

Analysis of Covariance

Analysis of covariance

- ▶ ANOVA: explanatory variables categorical (divide data into groups)
- ▶ traditionally, analysis of covariance has categorical x 's plus one numerical x ("covariate") to be adjusted for.
- ▶ `lm` handles this too.
- ▶ Simple example: two treatments (drugs) (a and b), with before and after scores.
- ▶ Does knowing before score and/or treatment help to predict after score?
- ▶ Is after score different by treatment/before score?

Data

Treatment, before, after:

a 5 20

a 10 23

a 12 30

a 9 25

a 23 34

a 21 40

a 14 27

a 18 38

a 6 24

a 13 31

b 7 19

b 12 26

b 27 33

b 24 35

b 18 30

b 22 31

b 26 34

Packages

```
library(tidyverse)
library(broom)
library(marginaleffects)
```

the last of these for predictions.

Read in data

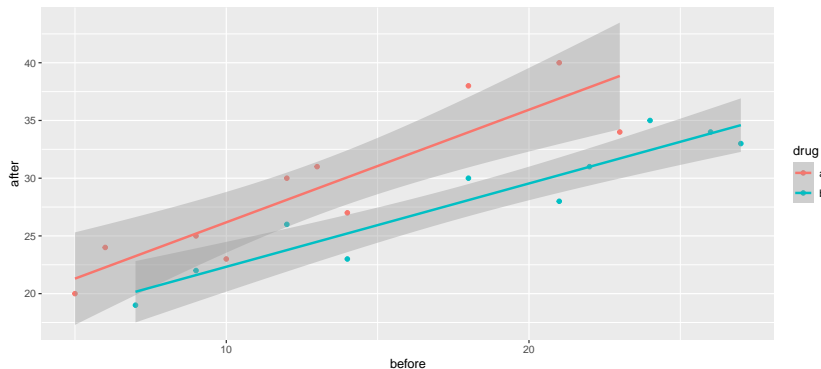
```
url <- "http://ritsokiguess.site/datafiles/ancova.txt"
prepost <- read_delim(url, " ")
prepost
```

```
# A tibble: 20 x 3
```

	drug	before	after
	<chr>	<dbl>	<dbl>
1	a	5	20
2	a	10	23
3	a	12	30
4	a	9	25
5	a	23	34
6	a	21	40
7	a	14	27
8	a	18	38
9	a	6	24
10	a	13	31
11	b	7	19
12	b	10	26

Making a plot

```
ggplot(prepost, aes(x = before, y = after, colour = drug))  
  geom_point() + geom_smooth(method = "lm")
```



Comments

- ▶ As before score goes up, after score goes up.
- ▶ Red points (drug A) generally above blue points (drug B), for comparable before score.
- ▶ Suggests before score effect *and* drug effect.

The means

```
prepost %>%  
  group_by(drug) %>%  
  summarize(  
    before_mean = mean(before),  
    after_mean = mean(after)  
  )
```

```
# A tibble: 2 x 3  
  drug   before_mean after_mean  
  <chr>         <dbl>         <dbl>  
1 a             13.1             29.2  
2 b             18              28.1
```

- ▶ Mean “after” score slightly higher for treatment A.
- ▶ Mean “before” score much higher for treatment B.
- ▶ Greater *improvement* on treatment A.

Testing for interaction

```
prepost.1 <- lm(after ~ before * drug, data = prepost)
anova(prepost.1)
```

Analysis of Variance Table

Response: after

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
before	1	430.92	430.92	62.6894	6.34e-07	***
drug	1	115.31	115.31	16.7743	0.0008442	***
before:drug	1	12.34	12.34	1.7948	0.1990662	
Residuals	16	109.98	6.87			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
summary(prepost.1)
```

Call:

```
lm(formula = after ~ before * drug, data = prepost)
```

Predictions

Set up values to predict for:

```
summary(prepost)
```

drug	before	after
Length:20	Min. : 5.00	Min. :19.00
Class :character	1st Qu.: 9.75	1st Qu.:23.75
Mode :character	Median :14.00	Median :29.00
	Mean :15.55	Mean :28.65
	3rd Qu.:21.25	3rd Qu.:33.25
	Max. :27.00	Max. :40.00

```
new <- datagrid(before = c(9.75, 14, 21.25),  
                 drug = c("a", "b"), model = prepost.1)  
new
```

	before	drug	rowid
1	9.75	a	1
2	9.75	b	2
3	14.00	a	3

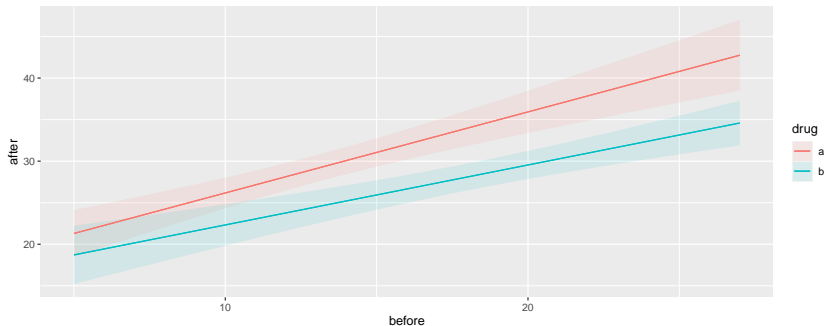
and then

