

Repeated Measures ANOVA

Repeated measures by profile analysis

- More than one response *measurement* for each subject. Might be
 - measurements of the same thing at different times
 - measurements of different but related things
- Generalization of matched pairs (“matched triples”, etc.).
- Variation: each subject does several different treatments at different times (called *crossover design*).
- Expect measurements on same subject to be correlated, so assumptions of independence will fail.
- Called *repeated measures*. Different approaches, but *profile analysis* uses Manova (set up right way).
- Another approach uses *mixed models* (random effects).

Packages

```
library(car)  
library(tidyverse)  
library(lme4) # for mixed models later
```

Example: histamine in dogs

- 8 dogs take part in experiment.
- Dogs randomized to one of 2 different drugs.
- Response: log of blood concentration of histamine 0, 1, 3 and 5 minutes after taking drug. (Repeated measures.)
- Data in `dogs.txt`, column-aligned.

Read in data

```
my_url <- "http://ritsokiguess.site/datafiles/dogs.txt"
dogs <- read_table(my_url)
```

```
##
## -- Column specification -----
## cols(
##   dog = col_character(),
##   drug = col_character(),
##   x = col_character(),
##   lh0 = col_double(),
##   lh1 = col_double(),
##   lh3 = col_double(),
##   lh5 = col_double()
## )
```

Setting things up

dogs

dog	drug	x	lh0	lh1	lh3	lh5
A	Morphine	N	-3.22	-1.61	-2.30	-2.53
B	Morphine	N	-3.91	-2.81	-3.91	-3.91
C	Morphine	N	-2.66	0.34	-0.73	-1.43
D	Morphine	N	-1.77	-0.56	-1.05	-1.43
E	Trimethaphan	N	-3.51	-0.48	-1.17	-1.51
F	Trimethaphan	N	-3.51	0.05	-0.31	-0.51
G	Trimethaphan	N	-2.66	-0.19	0.07	-0.22
H	Trimethaphan	N	-2.41	1.14	0.72	0.21

```
response <- with(dogs, cbind(lh0, lh1, lh3, lh5))  
dogs.1 <- lm(response ~ drug, data = dogs)
```

The repeated measures MANOVA

Get list of response variable names; we call them times. Save in data frame.

```
times <- colnames(response)
times.df <- data.frame(times=factor(times))
dogs.2 <- Manova(dogs.1,
  idata = times.df,
  idesign = ~times
)
```

The output (some; there is a lot)

```
summary(dogs.2)
```

```
##
## Type II Repeated Measures MANOVA Tests:
##
## -----
##
## Term: (Intercept)
##
## Response transformation matrix:
## (Intercept)
## lh0          1
## lh1          1
## lh3          1
## lh5          1
##
## Sum of squares and products for the hypothesis:
## (Intercept)
## (Intercept)    285.366
##
## Multivariate Tests: (Intercept)
##
##      Df test stat approx F num Df den Df    Pr(>F)
## Pillai      1  0.763467  19.36642      1      6 0.0045648 **
## Wilks       1  0.236533  19.36642      1      6 0.0045648 **
## Hotelling-Lawley 1  3.227738  19.36642      1      6 0.0045648 **
## Roy        1  3.227738  19.36642      1      6 0.0045648 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## -----
##
## Term: drug
```


What there is here

- three sets of tests, for
 - times
 - drug
 - their interaction
- two *types* of test for each of these:
 - multivariate
 - univariate
- multivariate is the same as MANOVA
- univariate is more powerful *if* it applies

Sphericity

- The thing that decides whether the univariate tests apply is called “sphericity”.
- This holds if the outcomes have equal variance (to each other) and have the same (positive) correlation across subjects.
- Tested using Mauchly's test (part of output)
- If sphericity rejected, there are adjustments to the univariate P-values due to Huynh-Feldt and Greenhouse-Geisser. Huynh-Feldt better if responses not actually normal (safer). Or use multivariate test.

