Estimating the FOMC's Interest Rate Rule with Variable Selection and Partial Regime Switching

Adam Check*†
Assistant Professor
University of St. Thomas
Phone: (651) 962-5295
Email: ajc@stthomas.edu

*I can be contacted at ajc@stthomas.edu

†I would like to thank Jeremy Piger, Dean Croushore, Mark Kuperberg, and Eric Gaus, the participants in the Liberal Arts Macro, Midwest Economic Research Group, and WEAI conferences, and two anonymous referees for helpful comments and suggestions on previous versions of this paper.

Abstract

When studying the Federal Open Market Committee's (FOMC's) interest rate rule, some authors, such as Gonzalez-Astudillo (2018), find evidence for changes in inflation and output gap responses. Others, such as Sims and Zha (2006), only find evidence for a change in the variance of the interest rate rule. In this paper, I develop a new two-regime Markov-switching model that probabilistically performs variable selection and identification of parameter change for each variable in the model. I find substantial evidence that there have been changes in the response to unemployment and in the volatility of the rule. When the FOMC responds strongly to the unemployment, I find a bi-modal density for the inflation response coefficient. Despite the bi-modal density, there is a low probability that there have been changes in the FOMC's response to inflation.

Keywords: Monetary Policy, Interest Rate Rule, Markov-switching, Model Averaging
JEL Codes: C22, C24, C51, C52, E52

1 Introduction

The modern view of monetary policy is that the Federal Open Market Committee (FOMC) adjusts the Federal Funds rate based on measures of economic performance. Mathematically, this is typically formulated as a version of the the Taylor Rule, first described in Taylor (1993), in which the target nominal Federal Funds rate is a linear function of output and inflation. This policy rule, and others taking very similar forms, are the foundation of the past two decades of empirical analysis of historical FOMC behavior.

Many researchers allow for the possibility that the coefficients of the policy rule change over time. In part, this is due to observed macroeconomic variables such as inflation changing substantially over time, suggesting possible changes in FOMC priorities. Additionally, narrative approaches such as Romer and Romer (2004) have found systematic differences in policy implementation under different Federal Reserve chairs, and text analysis such as that performed in Kaya et al. (2019) have found that the topics of discussion at the FOMC meetings have shifted over time. The possibility of changing coefficients in the policy rule has been modeled many different ways, many different measures of inflation, output, and employment have been used when estimating the policy rule, and many different time samples have been used. This has lead to a proliferation of results, with four different findings appearing commonly in this literature:

1. The FOMC's inflation response has changed over time, becoming more aggressive against inflation after the appointment of Paul Volker in 1979 (e.g. Clarida et al. (2000)).

2. The FOMC's output response has changed over time, becoming less responsive to changes in the output gap after the 1970's (e.g. Orphanides (2004)).

3. The FOMC's inflation and output response have both changed over time, typically according to the patterns identified in (1) and (2) (e.g. Gonzalez-Astudillo (2018)).

4. The coefficients of the FOMC's interest rate rule have not changed over time, but the variance has (e.g. Sims and Zha (2006)).

Addressing the nature and timing of structural change in FOMC behavior is of importance to both academics and policymakers. In many different types of DSGE models, coefficients in the interest rate rule help to determine inflation volatility and persistence, as well as short-term output growth rates and volatility. If monetary policy in the United States has changed, it is crucial that we document how, as it will eventually allow us to attribute changes in economic performance to changes in policy. This is especially important in light of the claim made by Taylor (2013) that a weak inflation response returned in the mid-2000s and engendered the housing bubble.

While the early papers in this literature such as Clarida et al. (2000) and Orphanides (2004) used split-sample regressions to model possible coefficient change, later work has used models explicitly designed to detect structural change. Some authors have used a time-varying parameter model, allowing the interest rate rule parameters to slowly change over time.[1] In more recent work, it has been common to model changes in the FOMC's interest rate rule as a Markov-Switching model.[2] However, the exact nature and timing of coefficient change is disputed, with many of these studies using different specifications of the interest rate rule, different time periods, or different information sets. In addition, in some studies such as Bennani et al. (2018) and Gonzalez-Astudillo (2018), the authors have searched for parsimonious models by "pre-testing" — performing frequentist testing of coefficient constancy in an unrestricted Markov-switching model, and then re-estimating a smaller model with a subset of coefficients restricted to be identical across regimes. Other studies, such as Murray et al. (2015), have simply estimated the unrestricted Markov-switching model.

The contribution of the current paper to this literature is threefold. First, I develop a new two-regime Markov-Switching model that endogenously determines whether variables

[1] Examples include Boivin (2006), Kim and Nelson (2006), and Primiceri (2005). A somewhat different approach that also allows the FOMC to shift its policy horizon is considered in Lee et al. (2015).

[2] Examples include Soques (2019), Bennani et al. (2018), Gonzalez-Astudillo (2018), Alba and Wang (2017), Murray et al. (2015), Castelnuovo et al. (2014), Bianchi (2013), and Davig and Leeper (2011).

belong in a regression and if so, whether they switch across regimes. Second, I document substantial evidence of parameter change in the volatility of the interest rate rule and in the FOMC's response to the unemployment gap. Third, I find that there is little-to-no evidence of parameter change in any of the other coefficients of the model: the intercept, the degree of interest rate smoothing, the response to the inflation gap, or the response to the response to the change in the unemployment rate. However, I do find evidence of a bi-modal density for the inflation response coefficient in one regime, which highlights the importance of averaging over different specifications rather than estimating a single Markov-Switching model.

In this paper, I introduce a novel Bayesian econometric model that is able to probabilistically determine the specification of the FOMC's interest rate rule in the presence of a two-regime Markov-Switching process. This Markov-Switching Stochastic Search Variable Selection (MS-SSVS) model nests both a constant coefficient model, consistent with the findings of Sims and Zha (2006), and a full Markov-Switching model, consistent with Murray et al. (2015) as special cases. In addition, the MS-SSVS model can probabilistically restrict a subset of coefficients to remain constant across regimes, nesting FOMC behavior like that identified by Orphanides (2004), Bennani et al. (2018), or Gonzalez-Astudillo (2018) where only a subset of coefficients change over time. The MS-SSVS model can also restrict coefficients in either one or both regimes to be zero, so that a variable may be completely excluded from the regression in either one regime or in both regimes. In short, for each coefficient in each regime, there are three possibilities: (1) the coefficient is restricted to zero; (2) the coefficient is restricted to be the same as the coefficient in the other regime; (3) the coefficient is freely estimated independently of the coefficient in the other regime. The restrictions and coefficients are estimated at the same time in a unified model, which avoids pre-testing and makes the estimated coefficients easily interpretable.[3] In addition, MS-SSVS averages over the uncertainty associated with model choice, so rather than only estimating a final "best" model, inference is performed by weighing estimates across models according

[3]See Giles and Giles (1993) for an overview of some of the problems associated with pre-testing.

to their posterior probabilities.

The MS-SSVS model builds on the work of George and McCulloch (1993) and George et al. (2008), who developed Stochastic Search Variable Selection (SSVS) in order to perform variable selection in linear regression models and linear Vector Autoregressions (VARs).[4] SSVS has some differences with competing methodologies such as Bayesian Model Averaging (BMA) that make it especially attractive in a Markov-Switching environment. One major advantage of SSVS is that it is not necessary to directly compute or approximate the marginal likelihood, which is a computationally intensive task in Markov-Switching models. Instead, the uncertainty associated with variable inclusion and variable switching is nested within a unified hierarchical model.

Using Monte-Carlo exercises with simulated data, I show that the newly developed MS-SSVS model is able to correctly identify when coefficients should be freely estimated and when they should be restricted — this is true both when the actual coefficients are zero, and when the coefficients are the same across regimes. As expected, the MS-SSVS model is better able to identify coefficient restrictions as the signal to noise ratio increases. This increase in signal is modeled in two ways, either as a reduction in error volatility or as an increase in sample size. The MS-SSVS model is particularly adept at identifying linear cross-regime restrictions with a high degree of accuracy, even as the amount of noise increases.

When I apply this new methodology to Federal Funds Rate data from 1970-2007, I find three things. First, I find evidence for two distinct regimes in the unemployment gap response coefficient. I find that there was a relatively strong unemployment gap response coefficient in the mid 1970s, the late 1980s and early 1990s, and between 2004-2006, with the weakest unemployment response coming during the leadership of Paul Volcker. Second, I find that the point-estimates of the other parameters in the rule are roughly constant, regardless of regime. This includes the FOMC's response to inflation, although I do find that the density

[4]SSVS restrictions are also utilized in Koop and Korobilis (2018), who allow for parameter restrictions in time-varying parameter models. The more general class of priors of this type is called "spike and slab" priors.

of the inflation response coefficient is bi-modal in the strong unemployment response regime. Finally, similar to Sims and Zha (2006), I find strong evidence of changes in the volatility of the Federal Funds rate.

I also consider an extension with a sample from 1970-2015, since 2015 is the last year for which FOMC Greenbook forecasts are currently available. This extension introduces the period covering the zero-lower-bound (ZLB) episode during and after the Great Recession, which lasted from 2008-2015. In order to avoid econometric issues that arise when using a censored dependent variable, for the period 2008-2015 I replace the Federal Funds rate with an estimate of the shadow rate from Johannsen and Mertens (2021). The results are qualitatively similar to those described above. However, the model now prefers shifts in both the FOMC's response to the change in unemployment and the unemployment gap. For the period 2008-2015, I find a high probability that the FOMC was responding relatively weakly to the change in the unemployment rate, instead putting relatively more emphasis on changes in the inflation gap. This makes intuitive sense, as the unemployment rate was steadily decreasing over this period, but inflation remained persistently below target.

2 Previous Studies of the FOMC Interest Rate Rule

In an attempt to distill previous findings on changes in the FOMC's interest rate rule, I have grouped papers according to their main conclusion as it relates to monetary policy. Broadly speaking, these papers typically fall into one of four categories:

1. The FOMC's inflation response has changed over time, but the output response has not.

2. The FOMC's output response has changed over time, but the inflation response has not.

3. The FOMC's inflation and output response have both changed over time.

4. None of the coefficients of the FOMC's interest rate rule have changed over time, but the variance has.

The findings of some recent and well-known studies of the FOMC's interest rate rule are highlighted in Table 1. This table is not meant to be an exhaustive list of all papers in this strand of literature, but instead it is meant to provide enough examples that the reader can get a sense of the types of differences seen across studies. In some of these studies, the findings are more subtle; for instance, Boivin (2006) finds that the point-estimates of both the output and inflation response coefficients change substantially over time, but that the confidence interval at any given point in time is very wide. In ambiguous cases like this I did my best to summarize the author's views about their findings.

[Table 1 about here]

From Table 1, we see that conclusions about changes in the policy rule are mixed. For example, two papers find strong evidence for changes in the interest rate smoothing parameter, while other papers find weak or no evidence for changes in this parameter. In many papers, the exact variables used (e.g. output gap vs. unemployment gap) affect the results, even when the methodology is the same. Furthermore, even when two papers find that the same parameter switched, the qualitative conclusions could differ. For instance, Clarida et al. (2000) and Alba and Wang (2017) both find evidence that the FOMC's inflation response has changed over time. However Clarida et al. (2000) find that the Taylor principle was likely not satisfied during the 1970's, while Alba and Wang (2017) find that the Taylor principle was always satisfied.

The source of the conflicting results is unclear. Bae et al. (2012) explored this issue within Chib's (1998) model of structural breaks, with their main area of interest being distinguishing the importance of sample start date and the use of real-time data. With a sample that begins in 1960, similar to the sample used by Clarida et al. (2000), they find that there was an abrupt increase in the inflation response coefficient in 1968, with the pre-1968 regime not satisfying the Taylor principle, but the 1968-1979 regime doing so. Since the sample in Orphanides (2004) begins in 1966, Bae et al. (2012) attribute the higher inflation response found by Orphanides (2004) to the sample start date rather than the use of real-time data. However, they did not consider all possible sources of difference between prior studies, as they did not analyze the impact of using monthly or quarterly data, or the impact of using different modeling techniques. While fully addressing the cause of the discrepancy in results among previous studies is beyond the scope of the current paper, I aim to shed light on the impact of modeling assumptions made within regime-switching models in this literature. Namely, I develop a model that takes uncertainty about the nature of regime-switching in each coefficient into account.

3 Data and Model Outline

For simplicity, I will first proceed as if the fully unrestricted Markov-switching model is being estimated. In that case, the model set-up is similar to others in the recent literature:

$$
\begin{aligned}
i_t &= \mu_{s_t} + \rho_{s_t} i_{t-1} + \phi_{\pi_{s_t}}(\pi_t^e - \pi_t^T) + \phi_{u_{s_t}} u_t^e + \phi_{\Delta u_{s_t}} \Delta u_t^e + \sigma_t \varepsilon_t \\
s_t &\in \{0, 1\} \\
P(s_t = j | s_{t-1} = i) &= q_{ij} \\
\varepsilon_t &\sim \mathcal{N}(0, 1)
\end{aligned}
$$

where i_t is the nominal Federal Funds rate in period t, π_t^e is the average three-quarter-ahead expected inflation rate at time t, π^T is the FOMC's inflation target estimated by Chan et al. (2013), u_t^e is the average three-quarter-ahead expected unemployment gap at time t, and Δu_t^e is the total expected change in the unemployment rate between time t and time $t+3$, and q_{ij} is the transition probability between regime i and regime j.

Finally, I assume that the volatility of the error term follows a random walk (i.e. the model exhibits "stochastic volatility"). Let $\sigma_t = \exp(\frac{h_t}{2})$. Then:

$$h_t = h_{t-1} + v_t$$

$$v_t \sim \mathcal{N}(0, Q)$$

There are a few differences between the model above and some of the other models in the literature. First, while most papers have included interest rate smoothing ($\rho_{s_t} i_{t-1}$), not all have.[5] Second, most papers in the literature use a measure of the output gap rather than the unemployment rate. However, as argued in Kozicki and Tinsley (2006) and Kozicki and Tinsley (2009), the historical narrative evidence from the 1970's and 1980's is much more consistent with a FOMC that responded to unemployment rather than output. In addition, Kozicki and Tinsley (2009) note that the Federal Funds rate target may depend on *changes* in real activity rather than (or in addition to) *gaps* in real activity. Therefore I include measures of both the real-time unemployment gap and the change in the unemployment rate. Since the MS-SSVS model performs variable selection, the model should be able to identify if one or more of these variables does not belong in the policy rule.[6]

In addition to differences in data and timing, I believe that the use of a stochastic volatility process is unique in the single-equation interest rate rule literature.[7] Since the main focus of this study is to identify possible change in coefficient values, it is important to allow the variance to evolve separately from the coefficient regimes. If the regimes covered both the coefficients as well as the variance, relatively large changes in the variance could drive the estimated regimes, which would then mainly identify periods of high and low volatility,

[5]For example, Alba and Wang (2017) does not allow for interest rate smoothing.

[6]Additionally, since I am using three-quarter-ahead forecasts from the Greenbook, if I were to use the output gap instead of the unemployment gap, I would need estimates of the FOMC's beliefs about both the level and growth rate of potential output. Constructing real-time estimates of both the level and growth rate of potential output that was available to the FOMC at the time of their decisions is non-trivial. These were typically not published in the Greenbook, and estimates of both output and potential output tend to be heavily revised relative to the much smaller revisions in the unemployment rate, which typically only occur if there are changes in the estimated seasonal factors.

[7]Primiceri (2005) allowed for stochastic volatility in a VAR set-up.

rather than identifying changes in coefficients. While modeling the variance as a separate regime switching process would also break this link, identifying variance regimes is not the main focus of the study, and stochastic volatility is robust to different types of parameter change.[8]

The data is recorded at each FOMC meeting and is gathered from historical FOMC Greenbooks. For inflation I use the relevant GNP or GDP Deflator inflation. I use "meeting-based timing", developed in Check (2019), in which the Federal Funds rate is averaged between meeting dates. In the baseline estimation, I assume a constant natural rate of unemployment and I use the methodology of Chan et al. (2013) to estimate the inflation target, using GDP Deflator data.[9] In baseline estimation, I restrict the sample from late 1969 through the end of 2007. The start date coincides with when the FOMC began regularly reporting three-quarter-ahead forecasts, and the end date is chosen to avoid the zero-lower-bound.[10] In an extended sample that goes through 2015, I instead replace the Federal Funds rate with an estimate of the shadow rate.

