ETC3250

Practice exam 2016

Instructions

- In the actual exam, there are 11 questions worth a total of 100 marks. You should attempt them all.
- Open book
- You can use the approved calculator

This practice exam has a range of questions that show possible range of toics that are examined. They come from last year's exam and practice exam, and there are a few additional new questions. Some of these questions are more open ended than the questions on the actual exam, there are fewer, "what is XXX, write a few sentences explaining XXX" on the actual exam.

QUESTION 1

(a) Briefly explain why best subset selection is not computationally feasible when the number of predictors p is large.

[2 marks]

(b) True or false. By constraining the L_1 norm of the vector of coefficients, it is possible to obtain sparse estimate for the coefficients. Explain your answer.

(c) When is Multidimensional scaling (MDS) is equivalent to Principal Component Analysis (PCA)?

(d) What is the difference between 2-fold cross-validation and 10-fold cross-validation? Which one would you use in practice?

[2 marks]

(e) Write the formula to compute the prediction of a K-nn classifier for the new data point x_0.

[2 marks]

[Total: 6 marks]

— END OF QUESTION 1 —

QUESTION 2

Suppose you estimate the coefficients of a linear regression by solving the following optimization problem:

$$\underset{\boldsymbol{\beta}}{\text{minimize}} \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{p} \beta_j x_{ij} \right)^2 + \lambda \sum_{j=1}^{p} \beta_j^2$$

where $\lambda \geq 0$ is a **tuning parameter**.

(a) Briefly explain how λ affects the bias and variance tradeoff of your estimate $\hat{\boldsymbol{\beta}}$.

[2 marks]

(b) Explain why is it important to standardize the predictors in this case

[2 marks]

(c) Sparsity is very useful in high-dimensional regression. Explain why.

(d) Does the previous optimization allows sparse estimates? Explain your answer.

[Total: 4 marks]

— END OF QUESTION 2 —

QUESTION 3

This question is about bootstrapping.

(a) Give an example of algorithm that uses bootstrapping and why.

(b) Bootstrapping can be used to estimate the sampling distribution of a statistic. Explain this procedure.

(c) Can we use the previous bootstrap procedure when the data is a time series? Explain.

[Total: 0 marks]

— END OF QUESTION 3 —

QUESTION 4

(a) Briefly explain why the K-means algorithm is guaranteed to decrease the value of the objective at each step.

(b) True or false. For any starting values of the assignment of the observations, the K-means algorithm will always converge to the same solution.

(c) For the following data.

	X1	X2	X3	X4
A	-1.02	0.27	-0.81	-0.34
B	-0.61	0.97	0.76	0.71
C	0.70	-0.38	0.88	-0.32
D	-0.82	0.48	-0.71	-0.98
E	-0.72	0.97	-0.33	0.04

and the associated distance matrix:

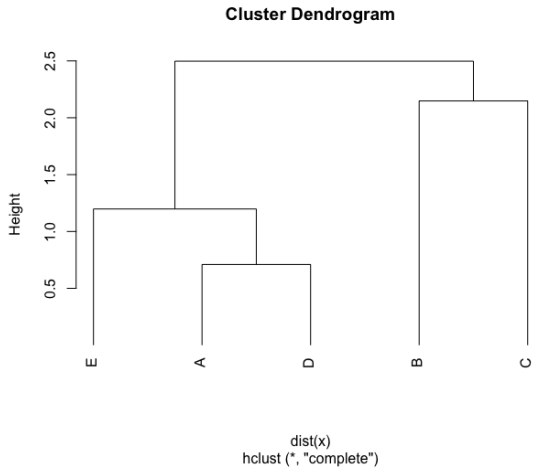
	A	B	C	D	E
A	0.00	2.06	2.50	0.71	0.98
B	2.06	0.00	2.15	2.30	1.28
C	2.50	2.15	0.00	2.45	2.33
D	0.71	2.30	2.45	0.00	1.20
E	0.98	1.28	2.33	1.20	0.00

(i) Compute the Euclidean distance between observations A and E.

[2 marks]

(ii) The dendrogram shows hierarchical clustering with complete linkage. Which two points were fused at the first step of hierarchical clustering with complete linkage?

[2 marks]



(iiI) What is the intercluster distance (linkage) values between the new cluster and the remaining four points? (There are three.)

