example: numbers 1–10 in various European languages. Define similarity between two languages by counting how often the same number has a name starting with the same letter (and dissimilarity by how often number has names starting with different letter).
- Crude (doesn't even look at most of the words), but see how effective.

Two kinds of cluster analysis

- Looking at process of forming clusters (of similar languages): **hierarchical cluster analysis** (`hclust`).
- Start with each individual in cluster by itself.
- Join "closest" clusters one by one until all individuals in one cluster.
- How to define closeness of two *clusters*? Not obvious, investigate in a moment.
- Know how many clusters: which division into that many clusters is "best" for individuals? **K-means clustering** (`kmeans`).

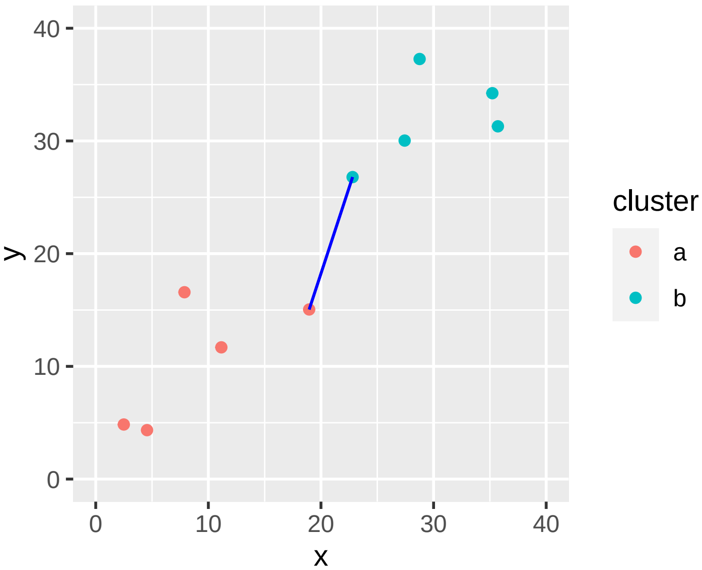
Two made-up clusters



How to measure distance between set of red points and set of blue ones?

Single-linkage distance

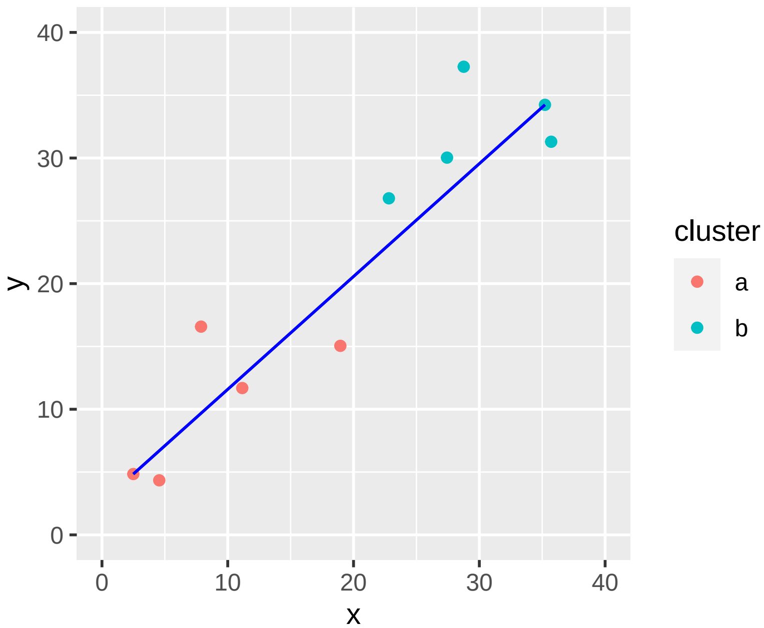
Find the red point and the blue point that are closest together:



Single-linkage distance between 2 clusters is distance between their closest points.

Complete linkage

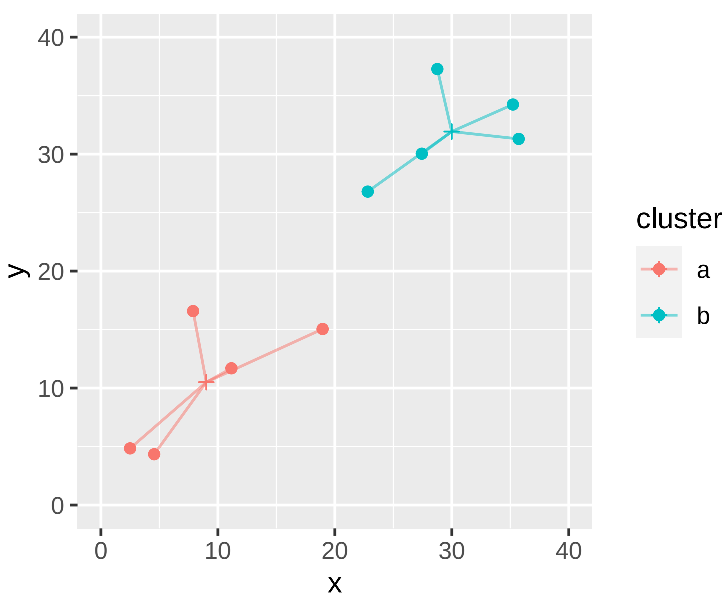
Find the red and blue points that are farthest apart:



Complete-linkage distance is distance between farthest points.

Ward's method

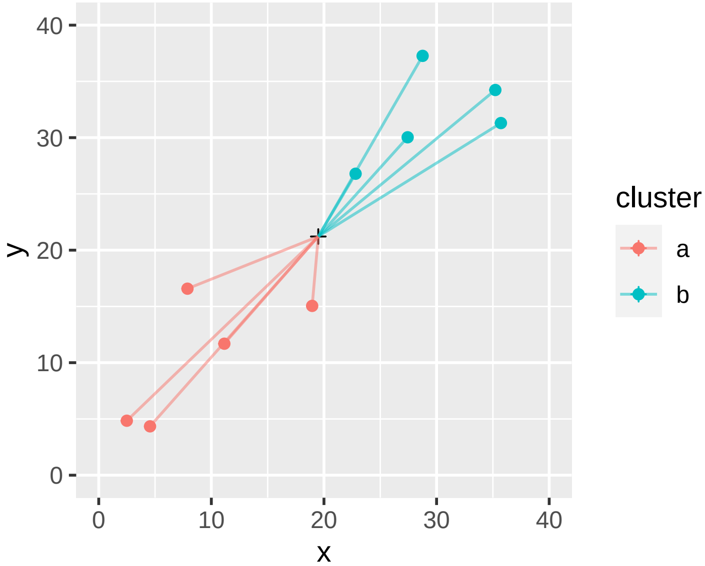
Work out mean of each cluster and join point to its mean:



Work out (i) sum of squared distances of points from means.

Ward's method part 2

Now imagine combining the two clusters and working out overall mean. Join each point to this mean:



Calc sum of squared distances (ii) of points to combined mean.

Ward's method part 3

- Sum of squares (ii) will be bigger than (i) (points closer to own cluster mean than combined mean).
- Ward's distance is (ii) minus (i).
- Think of as "cost" of combining clusters:
- if clusters close together, (ii) only a little larger than (i)
- if clusters far apart, (ii) a lot larger than (i) (as in example).

Hierarchical clustering revisited

- Single linkage, complete linkage, Ward are ways of measuring closeness of clusters.
- Use them, starting with each observation in own cluster, to repeatedly combine two closest clusters until all points in one cluster.
- They will give different answers (clustering stories).

- Single linkage tends to make “stringy” clusters because clusters can be very different apart from two closest points.
- Complete linkage insists on whole clusters being similar.
- Ward tends to form many small clusters first.