The choice of FOMC Greenbook data is consequential, as it helps avoid possible endogeneity but limits the available sample. In the standard Taylor rule framework, there is an endogeneity concern — while the FOMC adjusts the Federal Funds rate based on changes in macroeconomic variables such as inflation and unemployment, these variables also respond to changes, or expected changes, in the Federal Funds rate. However, since the Greenbook forecasts are created prior to each FOMC meeting under the assumption of no change in the Federal Funds rate at that meeting, the endogeneity concern is greatly alleviated. In other

[8] While policy changes by the FOMC may impact the volatility of a monetary policy rule, it is also likely impacted by structural changes in the banking system and policy implementation. Changes in volatility are therefore less likely to be well described by a two-regime Markov-Switching process.

[9] In a robustness exercise presented in Appendix C, I explore alternative assumptions concerning the natural rate of unemployment and inflation target.

[10] While using an averaged three-quarter-ahead forecast is somewhat non-standard in this literature, its choice was a practical one. The FOMC Greenbook started regularly providing three-quarter-ahead forecasts in the 1970's, but did not regularly provide four-quarter-ahead forecasts until the 1980's. In addition, averaging the forecasts through three-quarters-ahead has the effect of dampening the impact of shocks that are expected to be transitory. Intuitively, the FOMC would probably not want to respond to shocks they expect to be transitory, given that monetary policy impacts the economy with a lag.

words, the forecasts may cause the FOMC to adjust the Federal Funds rate, but the potential for a Federal Funds rate adjustment is ignored when the forecasts are created, breaking the link of possible endogeneity. This feature of the Greenbook forecasts was summarized nicely by Romer and Romer (2004),

> "The Greenbook forecast is almost always predicated on the assumption of no change in monetary policy in the very short run, where the very short run means at least until the FOMC meeting after the one for which the forecast is being made. This characteristic, along with the usual assumption of some lag in the effects of monetary policy, makes it unlikely that forecasts zero, one, and two quarters ahead are contaminated by assumptions or inside information about the course of monetary policy. As a result, these near-term forecasts provide information about what the Federal Reserve expected to happen to the economy in the absence of changes in monetary policy."

For this reason, along with the fact that Greenbook data provides insights into the real-time information available to the FOMC at the time of their decisions, use of Greenbook data is popular in this literature. In studies that specifically investigate endogeneity in these types of regressions, Coibion and Gorodnichenko (2011) use Greenbook data and find support for standard OLS rather than IV estimation, and Carvalho et al. (2019) use OLS with non-Greenbook data as an example of a regression that may suffer from endogeneity, and then compare this to OLS using Greenbook data which they argue should not suffer from endogeneity.

While the benefit of the Greenbook data is that it breaks the endogeneity link, a major drawback is that it is released with a five-year lag. As of writing, this limits the potential sample to the end of 2015, which covers a portion of the zero-lower-bound episode but does not cover the more recent liftoff period. In my main results, I limit the sample through the end of 2007 to avoid the period of the zero-lower-bound. In a robustness exercise, I extend my sample, replacing the Federal Funds rate with the shadow rate of Johannsen and Mertens's (2021). This shadow rate is produced at a quarterly frequency, so I am unable to map this directly into the meeting-based-timing used in my baseline results. Nevertheless, the results of this extension are qualitatively similar to the results for the 1970-2007 period.

4 Full Econometric Model

The model I introduce is based on a Markov-Switching model with switching in coefficients:

$$
\begin{aligned}
&y_t = X_t \beta_{St} + \varepsilon_t \\
&\varepsilon_t \sim \mathcal{N}(0, \sigma^2) \\
&S_t \in \{0, 1\}
\end{aligned}
$$

The regime, S_t follows a first order Markov process:

$$
\begin{aligned}
&Pr(S_t = j | S_{t-1} = i) = q_{ij} \\
&\qquad\qquad\qquad i, j \in \{0, 1\}
\end{aligned}
$$

The model as detailed above has been well studied, and there exist well known frequentist and Bayesian procedures to estimate it. These methods are described in Hamilton (1989), Kim and Nelson (1999), Frühwirth-Schnatter (2006), and Piger (2009), among others. While these techniques make it feasible to estimate the model, model comparison remains relatively cumbersome. The estimation process itself can be time consuming, making the estimation of more than a handful of models potentially infeasible. This problem is only exacerbated when performing Bayesian Model Averaging, as this requires estimation of the marginal likelihood of each model — this is an extra, complicated, and time consuming step that needs to be undertaken after estimation of the model.[11] Finally, the total number of models to consider grows more rapidly in this class of models than in linear models. In linear models, there are only two possibilities for each regressor — either it belongs in the model or it does not. However, in a Markov-switching model with two regimes, there are five possibilities for each

[11] Additionally, some authors such as Kruschke (2015) argue against using the marginal likelihood for model comparison due to its high sensitivity to parameter priors. This problem was originally pointed out in Jeffreys (1939) and received more attention after Lindley (1957) named it a "paradox."

regressor: (1) it does not belong in either regime, (2) it belongs in each regime, and the true effect is distinct under each regime, (3) it belongs in each regime but the true effect is the same regardless of regime, (4) the variable belongs in the first regime but not the second, or (5) the variable belongs in the second regime but not the first. This only increases the burden of model comparison, as the number of models to consider expands more rapidly when the possibility of switching is properly accounted for.

Despite the numerical difficulties with marginal likelihood calculations, model comparison in this class of models remains important. In this paper, I build an econometric model that allows for the five possibilities described above. It does so in a computationally feasible manner by utilizing a hierarchical prior to nest all of these possibilities within a single model. To do this, I build on Stochastic Search Variable Selection (SSVS), which was developed in George and McCulloch (1993) and further studied and implemented in George et al. (2008) and Koop and Korobilis (2010). In SSVS, a mixture distribution of Normal priors is used on the regression coefficients to allow researchers to place prior weight on the possibility that the coefficient may be exactly equal to zero (i.e. that the variable does not belong in the model). This prior impacts the model likelihood and allows the data to inform whether the variable belongs in the model. Since variable selection is built into the model likelihood, only one model needs to be estimated. This framework is therefore simpler and more efficient than performing Bayesian model averaging by estimating hundreds, thousands, or more models and then comparing them based on their marginal likelihoods.

To help elicit the mathematical details of this model, let $\beta_k = [\beta_{k,i}\ \beta_{k,j}]$ be a vector that contains coefficient k in regime i and regime j. Under the MS-SSVS model the prior $p(\beta_k)$ is assumed to be a mixture of Normal distributions with $\ell = 5$ components:

$$
\begin{aligned}
\beta_k \sim\ & \gamma_{k,1}\mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \tau_0\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right) + \gamma_{k,2}\mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \tau_1\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right) + \gamma_{k,3}\mathcal{N}\left(\begin{bmatrix}\hat{\beta}_{k,OLS}\\ \hat{\beta}_{k,OLS}\end{bmatrix}, \tau_1\begin{bmatrix}1 & 1-\epsilon\\1-\epsilon & 1\end{bmatrix}\right)\\
& + \gamma_{k,4}\mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}\tau_1 & 0\\0 & \tau_0\end{bmatrix}\right) + \gamma_{k,5}\mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}\tau_0 & 0\\0 & \tau_1\end{bmatrix}\right)\\
\gamma^k =\ & (\gamma_1^k, \gamma_2^k, \gamma_3^k, \gamma_4^k, \gamma_5^k) \in \{(1,0,0,0,0), (0,1,0,0,0), (0,0,1,0,0), (0,0,0,1,0), (0,0,0,0,1)\}\\
\tau_0 =\ & 0.1 \times \sqrt{\mathrm{Var}\left(\hat{\beta}_{k,OLS}\right)}\\
\tau_1 =\ & 15 \times \sqrt{\mathrm{Var}\left(\hat{\beta}_{k,OLS}\right)}
\end{aligned}
$$

where $\hat{\beta}_{k,OLS}$ is the OLS estimate of β_k under the assumption of no regime-switching, $\mathrm{Var}\left(\hat{\beta}_{k,OLS}\right)$ is the OLS estimate of the variance, and τ_0, τ_1, and ϵ are parameters chosen by the researcher that control the variances of each distribution in the prior.[12]

The mixture distribution described above represents five distinct possibilities:

1. The coefficient is restricted to be approximately zero in each regime, i.e. the variable is excluded in both regimes.

2. The coefficient estimates are freely estimated, independently of each other, i.e. the variable is included in each regime, and the effect in each regime is different.

3. The coefficient estimates are freely estimated, but identical, i.e. the variable is included in each regime, and the effect in each regime is the same.

4. The coefficient in regime 0 is freely estimated, but the coefficient in regime 1 is restricted to be near zero, i.e. the variable is excluded from regime 1.

[12] The choices of τ_0, τ_1, and ϵ are similar in spirit to Bayesian linear regression, where the researcher typically chooses the variance of the prior distribution for the regression coefficients. Note that SSVS models are not technically fully Bayesian since they rely on the data to inform the priors. This can be avoided by running OLS on a pre-sample of data and using those pre-sample estimates instead. In addition, in SSVS, model restrictions can only be enforced approximately. For more details, see the Technical Appendix.

5. The coefficient in regime 1 is freely estimated, but the coefficient in regime 0 is restricted to be near zero, i.e. the variable is excluded from regime 0.

For the coefficient on each variable, the prior distribution over the five indicator vectors, γ_k, I assume that each comes from the following Categorical distribution:

$$\gamma_k = \begin{cases} (1,0,0,0,0) & \text{with probability } p_{k,1} \\ (0,1,0,0,0) & \text{with probability } p_{k,2} \\ (0,0,1,0,0) & \text{with probability } p_{k,3} \\ (0,0,0,1,0) & \text{with probability } p_{k,4} \\ (0,0,0,0,1) & \text{with probability } p_{k,5} \end{cases}$$

$$\sum_{\ell=1}^{5} p_{k,\ell} = 1$$

$$0 < p_{k,\ell} < 1 \ \forall \ \ell \in \{1,2,3,4,5\}$$

where each prior mixture probability, $p_{k,\ell}$, is a fixed constant set by the researcher, and the prior probabilities can differ across different coefficients, k. Once a researcher has set the five relevant prior probabilities, this Categorical distribution is a valid discrete distribution over the indicator vectors representing the mixture components.[13]

In many cases, researchers may not be interested in exactly which of these five distributions is most probable. Instead, they may be interested in some type of combination of these five distributions. For example, in some applications, for each parameter in each regime, they may be interested instead in three possibilities: (1) the parameter is restricted to zero, (2) the parameter is freely estimated, (3) the parameter is non-zero but restricted to be identical to the parameter estimate in the other regime. These could be found by combining the relevant probabilities of the Categorical distribution. For example, in regime 0, the probability of parameter k being restricted to zero is $p_{k,1} + p_{k,5}$, since $p_{k,1}$ is the probability of the coefficient being zero in both regimes and $p_{k,5}$ is the probability of the coefficient being

[13] For more details on the exact estimation procedure, please refer to the Technical Appendix.

zero in regime 0 but non-zero in regime 1.

5 Monte-Carlo Analysis

To test the power of this procedure to identify coefficient restrictions, I perform a Monte-Carlo analysis. I consider two data generating processes (DGPs): one in which data is generated from a process that has several types of restrictions, and another that corresponds to linear regression. The first Monte-Carlo exercise is analogous to a situation in which some parameters in the Taylor rule switch, some remain constant across regimes, and others are zero in both regimes. The second Monte-Carlo exercise is analogous to a situation in which the Taylor rule variables are correctly specified, but that there is actually no switching — the parameter values are all identical across regimes.

In both cases, the true model can be written in matrix notation as:

$$
\begin{aligned}
y_t &= X_t \beta_{St} + \varepsilon_t \\
\varepsilon_t &\sim \mathcal{N}(0, \sigma^2) \\
Pr(S_t = j | S_{t-1} = i) &= q_{ij}
\end{aligned}
$$

I assume that there is an intercept and two independent regressors with mean zero:

$$
\begin{aligned}
X_t &= \begin{bmatrix} 1 & X_{2,t} & X_{3,t} \end{bmatrix} \\
\begin{bmatrix} X_{2,t} \\ X_{3,t} \end{bmatrix} &\sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)
\end{aligned}
$$

Since there are three columns in X_t, $K = 3$. Recall that the pair of coefficients multiplying the variable k but under regime 0 and regime 1 is given by $\beta_k = [\beta_{k,0}\ \beta_{k,1}]'$ for $k \in \{1, \cdots, K\}$.

5.1 DGP 1: Restricted Coefficients and Markov-Switching

For the first Monte-Carlo exercise, I assume that the intercept is different in each regime, the coefficient on the first regressor is identical across regimes, and the coefficient on the second regressor is zero in both regimes. This set-up is analogous to a situation where, for example, the real interest rate switches, the FOMC has the same response to inflation (x_1) across regimes, but does not respond to the output gap (x_2) in either regime. As defined above, we have:

$$\beta_1 = \begin{bmatrix} 1.0 \\ -0.5 \end{bmatrix} \quad \beta_2 = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix} \quad \beta_3 = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}$$

Finally, the transition probabilities for each regime are given by:

$$P = \begin{bmatrix} q_{00} & q_{01} \\ q_{10} & q_{11} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$$

which implies that the system will spend roughly 60% of time periods in regime 0 and 40% of time periods in regime 1.

5.1.1 Priors

In this exercise, I assume that the variance term is constant. Therefore, I set an inverse-Gamma prior on the variance term. The priors are are presented in Table 2.

[Table 2 about here]

5.1.2 Results

In the Monte-Carlo exercise, I vary both the number of observations, T, and the standard deviation of the error term, σ. I consider $T \in \{50, 100, 150, 200, 250\}$ and $\sigma \in \{0.1, 0.5, 1.0\}$.

Models estimated on DGPs with large T and small σ are most able to pick out the correct restrictions, as these models have the largest sample size and smallest variance.

In Tables 3-5, I present the average accuracy of identification of the correct restriction by the estimation procedure. This number has been averaged over the results across 200 separate data generation and estimation procedures. For example, for the first column of table (2), I set $\sigma = 0.1$ and $T = 50$. I then generate 200 data sets and run the estimation procedure on each. For each data set, I calculate the percentage of draws in which the appropriate restriction was chosen, and I average this percentage across all 200 data sets. For the estimation procedure, I use 15,000 burn-in draws and 20,000 posterior draws.

[Table 3 about here]

[Table 4 about here]

[Table 5 about here]

Three things become apparent when looking at these Tables 3-5. First, the MS-SSVS model is able to correctly identify all types of restrictions when the data has a high signal to noise ratio. This shows that the estimation procedure is well-behaved when the amount of noise in the data generating process is relatively small. Second, the model performs very well at detecting the linear regression restriction, $\beta_{2,0} = \beta_{2,1}$, and fairly well at detection of the zero-restriction for the coefficients $\beta_{3,0}$ and $\beta_{3,1}$. Third, as the noise increases, the model has a relatively more difficult time detecting that $\beta_{1,1}$ is actually different than zero compared to $\beta_{1,0}$. This is likely due to the fact that the absolute value of $\beta_{1,1}$ is smaller than the absolute value of $\beta_{1,0}$.

5.2 DGP 2: Linear Model with no switching

I repeat the same exercise as above, except I change the true value of the coefficients so that all regressors belong in the model, the effect is different than zero for all regressors, and the effect is the same in both regimes for all regressors. This corresponds to linear regression in which there is no misspecification — all the variables included in the model have a non-zero effect, and there are no excluded relevant variables. This set-up is analogous a situation in which the Taylor rule was correctly specified and there was no parameter change over time, similar to what was found by Sims and Zha (2006).

$$\beta_1 = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix} \quad \beta_2 = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix} \quad \beta_3 = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$$

I leave everything else, including the priors, unchanged and conduct the same analysis as above. I present the results in Tables 6-8.

[Table 6 about here]

[Table 7 about here]

[Table 8 about here]

Under the linear regression DGP, the model performs remarkably well under all sample sizes and all variance sizes. This may be partially driven by the fact that there is a 50% prior probability placed on each of these coefficients being identical. It is important to note that while I have fixed the prior mixture probabilities to be identical for all sets of parameters, in general a researcher could relax this assumption, placing different prior mixture probabilities on each pair of coefficients.[14]

[14]For example, if a researcher suspected that one pair of coefficients would be different in each regime, she could increase the prior probability on mixture 2 and reduce the prior probability on mixture 3.