[2 marks]

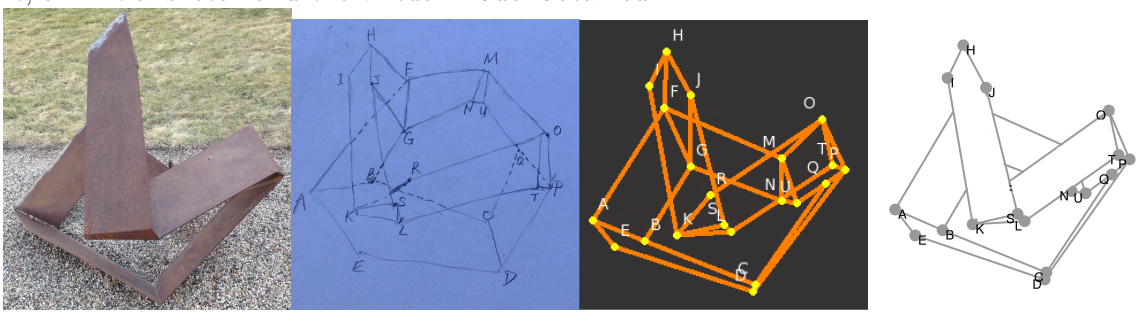
[Total: 6 marks]

— END OF QUESTION 4 —

QUESTION 5

This question is about dimension reduction.

You are asked to curate an art exhibit, containing 15 identical sculptures, that has one rule: the sculptures must all be oriented and resting on the ground uniquely. As you walk around the current exhibit you notice that there are several pairs where the sculptures are actually identically placed. To solve the problem you decide to make a virtual model by measuring the distances between each pair of vertices, and using multi-dimensional scaling to produce a 3D layout. Below are some photos of the exhibit, an initial sketch and the virtual model obtained.

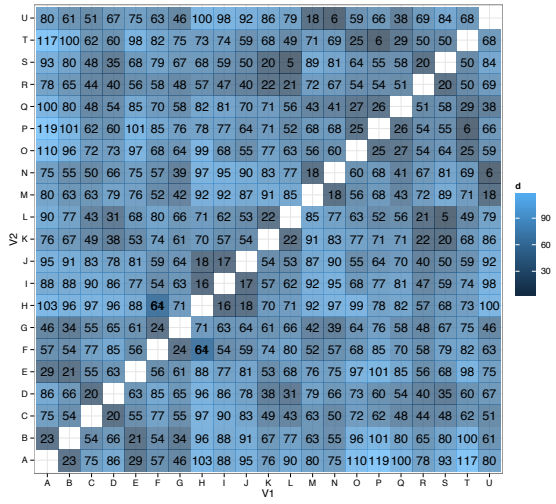


(a) Sketch the three point shape that would perfectly minimise the MDS classical stress function of this distance matrix.

[2 marks]

$$\begin{bmatrix} 0 & 3 & 4 \\ 3 & 0 & 5 \\ 4 & 5 & 0 \end{bmatrix}$$

(b) Below is a display of the inter-vertex distance matrix for one sculpture.



The eigenvalues are:

```
> dm_pca$sdev^2
 [1] 5.614e+03 4.284e+03 2.654e+03 7.438e+02 4.318e+02 1.349e+02 1.157e+02
 [8] 7.505e+01 4.341e+01 2.743e+01 2.479e+01 1.757e+01 1.699e+01 1.299e+01
[15] 1.058e+01 9.287e+00 5.996e+00 1.907e+00 1.634e+00 9.388e-01 2.307e-29
```

and the coefficients of the first eigenvector are:

```
    A     B     C     D     E     F     G     H     I     J     K     L     M
 0.35  0.33  0.02 -0.09  0.23  0.15  0.18 -0.22 -0.19 -0.26 -0.15 -0.24  0.18
    N     O     P     Q     R     S     T     U
 0.19 -0.20 -0.29 -0.12 -0.17 -0.26 -0.30  0.18
```

(i) What does this tell you about the main axis of variation of the sculpture?

[2 marks]

(ii) Sketch the scree plot. How many principal components would the scree plot suggest should be used to summarise the distances?

[2 marks]

(iii) What proportion of total variation do three principal components explain?

[2 marks]

(iv) Explain how multidimensional scaling is related to principal components, in the context of this problem?

[2 marks]

[Total: 10 marks]

— END OF QUESTION 5 —

QUESTION 6

A principal component analysis is conducted on a subset of Mexico City data, health and pollution where missing values have been imputed using regression methods. There are five variables in the subset: deaths (number of deaths each day), temp_mean (average temperature), humidity, NOX (nitrogen oxide, pollutant), O3 (ozone, pollutant).

```
> mexico.pca1 <- prcomp(mexico[,c("deaths", "temp_mean",
    "humidity", "NOX", "O3")], scale=T, retx=T)
> mexico.pca1
Standard deviations:
[1] 1.37 1.27 0.86 0.64 0.62

Rotation:
            PC1    PC2    PC3    PC4    PC5
deaths    -0.30  0.573 -0.471 -0.597  0.077
temp_mean -0.33 -0.578  0.338 -0.631  0.213
humidity   0.64 -0.074  0.003 -0.468 -0.608
NOX       -0.25  0.510  0.766 -0.028 -0.300
O3        -0.58 -0.268 -0.279  0.161 -0.699

> summary(mexico.pca1)
Importance of components:
                          PC1   PC2   PC3    PC4   PC5
Standard deviation     1.367 1.267 0.856 0.6362 0.624
Proportion of Variance 0.374 0.321 0.146 0.0809 0.078
Cumulative Proportion  0.374 0.695 0.841 0.9220 1.000
```

(a) Compute the total variance.

