Spring Level 4 — 06/26/2010

(1) ________ Sarah will have seven test scores. Each of the scores is an integer value. The first five scores are 82, 87, 92, 96 and 98. How many distinct values are possible for the median of Sarah's seven scores once she takes the last two tests?

(2) ________ A caterer offers five different appetizers, three different drinks and four different sandwiches. How many combinations of two appetizers, two drinks and two sandwiches can Scott choose for his party?

(3) ________ A television remote control has eleven buttons: 10 buttons with the numbers 0-9 on them and an on/off button. An infant is playing with the remote control and presses three buttons at random. Given that the television is initially off and when turned on it is on channel 03, what is the probability, expressed as a common fraction, that the infant will turn the television on and then turn the television to channel 12?

(4) ________ How many integers between 1 and 200 are multiples of both 3 and 5 but not of either 4 or 7?

(5) ________ The mean of six positive integers is 5, and the median is 6. What is the largest the mode could be, given that the mode is unique?

(6) ________ Hazel and Basil are playing a game in which either is equally likely to win any given point. Basil has 4 points to Hazel's 3 points. If the first person to get 5 points is the winner, what is the probability that Basil will win?

(7) ________ In how many ways can 81 be written as the sum of three positive perfect squares if the order of the three perfect squares does not matter?

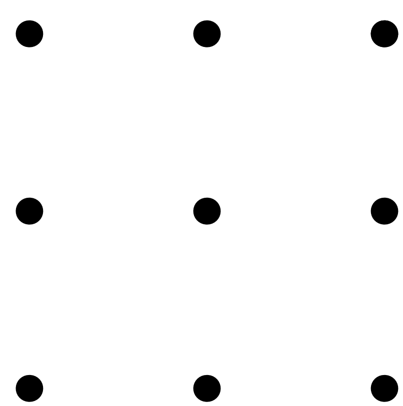
(8) ________ How many four-digit odd integers greater than 6000 can be formed from the digits 0, 1, 3, 5, 6 and 8, if no digit may be used more than once?

(9) __________ How many of the numbers from the set $\{1, 2, 3, \ldots, 50\}$ have a perfect square factor other than one?

(10) __________ Julie baked cupcakes for her family at home and for a party at school. She iced 4 cupcakes with red frosting, 2 cupcakes with orange frosting, 2 with yellow, 2 with green, 3 with blue and the last 3 with violet frosting. Each cupcake is iced with exactly one color of frosting. Julie plans to take exactly 10 of the cupcakes to her party, and will take either all of the cupcakes of a particular color or none of the cupcakes of that color. How many different combinations of cupcakes could she take to her party?

(11) __________ Four points are arranged on a plane such that a maximum number of lines can be determined by them. What is the maximum number of lines?

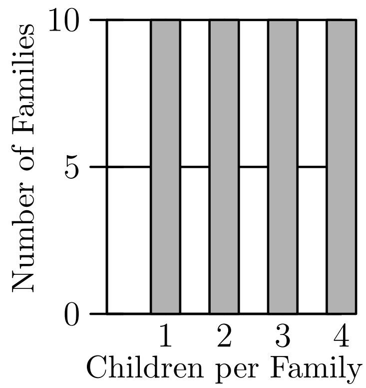
(12) __________ How many pairs of perpendicular line segments can be drawn using the points of this 3 by 3 grid as endpoints? (Segments must intersect at one point; either at an endpoint or within the segment.)



(13) __________ The mean of a sequence of r numbers is 10. When the sequence is extended to $r + 2$ terms to include the numbers 20 and 30, the mean of the sequence increases to 12. What is the value of r?

(14) __________ Out of 22 students surveyed on ice cream flavors, 12 liked chocolate, 5 liked only strawberry, and 6 liked vanilla. If 3 liked chocolate and vanilla, how many students did not like any of these flavors?

(15) ________ The results of a survey of the 40 families at West ES are shown in the graph. During the survey, each family was asked how many children were in their family. What was the average of the families' responses? Express your answer as a decimal to the nearest tenth.



(16) ________ Suppose x and y are two distinct two-digit positive integers such that y is the reverse of x. (For example, $x = 12$ and $y = 21$ is one such combination.) How many different sums $x + y$ are possible?

(17) ________ The probability of snow for each of the next three days is $\frac{3}{4}$. What is the probability that it will not snow at all during the next three days? Express your answer as a common fraction.

(18) ________ Sallie earned a grade of exactly 90% for a marking period based on six equally weighed tests. The four test papers she can find have grades of exctly 83%, 96%, 81%, and 82%. What is the sum of the percent scores of the two missing grades?

(19) ________ Given a list of 5 distinct nonnegative integers with mean of 18 and median of 19, what is the greatest possible range?

(20) ________ Jared has an average of 86% in his math class before the final exam. The final exam is 20% of his total grade. There are 55 points possible on the final exam and partial points are not given. If Jared wants to get an average of at least 88% in the class, what is the least number of points he needs to earn on the final exam?