6 Application: Interest Rate Rules

6.1 Baseline Sample: 1970-2007

Next, I apply the MS-SSVS model to the interest rate rule equation and data described in section three. Recall that I estimate a rule of the form:

$$\begin{aligned}
i_t &= \mu_{s_t} + \rho_{s_t} i_{t-1} + \phi_{\pi_{s_t}}(\pi_t^e - \pi_t^T) + \phi_{u_{s_t}} u_t^e + \phi_{\Delta u_{s_t}} \Delta u_t^e + \sigma_t \varepsilon_t \\
\varepsilon_t &\sim \mathcal{N}(0,1) \\
\sigma_t &= \exp\left(\frac{h_t}{2}\right) \\
h_t &= h_{t-1} + v_t \\
v_t &\sim N(0,Q) \\
s_t &\in \{0,1\} \\
P(s_t = j | s_{t-1} = i) &= q_{ij}
\end{aligned}$$

while also allowing for MS-SSVS coefficient restrictions on μ_{s_t}, ρ_{s_t}, $\phi_{\pi_{s_t}}$, $\phi_{u_{s_t}}$, and $\phi_{\Delta u_{s_t}}$.

I have three major findings: (1) I find evidence of distinct regimes in monetary policy, with the response to the unemployment gap changing substantially between regimes; (2) at the posterior mean, the FOMC's response to the inflation gap is similar across both regimes; (3) the volatility of the error term varies substantially across time and the timing does not appear to coincide with the estimated regimes.

Detailed findings for the regression parameters are presented in Tables 9, 10 and 11, as well as in Figure 1. In the last column of Tables 9 and 10, we see that there is substantial evidence that four of the five regression coefficients have entered the Taylor rule linearly, remaining constant regardless of regime. The only coefficient that exhibits strong evidence of regime switching is the unemployment gap response coefficient, which has over a 90% posterior probability of being distinct across regimes. This is further evidenced in Table 11, which shows that the posterior mean for all coefficients are very similar across regimes, with the exception of the unemployment gap response coefficient. Finally, this is visualized in Figure 1, which plots the entire estimated density of coefficients in each regime. While most densities lie largely on top of each other, the densities of the unemployment gap response coefficient are largely separated, indicating that it is distinct across regimes.

[Table 9 about here]

[Table 10 about here]

[Table 11 about here]

[Figure 1 about here]

To get a better sense of the magnitude of the difference across regimes, Figure 2 plots the FOMC's expected response to a persistent one-percentage-point unemployment gap up to 40 meetings (roughly 4–5 years) in the future. Clearly, the responses differ across the regimes. In the strong unemployment response regime, if this unemployment gap persisted for 40 meetings, the FOMC would have decreased the Federal Funds rate by a total of roughly 4.2 percentage points, vs. only a 0.9 percentage point reduction in the weak unemployment response regime.

On the contrary, in Figure 3, we see that even in the long-run, the FOMC's response to expected inflation is roughly constant across regimes. After a one-percentage-point inflation gap lasting 40 meetings, the FOMC would likely increase the Federal Funds rate by between roughly 2.0 and 2.5 percentage-points under either regime. For all periods, the credible intervals from each regime overlap substantially, indicating that there is very weak evidence of any meaningful difference between the inflation response across regimes.

[Figure 2 about here]

[Figure 3 about here]

Despite the fact that the posterior mean for the inflation response is roughly equal across regimes, the inflation response coefficient has a bimodal density in the strong unemployment response regime, as can be seen in Figure 1. In this regime, there is roughly a 13% probability that the FOMC was responding only very weakly to the inflation gap. This finding would be missed if parameter constancy had been "pre-tested" in an unrestricted model, with the final estimated model only allowing for changes in the unemployment gap. The bimodal inflation response shows the importance of averaging over uncertainty with respect to parameter switching, and underlines a strength of the MS-SSVS approach taken in this paper.

I also find strong evidence of change in the volatility of the interest rate rule. This change can be seen in Figure 4, with the standard deviation of the error in the interest rate rule peaking in the early 1980's at around 150 basis points and falling substantially since then, to below 25 basis points. The spike in volatility in the interest rate rule in the late 1970's and early 1980's is expected, as the Federal Reserve was on record as targeting the money supply rather than the interest rate rule during this time period. Once they returned to targeting the Federal Funds rate, the volatility steadily declined, as the interest rate rule matched actual FOMC behavior much more accurately.

[Figure 4 about here]

Along with the estimated parameter values, the timing of regimes and regime changes are of interest. These are presented in Figure 5, which plots the posterior mean of being in the strong unemployment gap response at each meeting date. Also displayed in Figure 5 are the dates of changes in Fed chair, and for each Fed chair, the average probability that the FOMC was in the strong unemployment gap response regime during their time as chair. While there are certainly swings in estimated regime probabilities within each chair's tenure, especially under Burns and Greenspan, the transition period between chairs also seems to correspond to regime change. For instance, the probability of being in the strong unemployment response regime begins declining from over 80% very late in Burns' tenure, continues sliding throughout Miller's brief leadership, and bottoms out below 20% as Volcker takes control in 1979. Interestingly, despite the fact that I am using a very different proxy of the output gap than Orphanides (2004), my findings are largely consistent with his — the FOMC under Burns was more likely to respond strongly to the output gap, with the FOMC under Volcker much less likely to do so. Under Burns, the average probability of being in the strong unemployment response regime was roughly 60%, while under Volcker it was roughly 20%. Because the inflation response coefficient is largely similar across regimes, this means that the FOMC under Volcker put *relatively* more weight on its inflation response, which is consistent with narrative historical evidence (e.g. Kaya et al. (2019)).

[Figure 5 about here]

While my results are consistent with some of the previous literature, such as Orphanides (2004) and Sims and Zha (2006), they stand in contrast to some others. For example, in a recent application of a Markov-Switching model to interest rate rules in Murray et al. (2015) the authors find that the inflation response in one regime was much lower than in the other regime, and there were periods in which it was highly probable that the FOMC failed to satisfy the "Taylor Principle." However, they find that this weak inflationary response occurred during the Volcker years, 1979-1985 (among other times). This seems highly counterfactual, as it goes against historical accounts, narrative evidence from the FOMC meetings during this time period, and other statistical evidence found by Clarida et al. (2000) among others, all of which attribute the fall in inflation after 1980 to the *strong* inflation response during Volcker's tenure. My findings are more consistent with this latter strand of literature, Although I do not find evidence of a direct change in the inflation response coefficient, I do find a reduction in the unemployment gap response during the Volcker years. Therefore, like Orphanides (2004), I find that the FOMC under Volcker responded *relatively* more to inflation than it had previously.

6.2 Extended Sample: 1970-2015

As of writing, FOMC Greenbook forecasts – incorporated into the newly created "Tealbook" in 2010 – are available through the last FOMC meeting of 2015. The additional period 2008-2015 spans the zero-lower-bound (ZLB) episode and coincides with other changes in monetary policy, such as the introduction of the Fed's official 2% inflation target, which was first announced in early 2012. Most importantly for this study, the ZLB period violates assumptions of the MS-SSVS model developed in section four. The model assumes that the residuals are Normally distributed, independent, and identically distributed. These assumptions are violated when the dependent variable is censored, like the Federal Funds rate was during the ZLB period.

Therefore, during the period for which the ZLB is binding, nearly all of the 2008-2015 period, I replace the observed Federal Funds rate with an estimate of the "shadow rate" from Johannsen and Mertens (2021). This estimate of the shadow rate, over the ZLB period, can be seen in Figure 6.

[Figure 6 about here]

This shadow rate, like those constructed in Aruoba et al. (2021a) and Aruoba et al. (2021b), comes from the output of a structural model for the U.S. economy. This is conceptually different from the shadow rate in Wu and Xia (2016), which is instead constructed based on the term structure of interest rates (i.e., the yield curve). As discussed in Aruoba et al. (2021a), VAR or DSGE based estimates tend to be qualitatively similar to each other, but distinct from the Wu and Xia (2016) estimate. As shown in Figure 7, the VAR based estimate from Johannsen and Mertens (2021) tends to show monetary policy at its most accommodative early in the Great Recession, and then roughly constant around -1% until mid-late 2015. In contrast, the Wu and Xia (2016) estimate is only marginally accommodative (around -0.5%) during 2009, and continues becoming more accommodative throughout the economic recovery, bottoming out in the second quarter of 2014 around -3%, before sharply rising right before liftoff at the end of 2015. As noted by Aruoba et al. (2021a), this seems like an unrealistic path of the stance of monetary policy given the narrative evidence from the time, which suggests that monetary policy was at its most accommodative during and just following the Great Recession.[15]

[Figure 7 about here]

While there are limitations to this approach, the results in this extended sample are qualitatively similar to those described in section 6.1. One limitation of using a shadow rate estimate to extend the sample in this study is that it changes the construction of the dependent variable. For periods in which the Federal Funds rate is above the ZLB, the dependent variable is constructed by averaging the daily FF rate between meeting dates. Once the ZLB is binding, I use the estimated shadow rate, which is only available quarterly. Therefore, when the FOMC meeting date falls within a given quarter, I use that quarter's shadow rate estimate. Nevertheless, the results over 1970-2015 sample are qualitatively similar to the estimates produced using only the 1970-2007 data. I still find that the intercept, AR(1) parameter and inflation gap response are roughly constant between regimes. In addition to switching occurring in the unemployment gap response coefficient, switching also appears to occur in the FOMC's response to the change in the unemployment rate. This is qualitatively similar to the findings in the limited sample, as switching occurs in unemployment response but not inflation response. The estimated regimes throughout the 1970-2007 period are similar to the baseline results.[16] The new 2008-2015 period is characterized by a very high probability of being in the regime with the more muted response to the unemployment rate. This makes intuitive sense, since the unemployment rate was steadily falling during this period, without a corresponding increase in interest rates. Meanwhile, inflation remained

[15] Consider one example of why these estimates may diverge. Movement in the Wu and Xia shadow rate, driven by a decrease in the yields on longer-term bonds over the 2010-2014 period, could have been caused by investors coming to believe a "secular stagnation" hypothesis, rather than from more accommodative monetary policy. As unemployment was steadily recovering during this period and inflation fairly stable, the values imputed in a VAR or DSGE model would be more stable.

[16] One exception is that mean probability of responding strongly to unemployment now appears to be roughly equal across the Burns and Volcker periods.

below target throughout most of this period, which would be consistent with the low or negative target Federal Funds rate that actually occurred.

7 Conclusion

Over the past 15 years, there has been considerable disagreement about the existence and nature of changes in the coefficients in the FOMC's interest rate rule. In an attempt to clarify the nature of these changes, I build a Markov-Switching model that can endogenously determine the existence of two types of restrictions: (1) zero-restrictions, in which a variable may be excluded from one or both of the regimes and (2) identity-restrictions, in which the regression coefficient on the same variable may be restricted to be identical across both regimes. The estimation procedure blends and extends the Gibbs samplers that were previously derived for estimation of Markov-Switching models and Stochastic Search Variable Selection models. I call this unified model an MS-SSVS model.

I find that the MS-SSVS model performs well at identifying true restrictions in a Monte-Carlo exercise using simulated data. In general, the MS-SSVS model performs best in data-sets that have a relatively small amount of noise. In these data sets, it is able to detect zero-restrictions, "identical" restrictions, and switching in the coefficients with high probability. The MS-SSVS model is still able to identify these restrictions as the amount of noise grows, and it is able to detect linear cross-regime restrictions with a surprisingly high degree of accuracy in even the noisiest data sets.

When I apply this model to an interest rate rule for the Federal Funds rate, I find three things. First, I find strong evidence that there have been changes in the volatility of interest rate rule over time. Second, consistent with Orphanides (2004), I find substantial evidence that there have been changes in the FOMC's unemployment response over time. I find that the periods least likely to have had a strong response to unemployment are the 1980s, the period from roughly 1995-2004, and in the extended sample, the period from 2008-2015.

Finally, there is relatively little evidence that there have been economically meaningful shifts in the FOMC's inflation response. Despite the fact that the point estimate of the response to the inflation gap is roughly equivalent across regimes, I do find a heightened probability of a weak inflation gap response in the 1970's. This last finding highlights the importance of averaging over the uncertainty regarding parameter regimes, as is done in the MS-SSVS model.

Table 1: Evidence of Parameter Change in Previous Studies

Study	Smoothing	Output	Inflation	Variance
Clarida et al. (2000)	–	–	Y	–
Orphanides (2004)	–	Y	–	–
Boivin (2006)	–	Y	Y	–
Sims and Zha (2006)	–	–	–	Y
Murray et al. (2015)	Y	Y	Y	Y
Alba and Wang (2017)	–	Y	Y	Y
Gonzalez-Astudillo (2018)	–	Y	Y	Y
Bennani et al. (2018)	Y	Y	–	–

Note: Smoothing, output, inflation, and variance indicate whether coefficients in each respective variable were found to change across regimes. "Y" means that there was evidence that it switched, "–" means that the variable was excluded from consideration, was not allowed to switch, or was not found to switch.

Table 2: Monte-Carlo Priors

Parameter	Prior Distribution	Prior Mean	Prior S.D.
c_0	Constant	0.1	–
c_1	Constant	15	–
τ_0^k	Constant	$c_0\sqrt{\mathrm{Var}\left(\hat{\beta}_{OLS}^k\right)}$	–
τ_1^k	Constant	$c_1\sqrt{\mathrm{Var}\left(\hat{\beta}_{OLS}^k\right)}$	–
p_1	Constant	0.25	–
p_2	Constant	1/12	–
p_3	Constant	0.50	–
p_4	Constant	1/12	–
p_5	Constant	1/12	–
γ_j^k	Categorical	$\begin{bmatrix} p_1 & p_2 & p_3 & p_4 & p_5 \end{bmatrix}$	$\begin{bmatrix} 0.432 & 0.275 & 0.5 & 0.275 & 0.275 \end{bmatrix}$
q_{00}	Beta	0.8	0.16
q_{11}	Beta	0.8	0.16
σ^2	Inverse Gamma	improper	infinite

Table 3: DGP1 – Low Variance ($\sigma = 0.1$)

		$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 250$
$\beta_{1,0} \neq 0$	Prior	66.7%	66.7%	66.7%	66.7%	66.7%
	Posterior	100%	100%	100%	100%	100%
$\beta_{1,1} \neq 0$	Prior	66.7%	66.7%	66.7%	66.7%	66.7%
	Posterior	100%	100%	100%	100%	100%
$\beta_{2,0} = \beta_{2,1}$	Prior	50.0%	50.0%	50.0%	50.0%	50.0%
	Posterior	99.4%	99.7%	99.8%	99.9%	99.9%
$\beta_{3,0} = 0$	Prior	33.3%	33.3%	33.3%	33.3%	33.3%
	Posterior	94.4%	95.4%	95.8%	95.8%	95.9%
$\beta_{3,1} = 0$	Prior	33.3%	33.3%	33.3%	33.3%	33.3%
	Posterior	94.4%	95.4%	95.8%	95.8%	95.9%

Note: $\beta_{1,i} \neq 0$ refers to the total probability that the intercept is (correctly) not restricted to be zero in regime i. For $\beta_{1,0}$ this is given by $p_{1,2} + p_{1,3} + p_{1,4}$. For $\beta_{1,0}$ this is given by $p_{1,2} + p_{1,3} + p_{1,5}$. $\beta_{2,0} = \beta_{2,1}$ refers to the probability that the coefficient on $X_{2,t}$ is (correctly) restricted to be very close to equal across regimes. This is given by $p_{2,3}$. $\beta_{3,i} = 0$ refers to the total probability that the coefficient on $X_{3,t}$ is (correctly) restricted to be very close to zero. For $\beta_{3,0} = 0$ this is given by $p_{3,1} + p_{3,5}$, while for $\beta_{3,1} = 0$ this is given by $p_{3,1} + p_{3,4}$.