Dissimilarity data in R

Dissimilarities for language data were how many number names had *different* first letter:

```
my_url <- "http://ritsokiguess.site/datafiles/languages.txt"
(number.d <- read_table(my_url))
```

```
# A tibble: 11 x 12
   la       en    no    dk    nl    de    fr    es    it
   <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
 1 en        0     2     2     7     6     6     6     6
 2 no        2     0     1     5     4     6     6     6
 3 dk        2     1     0     6     5     6     5     5
 4 nl        7     5     6     0     5     9     9     9
 5 de        6     4     5     5     0     7     7     7
 6 fr        6     6     6     9     7     0     2     1
 7 es        6     6     5     9     7     2     0     1
 8 it        6     6     5     9     7     1     1     0
 9 pl        7     7     6    10     8     5     3     4
10 hu        9     8     8     8     9    10    10    10
11 fi        9     9     9     9     9     9     9     8
# i 3 more variables: pl <dbl>, hu <dbl>, fi <dbl>
```

Making a distance object

```
number.d %>%
  select(-la) %>%
  as.dist() -> d
d
```

```
   en no dk nl de fr es it pl hu
no  2
dk  2  1
nl  7  5  6
de  6  4  5  5
fr  6  6  6  9  7
```

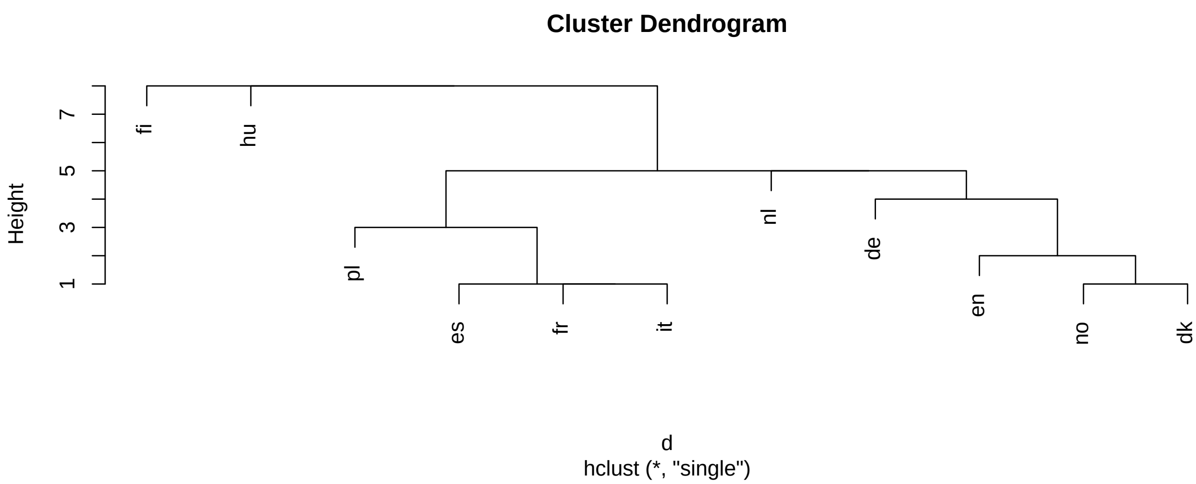
```
es  6  6  5  9  7  2
it  6  6  5  9  7  1  1
pl  7  7  6 10  8  5  3  4
hu  9  8  8  8  9 10 10 10 10
fi  9  9  9  9  9  9  9  8  9  8
```

```
class(d)
```

```
[1] "dist"
```

Cluster analysis and dendrogram

```
d.hc <- hclust(d, method = "single")
plot(d.hc)
```



Comments

- Tree shows how languages combined into clusters.
- First (bottom), Spanish, French, Italian joined into one cluster, Norwegian and Danish into another.
- Later, English joined to Norse languages, Polish to Romance group.
- Then German, Dutch make a Germanic group.
- Finally, Hungarian and Finnish joined to each other and everything else.

Clustering process

```
d.hc$labels
```

```
 [1] "en" "no" "dk" "nl" "de" "fr" "es" "it" "pl" "hu" "fi"
```

```
d.hc$merge
```

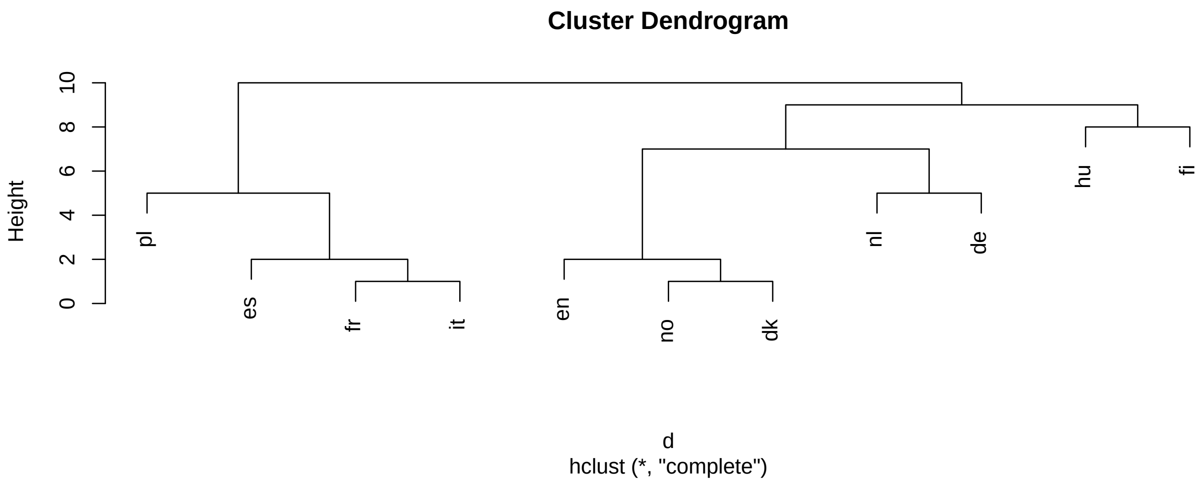
```
      [,1] [,2]
 [1,]   -2   -3
 [2,]   -6   -8
 [3,]   -7    2
 [4,]   -1    1
 [5,]   -9    3
 [6,]   -5    4
 [7,]   -4    6
 [8,]    5    7
 [9,]  -10    8
[10,]  -11    9
```

Comments

- Lines of `merge` show what was combined
  - First, languages 2 and 3 (`no` and `dk`)
  - Then languages 6 and 8 (`fr` and `it`)
  - Then #7 combined with cluster formed at step 2 (`es` joined to `fr` and `it`).
  - Then `en` joined to `no` and `dk` …
  - Finally `fi` joined to all others.

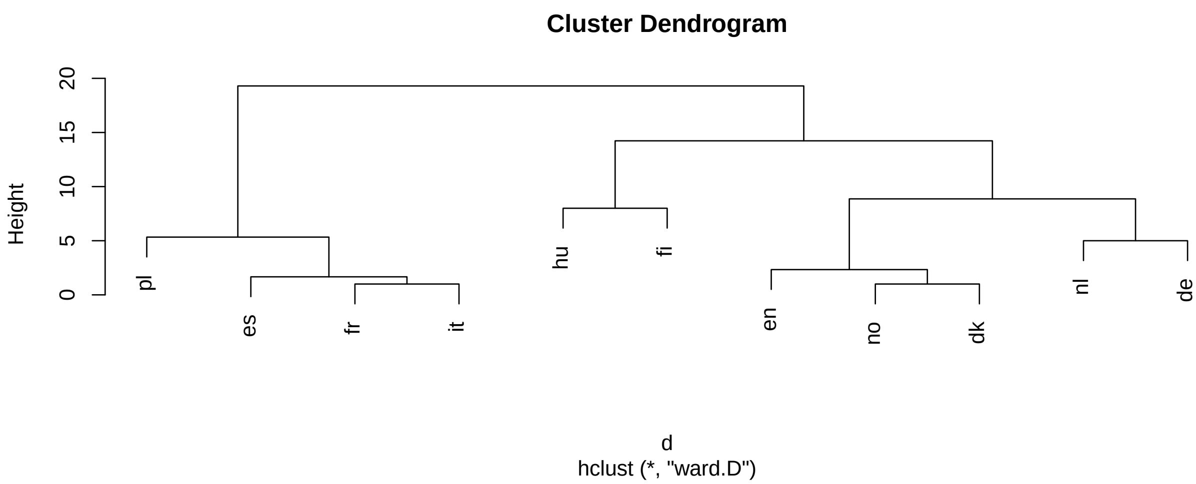
Complete linkage

```
d.hc <- hclust(d, method = "complete")
plot(d.hc)
```



Ward

```
d.hc <- hclust(d, method = "ward.D")
plot(d.hc)
```



Chopping the tree

- Three clusters (from Ward) looks good:

```
cutree(d.hc, 3)
```

```
en no dk nl de fr es it pl hu fi
 1  1  1  1  1  2  2  2  2  3  3
```