[1 marks]

(b) Make a sketch of the scree plot. Label your axes.

[2 marks]

(c) The PCA was conducted on the correlation matrix. Why do you think that this was necessary?

[1 marks]

(d) What proportion of variance is explained by four PCs?

[1 marks]

(e) With the help of the summary statistics below, compute the principal component score for this data value: $deaths = 296, temp\_mean = 12, humidity = 53, NOX = 0.028, O3 = 0.026$.

[2 marks]

```
> apply(mexico[,c("deaths", "temp_mean", "humidity", "NOX", "O3")],2,mean)
   deaths temp_mean  humidity       NOX        O3
  273.951    16.767    52.969     0.055     0.022
> apply(mexico[,c("deaths", "temp_mean", "humidity", "NOX", "O3")],2,sd)
   deaths temp_mean  humidity       NOX        O3
   28.990     2.652    14.544     0.024     0.009
```

(f) Interpret the first principal component.

[2 marks]

(g) How many principal components would you use to summarize the variation of this data? Why? [2 marks]

[Total: 11 marks]

— END OF QUESTION 6 —

QUESTION 7

(a) Select the propositions that are true, and briefly explain your answer:

[4 marks]

- ☐ The test error can be smaller than the training error.
- ☐ The model is underfitting when the training error is very large.
- ☐ Increasing the number of neighbours in the K-nearest neighbours algorithm will increase the flexibility of the model.
- ☐ Using cross-validation for model selection will necessarily provide models with better prediction accuracy compared to models selected by AIC and BIC.

(b) Consider a simple classification procedure applied to a two-class dataset with 5000 predictors and 50 samples:

Step 1. Find the 100 predictors having the largest correlation with the class labels.

Step 2. Apply logistic regression using only these 100 predictors .

How do we estimate the test set performance of this classification procedure?

[2 marks]

Can we use cross-validation? If yes, describe the step-by-step cross-validation procedure.

[3 marks]

(c) If we have n data points, what is the probability that a given data point does not appear in a bootstrap sample?

[3 marks]

[Total: 12 marks]

— END OF QUESTION 7 —

QUESTION 8

(a) Suppose we have a data set with five predictors, X_1 = GPA, X_2 = IQ, $X3$ = Gender (1 for Female and 0 for Male), X_4 = GPA×IQ, and X_5 = GPA×Gender. The response Y is starting salary after graduation (in thousands of dollars). Suppose we use least squares to fit the model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + e$$

where e is a random error term, and we obtain estimates $\hat{\beta}_0 = 50$, $\hat{\beta}_1 = 20$, $\hat{\beta}_2 = 0.07$, $\hat{\beta}_3 = 35$, $\hat{\beta}_4 = 0.01$, $\hat{\beta}_5 = -10$.

Which answer is correct, and why?

[4 marks]

(a) For a fixed value of IQ and GPA, males earn more on average than females.

(b) For a fixed value of IQ and GPA, females earn more on average than males.

(c) For a fixed value of IQ and GPA, males earn more on average than females provided that the GPA is high enough.

(d) For a fixed value of IQ and GPA, females earn more on average than males provided that the GPA is high enough.

(b) Predict the salary of a female with IQ of 110 and a GPA of 4.0.

[2 marks]

(c) True or false: Since the coefficient for the GPA×IQ interaction term is very small, there is very little evidence of an interaction effect. Justify your answer.

[3 marks]

(d) Suppose that the true relationship between starting salary and IQ is nonlinear for fixed values of GPA and Gender. You wish to compare the cubic model (including X_2, X_2^2 and X_2^3) to the linear model (including X_2 but not the quadratic or cubic terms). You split the available data into two forming a training set and test set, and you estimate both models using only the training data. Which of the two models would you expect to have larger mean squared error computed on the training data, or is there not enough information to say? Which of the two models would you expect to have larger mean squared error computed on the test data, or is there not enough information to say? Justify your answer.

[4 marks]

(e) You need to decide whether to add the quadratic and cubic terms to the model. Explain how you would do this using a test set, using cross-validation, and using the AIC. Comment on which of these three methods you would prefer and why.