Table 4: DGP1 – Medium Variance ($\sigma = 0.5$)

		$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 250$
$\beta_{1,0} \neq 0$	Prior	66.7%	66.7%	66.7%	66.7%	66.7%
	Posterior	75.3%	98.8%	100%	100%	100%
$\beta_{1,1} \neq 0$	Prior	66.7%	66.7%	66.7%	66.7%	66.7%
	Posterior	57.7%	95.2%	99.4%	100%	100%
$\beta_{2,0} = \beta_{2,1}$	Prior	50.0%	50.0%	50.0%	50.0%	50.0%
	Posterior	93.3%	97.8%	98.5%	98.9%	99.0%
$\beta_{3,0} = 0$	Prior	33.3%	33.3%	33.3%	33.3%	33.3%
	Posterior	79.5%	82.5%	83.9%	83.1%	82.7%
$\beta_{3,1} = 0$	Prior	33.3%	33.3%	33.3%	33.3%	33.3%
	Posterior	78.8%	81.8%	84.0%	83.3%	83.7%

Note: See the footnote under Table 3.

Table 5: DGP1 – High Variance ($\sigma = 1.0$)

		$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 250$
$\beta_{1,0} \neq 0$	Prior	66.7%	66.7%	66.7%	66.7%	66.7%
	Posterior	31.5%	55.6%	71.0%	81.2%	90.3%
$\beta_{1,1} \neq 0$	Prior	66.7%	66.7%	66.7%	66.7%	66.7%
	Posterior	13.0%	25.7%	42.1%	52.6%	64.8%
$\beta_{2,0} = \beta_{2,1}$	Prior	50.0%	50.0%	50.0%	50.0%	50.0%
	Posterior	80.1%	88.1%	93.0%	94.2%	96.4%
$\beta_{3,0} = 0$	Prior	33.3%	33.3%	33.3%	33.3%	33.3%
	Posterior	74.3%	76.6%	78.2%	76.3%	76.6%
$\beta_{3,1} = 0$	Prior	33.3%	33.3%	33.3%	33.3%	33.3%
	Posterior	74.4%	75.9%	78.6%	76.3%	77.4%

Note: See the footnote under Table 3.

Table 6: DGP2 – Low Variance ($\sigma = 0.1$), Linear Regression

		$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 250$
$\beta_{1,0} = \beta_{1,1}$	Prior	50.0%	50.0%	50.0%	50.0%	50.0%
	Posterior	100%	100%	100%	100%	100%
$\beta_{2,0} = \beta_{2,1}$	Prior	50.0%	50.0%	50.0%	50.0%	50.0%
	Posterior	100%	100%	100%	100%	100%
$\beta_{3,0} = \beta_{3,1}$	Prior	50.0%	50.0%	50.0%	50.0%	50.0%
	Posterior	100%	100%	100%	100%	100%

Note: $\beta_{1,0} = \beta_{1,1}$ refers to the total probability that the intercept is (correctly) restricted to be very close to equal across regimes. This is given by $p_{1,3}$. $\beta_{2,0} = \beta_{2,1}$ refers to the total probability that the coefficient on $X_{2,t}$ is (correctly) restricted to be very close to equal across regimes. This is given by $p_{2,3}$. $\beta_{3,0} = \beta_{3,1}$ refers to the total probability that the coefficient on $X_{3,t}$ is (correctly) restricted to be very close to equal across regimes. This is given by $p_{3,3}$.

Table 7: DGP2 – Medium Variance ($\sigma = 0.5$), Linear Regression

		$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 250$
$\beta_{1,0} = \beta_{1,1}$	Prior	50.0%	50.0%	50.0%	50.0%	50.0%
	Posterior	92.8%	96.5%	97.9%	99.4%	99.3%
$\beta_{2,0} = \beta_{2,1}$	Prior	50.0%	50.0%	50.0%	50.0%	50.0%
	Posterior	92.4%	96.0%	98.1%	99.0%	99.7%
$\beta_{3,0} = \beta_{3,1}$	Prior	50.0%	50.0%	50.0%	50.0%	50.0%
	Posterior	92.8%	96.6%	97.6%	99.2%	99.6%

Note: See the footnote under Table 6.

Table 8: DGP2 – High Variance ($\sigma = 1.0$), Linear Regression

		$T = 50$	$T = 100$	$T = 150$	$T = 200$	$T = 250$
$\beta_{1,0} = \beta_{1,1}$	Prior	50.0%	50.0%	50.0%	50.0%	50.0%
	Posterior	81.8%	85.7%	87.5%	90.6%	91.8%
$\beta_{2,0} = \beta_{2,1}$	Prior	50.0%	50.0%	50.0%	50.0%	50.0%
	Posterior	84.1%	86.1%	89.3%	92.4%	93.4%
$\beta_{3,0} = \beta_{3,1}$	Prior	50.0%	50.0%	50.0%	50.0%	50.0%
	Posterior	84.2%	88.3%	90.3%	92.3%	93.3%

Note: See the footnote under Table 6.

Table 9: Estimated Restrictions in the "Strong" Unemployment Response Regime

	Zero-Restriction	Freely Estimated	Identical Restriction
μ_1	0.0%	0.0%	100%
ρ_1	0.0%	0.0%	100%
ϕ_{π_1}	13.1%	4.5%	82.3%
ϕ_{u_1}	0.0%	89.9%	9.4%
$\phi_{\Delta u_1}$	0.3%	7.4%	89.9%

Note: Here, "Zero-restriction" is the percentage of posterior draws in which a prior mixture distribution centered on zero and with a small variance was selected, and the parameter was restricted to be very close to zero. "Freely Estimated" is the percentage of posterior draws in which a prior mixture distribution centered on zero with a very wide variance was selected, so the parameter estimate would likely differ from zero and from its value in the other regime. "Identical Restriction" is the percentage of posterior draws in which the prior mixture distribution that restricts the parameters to be equal across regimes, but different from zero, was selected.

Table 10: Estimated Restrictions in the "Weak" Unemployment Response Regime

	Zero-Restriction	Freely Estimated	Identical Restriction
μ_0	0.0%	0.0%	100%
ρ_0	0.0%	0.0%	100%
$\phi_{\pi,0}$	4.0%	13.7%	82.3%
$\phi_{UN,0}$	67.2%	23.5%	9.4%
$\phi_{\Delta UN,0}$	4.5%	5.5%	89.9%

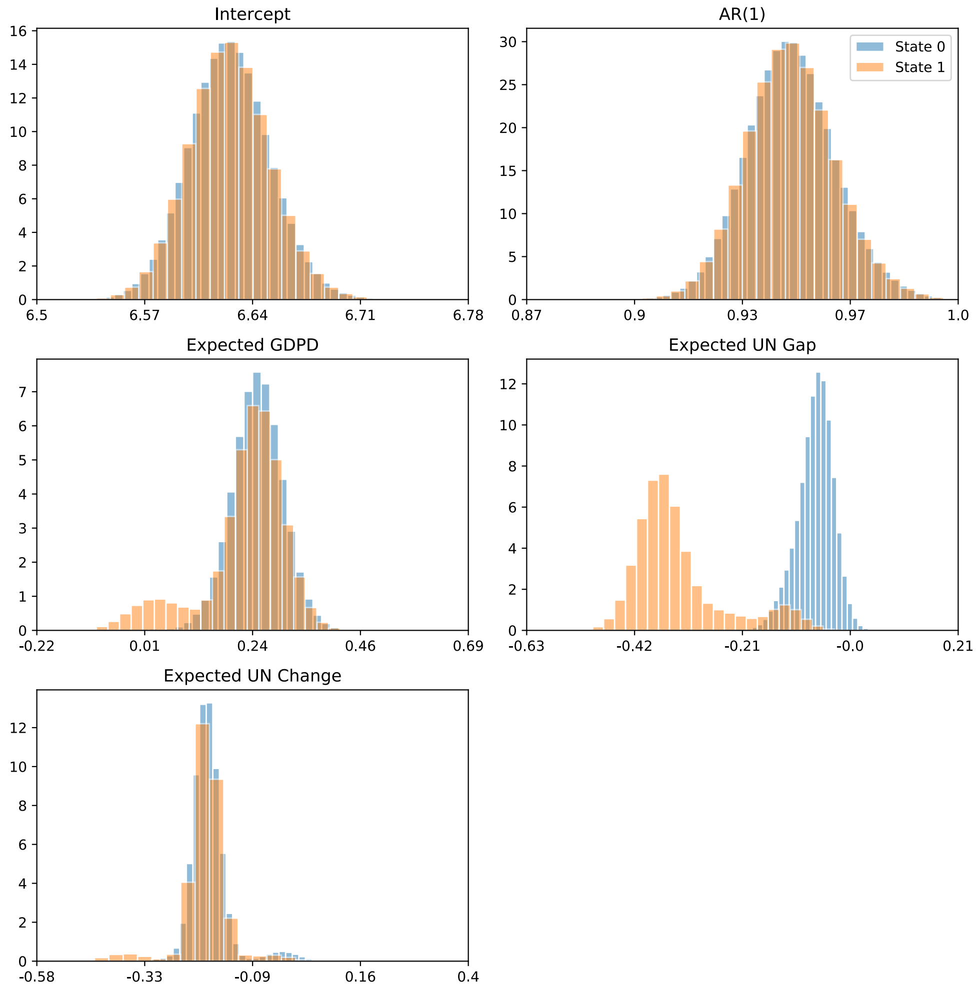
Note: See the footnote under Table 9.

Table 11: Mean Coefficient Values in Each Regime

	"Weak" UN Regime	"Strong" UN Regime
μ	6.622	6.622
ρ	0.949	0.949
ϕ_π	0.245	0.218
ϕ_{UN}	-0.070	-0.331
$\phi_{\Delta UN}$	-0.185	-0.195

Note: The columns here represent the posterior mean for each regression coefficient, conditional on being in either the "weak" unemployment response regime or the "strong" unemployment response regime. In this table, ϕ_π, ϕ_{UN}, and $\phi_{\Delta UN}$ represent one-period responses. The long-run responses are a function of these coefficients and ρ.

Figure 1: Regression Parameters



Note: Each panel of this plot contains two normalized histograms. In each, the y-axis is scaled so the area covered by the bars in the histogram sums to one. The the orange bars correspond to the posterior densities in the “strong” unemployment response regime, while the blue bars correspond to the posterior densities in the “weak” unemployment response regime. For most parameters the normalized histograms are closely aligned, suggesting no major differences in coefficient values between the two regimes. The major exception is the posterior densities for the FOMC’s response to the unemployment gap – hence the two regimes can be categorized as “strong” or “weak” unemployment response regimes.

Figure 2: Response to 100 Basis Point Unemployment Gap

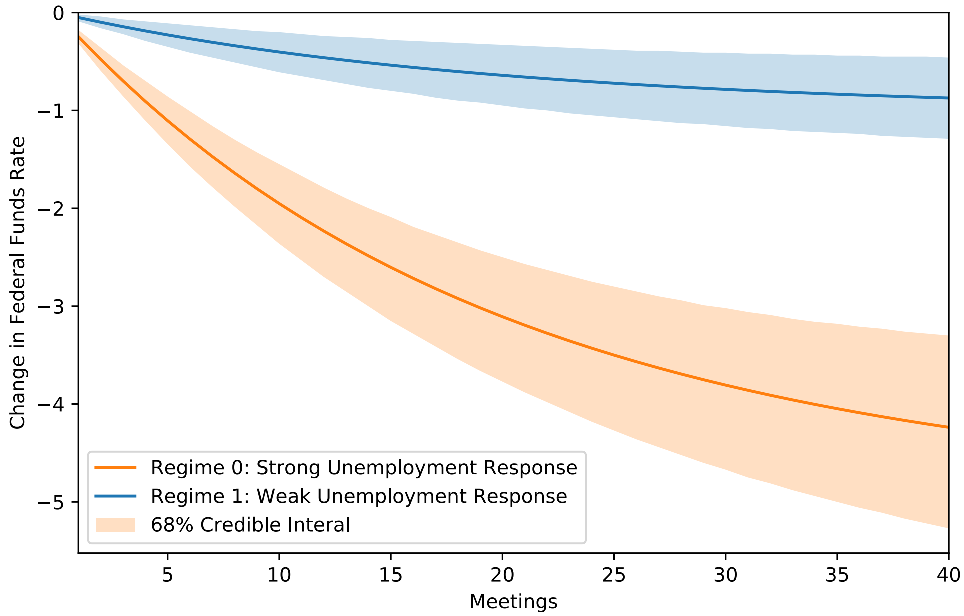


Figure 3: Response to 100 Basis Point Inflation Gap

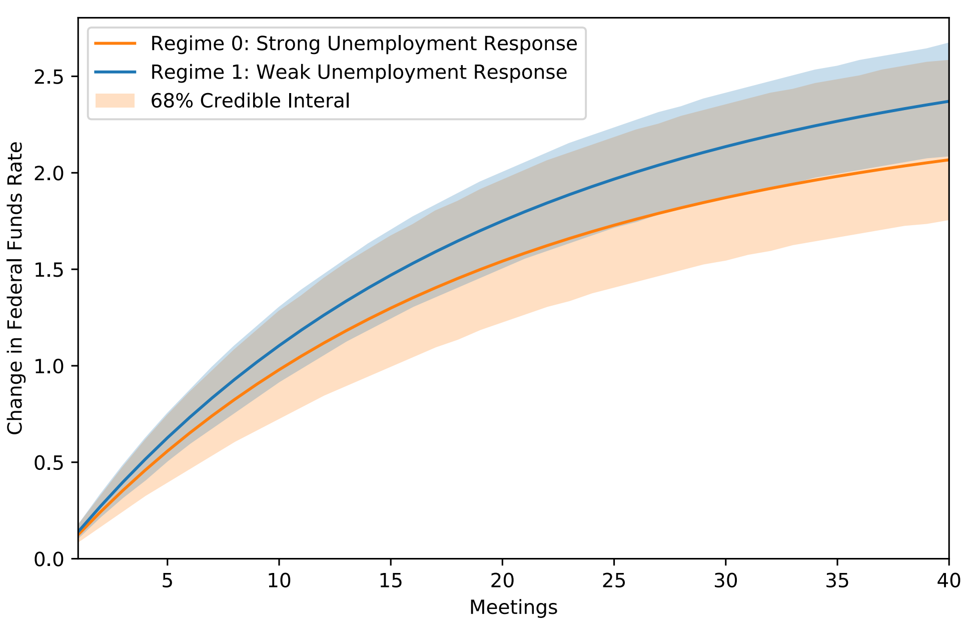


Figure 4: Stochastic Volatility

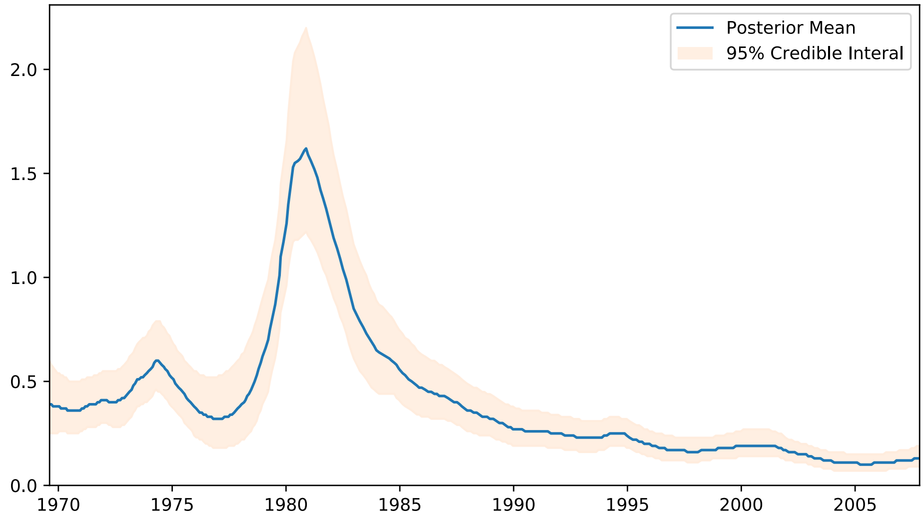
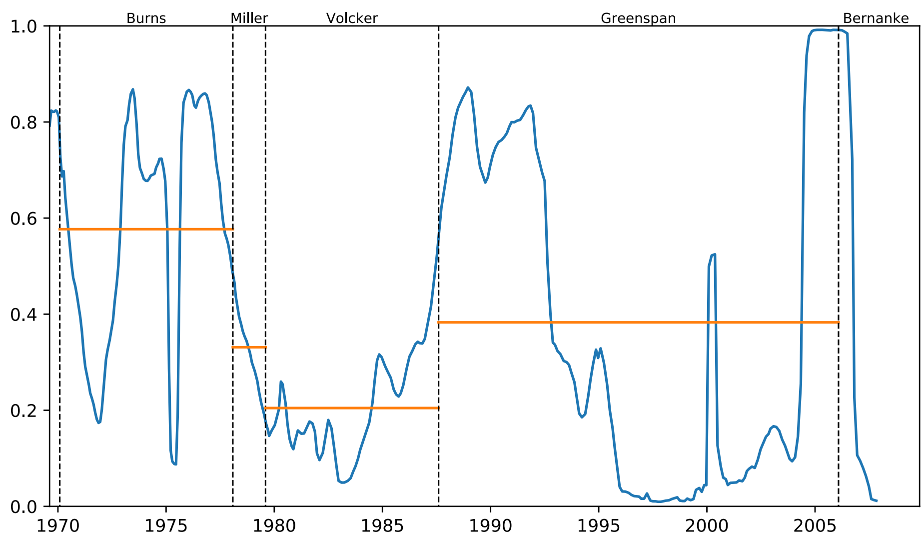


Figure 5: Probability of Strong Unemployment Response Regime



Note: The solid blue line represents the probability of the "strong unemployment response." The vertical dashed lines denote changes in Fed chair, and the orange horizontal lines represent the average probability of being in the "strong unemployment response" regime under the different chairs.