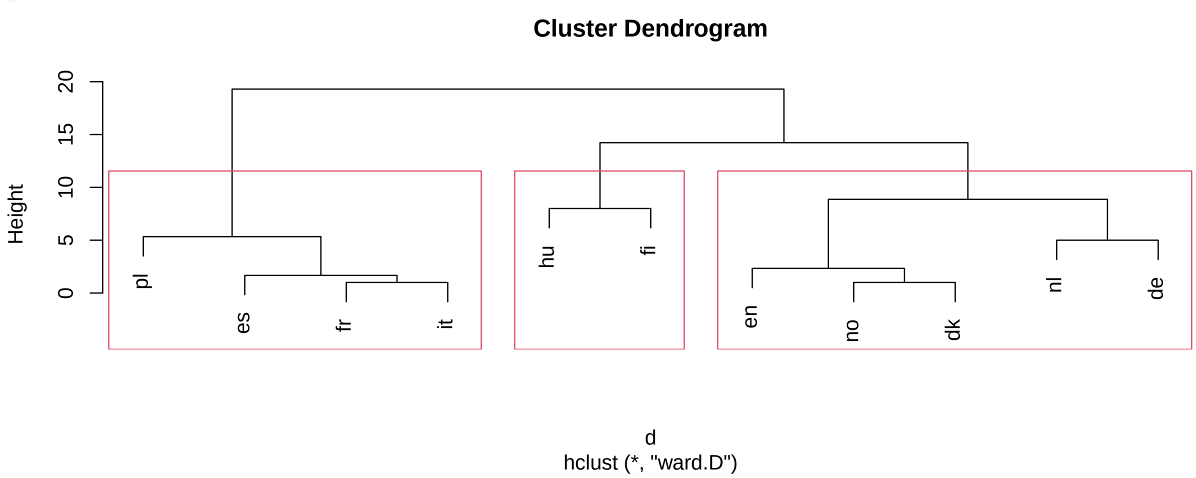
Turning the “named vector” into a data frame

```
cutree(d.hc, 3) %>% enframe(name="country", value="cluster")
```

```
# A tibble: 11 x 2
   country cluster
   <chr>     <int>
 1 en            1
 2 no            1
 3 dk            1
 4 nl            1
 5 de            1
 6 fr            2
 7 es            2
 8 it            2
 9 pl            2
10 hu            3
11 fi            3
```

Drawing those clusters on the tree

```
plot(d.hc)
rect.hclust(d.hc, 3)
```



Comparing single-linkage and Ward

- In Ward, Dutch and German get joined earlier (before joining to Germanic cluster).
- Also Hungarian and Finnish get combined earlier.

Making those dissimilarities

Original data:

```
my_url <- "http://ritsokiguess.site/datafiles/one-ten.txt"
lang <- read_delim(my_url, " ")
lang
```

```
# A tibble: 10 x 11
   en    no    dk    nl    de     fr     es     it     pl
   <chr> <chr> <chr> <chr> <chr>  <chr>  <chr>  <chr>  <chr>
 1 one   en    en    een   eins   un     uno    uno    jeden
 2 two   to    to    twee  zwei   deux   dos    due    dwa
 3 three tre   tre   drie  drei   trois  tres   tre    trzy
 4 four  fire  fire  vier  vier   quatre cuatro quatt~ czte~
 5 five  fem   fem   vijf  funf   cinq   cinco  cinque piec
 6 six   seks  seks  zes   sechs  six    seis   sei    szesc
 7 seven sju   syv   zeven sieben sept   siete  sette  sied~
 8 eight atte  otte  acht  acht   huit   ocho   otto   osiem
 9 nine  ni    ni    negen neun   neuf   nueve  nove   dzie~
10 ten   ti    ti    tien  zehn   dix    diez   dieci  dzie~
# i 2 more variables: hu <chr>, fi <chr>
```

It would be a lot easier to extract the first letter if the number names were all in one column.

Tidy, and extract first letter

```
lang %>% mutate(number=row_number()) %>%
    pivot_longer(-number, names_to="language", values_to="name") %>%
    mutate(first=str_sub(name, 1, 1)) -> lang.long
lang.long
```

```
# A tibble: 110 x 4
   number language name  first
    <int> <chr>    <chr> <chr>
 1      1 en       one   o
 2      1 no       en    e
 3      1 dk       en    e
 4      1 nl       een   e
 5      1 de       eins  e
 6      1 fr       un    u
 7      1 es       uno   u
 8      1 it       uno   u
 9      1 pl       jeden j
10      1 hu       egy   e
# i 100 more rows
```

Calculating dissimilarity

- Suppose we wanted dissimilarity between English and Norwegian. It's the number of first letters that are different.
- First get the lines for English:

```
english <- lang.long %>% filter(language == "en")
english
```

```
# A tibble: 10 x 4
   number language name  first
    <int> <chr>    <chr> <chr>
 1      1 en       one   o
 2      2 en       two   t
 3      3 en       three t
 4      4 en       four  f
 5      5 en       five  f
 6      6 en       six   s
 7      7 en       seven s
 8      8 en       eight e
 9      9 en       nine  n
10     10 en       ten   t
```

And then the lines for Norwegian

```
norwegian <- lang.long %>% filter(language == "no")
norwegian
```

```
# A tibble: 10 x 4
   number language name  first
    <int> <chr>    <chr> <chr>
 1      1 no       en    e
 2      2 no       to    t
 3      3 no       tre   t
 4      4 no       fire  f
 5      5 no       fem   f
 6      6 no       seks  s
 7      7 no       sju   s
 8      8 no       atte  a
 9      9 no       ni    n
10     10 no       ti    t
```

And now we want to put them side by side, matched by number. This is what `left_join` does. (A "join" is a lookup of values in one table using another.)

The join

```
english %>% left_join(norwegian, by = "number")
```

```
# A tibble: 10 x 7
   number language.x name.x first.x language.y name.y first.y
    <int> <chr>      <chr>  <chr>   <chr>      <chr>  <chr>
 1      1 en         one    o       no         en     e
 2      2 en         two    t       no         to     t
 3      3 en         three  t       no         tre    t
 4      4 en         four   f       no         fire   f
 5      5 en         five   f       no         fem    f
 6      6 en         six    s       no         seks   s
 7      7 en         seven  s       no         sju    s
 8      8 en         eight  e       no         atte   a
 9      9 en         nine   n       no         ni     n
10     10 en         ten    t       no         ti     t
```

`first.x` is 1st letter of English word, `first.y` 1st letter of Norwegian word.

Counting the different ones

```
english %>%
  left_join(norwegian, by = "number") %>%
  count(different=(first.x != first.y))
```

```
# A tibble: 2 x 2
  different     n
  <lgl>     <int>
1 FALSE         8
2 TRUE          2
```

or

```
english %>%
  left_join(norwegian, by = "number") %>%
  count(different=(first.x != first.y)) %>%
  filter(different) %>% pull(n) -> ans
ans
```

```
[1] 2
```

Words for 1 and 8 start with different letter; rest are same.

A language with itself

The answer should be zero:

```
english %>%
  left_join(english, by = "number") %>%
  count(different=(first.x != first.y)) %>%
  filter(different) %>% pull(n) -> ans
ans
```

```
integer(0)
```

- but this is "an integer vector of length zero".
- so we have to allow for this possibility when we write a function to do it.

Function to do this for any two languages

```
countdiff <- function(lang.1, lang.2, d) {
  d %>% filter(language == lang.1) -> lang1d
  d %>% filter(language == lang.2) -> lang2d
  lang1d %>%
    left_join(lang2d, by = "number") %>%
    count(different = (first.x != first.y)) %>%
    filter(different) %>% pull(n) -> ans
  # if ans has length zero, set answer to (integer) zero.
  ifelse(length(ans)==0, 0L, ans)
}
```

Testing

```
countdiff("en", "no", lang.long)
```

```
[1] 2
```

```
countdiff("en", "en", lang.long)
```

```
[1] 0
```

English and Norwegian have two different; English and English have none different.

Check.

For all pairs of languages?