```
cbind(predictions(prepost.1, newdata = new)) %>%  
  select(drug, before, estimate, conf.low, conf.high)
```

	drug	before	estimate	conf.low	conf.high
1	a	9.75	25.93250	24.05059	27.81442
2	b	9.75	22.14565	19.58681	24.70450
3	a	14.00	30.07784	28.43296	31.72271
4	b	14.00	25.21304	23.32649	27.09959
5	a	21.25	37.14929	34.32557	39.97300
6	b	21.25	30.44565	28.64373	32.24758

Predictions (with interaction included), plotted

```
plot_predictions(model = prepost.1, condition = c("before",
```



Lines almost parallel, but not quite.

Taking out interaction

```
prepost.2 <- update(prepost.1, . ~ . - before:drug)
summary(prepost.2)
```

Call:

```
lm(formula = after ~ before + drug, data = prepost)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-3.6348	-2.5099	-0.2038	1.8871	4.7453

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	18.3600	1.5115	12.147	8.35e-10 ***
before	0.8275	0.0955	8.665	1.21e-07 ***
drugb	-5.1547	1.2876	-4.003	0.000921 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.682 on 17 degrees of freedom

Multiple R-squared: 0.817, Adjusted R-squared: 0.7955

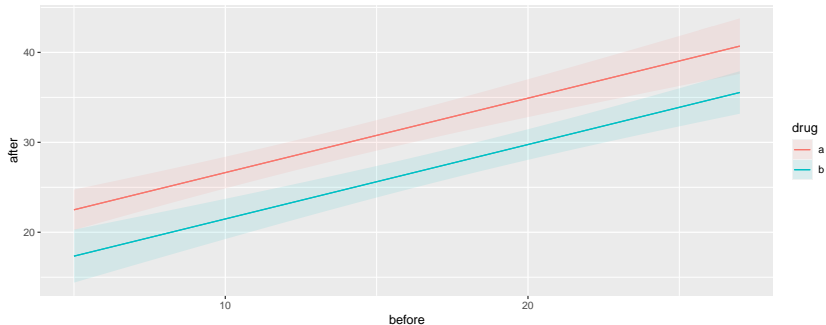
Predictions

```
cbind(predictions(prepost.2, newdata = new)) %>%  
  select(drug, before, estimate)
```

	drug	before	estimate
1	a	9.75	26.42794
2	b	9.75	21.27328
3	a	14.00	29.94473
4	b	14.00	24.79007
5	a	21.25	35.94397
6	b	21.25	30.78931

Plot of predicted values

```
plot_predictions(prepost.2, condition = c("before", "drug"))
```



This time the lines are *exactly* parallel. No-interaction model forces them to have the same slope.

Different look at model output

- ▶ `anova(prepost.2)` tests for significant effect of before score and of drug, but doesn't help with interpretation.
- ▶ `summary(prepost.2)` views as regression with slopes:

```
summary(prepost.2)
```

Call:

```
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```

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Residual standard error: 2.682 on 17 degrees of freedom

Multiple R-squared: 0.817, Adjusted R-squared: 0.7955

F-statistic: 37.96 on 2 and 17 DF, p-value: 5.372e-07

Understanding those slopes

```
tidy(prepost.2)
```

```
# A tibble: 3 x 5
  term          estimate std.error statistic  p.value
<chr>         <dbl>     <dbl>     <dbl>    <dbl>
1 (Intercept)   18.4        1.51      12.1 8.35e-10
2 before        0.827      0.0955     8.66 1.21e- 7
3 drugb        -5.15       1.29     -4.00 9.21e- 4
```

- ▶ before ordinary numerical variable; drug categorical.
- ▶ lm uses first category *druga* as baseline.
- ▶ Intercept is prediction of after score for before score 0 and *drug A*.
- ▶ before slope is predicted change in after score when before score increases by 1 (usual slope)
- ▶ Slope for *drugb* is *change* in predicted after score for being on drug B rather than drug A. Same for *any* before score (no interaction).

Summary

- ▶ ANCOVA model: fits different regression line for each group, predicting response from covariate.
- ▶ ANCOVA model with interaction between factor and covariate allows different slopes for each line.
- ▶ Sometimes those lines can cross over!
- ▶ If interaction not significant, take out. Lines then parallel.
- ▶ With parallel lines, groups have consistent effect regardless of value of covariate.