Univariate tests

```
summary(dogs.2)$sphericity.tests
```

```
##              Test statistic  p-value
## times              0.12334 0.084567
## drug:times         0.12334 0.084567
```

```
summary(dogs.2)$pval.adjustments
```

```
##              GG eps  Pr(>F[GG])  HF eps  Pr(>F[HF])
## times          0.5261798 3.744618e-06 0.6822614 1.843418e-07
## drug:times     0.5261798 2.348896e-03 0.6822614 7.307096e-04
## attr(,"na.action")
## (Intercept)      drug
##              1      2
## attr(,"class")
## [1] "omit"
```

```
summary(dogs.2)$univariate.tests
```

```
##              Sum Sq num Df Error SS den Df F value    Pr(>F)
## (Intercept)  71.342      1  22.1026      6 19.3664 0.004565 **
## drug         11.520      1  22.1026      6  3.1272 0.127406
## times        26.160      3   2.2534     18 69.6546 4.215e-10 ***
## drug:times    5.111      3   2.2534     18 13.6095 7.050e-05 ***
##
```

Comments

- The sphericity test for the interaction is almost significant
- The H-F adjusted P-value for the interaction is a bit bigger than the univariate one, but still strongly significant.
- Therefore any lack of sphericity does not affect our conclusion: there is an interaction between drug and time
- ie that the effect of time on log-histamine is different for the two drugs.

The multivariate test for interaction

```
summary(dogs.2)$multivariate.tests$`drug:times`
```

```
##
## Response transformation matrix:
##      times1 times2 times3
## lh0      1      0      0
## lh1      0      1      0
## lh3      0      0      1
## lh5     -1     -1     -1
##
## Sum of squares and products for the hypothesis:
##      times1      times2      times3
## times1  7.60500  2.0572500 -0.0292500
## times2  2.05725  0.5565125 -0.0079125
## times3 -0.02925 -0.0079125  0.0001125
##
## Sum of squares and products for error:
##      times1      times2      times3
## times1  1.34480 -0.311350 -0.362450
## times2 -0.31135  1.131975  0.346000
## times3 -0.36245  0.346000  0.309175
##
## Multivariate Tests: drug:times
##      Df test stat approx F num Df den Df  Pr(>F)
## Pillai      1  0.894761 11.33619      3      4 0.020023 *
## Wilks       1  0.105239 11.33619      3      4 0.020023 *
## Hotelling-Lawley 1  8.502141 11.33619      3      4 0.020023 *
## Roy         1  8.502141 11.33619      3      4 0.020023 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Comments

- The (appropriately adjusted) univariate test and the multivariate test should give similar results.
- Here, univariate with Huynh-Feldt P-value was 0.00073, multivariate was 0.0200, rather different but both significant.
- Significant interaction *is* the conclusion here.
- If the interaction had not been significant:
 - cannot remove interaction with time
 - so look at univariate (better, especially if adjusted for sphericity) or multivariate tests of main effects in *this* model

Next

- Interaction significant. Pattern of response over time different for the two drugs.
- Want to investigate interaction.

The wrong shape

- But data frame has several observations per line (“wide format”):

```
dogs %>% slice(1:6)
```

dog	drug	x	lh0	lh1	lh3	lh5
A	Morphine	N	-3.22	-1.61	-2.30	-2.53
B	Morphine	N	-3.91	-2.81	-3.91	-3.91
C	Morphine	N	-2.66	0.34	-0.73	-1.43
D	Morphine	N	-1.77	-0.56	-1.05	-1.43
E	Trimethaphan	N	-3.51	-0.48	-1.17	-1.51
F	Trimethaphan	N	-3.51	0.05	-0.31	-0.51

- Plotting works with data in “long format”: one response per line.
- The responses are log-histamine at different times, labelled 1h-something. Call them all 1h and put them in one column, with the time they belong to labelled.

Running pivot_longer, try 1

```
dogs %>% pivot_longer(starts_with("lh"),  
                      names_to = "time", values_to = "lh")
```

dog	drug	x	time	lh
A	Morphine	N	lh0	-3.22
A	Morphine	N	lh1	-1.61
A	Morphine	N	lh3	-2.30
A	Morphine	N	lh5	-2.53
B	Morphine	N	lh0	-3.91
B	Morphine	N	lh1	-2.81
B	Morphine	N	lh3	-3.91
B	Morphine	N	lh5	-3.91
C	Morphine	N	lh0	-2.66
C	Morphine	N	lh1	0.34
C	Morphine	N	lh3	-0.73
C	Morphine	N	lh5	-1.43
D	Morphine	N	lh0	-1.77
D	Morphine	N	lh1	-0.56
D	Morphine	N	lh3	-1.05
D	Morphine	N	lh5	-1.43

Repeated Measures ANOVA

Getting the times

Not quite right: for the times, we want just the numbers, not the letters lh every time. Want new variable containing just number in time:

parse_number.