- First need all the languages:

```
languages <- names(lang)
languages
```

```
 [1] "en" "no" "dk" "nl" "de" "fr" "es" "it" "pl"
[10] "hu" "fi"
```

- and then all *pairs* of languages:

```
pairs <- crossing(lang = languages, lang2 = languages)
```

The pairs

```
pairs
```

```
# A tibble: 121 x 2
   lang  lang2
   <chr> <chr>
 1 de    de   
 2 de    dk   
 3 de    en   
 4 de    es   
 5 de    fi   
 6 de    fr   
 7 de    hu   
 8 de    it   
 9 de    nl   
10 de    no   
# i 111 more rows
```

Run `countdiff` for all those language pairs

```
pairs %>% rowwise() %>%
  mutate(diff = countdiff(lang, lang2, lang.long)) -> thediff
thediff
```

```
# A tibble: 121 x 3
# Rowwise:
   lang  lang2  diff
   <chr> <chr> <int>
 1 de    de        0
 2 de    dk        5
 3 de    en        6
 4 de    es        7
 5 de    fi        9
 6 de    fr        7
 7 de    hu        9
 8 de    it        7
 9 de    nl        5
10 de    no        4
# i 111 more rows
```

Make square table of these

```
thediff %>% pivot_wider(names_from=lang2, values_from=diff)
```

```
# A tibble: 11 x 12
   lang     de    dk    en    es    fi    fr    hu    it
   <chr> <int> <int> <int> <int> <int> <int> <int> <int>
 1 de        0     5     6     7     9     7     9     7
 2 dk        5     0     2     5     9     6     8     5
 3 en        6     2     0     6     9     6     9     6
 4 es        7     5     6     0     9     2    10     1
 5 fi        9     9     9     9     0     9     8     9
 6 fr        7     6     6     2     9     0    10     1
 7 hu        9     8     9    10     8    10     0    10
 8 it        7     5     6     1     9     1    10     0
 9 nl        5     6     7     9     9     9     8     9
10 no        4     1     2     6     9     6     8     6
11 pl        8     6     7     3     9     5    10     4
# i 3 more variables: nl <int>, no <int>, pl <int>
```

and that was where we began.

Another example

Birth, death and infant mortality rates for 97 countries (variables not dissimilarities):

```
24.7  5.7  30.8 Albania         12.5 11.9  14.4 Bulgaria
13.4 11.7  11.3 Czechoslovakia  12   12.4   7.6 Former_E._Germany
11.6 13.4  14.8 Hungary         14.3 10.2    16 Poland
13.6 10.7  26.9 Romania           14    9  20.2 Yugoslavia
17.7   10    23 USSR            15.2  9.5  13.1 Byelorussia_SSR
```

```
13.4 11.6    13 Ukrainian_SSR   20.7  8.4  25.7 Argentina
46.6   18   111 Bolivia         28.6  7.9    63 Brazil
23.4  5.8  17.1 Chile           27.4  6.1    40 Columbia
32.9  7.4    63 Ecuador         28.3  7.3    56 Guyana
...
```

- Want to find groups of similar countries (and how many groups, which countries in each group).
- Tree would be unwieldy with 97 countries.
- More automatic way of finding given number of clusters?

Reading in

```
url <- "http://ritsokiguess.site/datafiles/birthrate.txt"
vital <- read_table(url)
vital
```

```
# A tibble: 97 x 4
   birth death infant country
   <dbl> <dbl>  <dbl> <chr>
 1  24.7   5.7   30.8 Albania
 2  13.4  11.7   11.3 Czechoslovakia
 3  11.6  13.4   14.8 Hungary
 4  13.6  10.7   26.9 Romania
 5  17.7  10     23   USSR
 6  13.4  11.6   13   Ukrainian_SSR
 7  46.6  18    111   Bolivia
 8  23.4   5.8   17.1 Chile
 9  32.9   7.4   63   Ecuador
10  34.8   6.6   42   Paraguay
# i 87 more rows
```

Standardizing

- Infant mortality rate numbers bigger than others, consequence of measurement scale (arbitrary).
- Standardize (numerical) columns of data frame to have mean 0, SD 1, done by `scale`.

```
vital %>%
  mutate(across(where(is.numeric), \(x) scale(x))) -> vital.s
```

Three clusters

Pretend we know 3 clusters is good. Take off the column of countries, and run `kmeans` on the resulting data frame, asking for 3 clusters:

```
vital.s %>% select(-country) %>%
  kmeans(3) -> vital.km3
names(vital.km3)
```

```
[1] "cluster"      "centers"      "totss"
[4] "withinss"     "tot.withinss" "betweenss"
[7] "size"         "iter"         "ifault"
```

A lot of output, so look at these individually.

What's in the output?

- Cluster sizes:

```
vital.km3$size
```

```
[1] 40 25 32
```

- Cluster centres:

```
vital.km3$centers
```

```
       birth      death      infant
1 -1.0376994 -0.3289046 -0.90669032
2  1.1780071  1.3323130  1.32732200
3  0.3768062 -0.6297388  0.09639258
```

- Cluster 2 has lower than average rates on everything; cluster 3 has much higher than average.

Cluster sums of squares and membership

```
vital.km3$withinss
```

```
[1] 17.21617 28.32560 21.53020
```

Cluster 1 compact relative to others (countries in cluster 1 more similar).

```
vital.km3$cluster
```

```
 [1] 3 1 1 1 1 1 2 1 3 3 1 2 1 1 1 1 1 1 1 1 1 2 2 1 3 3 3 2
[29] 1 3 1 3 3 1 1 3 3 3 2 2 3 3 2 2 3 2 2 2 3 1 1 1 1 1 1 3
[57] 3 3 3 3 1 1 1 1 1 1 1 1 1 1 3 3 3 3 3 1 2 1 3 3 2 3 1 3
[85] 2 2 2 2 3 2 2 2 2 2 3 2 2
```

The cluster membership for each of the 97 countries.

Store countries and clusters to which they belong

```
vital.3 <- tibble(
  country = vital.s$country,
  cluster = vital.km3$cluster
)
```

Next, which countries in which cluster?

Write function to extract them:

```
get_countries <- function(i, d) {
  d %>% filter(cluster == i) %>% pull(country)
}
```

Cluster membership: cluster 2

```
get_countries(2, vital.3)
```

```
 [1] "Bolivia"     "Mexico"      "Afghanistan"
 [4] "Iran"        "Bangladesh"  "Gabon"
 [7] "Ghana"       "Namibia"     "Sierra_Leone"
[10] "Swaziland"   "Uganda"      "Zaire"
[13] "Cambodia"    "Nepal"       "Angola"
[16] "Congo"       "Ethiopia"    "Gambia"
[19] "Malawi"      "Mozambique"  "Nigeria"
[22] "Somalia"     "Sudan"       "Tanzania"
[25] "Zambia"
```

Cluster 3

```
get_countries(3, vital.3)
```

```
 [1] "Albania"      "Ecuador"      "Paraguay"
 [4] "Kuwait"       "Oman"         "Turkey"
 [7] "India"        "Mongolia"     "Pakistan"
[10] "Algeria"      "Botswana"     "Egypt"
[13] "Libya"        "Morocco"      "South_Africa"
[16] "Zimbabwe"     "Brazil"       "Columbia"
[19] "Guyana"       "Peru"         "Venezuela"
[22] "Bahrain"      "Iraq"         "Jordan"
[25] "Lebanon"      "Saudi_Arabia" "Indonesia"
[28] "Malaysia"     "Philippines"  "Vietnam"
[31] "Kenya"        "Tunisia"
```

Cluster 1

```
get_countries(1, vital.3)
```

```
 [1] "Czechoslovakia"       "Hungary"
 [3] "Romania"              "USSR"
 [5] "Ukrainian_SSR"        "Chile"
 [7] "Uruguay"              "Finland"
 [9] "France"               "Greece"
[11] "Italy"                "Norway"
[13] "Spain"                "Switzerland"
[15] "Austria"              "Canada"
[17] "Israel"               "China"
[19] "Korea"                "Singapore"
[21] "Thailand"             "Bulgaria"
```

```
[23] "Former_E._Germany"    "Poland"
[25] "Yugoslavia"           "Byelorussia_SSR"
[27] "Argentina"            "Belgium"
[29] "Denmark"              "Germany"
[31] "Ireland"              "Netherlands"
[33] "Portugal"             "Sweden"
[35] "U.K."                 "Japan"
[37] "U.S.A."               "United_Arab_Emirates"
[39] "Hong_Kong"            "Sri_Lanka"
```

Problem!

