

Cointegration in the major European currency markets

Anthony Ori

Department of Economics, Birkbeck University of London

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Abstract

In the decade following the global financial crisis of 2007-2008 I find evidence of long-run price dependence between the major European currencies. Deviations from these values do occur, however the premium from trading on these deviations doesn't appear to be significant. The low returns can be interpreted as a validation of efficiency in these markets.

1. Introduction and market structure

According to the most recent triennial survey from the Bank of International Settlements (BIS), trading turnover in the Foreign Exchange Market (Forex) reached \$6.6 trillion per day in April 2019, which makes it the biggest in the world. Of the \$6.6 trillion, \$2.0 are traded in the Spot market. The US dollar is still the dominant currency in terms of circulation at 88% of all trades, but the focus of this study will be on the three most important European currencies, Euro (EUR), British Pound Sterling (GBP) and Swiss Franc (CHF), as we take the view of a European investor. The euro, the second most traded currency in the world, claims 32% of global trading, while GBP and CHF take 13% and 5% respectively. The primary objective of this study is to observe long-run behaviour of the three currencies of interest and their interrelation. Additionally, based on the huge trading volumes these markets will be very liquid, hence the secondary objective is to assess if there is any profitable trading opportunity in exploiting temporary deviations from the long-run values of the currency pairs, thus indirectly testing for market efficiency.

The BIS survey is important for statistics in this market because, unlike equity or futures markets, which are centralised in one or more exchanges, the spot Forex market is decentralised and transactions take place Over-The-Counter (OTC). There is little oversight of Forex activity, hence why gathering market-wide data is daunting. This means that participants rely on a network of brokers and dealers. [Fig. 1](#) illustrates the market structure.