Figure 6: Johannsen and Mertens (2021) Shadow Rate

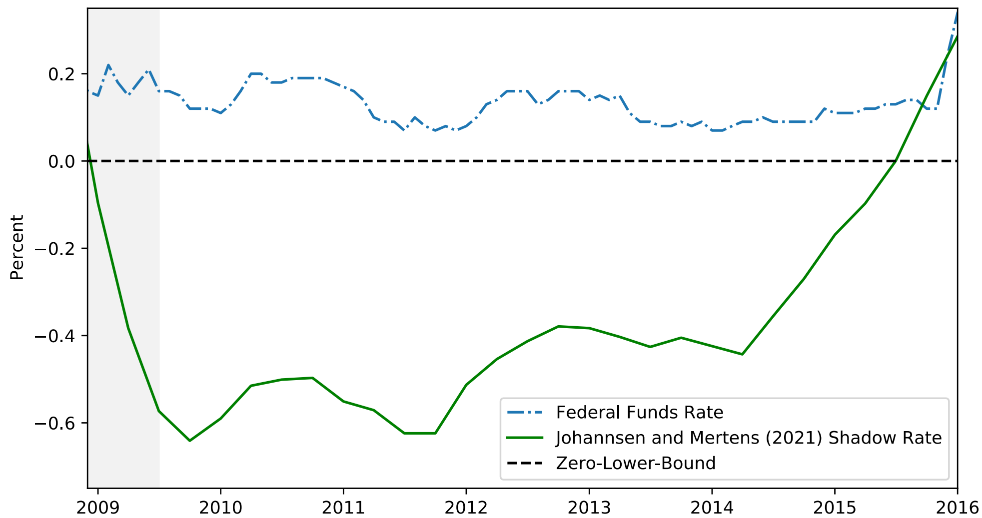


Figure 7: Johannsen and Mertens (2021) and Wu and Xia (2016) Shadow Rates

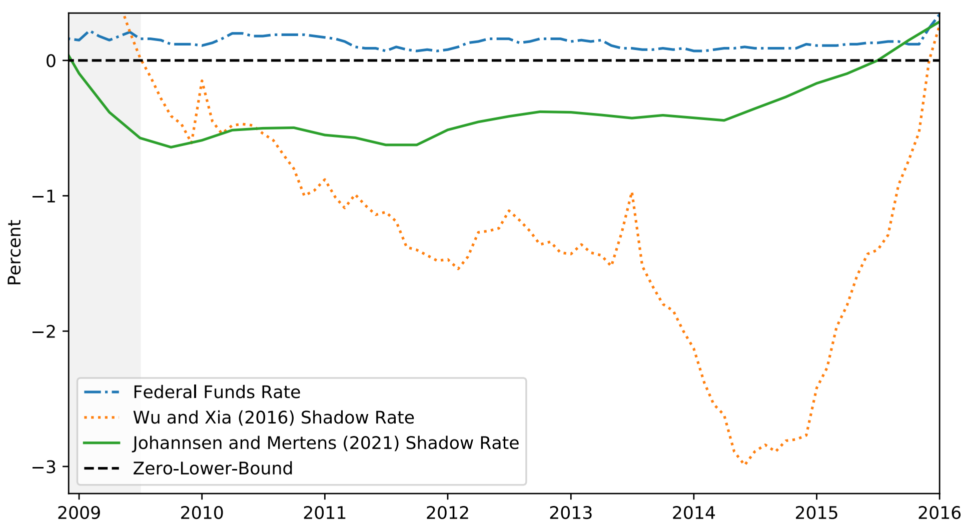
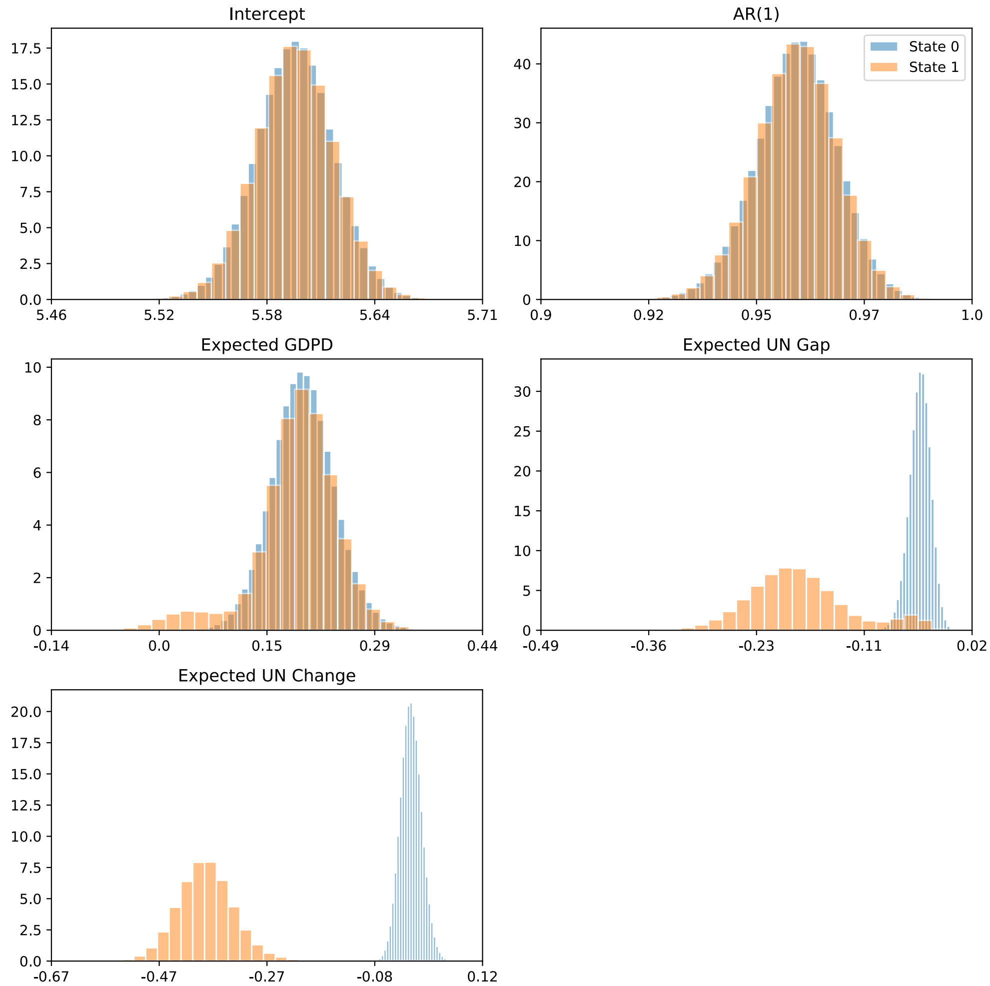
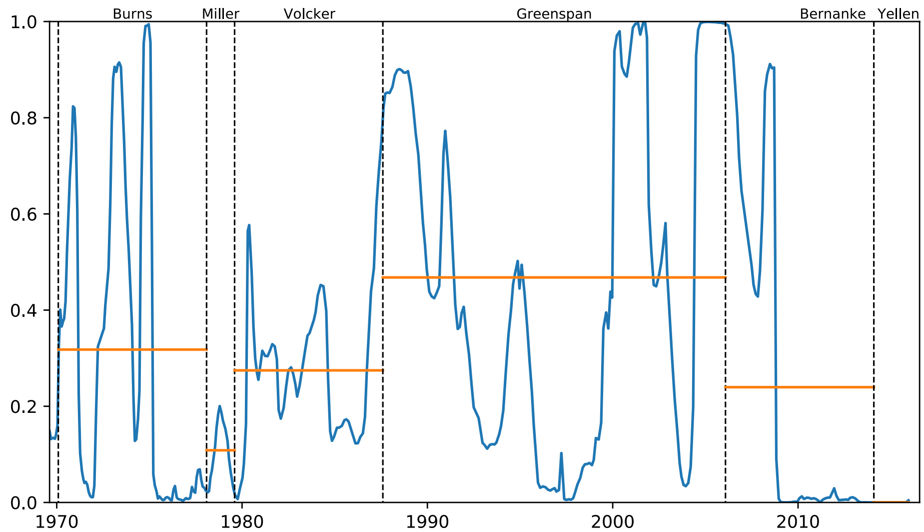


Figure 8: Regression Parameters (Extended Sample, 1970-2015)



Note: Each panel of this plot contains two normalized histograms. In each, the y-axis is scaled so the area covered by the bars in the histogram sums to one. The the orange bars correspond to the posterior densities in the "strong" unemployment response regime, while the blue bars correspond to the posterior densities in the "weak" unemployment response regime.

Figure 9: Probability of Strong Unemployment Response Regime (Extended Sample, 1970-2015)



Note: The solid blue line represents the probability of the "strong unemployment response." The vertical dashed lines denote changes in Fed chair, and the orange horizontal lines represent the average probability of being in the "strong unemployment response" regime under the different chairs.

References

Alba, J. D. and Wang, P. (2017). Taylor Rule And Discretionary Regimes In The United States: Evidence From A k-State Markov Regime-Switching Model. *Macroeconomic Dynamics*, 21(03):817–833.

Aruoba, B. S., Mlikota, M., Schorfheide, F., and Villalvazo, S. (2021a). Svars with occasionally-binding constraints. NBER Working Papers w28571, National Bureau of Economic Research, Inc.

Aruoba, S. B., Cuba-Borda, P., Higa-Flores, K., Schorfheide, F., and Villalvazo, S. (2021b). Piecewise-Linear Approximations and Filtering for DSGE Models with Occasionally Binding Constraints. *Review of Economic Dynamics*, Forthcoming.

Bae, J., Kim, C.-J., and Kim, D. (2012). The evolution of the monetary policy regimes in the U.S. *Empirical Economics*, 43(2):617–649.

Bennani, H., Kranz, T., and Neuenkirch, M. (2018). Disagreement between FOMC Members and the Fed's Staff: New Insights Based on a Counterfactual Interest Rate. *Journal of Macroeconomics*, 58:139–53.

Bianchi, F. (2013). Regime switches, agents' beliefs, and post-world war ii u.s. macroeconomic dynamics. *The Review of Economic Studies*, 80(2):463–490.

Bianchi, F. and Melosi, L. (2017). Escaping the Great Recession. *American Economic Review*, 107(4):1030–1058.

Boivin, J. (2006). Has U.S. Monetary Policy Changed? Evidence from Drifting Coefficients and Real-Time Data. *Journal of Money, Credit and Banking*, 38(5):1149–1173.

Carvalho, C., Nechio, F., and Tristao, T. (2019). Taylor Rule Estimation by OLS. *Federal Reserve Bank of San Francisco Working Paper 2018-11. Available at https://doi.org/10.24148/wp2018-11.*

Castelnuovo, E., Greco, L., and Raggi, D. (2014). Policy Rules, Regime Switches, And Trend Inflation: An Empirical Investigation For The United States. *Macroeconomic Dynamics*, 18(04):920–942.

Chan, J. C. C., Koop, G., and Potter, S. M. (2013). A New Model of Trend Inflation. *Journal of Business & Economic Statistics*, 31(1):94–106.

Check, A. (2019). Interest Rate Rules in Practice — A Taylor Rule or a Tailor-Made Rule? . *Unpublished Manuscript.*

Chib, S. (1998). Estimation and comparison of multiple change-point models. *Journal of Econometrics*, 86:221–241.

Clarida, R., Galí, J., and Gertler, M. (2000). Monetary Policy Rules And Macroeconomic Stability: Evidence And Some Theory. *The Quarterly Journal of Economics*, 115(1):147–180.

Cogley, T. and Sargent, T. J. (2005). Drift and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S. *Review of Economic Dynamics*, 8(2):262–302.

Coibion, O. and Gorodnichenko, Y. (2011). Monetary Policy, Trend Inflation, and the Great Moderation: An Alternative Interpretation. *American Economic Review*, 101(1):341–70.

Davig, T. and Leeper, E. M. (2011). Monetary-fiscal policy interactions and fiscal stimulus. *European Economic Review*, 55:211–27.

Del Negro, M., Giannoni, M. P., and Schorfheide, F. (2015). Inflation in the Great Recession and New Keynesian Models. *American Economic Journal: Macroeconomics*, 7(1):168–196.

Frühwirth-Schnatter, S. (2006). *Finite Mixture and Markov Switching Models*, volume 1 of *Spring Series in Statistics*. Springer Science & Business Media.

George, E. I. and McCulloch, R. E. (1993). Variable Selection Via Gibbs Sampling. *Journal of the American Statistical Association*, 88(423):881–89.

George, E. I., Sun, D., and Ni, S. (2008). Bayesian stochastic search for VAR model restrictions. *Journal of Econometrics*, 142(1):553–580.

Giles, J. A. and Giles, D. E. A. (1993). Pre-test Estimation and Testing in Econometrics: Recent Developments. *Journal of Economic Surveys*, 7(2):145–197.

Gonzalez-Astudillo, M. (2018). Indentifying the Stance of Monetary POlicy at the Zero Lower Bound: A Markov-Switching Estimation Exploiting Monetary-Fiscal Policy Interdependence. *Journal of Monetary, Credit, and Banking*, 50(1):115–154.

Hamilton, J. D. (1989). A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. *Econometrica*, 57(2):357–84.

Jeffreys, H. (1939). *Theory of Probability*. Oxford University Press, 4 edition.

Johannsen, B. K. and Mertens, E. (2021). A time-series model of interest rates with the effective lower bound. *Journal of Money, Credit and Banking*, Forthcoming.

Kaya, A., Golub, S., Kuperberg, M., and Lin, F. (2019). The federal reserve's dual mandate and the inflation-unemployment tradeoff. *Contemporary Economic Policy*, Forthcoming.

Kim, C.-J. and Nelson, C. R. (1999). *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications*, volume 1 of *MIT Press Books*. The MIT Press.

Kim, C.-J. and Nelson, C. R. (2006). Estimation of a forward-looking monetary policy rule: A time-varying parameter model using ex post data. *Journal of Monetary Economics*, 53(8):1949–1966.

Kim, S., Shephard, N., and Chib, S. (1998). Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models. *Review of Economic Studies*, 65(3):361–93.

Koop, G. and Korobilis, D. (2010). Bayesian Multivariate Time Series Methods for Empirical Macroeconomics. *Foundations and Trends(R) in Econometrics*, 3(4):267–358.

Koop, G. and Korobilis, D. (2018). Variational bayes inference in high-dimensional time-varying parameter models. *arXiv preprint arXiv:1809.03031*.

Kozicki, S. and Tinsley, P. (2006). Minding the Gap: Central Bank Estimates of the Unemployment Natural Rate. *Computational Economics*, 27(2):295–327.

Kozicki, S. and Tinsley, P. (2009). Perhaps the 1970s FOMC did what it said it did. *Journal of Monetary Economics*, 56(6):842–855.

Kruschke, J. K. (2015). *Doing Bayesian Data Analysis*. Academic Press / Elsevier, 2 edition.

Lee, K., Morley, J., and Shields, K. (2015). The Meta Taylor Rule. *Journal of Money, Credit and Banking*, 47(1):73–98.

Lindley, D. V. (1957). A statistical paradox. *Biometrica*, 44(1-2):187–192.

Murray, C. J., Nikolsko-Rzhevskyy, A., and Papell, D. H. (2015). Markov Switching And The Taylor Principle. *Macroeconomic Dynamics*, 19(04):913–930.