- `kmeans` uses randomization. So result of one run might be different from another run.
- Example: just run again on 3 clusters, `table` of results:

```
vital.s %>%
  select(-country) %>% kmeans(3) -> vital.km3a
table(
  first = vital.km3$cluster,
  second = vital.km3a$cluster
)
```

```
     second
first  1  2  3
    1 40  0  0
    2  0 24  1
    3  4  0 28
```

- Clusters are similar but *not same*.

Solution to this

- `nstart` option on `kmeans` runs that many times, takes best. Should be same every time:

```
vital.s %>%
  select(-country) %>%
  kmeans(3, nstart = 20) -> vital.km3b
```

How many clusters?

- Three was just a guess.
- Idea: try a whole bunch of #clusters (say 2–20), obtain measure of goodness of fit for each, make plot.
- Appropriate measure is `tot.withinss`.
- Run `kmeans` for each #clusters, get `tot.withinss` each time.

Function to get `tot.withinss`

…for an input number of clusters, taking only numeric columns of input data frame:

```
ss <- function(i, d) {
  d %>%
    select(where(is.numeric)) %>%
    kmeans(i, nstart = 20) -> km
  km$tot.withinss
}
```

Note: writing function to be as general as possible, so that we can re-use it later.

Constructing within-cluster SS

Make a data frame with desired numbers of clusters, and fill it with the total within-group sums of squares. `ss` expects a single number of clusters, not a vector of several, so run `rowwise`:

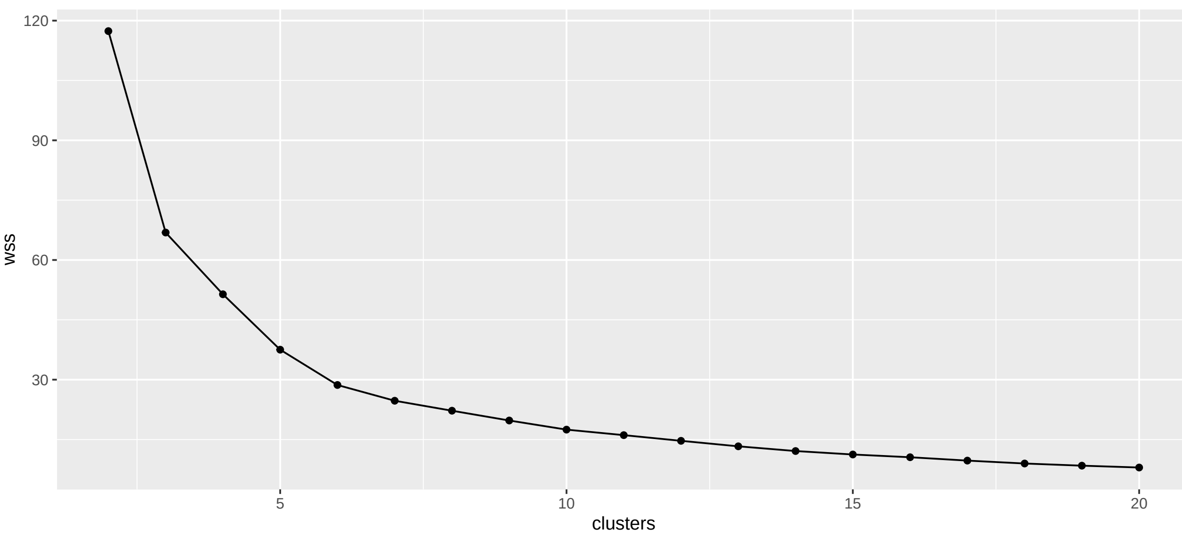
```
tibble(clusters = 2:20) %>%
  rowwise() %>%
  mutate(wss = ss(clusters, vital.s)) -> ssd
ssd
```

```
# A tibble: 19 x 2
# Rowwise:
   clusters    wss
      <int>  <dbl>
 1        2 117.
 2        3  66.9
 3        4  51.4
 4        5  37.5
```

```
5          6  28.7
6          7  24.7
7          8  22.2
8          9  19.8
9         10  17.5
10        11  16.1
11        12  14.7
12        13  13.3
13        14  12.1
14        15  11.2
15        16  10.5
16        17   9.71
17        18   8.98
18        19   8.44
19        20   7.97
```

Scree plot

```r
ggplot(ssd, aes(x = clusters, y = wss)) + geom_point() +
  geom_line()
```



Interpreting scree plot

- Lower `wss` better.

- But lower for larger #clusters, harder to explain.
- Compromise: low-ish `wss` and low-ish #clusters.
- Look for "elbow" in plot.
- Idea: this is where `wss` decreases fast then slow.
- On our plot, small elbow at 6 clusters. Try this many clusters.

Six clusters, using `nstart`

```
set.seed(457299)
```

```
vital.s %>%
  select(-country) %>%
  kmeans(6, nstart = 20) -> vital.km6
vital.km6$size
```

```
[1] 24 18 15  2  8 30
```

```
vital.km6$centers
```

```
       birth      death     infant
1  0.4160993 -0.5169988  0.2648754
2  1.2092406  0.7441347  1.0278003
3 -0.4357690 -1.1438599 -0.7281108
4 -0.2199722  2.1116577 -0.4544435
5  1.3043848  2.1896567  1.9470306
6 -1.1737104 -0.1856375 -0.9534370
```

Make a data frame of countries and clusters

```
vital.6 <- tibble(
  country = vital.s$country,
  cluster = vital.km6$cluster
)
vital.6 %>% sample_n(10)
```

```
# A tibble: 10 x 2
   country        cluster
   <chr>            <int>
 1 Ghana                2
 2 Ukrainian_SSR        6
 3 Ethiopia             5
 4 Somalia              5
 5 Oman                 1
 6 Botswana             2
 7 Paraguay             1
 8 Czechoslovakia       6
 9 Peru                 1
10 Afghanistan          5
```

Cluster 1

Below-average death rate, though other rates a little higher than average:

```
get_countries(1, vital.6)
```

```
 [1] "Ecuador"      "Paraguay"     "Oman"
 [4] "Turkey"       "India"        "Mongolia"
 [7] "Pakistan"     "Algeria"      "Egypt"
[10] "Libya"        "Morocco"      "South_Africa"
[13] "Zimbabwe"     "Brazil"       "Guyana"
[16] "Peru"         "Iraq"         "Jordan"
[19] "Lebanon"      "Saudi_Arabia" "Indonesia"
[22] "Philippines"  "Vietnam"      "Tunisia"
```

Cluster 2

High on everything:

```
get_countries(2, vital.6)
```

```
 [1] "Bolivia"    "Iran"       "Bangladesh" "Botswana"
 [5] "Gabon"      "Ghana"      "Namibia"    "Swaziland"
 [9] "Uganda"     "Zaire"      "Cambodia"   "Nepal"
[13] "Congo"      "Kenya"      "Nigeria"    "Sudan"
[17] "Tanzania"   "Zambia"
```

Cluster 3

Low on everything:

```
get_countries(3, vital.6)
```

```
 [1] "Albania"              "Chile"
 [3] "Israel"               "Kuwait"
 [5] "China"                "Singapore"
 [7] "Thailand"             "Argentina"
 [9] "Columbia"             "Venezuela"
[11] "Bahrain"              "United_Arab_Emirates"
[13] "Hong_Kong"            "Malaysia"
[15] "Sri_Lanka"
```

Cluster 4

Very high death rate, just below average on all else:

```
get_countries(4, vital.6)
```

```
[1] "Mexico" "Korea"
```

Cluster 5

Very high on everything:

```
get_countries(5, vital.6)
```

```
[1] "Afghanistan"  "Sierra_Leone" "Angola"
[4] "Ethiopia"     "Gambia"       "Malawi"
[7] "Mozambique"   "Somalia"
```

Cluster 6

A bit below average on everything:

```
get_countries(6, vital.6)
```

```
 [1] "Czechoslovakia"    "Hungary"
 [3] "Romania"           "USSR"
 [5] "Ukrainian_SSR"     "Uruguay"
 [7] "Finland"           "France"
 [9] "Greece"            "Italy"
[11] "Norway"            "Spain"
[13] "Switzerland"       "Austria"
[15] "Canada"            "Bulgaria"
[17] "Former_E._Germany" "Poland"
[19] "Yugoslavia"        "Byelorussia_SSR"
[21] "Belgium"           "Denmark"
[23] "Germany"           "Ireland"
[25] "Netherlands"       "Portugal"
[27] "Sweden"            "U.K."
[29] "Japan"             "U.S.A."
```

Comparing our 3 and 6-cluster solutions

```
table(three = vital.km3$cluster, six = vital.km6$cluster)
```

```
     six
three  1  2  3  4  5  6
    1  0  0  9  1  0 30
    2  0 16  0  1  8  0
    3 24  2  6  0  0  0
```

Compared to 3-cluster solution:

- most of (old) cluster 1 gone to (new) cluster 6
- cluster 2 split into clusters 2 and 5 (two types of "poor" countries)
- cluster 3 split into clusters 1 and 3 (two types of "intermediate" countries, divided by death rate).

