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CS 161 HW7

1) Base case:

$n=1$:

$Pr(\alpha_1|B) = Pr(\alpha_1|B)$ ✓

$n=2$:

$$Pr(\alpha_1,\alpha_2|B) = \frac{Pr(\alpha_1 \cap \alpha_2 \cap B)}{Pr(B)}$$

$$= \frac{Pr(\alpha_1|\alpha_2,B) \cdot Pr(\alpha_2 \cap B)}{Pr(B)}$$

$$= Pr(\alpha_1|\alpha_2,B) \cdot Pr(\alpha_2|B)$$ ✓

Inductive step:

Assume rule is true for $n=n$; $n+1$ works?

$$Pr(\alpha_1,\ldots,\alpha_{n+1}|B) = \frac{Pr(\alpha_1 \cap \alpha_2 \cap \ldots \alpha_{n+1} \cap B)}{Pr(B)}$$

$$= \frac{Pr(\alpha_1,\alpha_2,\ldots,\alpha_n|\alpha_{n+1},B) \cdot Pr(\alpha_{n+1},B)}{Pr(B)}$$

assume rule is true, use rule to expand

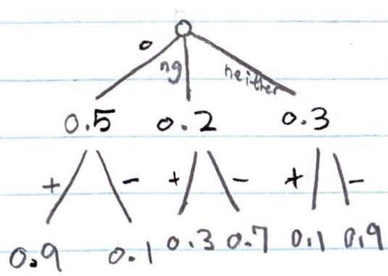
$$= Pr(\alpha_1|\alpha_2,\ldots,\alpha_{n+1},B) \cdot Pr(\alpha_2|\alpha_3,\ldots,\alpha_{n+1},B) \cdot \ldots$$
$$Pr(\alpha_n|\alpha_{n+1},B) \cdot Pr(\alpha_{n+1}|B)$$ ✓

Because the base case and inductive step have been proved, the identity is proved.

2) $P(o)=0.5$ $\quad P(o \cap ng)=0$ $\quad P(+|\neg o,\neg ng)=0.1$

$P(ng)=0.2$ $\quad P(+|o)=0.9$

$P(n)=0.3$ $\quad P(+|ng)=0.3$

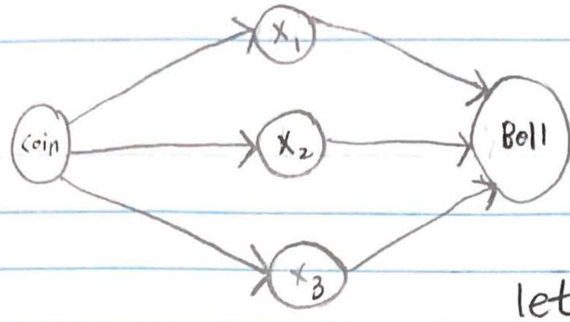


(disjoint)

$\boxed{Pr(o|+) = 83.3\%}$

$$P(o|+) = \frac{P(+|o) \cdot P(o)}{P(+)} = \frac{0.9 \cdot 0.5}{0.5 \cdot 0.9 + 0.2 \cdot 0.3 + 0.3 \cdot 0.1} = 0.833$$

3)



let H=heads, T=tails, B=bell

Coin	θ_{coin}
a	1/3
b	1/3
c	1/3

coin	X_1	$\theta_{X_1/coin}$
a	H	0.2
a	T	0.8
b	H	0.4
b	T	0.6
c	H	0.8
c	T	0.2

coin	X_2	$\theta_{X_2/coin}$
a	H	0.2
a	T	0.8
b	H	0.4
b	T	0.6
c	H	0.8
c	T	0.2

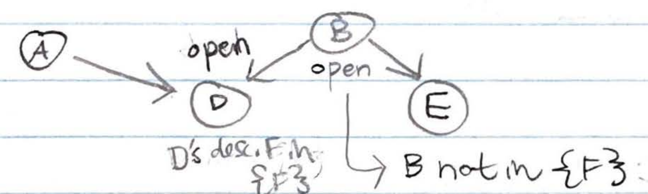
coin	X_3	$\theta_{X_3/coin}$
a	H	0.2
a	T	0.8
b	H	0.4
b	T	0.6
c	H	0.8
c	T	0.2

X_1	X_2	X_3	B	$\theta_{B \mid X_1, X_2, X_3}$
H	H	H	on	1
H	H	H	off	0
H	H	T	on	0
H	H	T	off	1
H	T	H	on	0
H	T	H	off	1
H	T	T	on	0
H	T	T	off	1
T	H	H	on	0
T	H	H	off	1
T	H	T	on	0
T	H	T	off	1
T	T	H	on	0
T	T	H	off	1
T	T	T	on	1
T	T	T	off	0