[3 marks]

[Total: 16 marks]

— END OF QUESTION 8 —

QUESTION 9

The OECD PISA Results in Focus report describes the survey as "the world's global metric for quality, equity and efficiency in school education". The goal of the PISA survey is to assess the workforce readiness of 15-year old students. Nearly 500,000 students were tested across 65 countries and economies. Students were examined on how well they can apply the knowledge they learned in school to applications outside of school. The reported scores range between 0–1000. Information about the students, parents, and schools is also collected. The students completed a questionnaire providing information about themselves, their homes, their schools, and a variety of psychological views regarding factors they believe affect their performance in school. School principals responded to a questionnaire covering their school system and learning experiences for their students. In some countries, parents completed a questionnaire requesting information about their perceptions regarding the school system, expectations for their child, and their involvement in their child's schooling.

You have a sample of this data for five countries: Australia, Germany, Japan, Jordan and South Korea. The variables are responses of the students to questions about how they use the internet, for purposes that include "One player games", "Collaborative games", "Use email", "Chat on line", "Social networks", "Browse the Internet for fun", "Read news", "Obtain practical information","Download music", "Upload content", "Internet for school", "Email students", "Email teachers", "Download from School", "Announcements", "Homework", "Share school material":

0 Never or hardly ever
1 Once or twice a month
2 Once or twice a week
3 Almost every day
4 Every day

In addition, you have information about gender, and schoolid. Missing values are codes as NA. For this question we will focus just on this subset of the variables:"One player games", "Collaborative games", "Download music", "Upload content". This means that there are 5 variables, and 37904 student responses, with the following breakdown by country:

Country	Count
Australia	14481
Germany	5001
Japan	6351
Jordan	7038
South Korea	5033

(a) This is a rough summary of the data, including summaries of missing values by cases and variables.

```
          name          Gender    One.player.games Collaborative.games Download.music
 Australia  :14481   Female:18515   Min.   :0.00    Min.   :0.00       Min.   :0.00
 Germany    : 5001   Male  :19389   1st Qu.:0.00    1st Qu.:0.00       1st Qu.:1.00
 Japan      : 6351                  Median :1.00    Median :0.00       Median :2.00
 Jordan     : 7038                  Mean   :1.27    Mean   :1.01       Mean   :1.93
 South Korea: 5033                  3rd Qu.:2.00    3rd Qu.:2.00       3rd Qu.:3.00
                                    Max.   :4.00    Max.   :4.00       Max.   :4.00
                                    NA's   :2567    NA's   :2612       NA's   :2613
```

Cases with missings	Count
0	37533
1	34
2	12
3	325

(a) What percentage of the internet usage variables had missing values?

[2 marks]

(b) How many cases (students) did not complete any of these survey questions at all?

[2 marks]

(c) Write a paragraph on how you would handle the missing values in an analysis of the data.

[2 marks]