Getting a picture from `kmeans`

- Use discriminant analysis on clusters found, treating them as "known" groups.

Discriminant analysis

- So what makes the groups different?
- Uses package MASS (loaded):

```
vital.lda <- lda(vital.km6$cluster ~ birth + death + infant,
                 data = vital.s)
vital.lda$svd
```

```
[1] 21.687195  8.851811  1.773006
```

```
vital.lda$scaling
```

```
             LD1        LD2        LD3
birth  2.6879695  1.1224202 -1.9483853
death  0.6652712 -2.7213044 -0.6049358
infant 2.1111801  0.7650912  2.3542296
```

- LD1 is some of everything (high=poor, low=rich).
- LD2 mainly death rate, high or low.

A data frame to make plot from

- Get predictions first:

```
vital.pred <- predict(vital.lda)
d <- data.frame(
  country = vital.s$country,
  cluster = vital.km6$cluster,
  vital.pred$x
)
d
```

```
                country cluster         LD1
1               Albania       3 -2.74034473
2        Czechoslovakia       6 -5.01874312
3               Hungary       6 -4.97189595
4               Romania       6 -4.40612396
5                  USSR       6 -3.87181416
6          Ukrainian_SSR       6 -4.95502329
7               Bolivia       2  7.04719692
```

8	Chile	3	-3.61284528
9	Ecuador	1	0.60813286
10	Paraguay	1	-0.09333631
11	Uruguay	6	-3.92003877
12	Mexico	4	1.17794263
13	Finland	6	-5.53992595
14	France	6	-5.48731510
15	Greece	6	-6.04517374
16	Italy	6	-6.23984314
17	Norway	6	-5.14396822
18	Spain	6	-6.20238403
19	Switzerland	6	-5.70503604
20	Austria	6	-5.48811665
21	Canada	6	-5.61852237
22	Afghanistan	5	9.15790453
23	Iran	2	5.17009665
24	Israel	3	-4.09921633
25	Kuwait	3	-3.52238895
26	Oman	1	2.12959716
27	Turkey	1	0.61384977
28	Bangladesh	2	6.18349326
29	China	3	-3.23659422
30	India	1	1.81800150
31	Korea	4	-1.46967404
32	Mongolia	1	1.67300272
33	Pakistan	1	2.24428372
34	Singapore	3	-5.25056428
35	Thailand	3	-3.05879372
36	Algeria	1	1.75779182
37	Botswana	2	4.48834877
38	Egypt	1	1.45515716
39	Gabon	2	5.07955834
40	Ghana	2	4.94529597
41	Libya	1	3.96906505
42	Morocco	1	2.33973207
43	Namibia	2	6.78839862
44	Sierra_Leone	5	10.11147508
45	South_Africa	1	1.22038392
46	Swaziland	2	6.62089564
47	Uganda	2	7.44759122
48	Zaire	2	5.01954548
49	Zimbabwe	1	2.90708292
50	Bulgaria	6	-5.02639603
51	Former_E._Germany	6	-5.36617200
52	Poland	6	-4.83913984
53	Yugoslavia	6	-4.87765104
54	Byelorussia_SSR	6	-4.89387951
55	Argentina	3	-3.38164344

```
56               Brazil        1 -0.17351090
57             Columbia        3 -1.72504134
58               Guyana        1 -0.64024371
59                 Peru        1  2.88979728
60            Venezuela        3 -2.71512137
61              Belgium        6 -5.61006438
62              Denmark        6 -5.36296636
63              Germany        6 -5.66618148
64              Ireland        6 -5.22803491
65          Netherlands        6 -5.69497185
66             Portugal        6 -5.54867384
67               Sweden        6 -5.14801099
68                 U.K.        6 -5.14080545
69                Japan        6 -6.74109078
70               U.S.A.        6 -4.98026088
71              Bahrain        3 -2.95751794
72                 Iraq        1  2.86550554
73               Jordan        1  0.78337204
74              Lebanon        1 -0.13242340
75         Saudi_Arabia        1  2.82947028
76 United_Arab_Emirates        3 -3.60965788
77             Cambodia        2  6.68714487
78            Hong_Kong        3 -6.56814921
79            Indonesia        1  0.59203984
80             Malaysia        3 -1.69768174
81                Nepal        2  5.98051592
82          Philippines        1 -0.11564367
83            Sri_Lanka        3 -3.86669807
84              Vietnam        1  0.73637772
85               Angola        5  8.67464018
86                Congo        2  4.71698904
87             Ethiopia        5  9.02400482
88               Gambia        5  9.16151584
89                Kenya        2  4.37728422
90               Malawi        5  9.25868839
91           Mozambique        5  8.17837256
92              Nigeria        2  6.80523311
93              Somalia        5  9.02055241
94                Sudan        2  6.19772237
95              Tunisia        1 -0.26827112
96             Tanzania        2  7.01894686
97               Zambia        2  5.90158841
           LD2          LD3
1   2.23114272  0.086392118
2  -2.54276395 -0.067491502
3  -3.62910309  0.149274227
4  -1.68114304  0.832426535
5  -0.99643221  0.134219177
```

6	-2.45593006	0.032543051
7	-1.82235340	-0.559232315
8	1.83697531	-0.440912763
9	2.45078735	0.333955737
10	2.72730980	-0.910115449
11	-0.75565723	0.086830641
12	-7.45661911	-2.185449656
13	-1.71396053	-0.111993227
14	-1.24432254	0.003489983
15	-1.35732249	0.717192252
16	-1.36850774	0.675127833
17	-1.94087447	-0.245927314
18	-0.77030865	0.612616941
19	-1.39900851	0.133327537
20	0.04445534	0.107554431
21	0.05655923	0.137152032
22	-1.57150002	3.855195867
23	1.59572127	0.728082136
24	1.32996325	-0.726569567
25	4.20168393	-0.538116615
26	2.88622987	-2.722021794
27	1.77493550	1.401386127
28	-0.58998184	0.808514931
29	1.37556784	0.521048584
30	1.07819827	1.747922987
31	-5.22549147	-1.651932383
32	1.97934176	-0.052585403
33	2.56907200	2.904858080
34	1.56461210	-0.048777986
35	0.81462641	0.027926012
36	2.32221009	0.405915880
37	1.35059836	-2.251690523
38	1.48375960	-1.484115076
39	-1.84934413	0.223023990
40	0.51518412	-0.679939548
41	2.51546738	-0.550299598
42	1.57697751	0.620167354
43	1.81616353	1.811177322
44	-4.13640327	0.708806858
45	1.07036311	0.584292394
46	1.53114576	0.486213522
47	-0.08614119	-1.461769491
48	-0.14593127	-1.354022712
49	1.53174980	-1.155638673
50	-2.68287368	0.194600531
51	-3.13019129	-0.146640390
52	-1.51169554	0.238889303
53	-0.76403413	0.653220196

54	-1.07548768	0.052116630
55	0.23391708	0.049204615
56	1.80173696	0.887331423
57	2.37367896	0.116914571
58	2.01175992	0.650267842
59	2.70398544	2.617485939
60	3.09958020	-0.531015134
61	-2.07122472	0.103010698
62	-2.80594142	-0.144208119
63	-2.48058100	0.085615217
64	-0.94271289	-0.168083173
65	-0.81402153	0.149795861
66	-1.34891130	0.526746892
67	-2.19511746	-0.439370072
68	-2.45732616	-0.218666957
69	-0.01816615	0.738650841
70	-0.19798546	-0.186143629
71	3.40403941	-0.956027685
72	3.12008079	-0.806112179
73	3.21739659	-1.371400507
74	1.34062918	-0.430471811
75	3.22903162	-0.605791417
76	3.10639879	0.361277059
77	-1.11737674	1.343455301
78	1.21156621	0.795955999
79	1.12304477	1.306331526
80	2.74828222	-1.241072895
81	-0.24581105	1.734265161
82	2.00054960	-0.669610455
83	1.46702228	-0.073210404
84	1.14664253	0.270008884
85	-2.62832040	0.398976877
86	-0.50507133	-1.989873503
87	-2.80509362	0.132536656
88	-3.21458937	0.521137723
89	1.48515306	-1.740964552
90	-5.46422766	-0.742329819
91	-1.74864060	1.141424236
92	-0.35943688	-0.827233974
93	-2.47121452	-0.274058230
94	-0.74977705	-0.138778600
95	2.17721533	0.042802627
96	0.75977710	-0.855439352
97	0.55264060	-2.233552693

