✱ Understanding Maclaurin Series for $f(x)=\sin(x)$ and graphing

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Taylor series of function $f(x)$:

$$f(x)=f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2\cdots$$

Maclaurin series is Taylor series for $a=0$

$$f(x)=f(0)+\frac{f'(0)}{1!}(x-0)+\frac{f''(0)}{2!}(x-0)^2\cdots$$

Finding $f(0)$, $f'(0)$, $f''(0)\cdots$

Differential Term	Function	$x=a=0$
0	$\sin(x)$	0
1	$\cos(x)$	1
2	$-\sin(x)$	0
3	$-\cos(x)$	-1
4	$\sin(x)$	0
⋮	⋮	⋮

Substituting into Maclaurin series form:

$$f(x)=0+\frac{1}{1!}(x)+\frac{0}{2!}(x-0)^2+\frac{-1}{3!}(x-0)^3+\frac{0}{4!}(x-0)^4$$

For every other term it is 0, to simplify:

$$f(x)=\frac{1}{1!}x+\frac{-1}{3!}x^3+\frac{1}{5!}x^5\cdots$$

Sum Representation: $f(x)=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$

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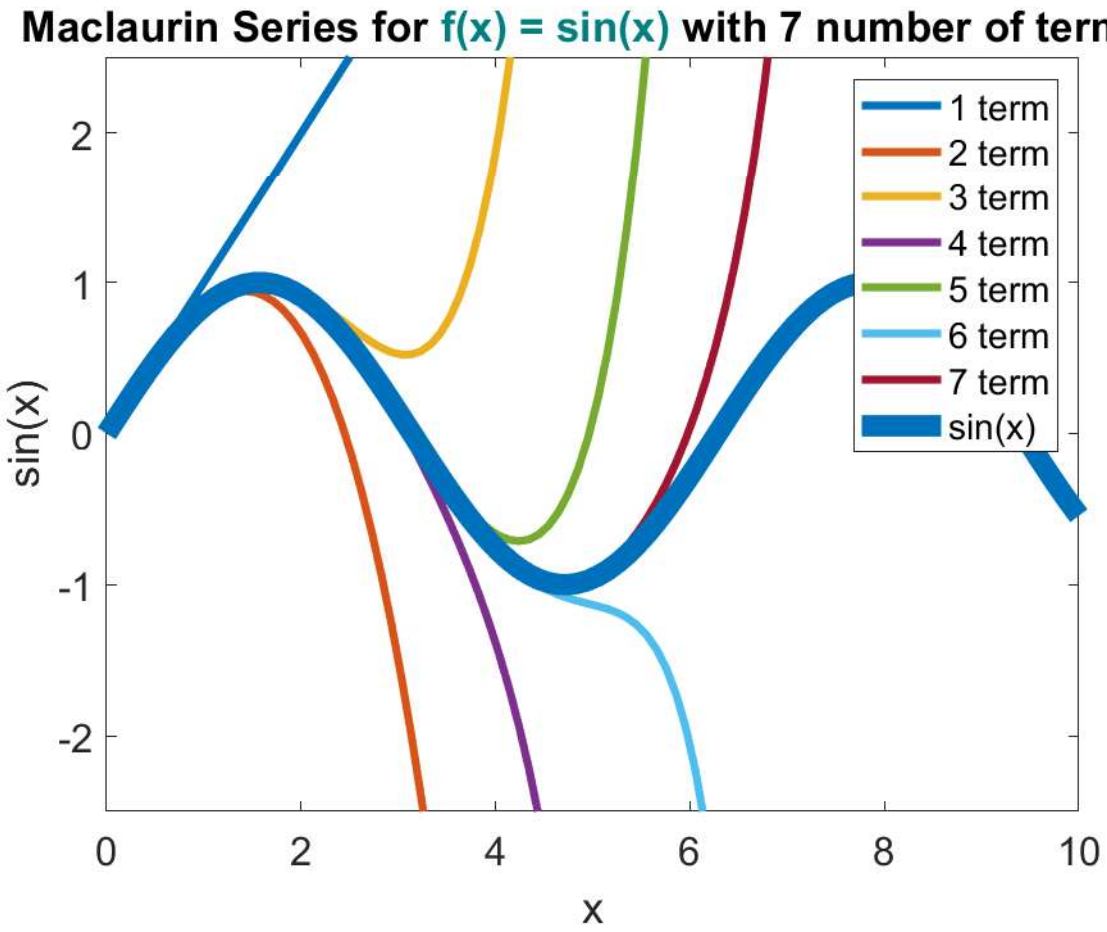
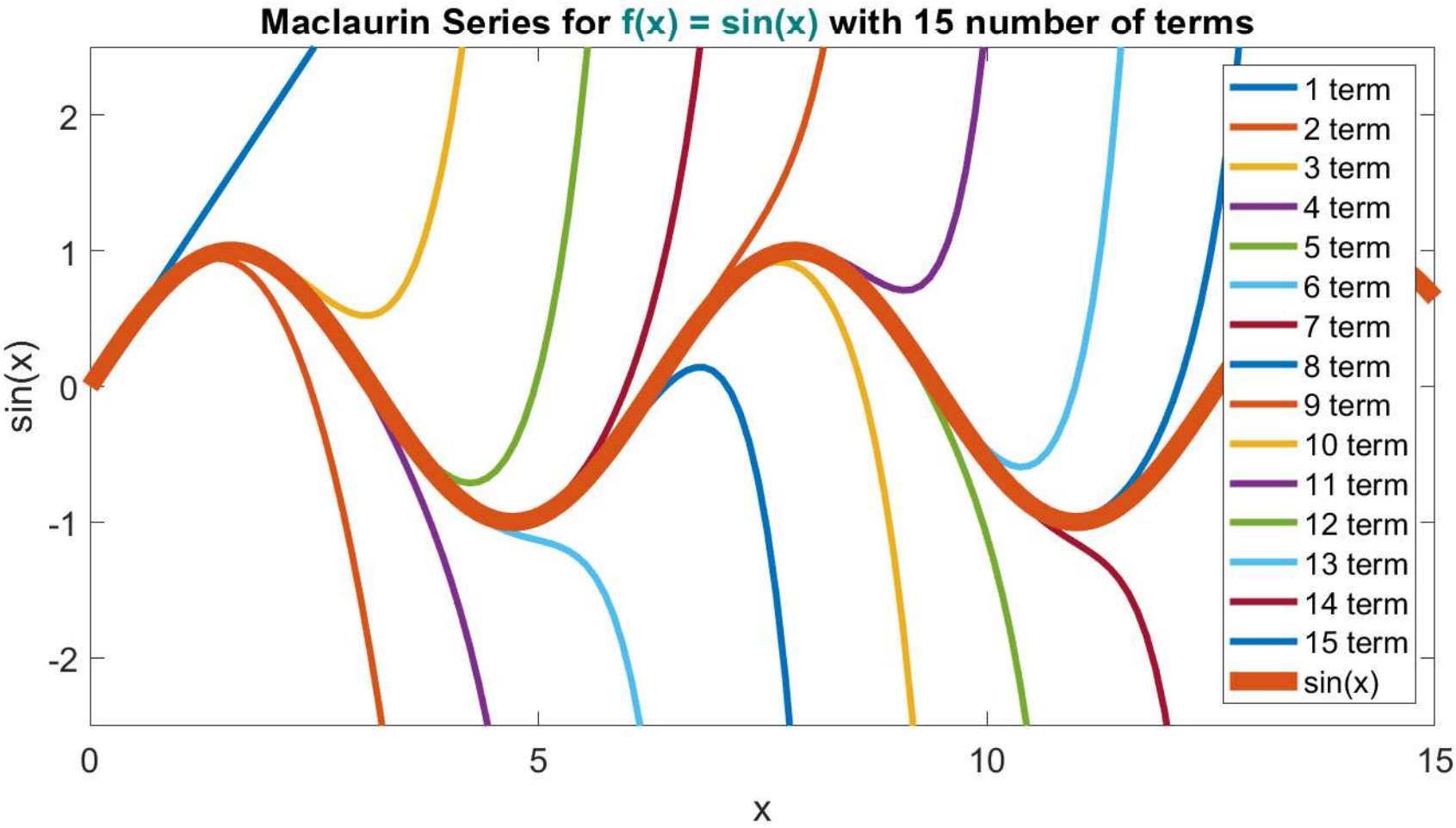
%---------------------------------Code Start-----------------------------
% Alan Chen
% Understanding Maclauring Series for f(x) = sin(x)
% Graphing for any given nth term
% 7/12/2019