```
dogs %>%  
  pivot_longer(starts_with("lh"),  
               names_to = "timex", values_to = "lh") %>%  
  mutate(time = parse_number(timex))
```

dog	drug	x	timex	lh	time
A	Morphine	N	lh0	-3.22	0
A	Morphine	N	lh1	-1.61	1
A	Morphine	N	lh3	-2.30	3
A	Morphine	N	lh5	-2.53	5
B	Morphine	N	lh0	-3.91	0
B	Morphine	N	lh1	-2.81	1
B	Morphine	N	lh3	-3.91	3
B	Morphine	N	lh5	-3.91	5
C	Morphine	N	lh0	-2.66	0

What I did differently

- I realized that `pivot_longer` was going to produce something like `lh1`, which I needed to do something further with, so this time I gave it a temporary name `timex`.
- This enabled me to use the name `time` for the actual numeric time.
- This works now, so next save into a new data frame `dogs.long`.

Saving the pipelined results

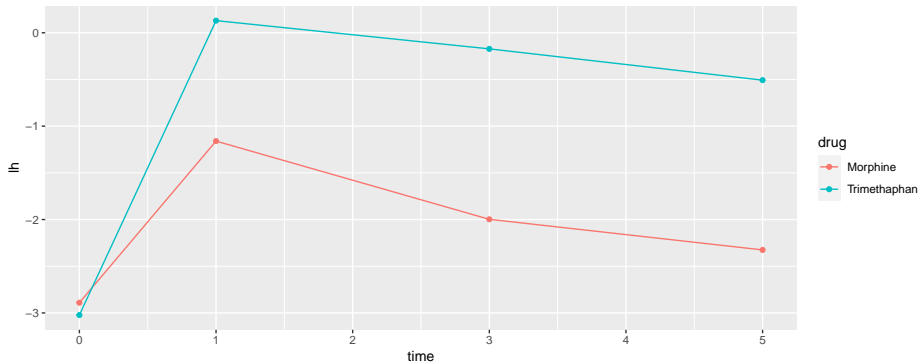
```
dogs %>%  
  pivot_longer(starts_with("lh"),  
               names_to = "timex", values_to = "lh") %>%  
  mutate(time = parse_number(timex)) -> dogs.long
```

This says:

- Take data frame `dogs`, and then:
- Combine the columns `lh0` through `lh5` into one column called `lh`, with the column that each `lh` value originally came from labelled by `timex`, and then:
- Pull out numeric values in `timex`, saving in `time` and then:
- save the result in a data frame `dogs.long`.

Interaction plot

```
ggplot(dogs.long, aes(x = time, y = lh,  
                      colour = drug, group = drug)) +  
  stat_summary(fun = mean, geom = "point") +  
  stat_summary(fun = mean, geom = "line")
```



Comments

- Plot mean 1h value at each time, joining points on same drug by lines.
- drugs same at time 0
- after that, Trimethaphan higher than Morphine.
- Effect of drug not consistent over time: significant interaction.

Take out time zero

- Lines on interaction plot would then be parallel, and so interaction should no longer be significant.
- Go back to original “wide” dogs data frame.

```
response <- with(dogs, cbind(lh1, lh3, lh5)) # excl time 0
dogs.1 <- lm(response ~ drug, data = dogs)
times <- colnames(response)
times.df <- data.frame(times=factor(times))
dogs.2 <- Manova(dogs.1,
  idata = times.df,
  idesign = ~times
)
```

Results (univariate)

```
summary(dogs.2)$sphericity.tests
```

```
##                Test statistic p-value
## times                0.57597 0.25176
## drug:times           0.57597 0.25176
```

```
summary(dogs.2)$pval.adjustments
```

```
##                GG eps   Pr(>F[GG])    HF eps   Pr(>F[HF])
## times          0.7022305 0.0003752847 0.8520467 0.0001117394
## drug:times     0.7022305 0.1078608639 0.8520467 0.0942573437
## attr("na.action")
## (Intercept)          drug
##                1          2
## attr("class")
## [1] "omit"
```