Orphanides, A. (2004). Monetary Policy Rules, Macroeconomic Stability, and Inflation: A View from the Trenches. *Journal of Money, Credit and Banking*, 36(2):151–75.

Orphanides, A. and Williams, J. C. (2005). The decline of activist stabilization policy: Natural rate misperceptions, learning, and expectations. *Journal of Economic Dynamics and Control*, 29(11):1927–1950.

Piger, J. (2009). *Models of Regime Changes*. Encyclopedia of Complexity and System Science. Springer.

Primiceri, G. E. (2005). Time Varying Structural Vector Autoregressions and Monetary Policy. *Review of Economic Studies*, 72(3):821–852.

Romer, C. D. and Romer, D. H. (2004). A New Measure of Monetary Shocks: Derivation and Implications. *American Economic Review*, 94(4):1055–1084.

Sims, C. A. and Zha, T. (2006). Were There Regime Switches in U.S. Monetary Policy? *American Economic Review*, 96(1):54–81.

Smets, F. and Wouters, R. (2003). An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, 1(5):1123–1175.

Soques, D. (2019). Timing and Signals of Monetary Regime Switching. *Unpublished Manuscript*.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39(1):195–214.

Taylor, J. B. (2013). A Review of Recent Monetary Policy. Economics Working Papers 13103, Hoover Institution, Stanford University.

Wu, J. C. and Xia, F. D. (2016). Measuring the macroeconomic impact of monetary policy at the zero lower bound. *Journal of Money Credit and Banking*, 48(2-3):253–291.

Technical Appendix — More information about SSVS and the Estimation Procedure

Enforcing parameter restrictions in SSVS

In an SSVS model, the posterior probability of each restriction is proportional to the prior probability of the restriction times the prior density for β^k under that restriction evaluated at the posterior draw of β^k. Therefore, the "restrictions" are only enforced approximately. To see why this is necessary, consider a prior density for the zero-restriction in which $\tau_0 = 0$, so that the coefficient was literally restricted to equal zero. Unless this restriction was chosen, our estimate of β^k will almost surely never be exactly zero. Therefore, the posterior density under this restriction will always be zero, since $\beta^k \neq 0$; therefore this restriction will never be enforced. Our goal when choosing τ_0 is to choose a sensible value that will enforce this zero-restriction when appropriate, while keeping the estimate of β^k near zero in the event that this restriction is chosen. Our goal when choosing ϵ is similar. We want to choose a number small enough that the prior density under this restriction will be high when both values of β^k are approximately equal, but we need to be careful to not choose a value for ϵ that is so small that the restriction will never be enforced.

Using Data to Inform SSVS Priors

The prior for the regression coefficients is data dependent since $\widehat{Var(b_k)}$ depends on the dependent variable. Therefore, it does not adhere to the requirement, in a Bayesian approach, that the prior be independent of the observed dependent data. However, as discussed in George and McCulloch (1993), the zero-restriction region depends on the values for both τ_0 and τ_1. George and McCulloch (1993) find that using $\widehat{Var(b_k)}$ in the choices of τ_0 and τ_1 helps to ensure that this zero-restriction region lies over a sensible space so that the coefficients are restricted to be close to zero only where appropriate. This prior, although not technically valid due to its dependence on the observed data, remains popular in the literature, as evidenced by its use in Koop and Korobilis (2010).[17]

Estimation Procedure

I set independent priors across the hierarchical parameters:

$$p(q_{00}, q_{11}, \sigma^2, \tau_0, \tau_1, p_1, p_2, p_3, p_4, p_5, \epsilon) = \\ p(q_{00})p(q_{11})p(\sigma^2)p(\tau_0)p(\tau_1)p(p_1)p(p_2)p(p_3)p(p_4)p(p_5)p(\epsilon)$$

I assume that the prior parameters $\tau_0, \tau_1, \epsilon, p_1, p_2, p_3, p_4, p_5$ are each set by the researcher, i.e. their prior is a point-mass at a particular value. This is a common assumption in

[17] Priors of this form are sometimes called "empirical Bayes" methods. One argument for their use, although not mathematically rigorous, is that the goal of empirics is to discover features of the data. Priors of this form should be considered if they can be shown to be well behaved and able to uncover features of the data, even if they do not technically adhere to proper Bayesian theory.

the SSVS literature. The parameters τ_0, τ_1, and ϵ control the variance of the each prior mixture distribution. The probabilities, p_1, p_2, p_3, p_4, and p_5, control the weights for each prior distribution, and are assumed to be constant across all coefficients, so the k subscript has been dropped for ease of notation.

For the other three hyper-parameters, q_{00}, q_{11}, and σ^2, I set prior distributions:

$$\begin{aligned} p(q_{00}) &= \text{Beta}(a_0, b_0) \\ p(q_{11}) &= \text{Beta}(a_1, b_1) \\ p(\sigma^2) &= \text{InverseGamma}(\alpha_Q, \beta_Q) \end{aligned}$$

Drawing from the full posterior directly is intractable. Instead, I draw from each of the conditional posteriors. This is called the Gibbs sampler. Let $\beta = [\beta_0, \beta_1]'$, $P_s = [q_{00}\ q_{11}]'$, $\tau = [\tau_0, \tau_1]'$, $P_\gamma = [p_1, p_2, p_3, p_4, p_5]'$, $\Gamma = \gamma^K$. The process is as follows:

1. Sample the indicators for the mixture of normals prior each variable:

$$\begin{aligned} p(\Gamma^{(z)}|Y, \beta^{(z-1)}, P_s^{(z-1)}, \sigma^{2,(z-1)}, S_T^{(z-1)}, \tau, P_\gamma) &= p(\Gamma^{(z)}|Y, \beta^{(z-1)}, S_T^{(z-1)}, \tau, P_\gamma) \\ p(\Gamma^{(z)}|Y, \beta^{(z-1)}, S_T^{(z-1)}, \tau, P_\gamma) &= \text{ Categorical} \\ \Gamma_k^{(z)} &= \text{ Categorical} \begin{pmatrix} \frac{p_1 f(N(0,\Sigma_1)|\beta^k)}{\sum_{i=1}^5 p_i f(N(0,\Sigma_i)|\beta^k)} \\ \frac{p_2 f(N(0,\Sigma_2)|\beta^k)}{\sum_{i=1}^5 p_i f(N(0,\Sigma_i)|\beta^k)} \\ \frac{p_3 f(N(0,\Sigma_3)|\beta^k)}{\sum_{i=1}^5 p_i f(N(0,\Sigma_i)|\beta^k)} \\ \frac{p_4 f(N(0,\Sigma_4)|\beta^k)}{\sum_{i=1}^5 p_i f(N(0,\Sigma_i)|\beta^k)} \\ \frac{p_5 f(N(0,\Sigma_5)|\beta^k)}{\sum_{i=1}^5 p_i f(N(0,\Sigma_i)|\beta^k)} \end{pmatrix} \end{aligned}$$

This procedure is based on George and McCulloch (1993). In the current paper, their procedure is slightly modified because I have a mixture of five normal distributions rather than two. Once the prior mixture distributions are selected, form the prior variance for β as:

$$D = \begin{bmatrix} \Sigma^{k=1} & 0 & \cdots & 0 & 0 \\ 0 & \Sigma^{k=2} & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \cdots & 0 \\ 0 & 0 & 0 & \cdots & \Sigma^{k=K} \end{bmatrix}$$

D is block diagonal, with the Cholesky decomposition of the two-by-two mixture variance for each pair of coefficients, k, Σ^k along the diagonals, with zeros everywhere else.

2. Sample the regression coefficients:

$$
\begin{aligned}
p(\beta^{(z)}|Y,\Gamma^{(z)},P_s^{(z-1)},\sigma^{2,(z-1)},S_T^{(z-1)},\tau,P_\gamma) &= p(\beta^{(z)}|Y,\Gamma^{(z)},\sigma^{2,(z-1)},S_T^{(z-1)},\tau) \\
p(\beta^{(z)}|Y,\Gamma^{(z)},\sigma^{2,(z-1)},S_T^{(z-1)},\tau) &\sim \mathcal{N}\Big(\hat{\beta},V\Big) \\
V &= ((DRD')^{-1}+X'X)^{-1} \\
\hat{\beta} &= VX'Y
\end{aligned}
$$

where R is a prior correlation matrix. In this application, R is set to the identity matrix.

3. Sample the variance of the regression error:

$$
\begin{aligned}
p(\sigma^{2,(z)}|Y,\Gamma^{(z)},\beta^{(z)},P_s^{(z-1)},S_T^{(z-1)},\tau,P_\gamma) &= p(\sigma^{2,(z)}|Y,\beta^{(z)},S_T^{(z-1)}) \\
p(\sigma^{2,(z)}|Y,\beta^{(z)},S_T^{(z-1)}) &= \text{ Inverse Gamma} \\
\sigma^{2,(z)} &\sim \text{ IG}\left(a_Q+\frac{T}{2},\beta_Q+\frac{SSE}{2}\right)
\end{aligned}
$$

where T is the sample size and $SSE=(Y-X\beta)'(Y-X\beta)$. This step is replaced by sampling stochastic volatility via Kim et al. (1998) in the interest rate rule application.

4. Sample the Markov Regime indicators:

$$
p(S_T^{(z)}|Y,\Gamma^{(z)},\beta^{(z)},\sigma^{2,(z)},P_s^{(z-1)},\tau,P_\gamma) = p(S_T^{(z)}|Y,\beta^{(z)},\sigma^{2,(z)},P_s^{(z-1)})
$$

using the procedure described in Kim and Nelson (1999).

5. Sample the Markov transition probabilities:

$$
\begin{aligned}
p(P_s^{(z)}|Y,\Gamma^{(z)},\beta^{(z)},\sigma^{2,(z)},S_T^{(z)},\tau,P_\gamma) &= p(P_s^{(z)}|Y,S_T^{(z)}) \\
p(P_s^{(z)}|Y,S_T^{(z)}) &= \text{ Beta} \\
P_s^{ii,(z)} &= \text{ Beta}(a_i+N_{ii},b_i+N_{ij})
\end{aligned}
$$

where N_{ij} is the number of times that the regime transitioned from regime i to regime j in $S_T^{(z)}$.

Overview of the prior distribution for the regression parameters

Helicopter Tour of Prior for β^k

Recall that the prior for $\beta^k = [\beta_0^k \ \beta_1^k]'$ is given by a mixture of five Normal distributions:

$$\beta^k \sim \gamma_1^k \mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \tau_0 \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right) + \gamma_2^k \mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \tau_1 \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right) + \gamma_3^k \mathcal{N}\left(\begin{bmatrix}\hat{\beta}^k_{\text{OLS}}\\\hat{\beta}^k_{\text{OLS}}\end{bmatrix}, \tau_1 \begin{bmatrix}1 & 1-\epsilon\\1-\epsilon & 1\end{bmatrix}\right)$$
$$+ \gamma_4^k \mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}\tau_1 & 0\\0 & \tau_0\end{bmatrix}\right) + \gamma_5^k \mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}\tau_0 & 0\\0 & \tau_1\end{bmatrix}\right)$$

In my application, I choose:

$$Pr(\gamma_1^k = 1) = 0.5$$
$$Pr(\gamma_2^k = 1) = \frac{.25}{3}$$
$$Pr(\gamma_3^k = 1) = 0.25$$
$$Pr(\gamma_4^k = 1) = \frac{.25}{3}$$
$$Pr(\gamma_5^k = 1) = \frac{.25}{3}$$

In addition, I choose:

$$\tau_0^k = c_0 \sqrt{\widehat{var(\beta^k)}}$$
$$\tau_1^k = c_1 \sqrt{\widehat{var(\beta^k)}}$$
$$c_0 = 0.1$$
$$c_1 = 15.0$$

where $\widehat{var(\beta^k)}$ is the OLS estimate of the variance of β^k under a no regime switching assumption. These priors are similar to ones suggested in George and McCulloch (1993) and Koop and Korobilis (2010). Finally, for the case of parameters restricted to be equal across regimes, I set $\epsilon = 1.0 - 0.99999$.

In figures (1)-(3), I plot the prior probability density function of β_0 and β_1, implicitly assuming that $\hat{\beta}^k_{\text{OLS}} = 0$. This prior density function has some striking features. It is strongly peaked near $\beta_0 = 0$ and $\beta_1 = 0$, so there is a relatively high prior probability that both coefficients are restricted to zero. If the estimated coefficients land in the orange region of figure 2 (or the yellow region of figure 3), it is almost a certainty that the priors for β_0 and β_1 will be centered on zero with a very tight prior variance. Additionally, there are three other regions which receive relatively large prior mass: both regions where one of the coefficients is restricted to be near zero, and the diagonal region representing coefficients that are (roughly) identical under each regime. Outside of these four relatively narrow but sharply peaked regions, the Normal distribution with the highest probability density function corresponds to both regimes being freely estimated.

Figure 10: Prior Probability Density Function for Different Values of β_0 and β_1

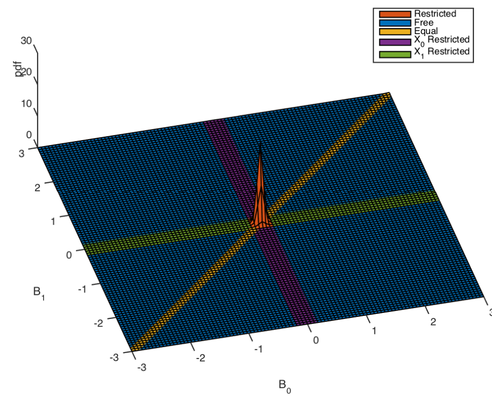


Figure 11: Prior Probability Density Function for Different Values of β_0 and β_1: View from Above

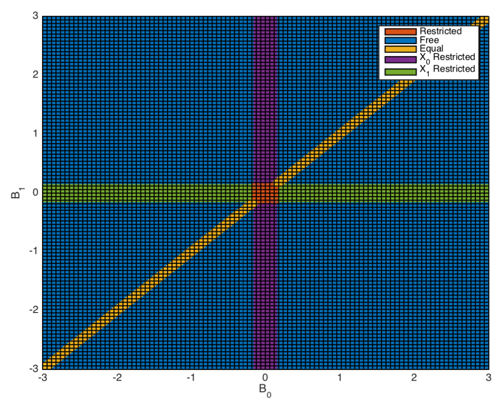
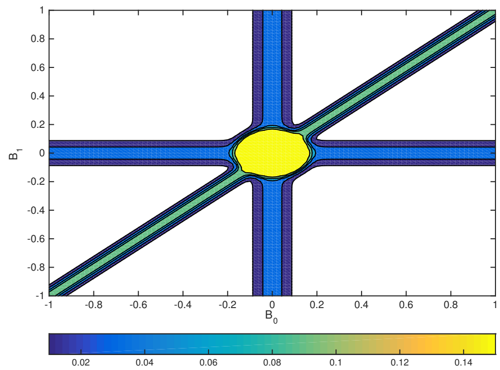


Figure 12: Contour Plot of the Prior Probability Density Function for Different Values of β_0 and β_1



Convergence Diagnostics

Below, I present evidence that the estimator converges to a unique stationary distribution. I first present running mean plots throughout the samples that are discarded. If the sampler is converging to a stationary distribution, then the means of all of the parameters of the model should converge to their means in the stationary distribution. If it is not, then these means will be trending up, down, or bouncing around. Next, I present the autocorrelation functions for the parameters of the model. These functions show the correlation between the draw of the parameter at one iteration and the draw of the same parameter t iterations later. If the sampler is well-behaved, then the autocorrelation functions should fall towards zero as the number of iterations increases. A simple rule-of-thumb is that the number of discarded "burn-in" draws should be at least ten times larger than the maximum number of iterations that it takes the autocorrelation of any parameter to drop to zero.