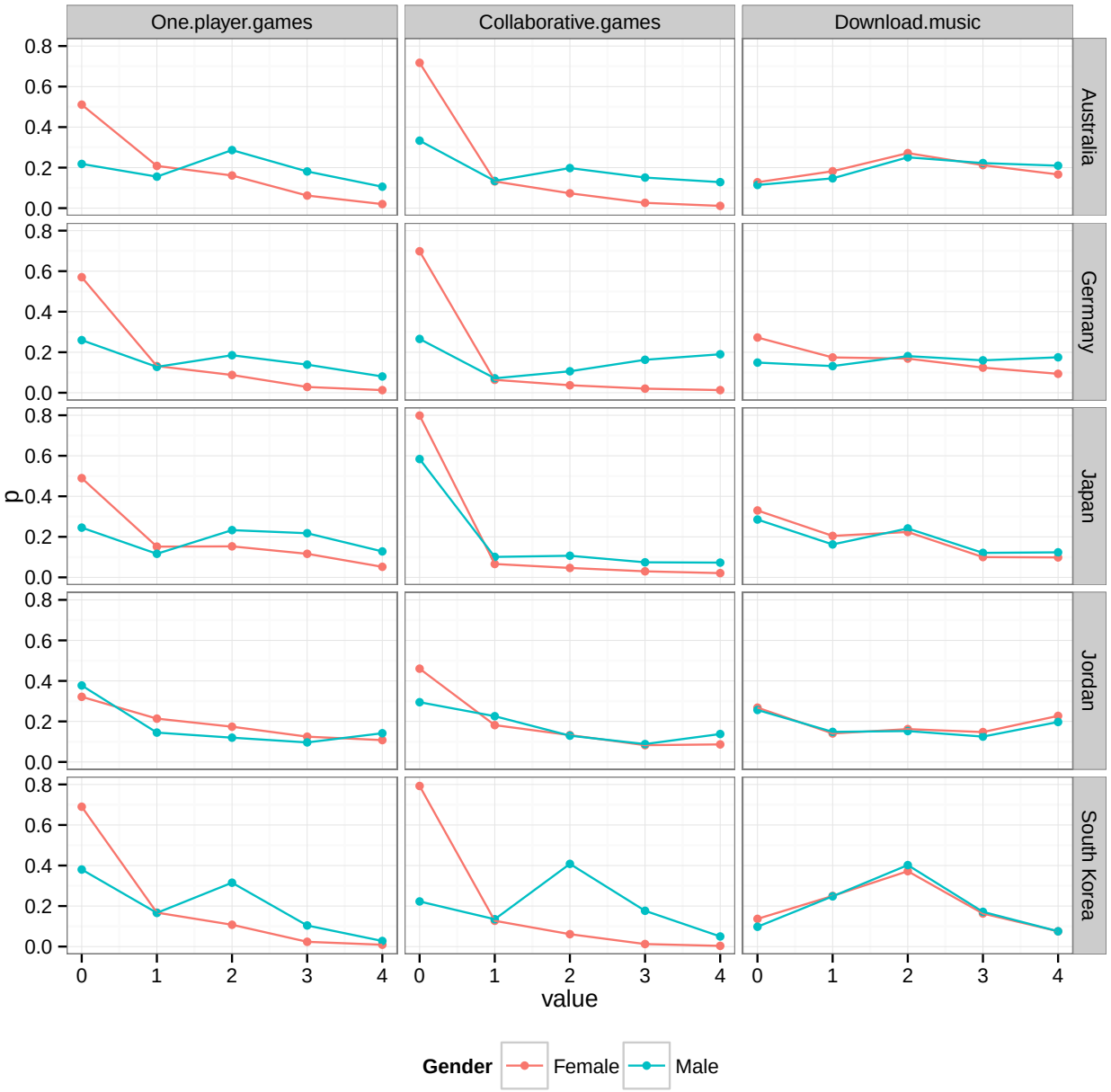
(b) We want to look at how students report using the internet, and in particular whether there are differences between countries and gender. To organize the data we need to rearrange it and summarise in order to make some plots. The following code was used to (partially) do this:

```
> internet.m.tb <- internet[,c(1,3,4,5,12,13)] %>%
+   gather(key=usage, value=value, -name, -Gender) %>%
+   group_by(name, Gender, usage, value) %>%
+   tally(sort=TRUE)
> head(internet.m.tb)
Source: local data frame [6 x 5]
Groups: name, Gender, usage [1]

       name Gender            usage value     n
     (fctr) (fctr)           (fctr) (int) (int)
1 Australia Female One.player.games     0  3613
2 Australia Female One.player.games     1  1474
3 Australia Female One.player.games     2  1140
4 Australia Female One.player.games     3   439
5 Australia Female One.player.games    NA   268
6 Australia Female One.player.games     4   141
...
> internet.m.tb.n <- summarise(group_by(internet.m.tb, name, Gender, usage), tot=sum(n))
> internet.m.tb <- merge(internet.m.tb, internet.m.tb.n)
> internet.m.tb.p <- summarise(group_by(internet.m.tb, name, Gender, usage, value), p=n/tot)
> head(internet.m.tb.p)
Source: local data frame [6 x 5]
Groups: name, Gender, usage [1]

       name Gender            usage value          p
     (fctr) (fctr)           (fctr) (int)      (dbl)
1 Australia Female One.player.games     0 0.51067138
2 Australia Female One.player.games     1 0.20833922
3 Australia Female One.player.games     2 0.16113074
4 Australia Female One.player.games     3 0.06204947
5 Australia Female One.player.games     4 0.01992933
6 Australia Female One.player.games    NA 0.03787986
...
qplot(value, p, data=internet.m.tb.p, geom=c("point","line"), color=Gender) +
  facet_grid(name~usage) +
  theme_bw() + theme(legend.position="bottom")
```

The plot shows the fully rearranged and summarised data of proportions of the level of usage, separately by country and purpose, colored by gender.



(a) What does the code block (shown below from the full set of commands) do?

[2 marks]

```
> internet.m.tb.n <- summarise(group_by(internet.m.tb, name,
                                        Gender, usage), tot=sum(n))
> internet.m.tb <- merge(internet.m.tb, internet.m.tb.n)
```

(b) Explain (in a few sentences) what the "wide" and "long" forms of the data are according to "tidyr", and how this can help when working with large data.

[2 marks]

(c) From the plot, would you say there is a different pattern of of one player gaming by boys and girls in Australia? And give your reasons.

[2 marks]

(d) From the plot, which country and what type of usage exhibits the biggest difference between boys and girls? Describe the difference.

[2 marks]

(e) When you reduce the counts to proportions there are various choices in how to compute the proportion, and this changes the way the results can be compared and constrasted. Explain how proportions were calculated here, and why you think this was a reasonable way to calculate them in order to answer the two qustions above.

[2 marks]

(f) One cognitive phenomenon to avoid when making plots is "change blindness". Explain what this means. Does facetting of plots potentially induce change blindness? if so, how?

[2 marks]

(g) How were "NA" or missings handled?