```
summary(dogs.2)$univariate.tests
```

```
##                Sum Sq num Df Error SS den Df F value    Pr(>F)
## (Intercept)  24.2607      1  20.1874      6  7.2106  0.03628 *
## drug         16.2197      1  20.1874      6  4.8207  0.07053 .
## times        3.3250      2   0.7301     12 27.3251 3.406e-05 ***
## drug:times   0.3764      2   0.7301     12  3.0929  0.08254 .
##
```


Comments

- sphericity: no problem (P-value 0.25)
- univariate test for interaction no longer significant (P-value 0.082)
- Huynh-Feldt adjusted P-value for interaction very similar, 0.094 (adjustment not really worth doing)
- look at main effects:
 - strong significance of time, even after taking out time 0
 - actually *not* significant drug effect, despite interaction plot

Multivariate tests, all at once

```
dogs.2
```

```
##
## Type II Repeated Measures MANOVA Tests: Pillai test statistic
##              Df test stat approx F num Df den Df    Pr(>F)
## (Intercept)   1   0.54582    7.2106      1     6 0.036281 *
## drug           1   0.44551    4.8207      1     6 0.070527 .
## times         1   0.85429   14.6569      2     5 0.008105 **
## drug:times     1   0.43553    1.9289      2     5 0.239390
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Same conclusions as for univariate tests, though with bigger P-values.

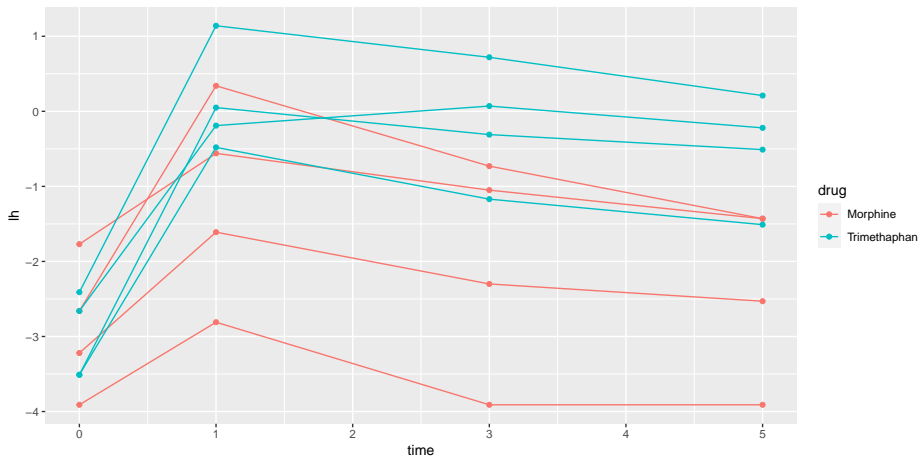
Is the non-significant drug effect reasonable?

- Plot *actual data*: lh against days, labelling observations by drug: “spaghetti plot”.
- Uses long data frame (confusing, yes I know):
- Plot (time,lh) points coloured by drug
- and connecting measurements for each *dog* by lines.
- This time, we want group=dog (want the measurements for each *dog* joined by lines), but colour=drug:

```
g <- ggplot(dogs.long, aes(  
  x = time, y = lh,  
  colour = drug, group = dog  
)) +  
  geom_point() + geom_line()
```

The spaghetti plot

g



Comments

- For each dog over time, there is a strong increase and gradual decrease in log-histamine. This explains the significant time effect.
- The pattern is more or less the same for each dog, regardless of drug. This explains the non-significant interaction.
- Most of the trimethaphan dogs (blue) have higher log-histamine throughout (time 1 and after), and some of the morphine dogs have lower.
- *But* two of the morphine dogs have log-histamine profiles like the trimethaphan dogs. This ambiguity is probably why the drug effect is not quite significant.

Mixed models

- Another way to fit repeated measures
- Subjects (on whom repeated measures taken) are *random sample of all possible subjects* (random effects)
- Times and treatments are *the only ones we care about* (fixed effects)
- Use package lme4 function lmer (like lm in some ways)
- Uses long-format “tidy” data

Fitting the model (uses lme4)

```
# dogs.long including time zero  
dogs.3 <- lmer(lh~drug*time+(1|dog), data=dogs.long)
```

- note specification of random effect: each dog has “random intercept” that moves log-histamine up or down for that dog over all times

What can we drop?

