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CS 161 HW 5

1) $P \Rightarrow \neg Q$: $\neg P \vee \neg Q$
$Q \Rightarrow \neg P$: $\neg Q \vee \neg P$

	P Q	$P \Rightarrow \neg Q$	$Q \Rightarrow \neg P$
0	T T	X	X
1	T F	✓	✓
2	F F	✓	✓
3	F T	✓	✓

let α be $P \Rightarrow \neg Q$ and B be $Q \Rightarrow \neg P$. Because $M(\alpha) = M(B) = \{1,2,3\}$, the two sentences are equivalent.

$P \Leftrightarrow \neg Q$: $P \Rightarrow \neg Q$: $\neg P \vee \neg Q$; $\neg Q \Rightarrow P$: $Q \vee P$: $(\neg P \vee \neg Q) \wedge (Q \vee P)$

	P	Q	$P \Leftrightarrow \neg Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
0	T	T	X	X
1	T	F	✓	✓
2	F	F	X	X
3	F	T	✓	✓

let α be $P \Leftrightarrow \neg Q$ and B be $((P \wedge \neg Q) \vee (\neg P \wedge Q))$. Because $M(\alpha) = M(B) = \{1,3\}$, the two sentences are equivalent.

2)

a. $\neg(\text{Smoke} \Rightarrow \text{fire}) \lor (\neg\text{Smoke} \Rightarrow \neg\text{fire})$

$\neg(\neg\text{Smoke} \lor \text{fire}) \lor (\text{Smoke} \lor \neg\text{fire})$

$(\text{Smoke} \land \neg\text{fire}) \lor (\text{Smoke} \lor \neg\text{fire})$

	Smoke	fire	$(\text{Smoke} \Rightarrow \text{fire}) \Rightarrow (\neg\text{Smoke} \Rightarrow \neg\text{fire})$
0	T	T	✓
1	T	F	✓
2	F	T	✗
3	F	F	✓

Because the set of worlds that the sentence is valid in is not all possible worlds, but not no worlds either, the sentence is neither valid nor unsat.

b. $\neg(\text{smoke} \Rightarrow \text{fire}) \lor ((\text{Smoke} \lor \text{heat}) \Rightarrow \text{fire})$

$\neg(\neg\text{Smoke} \lor \text{fire}) \lor (\neg(\text{Smoke} \lor \text{heat}) \lor \text{fire})$

$(\text{smoke} \land \neg\text{fire}) \lor ((\neg\text{Smoke} \land \neg\text{heat}) \lor \text{fire})$

	Smoke	fire	heat	$(\text{Smoke} \Rightarrow \text{fire}) \Rightarrow ((\text{Smoke} \lor \text{heat}) \Rightarrow \text{fire})$
0	T	T	T	✓
1	T	T	F	✓
2	T	F	T	✓
3	T	F	F	✓
4	F	T	T	✓
5	F	T	F	✓
6	F	F	T	✗
7	F	F	F	✓

Because the set of worlds that the sentence is valid in is not all possible worlds, but not empty, the sentence is neither valid nor unsat.

c) LHS: $(smoke \land heat) \Rightarrow fire$

$\neg(smoke \land heat) \lor fire$

RHS: $((smoke \Rightarrow fire) \lor (heat \Rightarrow fire))$

$((\neg smoke \lor fire) \lor (\neg heat \lor fire))$

1. $\neg(smoke \land heat) \lor fire \Rightarrow ((\neg smoke \lor fire) \lor (\neg heat \lor fire))$
1. $\neg(\neg(smoke \land heat) \lor fire) \lor ((\neg smoke \lor fire) \lor (\neg heat \lor fire))$
1. $(smoke \land \neg fire) \lor (\neg smoke \lor fire \lor \neg heat)$
2. $(\neg smoke \lor fire \lor \neg heat) \Rightarrow \neg(smoke \land heat) \lor fire$
2. $\neg(\neg smoke \lor fire \lor \neg heat) \lor (\neg(smoke \land heat) \lor fire)$
2. $(\neg(\neg smoke \lor fire) \land heat) \lor (\neg smoke \lor \neg heat \lor fire)$
2. $(smoke \land \neg fire \land heat) \lor \neg smoke \lor \neg heat \lor fire$