Running Mean Plots

Figure 13: Regression Parameters

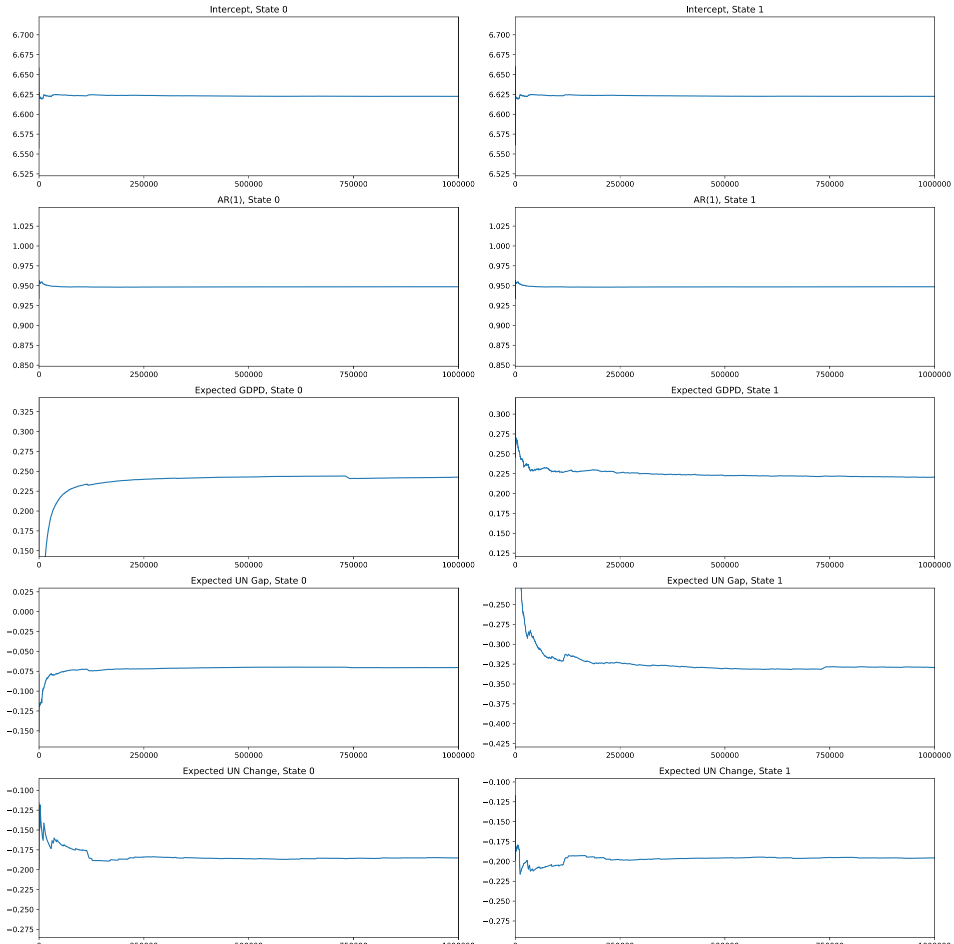
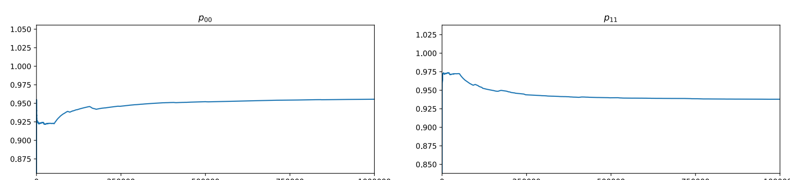


Figure 14: Regime Probabilities



Autocorrelation Functions

Figure 15: Regression Parameters

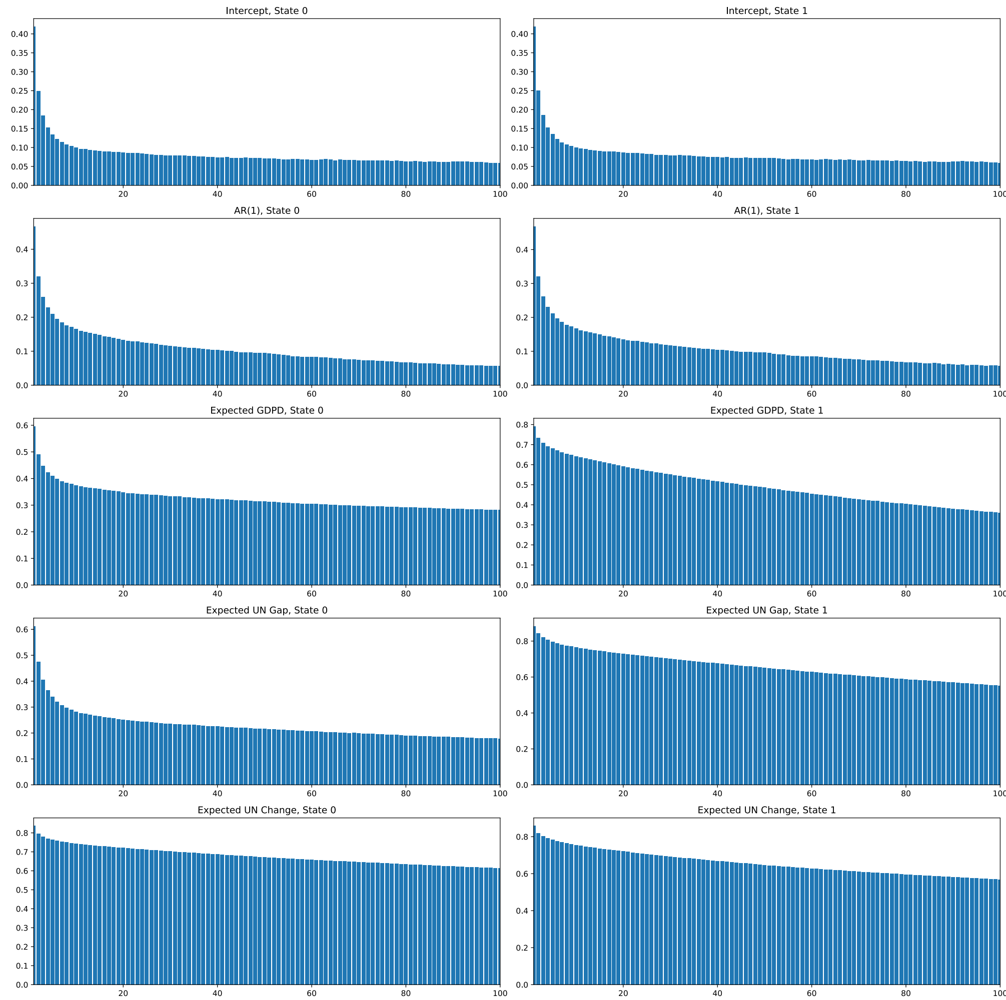
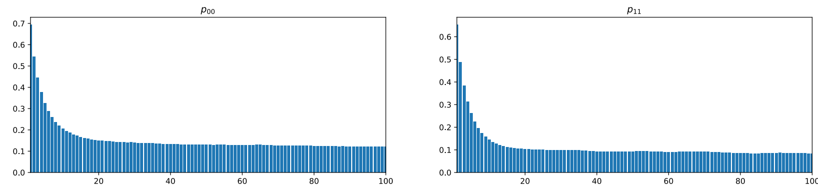


Figure 16: Regime Probabilities



Both of these metrics suggest that the sampler is well-behaved, although the ACF function is declining fairly slowly. The running mean plots become flat towards the end of the discarded draws, suggesting that the sampler has converged to a stationary distribution. The autocorrelation plots for a couple of the regression parameters is still fairly high at 100 draws, suggesting that more than 1,000 burn-in draws are needed. I perform 1,000,000 burn-in draws in an abundance of caution. I keep the next 1,000,000 draws and use them to form posterior inference.

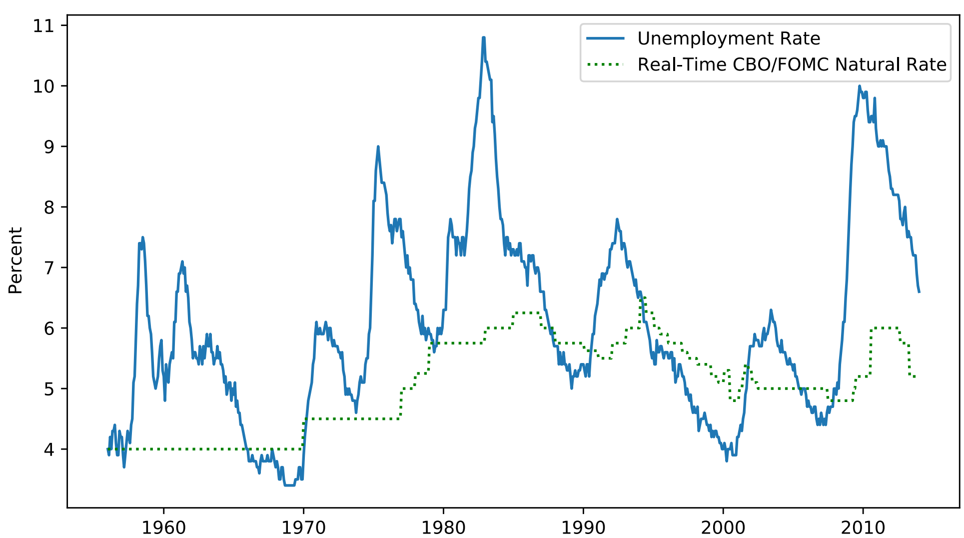
Appendix C – Robustness Exercises

In this appendix, I explore alternative formulations of the unemployment gap and the inflation gap. In the main body, for the unemployment gap, I assume a constant natural unemployment rate. For the inflation gap, I used an estimate of the FOMC's inflation target from Chan et al. (2013). However, it is worthwhile to explore alternatives to see if these assumptions are having a strong influence on the results.

Alternative Unemployment Gaps

For the unemployment gap, I consider two possible alternatives. First, I consider a real-time estimate of the natural rate that comes from various sources, but represents the information available to, and beliefs of, the FOMC. This is stitched together from the narrative evidence in Orphanides and Williams (2005) for the period before 1989, from the NAIRU estimates presented in the FOMC Bluebook between 1989-1997, and then from the FOMC Greenbook after 1997. This estimate of the natural rate is presented in Figure 17, below.

Figure 17: Real-Time CBO/FOMC Estimate of the Natural Rate



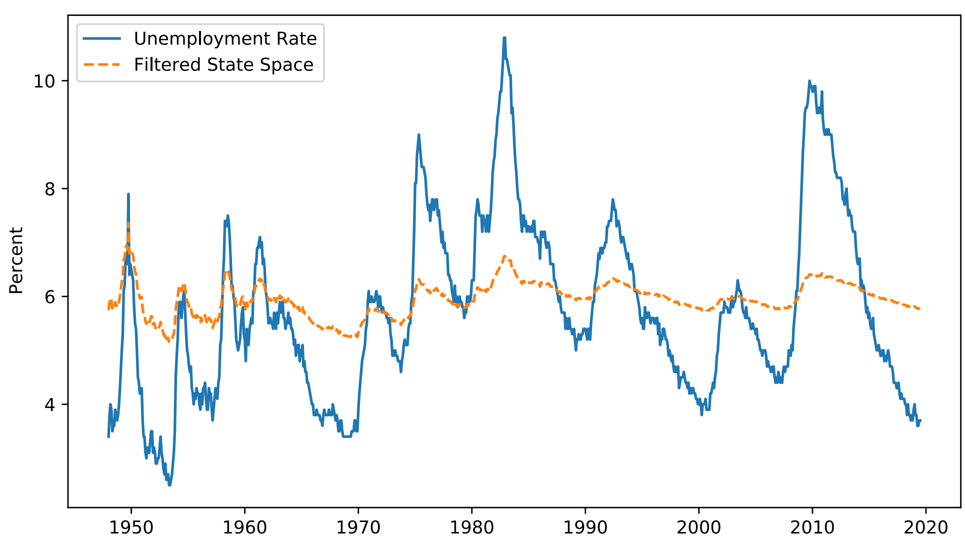
Next, I consider the estimate from a univariate state space model that allows for timevariation in the natural rate. The model is as follows:

$$
\begin{aligned}
u_t &= c_t + \tilde{u}_t \\
c_t &= \rho c_{t-1} + \omega_t \\
\tilde{u}_t &= \tilde{u}_{t-1} + v_t \\
0 \leq |\rho| &\leq 1 \\
\omega_t &\sim \mathcal{N}(0, \sigma_\omega) \\
v_t &\sim \mathcal{N}(0, \sigma_v)
\end{aligned}
$$

where u_t is the unemployment rate, c_t is the cyclical component of the unemployment rate, $\tilde{u}_t$ is the natural or long-run unemployment rate, ρ is the persistence of the cyclical component, ω_t is the error term in the cyclical component, v_t is the error term of the natural rate, σ_ω is the standard deviation of the error term of the cyclical component, and σ_v is the standard deviation of the error term of the natural rate.

To estimate this model, I place loose priors on ρ, σ_ω, and σ_v, and use Gibbs sampling to perform Bayesian estimation. One drawback of this approach is that since estimation uses all of the data, this model does not produce estimates of the unemployment gap that could have been produced in real-time by the FOMC. However, to alleviate this problem somewhat, after estimating the model parameters I set them to their posterior medians, and use one-sided Kalman filtering to produce estimates of the natural rate of unemployment. If the FOMC had access to the model parameters, this portion could have been done in real-time, since it is using filtered, rather than smoothed, estimates of the natural rate. After estimating this model, I find qualitatively similar results to the CBO/FOMC natural rate, especially after 1980.

Figure 18: Filtered State-Space Estimate of the Natural Rate



In total, I have three separate estimates of the unemployment gap. The first, used in the main body of the paper, assumes a constant natural rate. The second uses a reconstructed real-time estimate of the natural rate from the FOMC and CBO. The third uses a state-space model estimated over the full sample. While the first two are fully consistent with a real-time forecasting rule, the third is not since it uses the full sample for estimation. However, the state-space model was applied in such a manner that conditional on the model parameters, only "real-time" data is incorporated in the filtered estimates of the natural rate. In any case, the resulting unemployment gap from all three methods are similar and strongly correlated, as can bee seen in Figure 19 and Table 12.

Figure 19: All Three Estimates of the Unemployment Gap

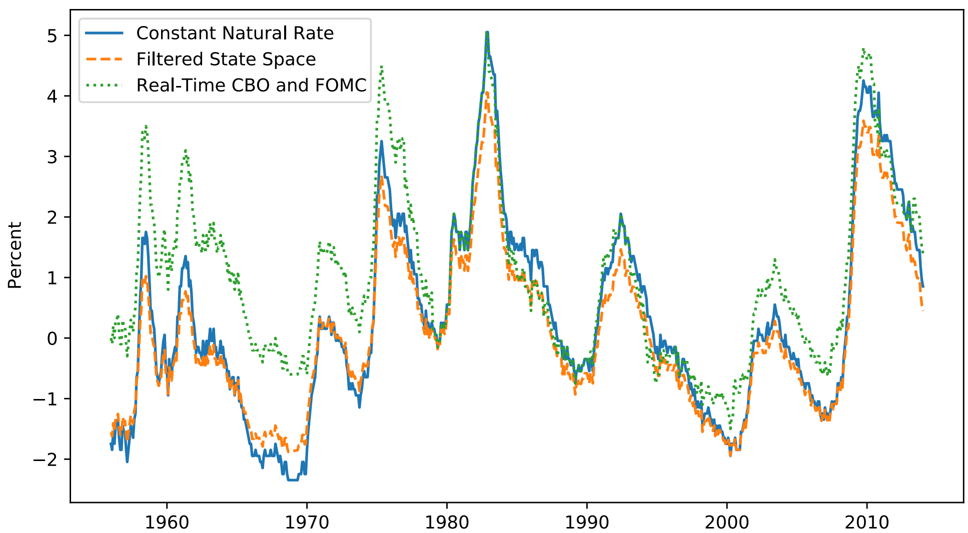


Table 12: Correlation Matrix for Various Unemployment Gaps

	Constant	CBO/FOMC	State Space
Constant	1.00	0.82	0.99
CBO/FOMC	0.82	1.00	0.83
State Space	0.99	0.83	1.00

Alternative Inflation Gap

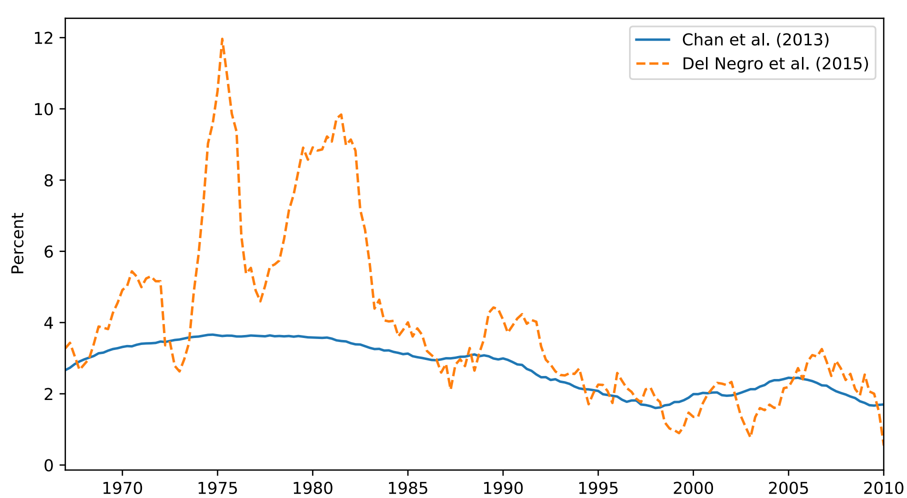
In the baseline results, I use an estimate of trend inflation from Chan et al. (2013) as my estimate of the FOMC's target inflation rate. However, there are other ways to estimate trend inflation. For instance, one could use an AR(1) model, an unobserved components model, or a state space model with persistence like the one I used for the natural rate of unemployment. In fact, the latter forms the basis of the trend inflation estimates in Chan et al. (2013), who also allow for bounds on trend inflation. I believe this is a reasonable course of action, as I share their view that there is,

> "no compelling reason for thinking that, even in high-inflation times, it is likely that central bankers desired high-trend inflation. Experiences such as the high-inflation period of the 1970s are better thought of as times in which deviations from the desired trend level of inflation were quite persistent." – Chan et al. (2013)

This is especially relevant in the present study, as I don't simply need a measure of *trend* inflation. Instead, I need an estimate of the FOMC's inflation *target* (i.e., their desired inflation rate).