- using drop1:

```
drop1(dogs.3, test="Chisq")
```

	npar	AIC	LRT	Pr(Chi)
	NA	113.2597	NA	NA
drug:time	1	114.2130	2.953368	0.0856988

- Interaction again not significant, but P-value smaller than before

Re-fit without interaction

```
dogs.4 <- update(dogs.3, .~.-drug:time)
drop1(dogs.4, test="Chisq")
```

	npars	AIC	LRT	Pr(Chi)
	NA	114.2130	NA	NA
drug	1	115.5691	3.356021	0.0669597
time	1	114.9631	2.750099	0.0972484

- This time neither drug nor (surprisingly) time is significant.
- MANOVA and `lmer` methods won't agree, but both valid ways to approach problem.

The exercise data

- 30 people took part in an exercise study.
- Each subject was randomly assigned to one of two diets (“low fat” or “non-low fat”) and to one of three exercise programs (“at rest”, “walking”, “running”).
- There are $2 \times 3 = 6$ experimental treatments, and thus each one is replicated $30/6 = 5$ times.
- Nothing unusual so far.
- However, each subject had their pulse rate measured at three different times (1, 15 and 30 minutes after starting their exercise), so have repeated measures.

Reading the data

Separated by *tabs*:

```
url <- "http://ritsokiguess.site/datafiles/exercise2.txt"
exercise.long <- read_tsv(url)
```

```
## Rows: 90 Columns: 5
```

```
## -- Column specification -----
```

```
## Delimiter: "\t"
```

```
## chr (3): diet, exertype, time
```

```
## dbl (2): id, pulse
```

```
##
```

```
## i Use `spec()` to retrieve the full column specification for
```

```
## i Specify the column types or set `show_col_types = FALSE`
```

The data (some)

```
exercise.long %>% slice(1:8)
```

id	diet	exertype	pulse	time
1	nonlowfat	atrest	85	min01
1	nonlowfat	atrest	85	min15
1	nonlowfat	atrest	88	min30
2	nonlowfat	atrest	90	min01
2	nonlowfat	atrest	92	min15
2	nonlowfat	atrest	93	min30
3	nonlowfat	atrest	97	min01
3	nonlowfat	atrest	97	min15

- This is “long format”, which is usually what we want.
- But for repeated measures analysis, we want *wide* format!
- `pivot_wider`.

Making wide format

- `pivot_wider` needs: a column that is going to be split, and the column to make the values out of:

```
exercise.long %>% pivot_wider(names_from=time,  
                             values_from=pulse) -> exercise.wide  
exercise.wide %>% sample_n(5)
```

id	diet	exertype	min01	min15	min30
2	nonlowfat	atrest	90	92	93
24	nonlowfat	running	87	132	120
5	nonlowfat	atrest	91	92	91
16	lowfat	walking	84	86	89
27	lowfat	running	100	126	140

- Normally `pivot_longer` min01, min15, min30 into one column called pulse labelled by the number of minutes. But Manova needs it the other way.

Setting up the repeated-measures analysis

- Make a response variable consisting of min01, min15, min30:

```
response <- with(exercise.wide, cbind(min01, min15, min30))
```

- Predict that from diet and exertype and interaction using lm:

```
exercise.1 <- lm(response ~ diet * exertype,  
  data = exercise.wide  
)
```

- Run this through Manova:

```
times <- colnames(response)  
times.df <- data.frame(times=factor(times))  
exercise.2 <- Manova(exercise.1,  
  idata = times.df,  
  idesign = ~times)
```

Sphericity tests

```
summary(exercise.2)$sphericity.tests
```

```
## Warning in summary.Anova.mlm(exercise.2): HF eps > 1 treated
```

##	Test statistic	p-value
## times	0.92416	0.40372
## diet:times	0.92416	0.40372
## exertype:times	0.92416	0.40372
## diet:exertype:times	0.92416	0.40372

No problem with sphericity; go to univariate tests.