$[(smoke \land \neg fire) \lor (\neg smoke \lor fire \lor \neg heat)] \land$
$[(smoke \land \neg fire \land heat) \lor \neg smoke \lor \neg heat \lor fire]$

	smoke	fire	heat	sentence
0	T	T	T	✓
1	T	T	F	✓
2	T	F	T	✓
3	T	F	F	✓
4	F	T	T	✓
5	F	T	F	✓
6	F	F	T	✓
7	F	F	F	✓

Because the set of all worlds the sentence is valid in is all possible worlds, the sentence is valid.

3)

a. $\Delta = \{ mythical \Rightarrow \neg mortal,\ \neg mythical \Rightarrow mortal \wedge mammal,$
$\neg mortal \vee mammal \Rightarrow horned,\ horned \Rightarrow magical \}$

b. mythical : a
mortal : b
mammal : c
horned : d
magical : e

$(\neg a \vee \neg b) \wedge (a \vee (b \wedge c)) \wedge (\neg(\neg b \vee c) \vee d) \wedge (\neg d \vee e)$

$(\neg a \vee \neg b) \wedge ((a \vee b) \wedge (a \vee c)) \wedge ((b \wedge \neg c) \vee d) \wedge (\neg d \vee e)$

$\boxed{(\neg a \vee \neg b) \wedge (a \vee b) \wedge (a \vee c) \wedge (b \vee d) \wedge (\neg c \vee d) \wedge (\neg d \vee e)}$

c. mythical

Δ is CNF from part b
α is a
$\Delta \models \alpha$?
$\Delta \wedge \neg\alpha$ unsat?

No more rules can be applied, and no contradictions have been found. Thus, $\Delta \wedge \neg\alpha$ is SAT, and Δ cannot be used to prove the unicorn is mythical.

0) $\neg a \vee \neg b$	Δ	
1) $a \vee b$		
2) $a \vee c$		
3) $b \vee d$		
4) $\neg c \vee d$		
5) $\neg d \vee e$		
6) $\neg a$	$\neg\alpha$	
7) b	1,6	
8) c	2,6	
9) d	4,8	
10) e	5,9	

α is e

0) $\neg a \vee \neg b$	Δ	
1) $a \vee b$		
2) $a \vee c$		
3) $b \vee d$		
4) $\neg c \vee d$		
5) $\neg d \vee e$		
6) $\neg e$	$\neg\alpha$	
7) $\neg d$	5,6	
8) $\neg c$	4,7	
9) a	2,8	
10) $\neg b$	0,9	
11) d	3,10	

7 and 11 contradict. $\neg d \wedge d$ never true, so $\Delta \wedge \neg\alpha$ is unSAT and $\Delta \models \alpha$. Thus, we can use Δ to prove the unicorn is magical.

3)

c. continued

α is d

horned

0) $\neg a \vee \neg b$
1) $a \vee b$
2) $a \vee c$
3) $b \vee d$
4) $\neg c \vee d$

Δ

5) $\neg d \vee e$
6) $\neg d$ $\quad \neg\alpha$

7) b $\quad$ 3,6
8) $\neg a$ $\quad$ 0,7
9) c $\quad$ 2,8
10) d $\quad$ 4,9

6 and 10 contradict, so $\Delta \wedge \neg\alpha$ is unsat and $\Delta \models \alpha$. Thus, we can use Δ to prove the unicorn is horned.