[2 marks]

[Total: 20 marks]

— END OF QUESTION 9 —

QUESTION 10

The OECD PISA Results in Focus report describes the survey as "the world's global metric for quality, equity and efficiency in school education". The goal of the PISA survey is to assess the workforce readiness of 15-year old students. Nearly 500,000 students were tested across 65 countries and economies. Students were examined on how well they can apply the knowledge they learned in school to applications outside of school. The reported scores range between 0–1000. Information about the students, parents, and schools is also collected. The students completed a questionnaire providing information about themselves, their homes, their schools, and a variety of psychological views regarding factors they believe affect their performance in school. School principals responded to a questionnaire covering their school system and learning experiences for their students. In some countries, parents completed a questionnaire requesting information about their perceptions regarding the school system, expectations for their child, and their involvement in their child's schooling.

You have a sample of this data for five countries: Australia, Germany, Japan, Jordan and South Korea. The variables are responses of the students to questions about how they use the internet, for purposes that include "One player games", "Collaborative games", "Use email", "Chat on line", "Social networks", "Browse the Internet for fun", "Read news", "Obtain practical information","Download music", "Upload content", "Internet for school", "Email students", "Email teachers", "Download from School", "Announcements", "Homework", "Share school material":

0 Never or hardly ever
1 Once or twice a month
2 Once or twice a week
3 Almost every day
4 Every day

In addition, you have information about gender, and schoolid. Missing values are codes as NA. For this question we will focus just on this subset of the variables: "Use email","Chat on line","Social networks","Browse the Internet for fun". This means that there are 6 variables, and 37904 student responses, with the following breakdown by country:

Country	Count
Australia	14481
Germany	5001
Japan	6351
Jordan	7038
South Korea	5033

This is a rough summary of the data, including summaries of missing values by cases and variables.

```
  Use.email      Chat.on.line   Social.networks Browse.the.Internet.for.fun
Min.   :0.000   Min.   :0.000   Min.   :0.00    Min.   :0.000
1st Qu.:1.000   1st Qu.:0.000   1st Qu.:1.00    1st Qu.:2.000
Median :2.000   Median :2.000   Median :3.00    Median :3.000
Mean   :1.908   Mean   :1.734   Mean   :2.43    Mean   :2.539
3rd Qu.:3.000   3rd Qu.:3.000   3rd Qu.:4.00    3rd Qu.:4.000
Max.   :4.000   Max.   :4.000   Max.   :4.00    Max.   :4.000
NA's   :2691    NA's   :2716    NA's   :2613    NA's   :2600
```

Cases with missings	Count
0	37525
1	35
2	8
3	6
4	330

(a) What percentage of the internet usage variables had missing values?

[1 marks]

(b) How many cases (students) did not complete any of these survey questions at all?

[1 marks]

(c) Write a paragraph on how you would handle the missing values in an analysis of the data.

[2 marks]