Univariate tests

```
summary(exercise.2)$univariate.tests
```

```
## Warning in summary.Anova.mlm(exercise.2): HF eps > 1 treated as 1
```

```
##              Sum Sq num Df Error SS den Df      F value    Pr(>F)
## (Intercept)    894608      1  2085.2    24 10296.6595 < 2.2e-16 ***
## diet           1262      1  2085.2    24   14.5238 0.0008483 ***
## exertype       8326      2  2085.2    24   47.9152 4.166e-09 ***
## diet:exertype   816      2  2085.2    24    4.6945 0.0190230 *
## times          2067      2  1563.6    48   31.7206 1.662e-09 ***
## diet:times       193      2  1563.6    48    2.9597 0.0613651 .
## exertype:times  2723      4  1563.6    48   20.9005 4.992e-10 ***
## diet:exertype:times 614      4  1563.6    48    4.7095 0.0027501 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- The three-way interaction is significant
 - the effect of diet on pulse rate over time is different for the different exercise types

Multivariate very similar

```
exercise.2
```

```
##
## Type II Repeated Measures MANOVA Tests: Pillai test statistic
##
##          Df test stat approx F num Df den Df      Pr(>F)
## (Intercept)      1   0.99767   10296.7      1    24 < 2.2e-16 ***
## diet              1   0.37701     14.5      1    24 0.0008483 ***
## exertype          2   0.79972     47.9      2    24 4.166e-09 ***
## diet:exertype      2   0.28120      4.7      2    24 0.0190230 *
## times             1   0.78182     41.2      2    23 2.491e-08 ***
## diet:times         1   0.25153      3.9      2    23 0.0357258 *
## exertype:times     2   0.83557      8.6      4    48 2.538e-05 ***
## diet:exertype:times 2   0.51750      4.2      4    48 0.0054586 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- Three-way interaction significant, so cannot remove anything.
- Pulse rate depends on diet and exercise type *combination*, and *that* is different for each time.

Making some graphs

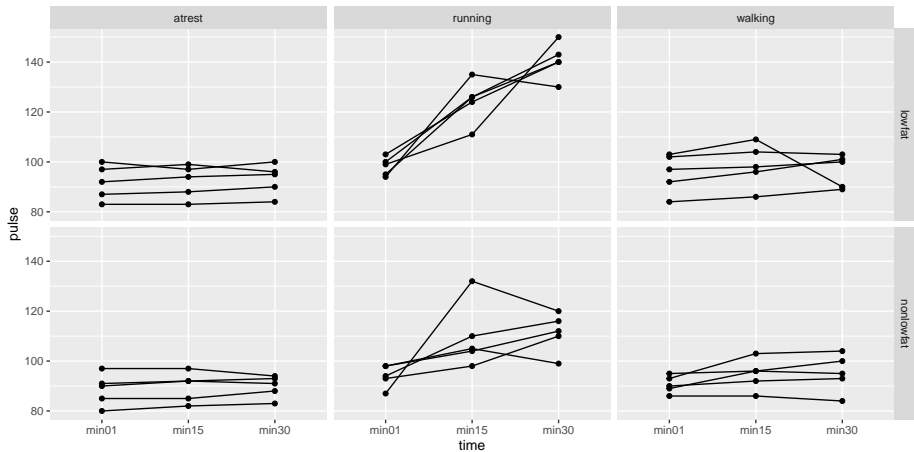
- Three-way interactions are difficult to understand. To make an attempt, look at some graphs.
- Plot time trace of pulse rates for each individual, joined by lines, and make *separate* plots for each diet-exertype combo.
- ggplot again. Using *long* data frame:

```
g <- ggplot(exercise.long, aes(  
  x = time, y = pulse,  
  group = id  
)) + geom_point() + geom_line() +  
  facet_grid(diet ~ exertype)
```

- `facet_grid(diet~exertype)`: do a separate plot for each combination of diet and exercise type, with diets going down the page and exercise types going across. (Graphs are usually landscape, so have the factor exertype with more levels going across.)

The graph(s)

α



Comments on graphs

- For subjects who were at rest, no change in pulse rate over time, for both diet groups.
- For walking subjects, not much change in pulse rates over time. Maybe a small increase on average between 1 and 15 minutes.
- For both running groups, an overall increase in pulse rate over time, but the increase is stronger for the lowfat group.
- No consistent effect of diet over all exercise groups.
- No consistent effect of exercise type over both diet groups.
- No consistent effect of time over all diet-exercise type combos.