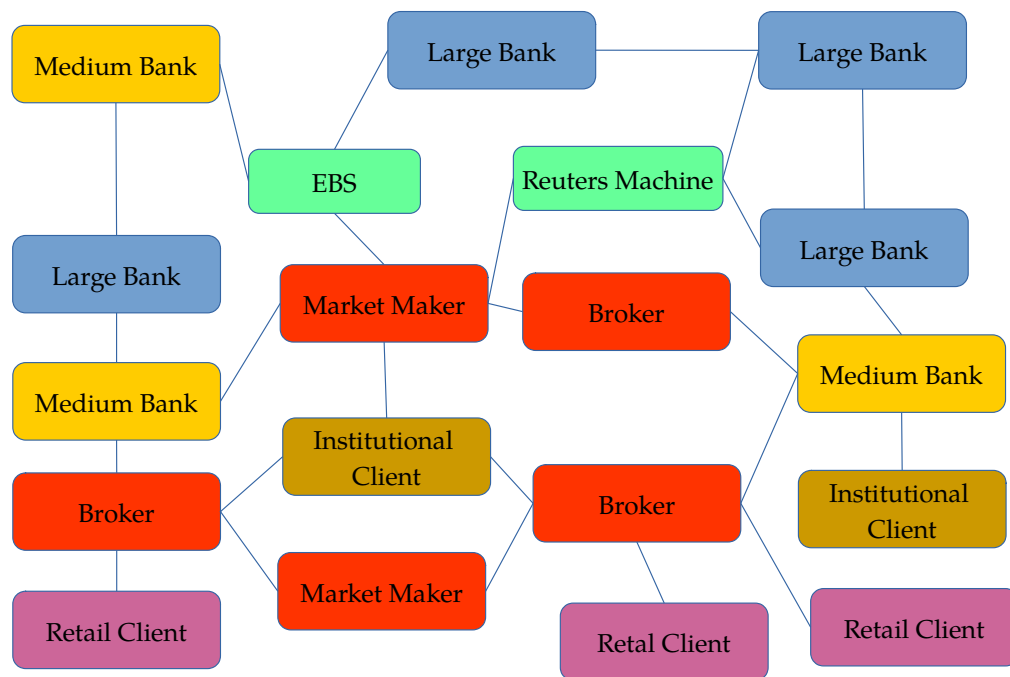


Figure 1. Forex market structure

In this market the key participants are banks, the larger ones in particular, as they represent the main dealers in the global network. Small and medium sized banks play an important role but lack the financial muscle to process the volumes of the bigger equivalents. Banks participate either for profit on their own accord (proprietary trading) or more commonly to process client orders. Central banks are of notable importance because they do not engage with the same frequency of commercial banks but their intervention dramatically affects exchange rates even if their operations are not profit-driven. A central bank engages in the market for operations that involve handling foreign reserves, borrowing or lending on behalf of the nation's government, when it sets the interest rate in order to control inflation, or when it buys or sells national currency if it deems it undervalued or overvalued.

Institutional clients are the most prominent clients of the dealers and brokers, and are either commercial companies who participate in the market for business reasons and not to directly profit from exchange rate fluctuations, or speculators who by contrast participate purely to profit from said fluctuations. A car manufacturer for example might need to process a payment for immediate delivery for parts coming from different countries and thus needs to buy items with foreign currencies. To practically achieve that the company needs sell the home currency and buy the equivalent in the foreign currencies. The manufacturer is not concerned with making profits by taking advantage of

exchange rate fluctuations but rather in securing the parts it needs to continue its manufacturing operation. Speculators on the other hand are solely concerned with profiting from exchange rate fluctuations. Brokers and market makers fulfil the role of trade facilitators and very much like the dealing banks are primarily concerned with volume, because higher volume implies higher transaction costs, which translates to higher fees. This market power cannot be abused for too long due to size of the market and competition. This implies that the market is efficient. By exploring the profitability of trading in a relative-value arbitrage framework I will also indirectly test for said efficiency and if an investor is adequately compensated for trading on temporary violations of the "fair" value of a currency.

The last category of participants in the market, in terms of volume and relevance, are retail traders. Due to smaller volumes, retail traders interact with the wider market solely via dedicated retail brokers, who act as liquidity providers, unlike institutional clients that can establish relationships directly with dealing banks, access professional-grade software like EBS or Reuters¹ Machines, or even act as market makers themselves. This superior market presence is the reason why institutional clients have access to better rates than retail equivalents.

When speaking of currencies and rates what is meant is that we price one currency in terms of another, so when we see a quote like EUR/USD this means that one unit of euro is expressed in terms of US dollars, i.e. $\text{EUR/USD} = 1.11$ means that €1 is equivalent to \$1.11. This clarification is important because the notion of relative-value arbitrage is even trickier in the currency markets because by default a currency is already priced in terms of another, in other words, it's already relatively priced, but what is meant in the context of trading is that a specific currency pair is "mispriced" relative to another pair, i.e., "EUR/USD is mispriced relative to GBP/USD" can be translated in simple English as "the euro in US dollar terms is mispriced relative to the sterling in US dollar terms". How those mispricings are identified varies but I chose cointegration because it allows for analysis of long-run dependence without loss of information from transforming the level series, the original data is used as is.

The rest of the paper is structured as follows: Section 2. will cover a selective literature review on pairs trading and cointegration; Section 3. will cover the methodology employed to identify pairs that "move together" and the trading strategy; Section 4. describes the data; Section 5. discusses the empirical results; and Section 6. the concluding remarks.

¹ Reuters machines are now rebranded as Refinitiv.

2. Literature Review

There's long standing literature in econometrics that covers cointegration since its formalisation in the seminal paper from [Engle and Granger \(1987\)](#), and how this technique addresses the shortcomings of correlation analysis and spurious regressions first discussed by [Yule \(1926\)](#) and later [Granger and Newbold \(1974\)](#), namely to find meaningful relationships between economic or financial variables. In the stream of literature concerned with commodities, [Malliaris and Urrutia \(1996\)](#) for example, identify co-movements between the agricultural commodities listed on the Chicago Board of Trade (CBOT) with pair-wise testing via Engle-Granger approach, and note that there are important economic, geographic and weather-related factors that suggest a priori the rejection of price independence. The results from their study confirm these expectations. In a similar vein, in the following section I explain how in the case of the major European currencies there are sound economic reasons to expect the rejection of price independence in favour of cointegration.