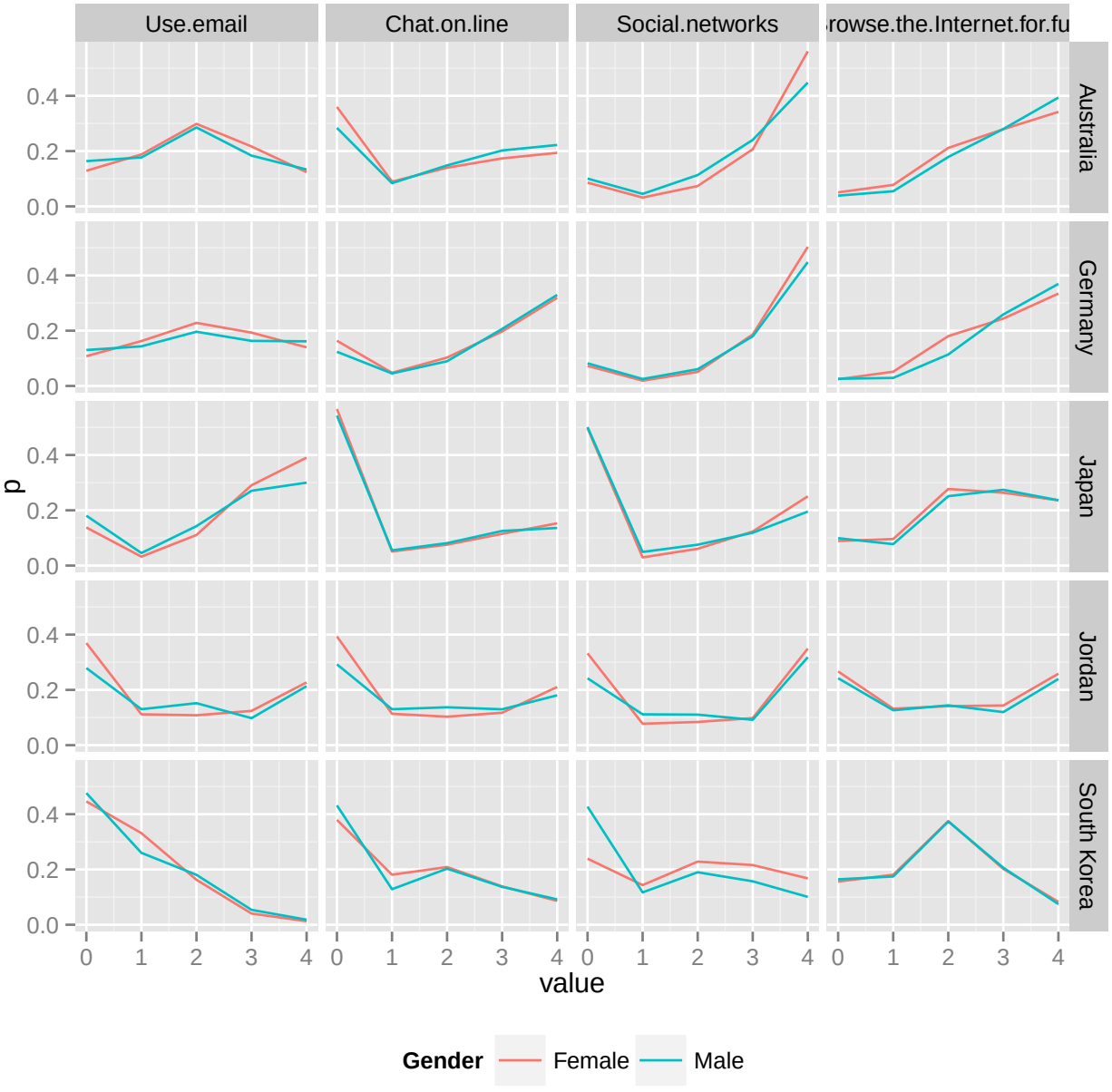
We want to look at how students report using the internet, and in particular whether there are differences between countries and gender. To organize the data we need to rearrange it and summarise in order to make some plots. The following code was used to (partially) do this:

```
> library(dplyr)
> library(tidyr)
> head(internet[,c(1,3,6,7,8,9)])
       name Gender Use.email Chat.on.line Social.networks Browse.the.Internet.for.fun
1 Australia Female         2            1               3                           1
2 Australia Female         1            2               4                           3
3 Australia Female         3            3               3                           3
4 Australia   Male         2            4               4                           4
5 Australia   Male         2            3               3                           3
6 Australia Female         0            2               4                           3
...
> internet.m.tb <- internet[,c(1,3,6,7,8,9)] %>%
                     gather(key=usage, value=value, -name, -Gender) %>%
                     group_by(name, Gender, usage, value) %>%
                     tally(sort=TRUE)
> head(internet.m.tb)
Source: local data frame [6 x 5]
Groups: name, Gender, usage [1]

       name Gender     usage value     n
     (fctr) (fctr)    (fctr) (int) (int)
1 Australia Female Use.email     2  2112
2 Australia Female Use.email     3  1533
3 Australia Female Use.email     1  1329
4 Australia Female Use.email     0   911
5 Australia Female Use.email     4   875
6 Australia Female Use.email    NA   315
...
> internet.m.tb.n <- summarise(group_by(internet.m.tb, name, Gender, usage), tot=sum(n))
> internet.m.tb <- merge(internet.m.tb, internet.m.tb.n)
> internet.m.tb.p <- summarise(group_by(internet.m.tb, name, Gender, usage, value), p=n/tot)
> head(internet.m.tb.p)
Source: local data frame [6 x 5]
Groups: name, Gender, usage [1]

       name Gender     usage value          p
     (fctr) (fctr)    (fctr) (int)      (dbl)
1 Australia Female Use.email     0 0.12876325
2 Australia Female Use.email     1 0.18784452
3 Australia Female Use.email     2 0.29851590
4 Australia Female Use.email     3 0.21667845
5 Australia Female Use.email     4 0.12367491
6 Australia Female Use.email    NA 0.04452297
...
qplot(value, p, data=internet.m.tb.p, geom="line", color=Gender) +
  facet_grid(name~usage) +
  theme(legend.position="bottom")
```

The plot shows the fully rearranged and summarised data of proportions of the level of usage, separately by country and purpose, colored by gender.



(d) What does the code block (shown below from the full set of commands) do?

[2 marks]

```
> internet.m.tb <- internet[,c(1,3,6,7,8,9)] %>%
                       gather(key=usage, value=value, -name, -Gender) %>%
                       group_by(name, Gender, usage, value) %>%
                       tally(sort=TRUE)
```

(e) Explain (in a few sentences) what the "Split-Apply-Combine" approach to working with large data is.