“Simple effects” of diet for the subjects who ran

- Looks as if there is only any substantial time effect for the runners. For them, does diet have an effect?
- Pull out only the runners from the wide data:

```
exercise.wide %>%  
  filter(exertype == "running") -> runners.wide
```

- Create response variable and do MANOVA. Some of this looks like before, but I have different data now:

```
response <- with(runners.wide, cbind(min01, min15, min30))  
runners.1 <- lm(response ~ diet, data = runners.wide)  
times <- colnames(response)  
times.df <- data.frame(times=factor(times))  
runners.2 <- Manova(runners.1,  
  idata = times.df,  
  idesign = ~times  
)
```

Sphericity tests

```
summary(runners.2)$sphericity.tests
```

```
## Warning in summary.Anova.mlm(runners.2): HF eps > 1 treated
```

```
##           Test statistic p-value
## times           0.81647  0.4918
## diet:times       0.81647  0.4918
```

- No problem, look at univariate tests.

Univariate tests

```
summary(runners.2)$univariate.tests
```

```
## Warning in summary.Anova.mlm(runners.2): HF eps > 1 treated as 1

##           Sum Sq num Df Error SS den Df   F value    Pr(>F)
## (Intercept) 383522     1   339.2     8 9045.3333 1.668e-13 ***
## diet         1920     1   339.2     8  45.2830 0.0001482 ***
## times        4714     2  1242.0    16  30.3644 3.575e-06 ***
## diet:times    789     2  1242.0    16   5.0795 0.0195874 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- Interaction still significant
 - dependence of pulse rate on time still different for the two diets

Multivariate results (comparison)

```
runners.2
```

```
##
## Type II Repeated Measures MANOVA Tests: Pillai test statistic
##           Df test stat approx F num Df den Df    Pr(>F)
## (Intercept) 1    0.99912   9045.3      1      8 1.668e-13 ***
## diet         1    0.84986    45.3      1      8 0.0001482 ***
## times        1    0.92493    43.1      2      7 0.0001159 ***
## diet:times   1    0.68950     7.8      2      7 0.0166807 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Same.

How is the effect of diet different over time?

- Table of means. Only I need long data for this:

```
runners.wide %>%  
  pivot_longer(starts_with("min"),  
               names_to = "time", values_to = "pulse") %>%  
  group_by(time, diet) %>%  
  summarize(  
    mean = mean(pulse),  
    sd = sd(pulse)  
  ) -> summ
```

```
## `summarise()` has grouped output by 'time'. You can override  
## the `groups` argument.
```

- Result of `summarize` is data frame, so can save it (and do more with it if needed).

Understanding diet-time interaction

- The summary:

summ

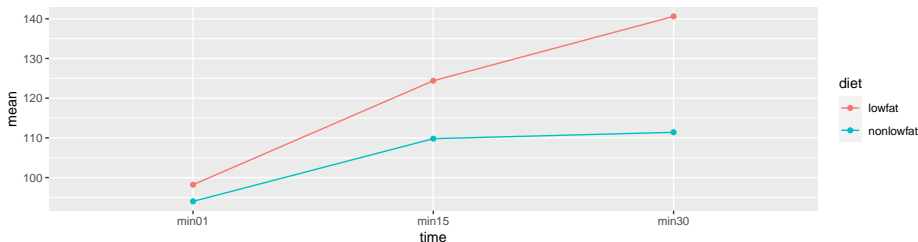
time	diet	mean	sd
min01	lowfat	98.2	3.701351
min01	nonlowfat	94.0	4.527693
min15	lowfat	124.4	8.619745
min15	nonlowfat	109.8	13.122500
min30	lowfat	140.6	7.197222
min30	nonlowfat	111.4	7.924645

- Pulse rates at any given time higher for lowfat (diet effect),
- Pulse rates increase over time of exercise (time effect),
- but the *amount by which pulse rate higher* for a diet depends on time: diet by time interaction.

Interaction plot

- We went to trouble of finding means by group, so making interaction plot is now mainly easy:

```
ggplot(summ, aes(x = time, y = mean, colour = diet,  
                 group = diet)) + geom_point() + geom_line()
```



Comment on interaction plot

- The lines are not parallel, so there is interaction between diet and time for the runners.
- The effect of time on pulse rate is different for the two diets, even though all the subjects here were running.