Answer Sheet

Number	Answer	Problem ID
1	10 values	34D1
2	180 ways	30C21
3	1/1331	D32C
4	9 integers	015C
5	8	0BC2
6	$\frac{3}{4}$	BA3A1
7	3 ways	C5D1
8	72	01101
9	19	34B5
10	5 combinations	54D5
11	6	4B2C
12	78	45B5
13	13	CA322
14	2 students	5BB41
15	2.5 children	1B551
16	15 sums	BA51
17	1/64	5223
18	198 percent	C15
19	50	10A21
20	53	3023

Solutions

(1) **10 values ID: [34D1]**
No solution is available at this time.

(2) **180 ways ID: [30C21]**
No solution is available at this time.

(3) **1/1331 ID: [D32C]**
No solution is available at this time.

(4) **9 integers ID: [015C]**
Integers that are multiples of both 3 and 5 must be multiples of 15. We can start by listing the multiples of 15 between 1 and 200:

$$15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165, 180, 195$$

Checking only the even numbers, we can eliminate those that are multiples of 4, leaving us with:

$$15, 30, 45, 75, 90, 105, 135, 150, 165, 195$$

Finally, we need to eliminate any remaining multiples of 7. The only multiples of 7 we need to be concerned with are those ending in 5 or 0. The only number on the list that is a multiple of 7 is 105. Our final list is:

$$15, 30, 45, 75, 90, 135, 150, 165, 195$$

This leaves us with $\boxed{9}$ integers.

(5) **8 ID: [0BC2]**
No solution is available at this time.

(6) $\frac{3}{4}$ **ID: [BA3A1]**
No solution is available at this time.

(7) **3 ways** **ID: [C5D1]**
Since we are partitioning 81 into sums of perfect squares, we proceed by subtracting out perfect squares and seeing which work: $81 - 64 = 17 = 16 + 1$. Further, $81 - 49 = 32 = 16 + 16$. And finally, $81 - 36 = 45 = 36 + 9$. Although there is more to check through, this sort of method should convince us that these are the only $\boxed{3}$ solutions: $1^2 + 4^2 + 8^2 = 81$, $4^2 + 4^2 + 7^2 = 81$, and $3^2 + 6^2 + 6^2 = 81$.

(8) **72** **ID: [01101]**
No solution is available at this time.

(9) 19 **ID: [34B5]**
The potential square factors are 4, 9, 16, 25, 36, and 49. 4 divides 12 of the numbers. 9 divides 5 of the numbers, but we've counted $4 \cdot 9 = 36$ twice, so we subtract 1. 16 divides 3 of the numbers, but each of those is also divisible by 4, so we don't count them. 25 divides 2. 36 divides 1, itself, but it's already been counted. Finally, 49 divides 1. Thus, our final answer is $12 + (5 - 1) + 2 + 1 = \boxed{19}$.

(10) **5 combinations** **ID: [54D5]**
If Julie includes one of the colors that cover three cupcakes, she must also include the other color that covers three cupcakes. This is because she must make ten cupcakes total, and all of the other colors cover an even number of cupcakes, so there is no way to make ten with three and some combination of even numbers. Thus, if she includes blue and violet, she has four cupcakes left to choose. There are three ways in which she can choose four cupcakes if she chooses colors that cover two (green and orange, green and yellow, or orange and yellow). Alternately, she can choose a color that covers four (red). Finally, if she doesn't include any colors that cover three cupcakes, she must choose all of the other cupcakes in order to make ten. Thus, Julie has $\boxed{5}$ different combinations of cupcakes.

(11) **6** **ID: [4B2C]**
No solution is available at this time.

(12) 78 **ID: [45B5]**
No solution is available at this time.

(13) **13 ID: [CA322]**

Let S denote the sum of the original r numbers. Since the mean of the r numbers is 10, we have $S/r = 10$. After 20 and 30 are added, the sum of the sequence becomes $S + 50$ and the number of terms becomes $r + 2$. Therefore $12 = \frac{S+50}{r+2}$. Clearing denominators, we find

$$S = 10r, \text{ and}$$
$$S + 50 = 12r + 24.$$

Substituting $10r$ for S in the second equation gives $10r + 50 = 12r + 24$, which may be solved to obtain $r = \boxed{13}$.

(14) 2 **students ID: [5BB41]**

No solution is available at this time.

(15) **2.5 children ID: [1B551]**

The total number of children is $10 \cdot 1 + 10 \cdot 2 + 10 \cdot 3 + 10 \cdot 4 = 100$ children. There are a total of $10 + 10 + 10 + 10 = 40$ families. So the average children per family is $\dfrac{100}{40} = \boxed{2.5 \text{ children}}$.

(16) **15 sums ID: [BA51]**

No solution is available at this time.

(17) **1/64 ID: [5223]**

The probability of it snowing on any one day is $\frac{3}{4}$ so the probability of it not snowing on any one day is $\frac{1}{4}$. So, the probability it not snowing on all three days is $\left(\frac{1}{4}\right)^3 = \boxed{\dfrac{1}{64}}$.

(18) **198 percent ID: [C15]**

Suppose each test is worth x points. Then on the tests Sallie found, she has a total of $.83x + .96x + .81x + .82x = 3.42x$ points. In all, she earned $6 \cdot .9x = 5.4x$ points. Thus, on the two remaining tests she scored $1.98x$ points, which is a total of $\boxed{198\%}$.

(19) **50 ID: [10A21]**

No solution is available at this time.

(20) **53** **ID: [3023]**

No solution is available at this time.