Nevertheless, use of the bounded model of trend inflation of Chan et al. (2013) is a choice, and it is important to investigate how consequential that choice is. In much of the DSGE literature, authors use a random walk (or unobserved components model) to model trend inflation (e.g., Smets and Wouters (2003) and Cogley and Sargent (2005)). One exception is Del Negro et al. (2015), who model trend inflation as an AR(1) process. After estimating this parameter, they find a posterior mode for the AR(1) coefficient of 0.99. Using this estimate on GDP Deflator inflation, I construct an alternative measure of trend inflation. The two estimates of trend inflation are presented in Figure 20. We can see that the estimates from Del Negro et al. (2015) and Chan et al. (2013) are much different. The Del Negro et al. (2015) estimate is highly volatile and rises to a peak of 12% while the Chan et al. (2013) estimate is much more stable, never exceeding 3.7%. While Del Negro et al.'s (2015) method is certainly a valid measure of *trend* inflation, it seems inconsistent with the narrative evidence from the 1970's to use it as a measure of *target* inflation, since the FOMC likely did not desire inflation to be as high as 12%, their preferences for inflation were probably much less volatile, and during the early Volcker years they likely did not desire for inflation to remain above 8%.

Figure 20: Two Estimates of Trend Inflation



Results

Next, I fit the MS-SSVS model using these alternative measures of the unemployment gap or inflation gap. As seen above in Figure 19 and Table 12, the unemployment gap constructed using the state space model is nearly identical to the unemployment gap that assumes a natural rate. Therefore, the results when using the unemployment gap constructed using the state space model are very similar to the baseline results. The results when using the CBO/FOMC real-time unemployment gap differ more from the baseline, but remain qualitatively similar. In both cases, there is substantial evidence of switching in the unemployment response coefficient(s), but little evidence of switching in the inflation response coefficient. Similarly, the estimated probability of being in the "strong" or "weak" unemployment response regime are qualitatively similar. Finally, the volatility of the rule changes in a similar way over time, regardless of which unemployment gap estimate is used.

On the other hand, when the alternative inflation gap is used, the differences compared to the baseline are more pronounced. This includes the posterior density of the inflation response in one regime having substantial mass in the negative region, suggesting that in this regime higher inflation would lead to a lower Federal Funds rate. As discussed above, the AR(1) estimate of the inflation target does not comport with the narrative evidence, especially during the 1970's, so I take these estimates with a grain of salt.

In Tables 13 and 14, I present summaries of the posterior mean parameter estimates, conditional on regime, for each of these alternatives.

Table 13: Posterior Mean Response Coefficients in the "Weak" Unemployment Response Regime

	Baseline	CBO/FOMC	State Space	Del Negro et al. (2015)
μ	6.622	6.606	6.622	6.606
ρ	0.949	0.938	0.956	0.988
ϕ_π	0.245	0.252	0.217	0.147
ϕ_{UN}	-0.070	-0.082	-0.091	-0.013
$\phi_{\Delta UN}$	-0.185	-0.081	-0.183	-0.108

Note: "Baseline" are the results in the body of the paper. "CBO/FOMC" are the results when the unemployment gap is replaced with the CBO/FOMC real-time unemployment gap. "State Space" are the results when the unemployment gap is replaced with the filtered state-space estimate. "Del Negro et al. (2015)" are the results when the inflation gap is replaced with the AR(1) estimate from Del Negro et al. (2015). In this table, ϕ_π, ϕ_{UN}, and $\phi_{\Delta UN}$ represent one-period responses. The long-run responses are a function of these coefficients and ρ.

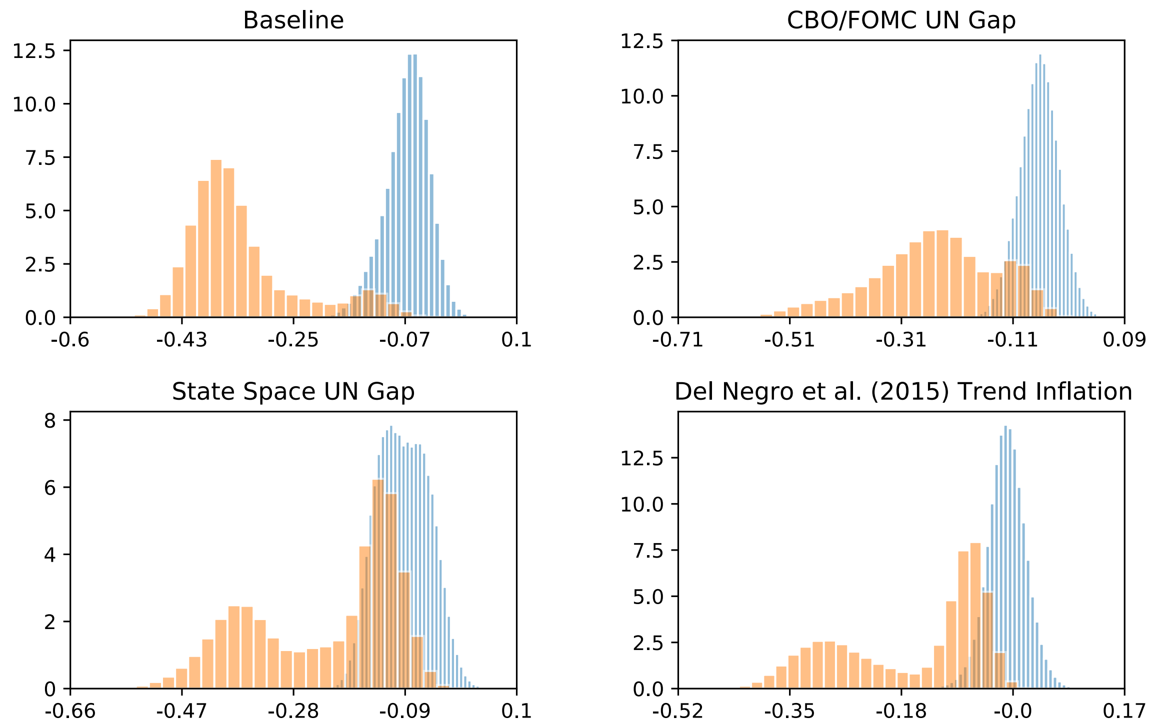
Table 14: Posterior Mean Response Coefficients in the "Strong" Unemployment Response Regime

	Baseline	CBO/FOMC	State Space	Del Negro et al. (2015)
μ	6.622	6.606	6.622	6.606
ρ	0.949	0.938	0.956	0.987
ϕ_π	0.218	0.178	0.123	-0.064
ϕ_{UN}	-0.331	-0.254	-0.217	-0.158
$\phi_{\Delta UN}$	-0.195	-0.271	-0.195	-0.310

Note: See the footnote under Table 13.

We can also examine the posterior densities more carefully. For instance, for all alternatives, we always see at least some evidence of switching across in the unemployment gap response:

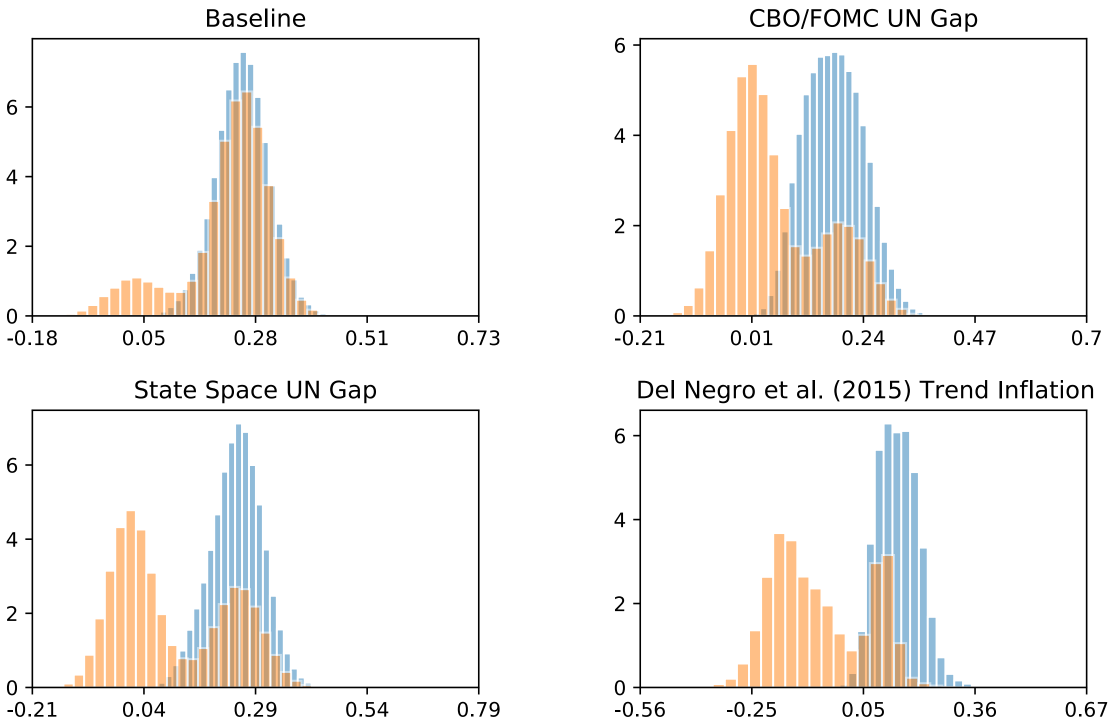
Figure 21: Unemployment Gap Response Coefficient Density



Note: The blue histogram is the posterior density in the "weak" UN response regime, and the orange histogram is the posterior density in the "strong" UN response regime.

For the inflation response coefficient, things are a bit different. Relative to the baseline, both models using an alternative measure of the unemployment gap show a higher probability of a muted inflation response in the "strong" unemployment response regime. When using the Del Negro et al. (2015) estimate of trend inflation, the inflation response in the "strong" unemployment response is more than just muted — it actually reverses sign. It now suggests that the FOMC had a negative inflation response (i.e., higher inflation implies a lower nominal Federal Funds rate target). This seems counterfactual, and is may be an artifact of this estimate of trend inflation being inconsistent with the FOMC's true underlying target.

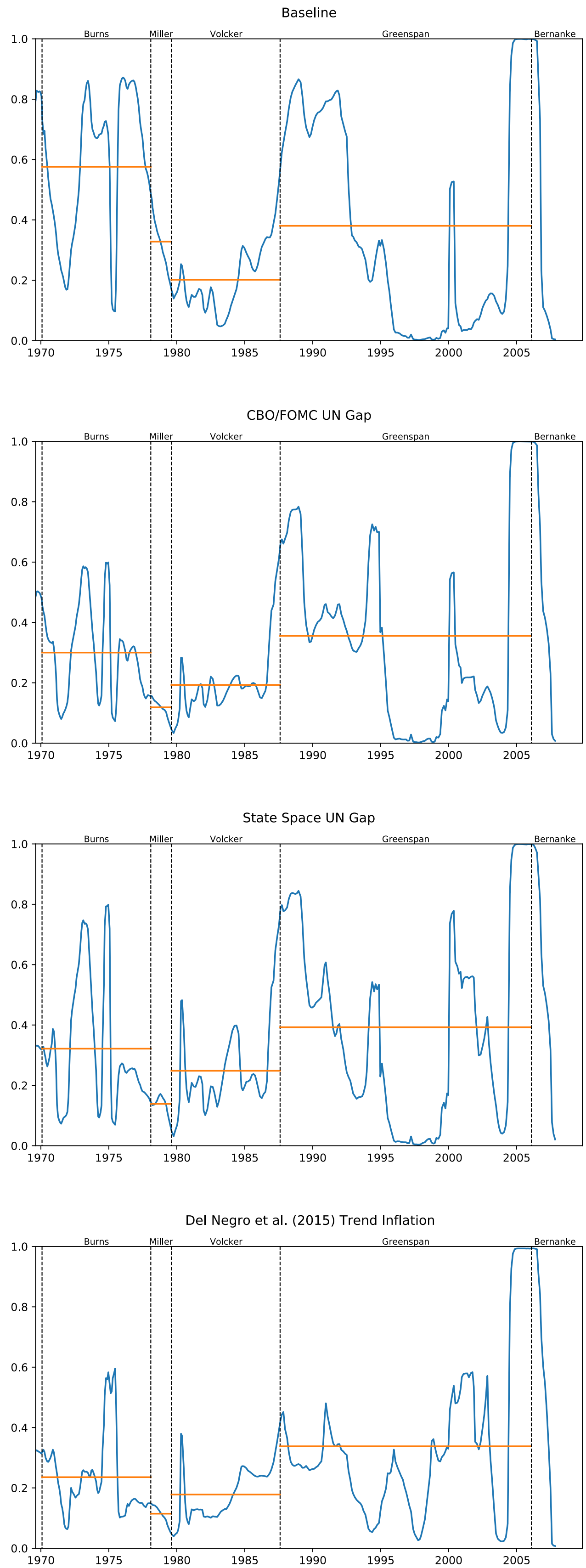
Figure 22: Inflation Gap Response Coefficient Density



Note: The blue histogram is the posterior density in the "weak" UN response regime, and the orange histogram is the posterior density in the "strong" UN response regime.

Finally, we can compare estimated regime probabilities over time. These are displayed below, in Figure 23.

Figure 23: Estimated Probability of "Strong" Unemployment Response Regime



Looking at the top three graphs, we can see that when the Chan et al. (2013) estimate of trend inflation is used, the regimes are qualitatively similar, with a relatively high probability of being in the "strong" unemployment response regime in the early 1970's, late 1980's to mid 1990's, around 2000, and from roughly 2005-2006. When the Del Negro et al. (2015) estimate of trend inflation is used, the probability of being in that regime rarely rises above 50%, with the exception of the 2005-2006 period, during which all four versions find a 100% probability of being in the "strong" unemployment response regime.

Conclusion

The major results of the baseline model are robust to alternative constructions of the unemployment gap: (1) there are two distinct regimes in unemployment response, (2) there is less evidence of change in the intercept, AR(1), or inflation response parameters, and (3) there is strong evidence of changes in volatility.

The same can not be said for the alternative measure of trend inflation considered, namely, the model from Del Negro et al. (2015). However, this estimate of trend inflation seems inconsistent with the narrative evidence of the FOMC's views during the 1970's and 1980's. For example, it estimates that trend inflation reached 12% in the 1970's and remained persistently above 8% throughout the early-to-mid 1980's. While this may be a perfectly valid description of the way inflation was behaving at the time, it seems inconsistent with the desires of the FOMC. Ultimately, I agree with the views expressed in Chan et al. (2013) that the high inflation of the 1970's was the result of a more persistent deviation from the FOMC's inflation target. Therefore, I believe that the Chan et al. (2013) estimate is more suited for use in this context.