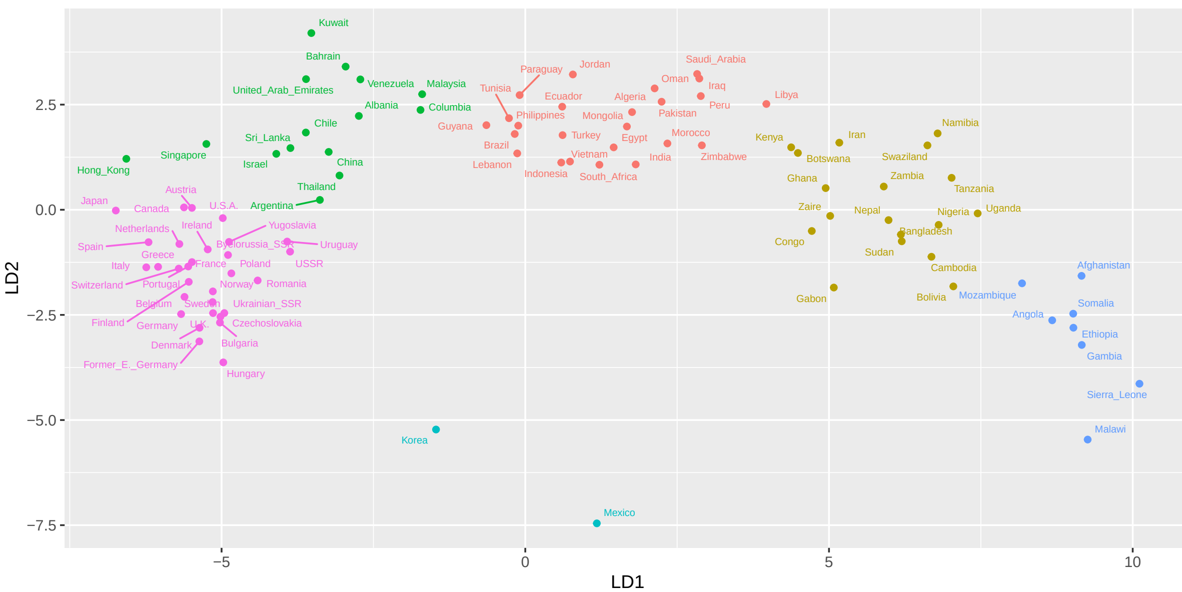
What’s in there; making a plot

- `d` contains country names, cluster memberships and discriminant scores.
- Plot `LD1` against `LD2`, colouring points by cluster and labelling by country:

```
g <- ggplot(d, aes(
  x = LD1, y = LD2, colour = factor(cluster),
  label = country
)) + geom_point() +
  geom_text_repel(size = 2, max.overlaps = Inf) + guides(colour = "none")
```

The plot

```
g
```



It would be better to zoom in on parts of this plot.

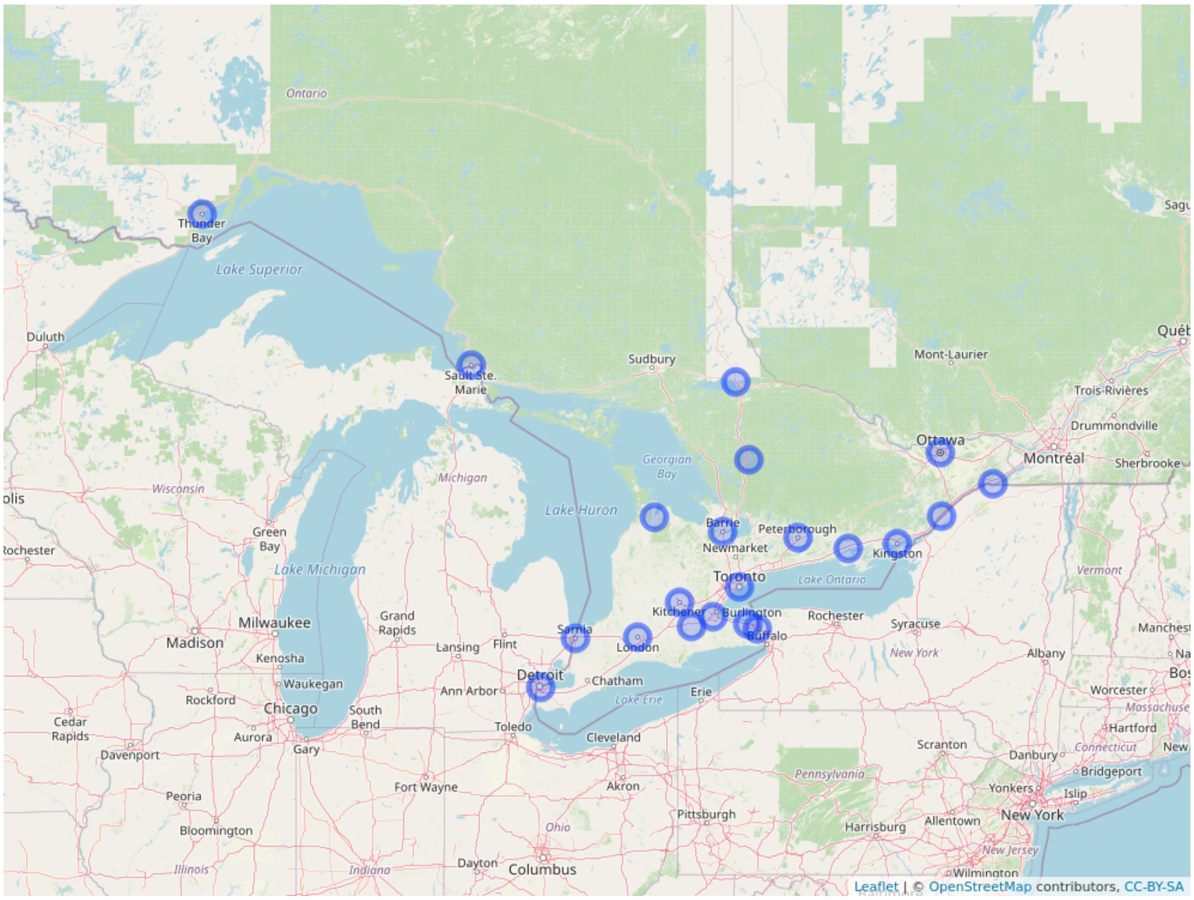
Final example: a hockey league

- An Ontario hockey league has teams in 21 cities. How can we arrange those teams into 4 geographical divisions?

- Distance data in spreadsheet.

- Take out spaces in team names.
- Save as "text/csv".
- Distances, so back to `hclust`.