4) Figure 1)

decomposable: yes, vars on each side of AND gates are different

determinism: no, top or gate inputs aren't mut. excl. (A= true, B=false, C=true, D=false)

① $(\neg A \wedge B) \vee (\neg B \wedge A)$: ✓

② $(C) \vee (\neg D \wedge \neg C)$: ✓

③ $(\neg A \wedge \neg B) \vee (A)$: ✓

④ $(C \wedge \neg D) \vee (D \wedge \neg C)$ ✓

⑤ ① ∧ ② ∨ ③ ∧ ④

smooth: No, with ②, inputs on rightside aren't all present on RHS (D).

Figure 2)

decomposable: yes, vars in each side of ANDs are diff.

determinism: no, or gate ① is same on each side (A= false, B= true leads to both sides being true)

① $(\neg A \wedge B) \vee (\neg A \wedge B)$: X

② $(C \wedge D) \vee (\neg D \wedge \neg C)$

③ $(\neg A \wedge B) \vee (\neg A \wedge B)$

④ $(C \wedge \neg D) \vee (D \wedge \neg C)$

⑤ ① ∧ ② ∨ ③ ∧ ④

smooth: yes, in each or gate, ①-⑤, all vars present in either side are also pres. on other side.

5)

a.

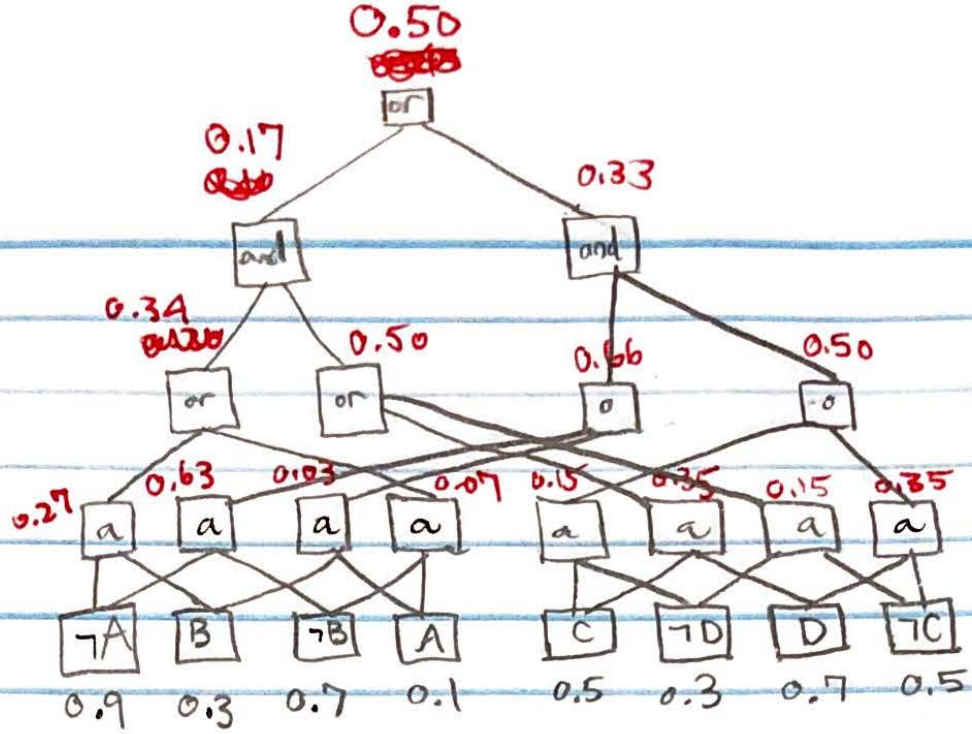
XOR

	A B	$(\neg A \wedge B) \vee (\neg B \wedge A)$
0	T T	X
1	T F	✓
2	F T	✓
3	F F	X

$$WMC = w(A, \neg B) + w(\neg A, B)$$
$$= w(A) \cdot w(\neg B) + w(\neg A) \cdot w(B)$$
$$= 0.1 \cdot 0.7 + 0.9 \cdot 0.3$$
$$\boxed{= 0.34}$$

b. The count on the root is same as the WMC for the formula. The same calculations are performed.

c.



$\boxed{WMC = 0.50}$