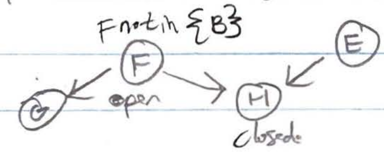
4)

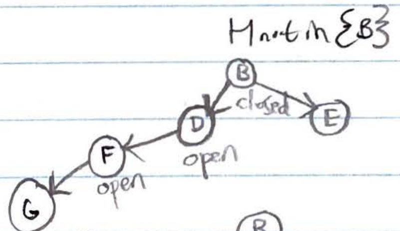
a. $I(A, \emptyset, BE)$
$I(B, \emptyset, AC)$
$I(C, A, BDE)$
$I(D, AB, CE)$
$I(E, B, ACDFG)$
$I(F, CD, ABE)$
$I(G, F, ABCDEH)$
$I(H, EF, ABCDG)$

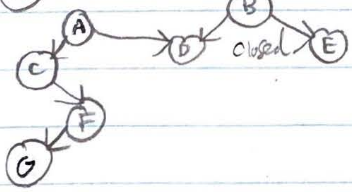
b. d_separated(A, F, E) : No, path A-D-B-E is open.



d_separated(G, B, E) : Yes, the different paths below are blocked.







d_separated(AB, CDE, GH):

check A to G, A to H, B to G, B to H

- A-C-F-G
  - C is closed
- A-D-F-G:
  - D is closed
- A-D-B-E-H-F-G:
  - D is open
  - B is open
  - E is closed
- A-C-F-H
  - C is closed
- A-C-F-D-B-E-H
  - C is closed
- A-D-B-E-H
  - D is open
  - B is open
  - E is closed
- A-D-F-H
  - D is closed
- B-D-F-G
  - D is closed
- B-D-A-C-F-G
  - D is open
  - A is open
  - C is closed
- B-E-H-F-G
  - E is closed
- B-E-H
  - E is closed
- B-D-F-H
  - D is closed
- B-D-A-C-F-H
  - D is open
  - A is open
  - C is closed

Every path between AB and GH is blocked by CDE, so AB and GH are separated given Z={CDE}.

c) $Pr(a,b,c,d,e,f,g,h) = Pr(a)\cdot Pr(b)\cdot Pr(c|a)\cdot Pr(d|a,b)\cdot Pr(e|b)\cdot Pr(f|c,d)\cdot Pr(g|f)\cdot Pr(h|e,f)$

Each nodes probability multiplied together.

d) $Pr(A=1,B=1) = Pr(A=1)\cdot Pr(B=1)$

$= 0.2\cdot 0.7$

$\boxed{= 0.14}$

A and B are independent because A and B are separated given Z={}, using I(A,∅,BE).

$Pr(E=0|A=0) = \frac{Pr(E=0,A=0)}{Pr(A=0)}$

A and E are ind. b/c they are seperated given Z={}, using I(A,∅,BE).

$= \frac{Pr(E=0)\cdot Pr(A=0)}{Pr(A=0)}$

$= Pr(E=0)$

$= Pr(E=0|B=0)\cdot Pr(B=0) + Pr(E=0|B=1)\cdot Pr(B=1)$

$= 0.1\cdot 0.3 + 0.9\cdot 0.7 = \boxed{0.66}$

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5)

a. $\neg A \vee B$

$$\boxed{\mathcal{M}(\alpha) = \{w_0, w_2, w_3\}}$$

	A	B	α
w_3	0	0	1
w_2	0	1	1
w_1	1	0	0
w_0	1	1	1

b. $Pr(\alpha) = Pr(w_0) + Pr(w_2) + Pr(w_3)$

$= 0.3 + 0.1 + 0.4$

$\boxed{= 0.8}$

c.

	A	B	$Pr(A,B)$	$Pr(A,B \mid \alpha)$
w_0	T	T	0.3	$0.3/0.8 = 0.375$
~~w_1~~	~~T~~	~~F~~	~~0.2~~	0
w_2	F	T	0.1	$0.1/0.8 = 0.125$
w_3	F	F	0.4	$0.4/0.8 = 0.500$

$$Pr(w \mid \alpha) = \begin{cases} 0 & \text{if } w \models \neg\alpha \\ Pr(w)/Pr(\alpha) & \text{if } w \models \alpha \end{cases}$$

d. $$Pr(\neg A \vee \neg B \mid \alpha) = \frac{Pr((\neg A \vee \neg B) \wedge \alpha)}{Pr(\alpha)}$$

$$= \frac{0.1 + 0.4}{0.8}$$

$$\boxed{= 0.625}$$