A map

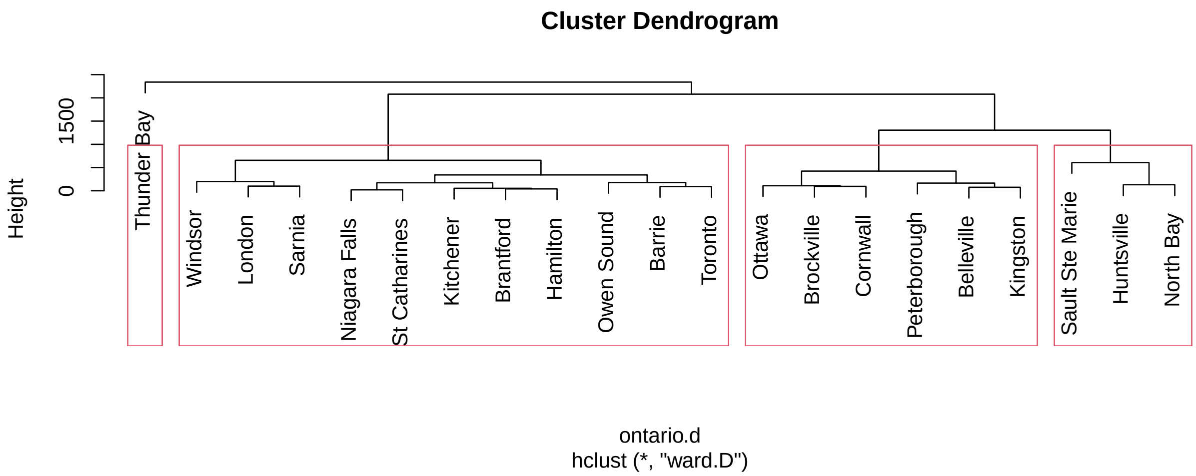


Attempt 1

```
my_url <-
  "http://ritsokiguess.site/datafiles/ontario-road-distances.csv"
ontario <- read_csv(my_url)
ontario.d <- ontario %>% select(-1) %>% as.dist()
ontario.hc <- hclust(ontario.d, method = "ward.D")
```

Plot, with 4 clusters

```
plot(ontario.hc)
rect.hclust(ontario.hc, 4)
```

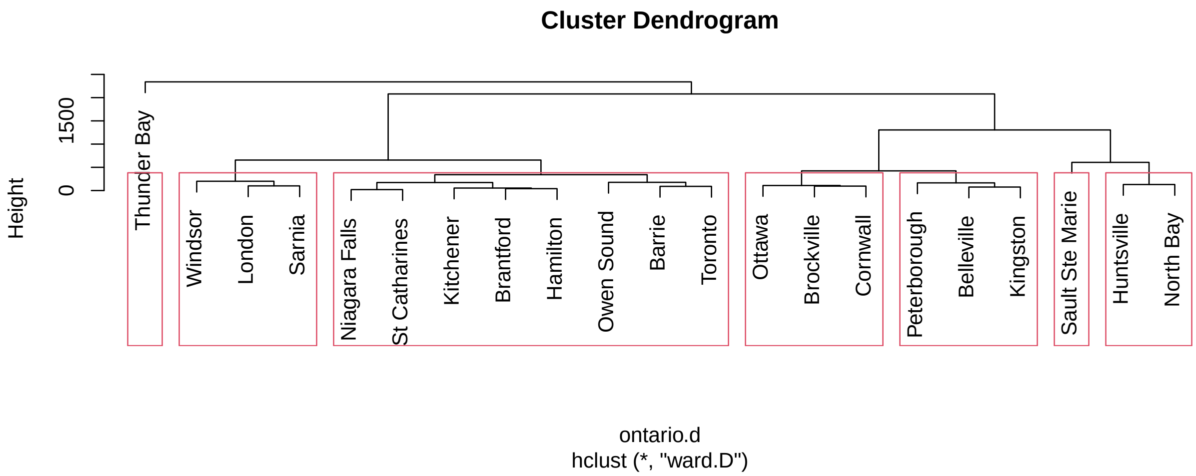


Comments

- Can't have divisions of 1 team!
- "Southern" divisions way too big!
- Try splitting into more. I found 7 to be good:

Seven clusters

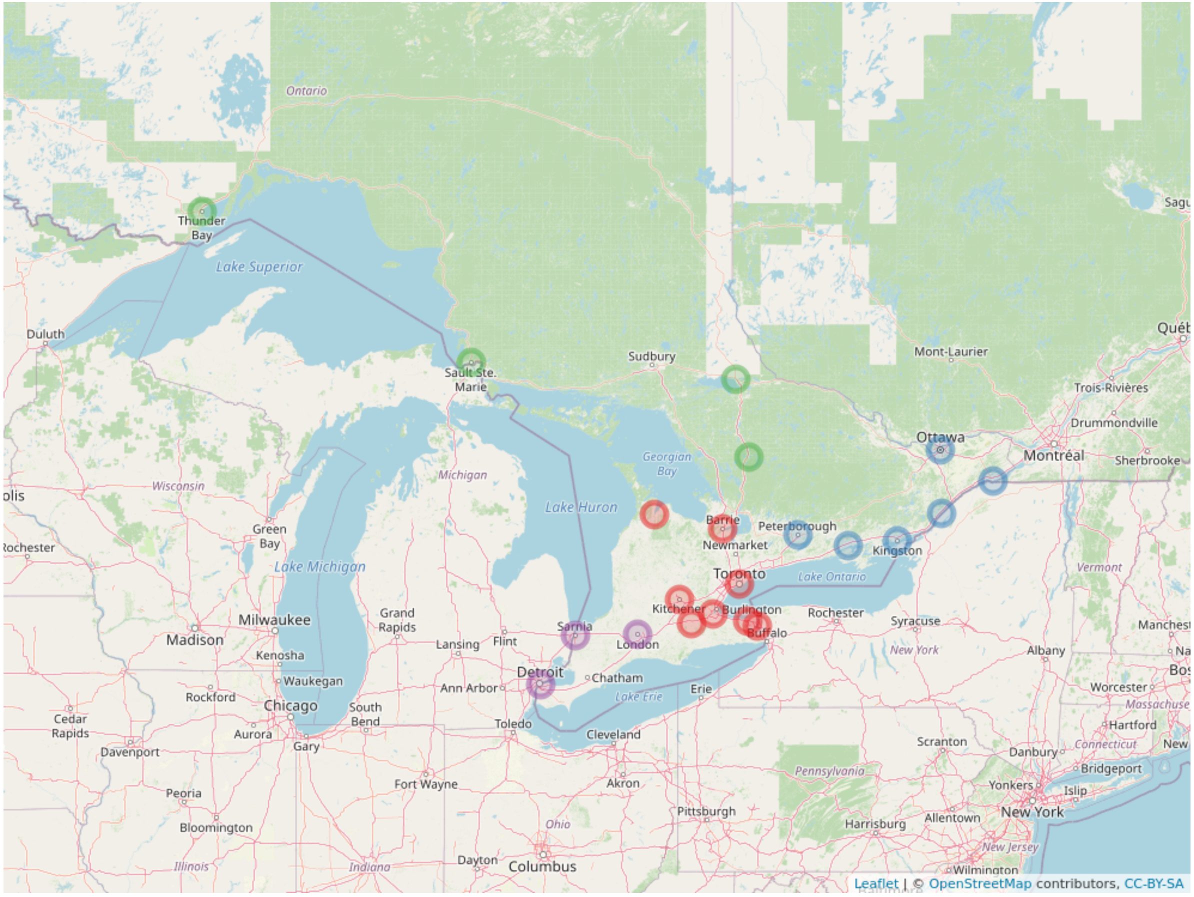
```
plot(ontario.hc)
rect.hclust(ontario.hc, 7)
```



Divisions now

- I want to put Huntsville and North Bay together with northern teams.
- I'll put the Eastern teams together. Gives:
- North: Sault Ste Marie, Sudbury, Huntsville, North Bay
- East: Brockville, Cornwall, Ottawa, Peterborough, Belleville, Kingston
- West: Windsor, London, Sarnia
- Central: Owen Sound, Barrie, Toronto, Niagara Falls, St Catharines, Brantford, Hamilton, Kitchener
- Getting them same size beyond us!

Another map


Ontario
Thunder Bay
Lake Superior
Duluth
Sault Ste. Marie
Sudbury
Mont-Laurier
Trois-Rivières
Drummondville
Ottawa
Montréal
Sherbrooke
Georgian Bay
Michigan
Lake Huron
Wisconsin
Barrie
Newmarket
Peterborough
Kingston
Green Bay
Lake Michigan
Toronto
Lake Ontario
Vermont
Milwaukee
Madison
Kitchener
Burlington
Rochester
Syracuse
Grand Rapids
Sarnia
London
Buffalo
Lansing
Flint
Kenosha
New York
Albany
Worcester
Detroit
Chatham
Waukegan
Ann Arbor
Rockford
Chicago
Lake Erie
Erie
South Bend
Toledo
Cedar Rapids
Aurora
Cleveland
Hartford
Connecticut
Scranton
Danbury
Davenport
Gary
Pennsylvania
Bridgeport
Fort Wayne
Akron
Yonkers
Islip
Peoria
Pittsburgh
Harrisburg
Allentown
Bloomington
Ohio
New Jersey
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Indiana
Dayton
Columbus
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