clear
clc
xmin = 0;
xmax = input("What is the x max for graph? Ex. Input 5 would graph function from 0 to 5 horizontally\n");
nterm = input("Number of terms, n, in sum representation of the Maclaurin Series\n");
x = linspace(xmin,xmax,xmax*10);
y = sin(x);
yn = 0;
table=[];
for n = 0 : nterm-1
    yn = yn + ((-1)^n)*((x).^(2*n+1))/factorial(2*n + 1);
    plot(x,yn,'LineWidth',3)
    table = [table num2str(n+1)+" term"];
    hold on;
end
plot(x,y,'LineWidth',8)
hold off;
xlim([xmin xmax])
ylim([-2.5 2.5])
xlabel('x')
ylabel('sin(x)')
set(gca,'FontSize',15)
title(['\fontsize{16}Maclaurin Series for {'...
'\color[rgb]{0 .5 .5}f(x) = sin(x)} with ',num2str(nterm),'\fontsize{16} number of terms '])
table = [table "sin(x)"];
legend(table);
%---------------------------------Code End-------------------------------
```


Maclaurin Series for f(x) = sin(x) with 7 number of terms
1 term
2 term
3 term
4 term
5 term
6 term
7 term
sin(x)
sin(x)
x
2
1
0
-1
-2
0
2
4
6
8
10


Maclaurin Series for f(x) = sin(x) with 15 number of terms
sin(x)
x
2
1
0
-1
-2
0
5
10
15
1 term
2 term
3 term
4 term
5 term
6 term
7 term
8 term
9 term
10 term
11 term
12 term
13 term
14 term
15 term
sin(x)