[2 marks]

(f) From the plot, which country and what type of usage exhibits the biggest difference between boys and girls? Describe the difference.

[2 marks]

(g) Something that is missing from the display is a representation of the variation. Without this, it is hard to support statements about differences between the boys and girls internet usage. In a few sentences describe how you might calculate a measure that represents the variability estimates of proportions. (There is more than one way, but only discuss one.) And for a bonus point explain how you would add these to the plot using `ggplot2` commands.

[2 marks]

(h) For ease of making comparisons, the "proximity principle" is important in constructing data plots. The arrangement used in this plot allows for the primary comparison of girls and boys usage. Suppose you really want to primarily compare usage between countries, and secondarily gender. How could you re-arrange the facets to facilitate that comparison?

[2 marks]

(i) There is no indication of "NA" or missings in the plot. Why not? What happened to the missings?

[2 marks]

[Total: 16 marks]

— END OF QUESTION 10 —

QUESTION 11

(a) Entropy is commonly used to measure the impurity of a subset generated by a split. In the following data we want to use a one-sided purity measure based on entropy to measure the quality of the split, which is defined as this, for two classes (0,1):

$$\text{minimum}\{(-\hat{p}_0^L \log \hat{p}_0^L - \hat{p}_1^L \log \hat{p}_1^L), (-\hat{p}_0^R \log \hat{p}_0^R - \hat{p}_1^R \log \hat{p}_1^R)\}$$

X_1	X_2	Class
-5	1	0
-3	2	1
0	-1	1
1	0	1
3	7	0
4	8	0
6	10	0

(i) Calculate the impurity for a split on X_1 between 1 and 3. [2 marks]

(ii) Would a split on X_2 between 1 and 2 yield a lower (or equal) impurity value? [2 marks]

(b) For a data set with 5 variables and 10000 cases, how many calculations of impurity would need to be made in order to decide on where the first split would be made? (Explain your working.) [2 marks]

(c) Look at the following tree fit:

```
> rpart(area~., data=subset(y, region==1, select=area:eicosenoic))
n= 323

node), split, n, loss, yval, (yprob)
      * denotes terminal node

 1) root 323 120 South-Apulia (0.1734 0.0774 0.1115 0.6378)
   2) linoleic< 9.5e+02 119  63 Calabria (0.4706 0.2017 0.2521 0.0756)
     4) palmitoleic>=96 74  20 Calabria (0.7297 0.0135 0.1351 0.1216)
       8) arachidic< 80 67  13 Calabria (0.8060 0.0149 0.0448 0.1343)
        16) linolenic>=37 53   2 Calabria (0.9623 0.0000 0.0189 0.0189) *
        17) linolenic< 37 14   6 South-Apulia (0.2143 0.0714 0.1429 0.5714) *
       9) arachidic>=80 7   0 Sicily (0.0000 0.0000 1.0000 0.0000) *
     5) palmitoleic< 96 45  22 North-Apulia (0.0444 0.5111 0.4444 0.0000)
      10) stearic< 2.6e+02 23   2 North-Apulia (0.0000 0.9130 0.0870 0.0000) *
      11) stearic>=2.6e+02 22   4 Sicily (0.0909 0.0909 0.8182 0.0000) *
   3) linoleic>=9.5e+02 204   7 South-Apulia (0.0000 0.0049 0.0294 0.9657) *
```

(i) How many cases are there in the terminal node labelled 11? [2 marks]

(ii) Calculate the error rate of the tree. [2 marks]

[Total: 10 marks]

— END OF QUESTION 11 —

QUESTION 12

(a) Linear and quadratic discriminant analysis: We fit a linear discriminant analysis model to the chocolates data with results as shown below.

```
> lda(Type~., data=subset(chocolates, select=Type:Protein), prior=c(0.5,0.5))
Call:
lda(Type ~ ., data = subset(chocolates, select = Type:Protein),
    prior = c(0.5, 0.5))

Prior probabilities of groups:
Dark Milk
 0.5  0.5

Group means:
     Calories CalFat TotFat SatFat Chol Na Carbs Fiber Sugars Protein
Dark      551    356     40     23  4.5 20    46   7.4     31     7.5
Milk      527    274     31     18 14.6 76    57   2.3     48     6.7

Coefficients of linear discriminants:
              LD1
Calories -0.00045
CalFat    0.00129
TotFat   -0.06475
SatFat   -0.00721
Chol      0.02427
Na        0.01394
Carbs    -0.00382
Fiber    -0.17421
Sugars    0.01599
Protein   0.12223
```

(a) Write down the classification rule.

[3 marks]

(b) Based on the plots of the data in the previous question would it be better to use LDA or QDA for this data, in order to satisfy the assumptions. Explain your answer.

[2 marks]

[Total: 5 marks]

— END OF QUESTION 12 —