# Understanding Maclaurin Series for $f(x)=\sin(x)$ and graphing

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Taylor series of function $f(x)$:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 \cdots$$

Maclaurin series is Taylor series for $a=0$

$$f(x) = f(0) + \frac{f'(0)}{1!}(x-0) + \frac{f''(0)}{2!}(x-0)^2 \cdots$$

Finding $f(0), f'(0), f''(0) \cdots$

| Differential Term | Function | $x=a=0$ |
|---|---|---|
| 0 | $\sin(x)$ | 0 |
| 1 | $\cos(x)$ | 1 |
| 2 | $-\sin(x)$ | 0 |
| 3 | $-\cos(x)$ | $-1$ |
| 4 | $\sin(x)$ | 0 |
| $\vdots$ | $\vdots$ | $\vdots$ |

Substituting into Maclaurin series form:

$$f(x) = 0 + \frac{1}{1!}(x) + \frac{0}{2!}(x-0)^2 + \frac{-1}{3!}(x-0)^3 + \frac{0}{4!}(x-0)^4$$

For every other term it is 0, to simplify:

$$f(x) = \frac{1}{1!}x + \frac{-1}{3!}x^3 + \frac{1}{5!}x^5 \cdots$$

$$\boxed{\text{Sum Representation: } f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}}$$