[Mohammadi and Jahan-Parvar \(2008\)](#) find in the energy markets the existence of long-run relationships between real oil prices and real exchange rates in fourteen oil-exporting countries. Other streams of literature mainly focus on applications of cointegration to equity markets ([Alexander and Dimitriu 2002](#), and [Dunis and Ho 2005](#)), which is not surprising because the concept of pairs trading was refined in the 1980s by researchers at Morgan Stanley, and was first applied to long/short stock strategies. Since then Hedge Funds have adopted this strategy with many complex variations since suitable pairs can be identified in several ways. Correlation analysis is one of them and the most immediate, but the crucial limitation of correlation is that it applies to stationary data, which is hardly found in economic or financial variables. Hence what is done in practice do to achieve stationary is differencing the data, however that can be problematic due to the potential loss of valuable information in the conversion process. That is particularly evident when converting non-stationary price series to stationary return series: there is loss of long memory information contained in the level series ([Alexander and Dimitriu 2004](#)).

More interestingly, [Blazquez et al \(2018\)](#) performs an empirical comparison of the different techniques used to form pairs in pairs-trading strategies, which are: correlation, distance, stochastic, stochastic differential residual and cointegration; and conclude that cointegration is the most efficient in terms of forming pairs with the highest probability of generating profits. Surprisingly, there have been few attempts to transfer those ideas of cointegration and pairs-trading over to currency markets, and according to [Dunis et al \(2010\)](#) a primary reason for that is the lack of standardised benchmarks to compare the currency portfolios with. In the equity or commodity world this issue doesn't apply because a given portfolio can be compared to widely recognised indices like the S&P 500, FTSE 100, DAX, GSCI , etc. To address this, [Dunis et al \(2010\)](#) construct a currency index by

building on the work of [Acar and Lequeux \(1998\)](#). I will not replicate the currency index in this study because it's not the primary focus, however the sparse coverage of currency markets is also another motivating factor for this study.

3. Methodology

The primary objective of this paper is to first identify if the three major European currency pairs are cointegrated, and secondarily if they are, implement a simple trading strategy to take advantage of deviations from this long-run relationship. There is good reason to expect cointegration of the trio from an economic perspective because the three currencies are subject to similar economic factors. External shocks like developments in the US, should enter those exchange rates almost symmetrically, as they are currencies of countries in the the European Union (EU), with the exception of Switzerland. Even if Switzerland is not a member of the EU its largest trading partners are countries from within the bloc.

To keep some degree of continuity with existing literature in pairs trading ([Andrade et al., 2005](#)), the implementation of the strategy is divided in two phases. First phase, or formation period, the three pairs are examined for cointegrating relationships; in the second phase, or trading period, the cointegrated combinations are traded. But there are two differences, the first being that I don't use overlapping formation and trading cycles but rather sequential formation and trading periods, each lasting one year. So there is no overlap between trading cycles. The other difference is that in literature, generally, pairs are formed based on minimum price distance criteria, while I use cointegration, very much like the approach in [Hain et al. \(2017\)](#). The benefit from doing this is that the strategy can be applied to more than two price series, in fact, in our case we have a three-way relationship of the three variables under analysis. Specifically, I apply cointegration in the Johansen sense, and not Engle-Granger, which is limited to the bi-variate cases.

Johansen tests are performed at the 5% significance level and the test results will be presented in the next section. The test performs estimates with a Vector Error Correction Model (VECM) and establishes the number of cointegrating relationships and vectors, if any. Following [Hain et. al \(2017\)](#), I use the cointegrating vectors as weights in the portfolio, with the difference that I do not use the historical spreads crossings as an in-sample profitability ranking metric since the number of variables is only three, all three pairs are traded regardless of their number of crossings. A crossing is defined as a deviation by more than one standard deviation in the previous period followed by a convergence to the mean of the spread in the next one. Spread refers to the difference between the current price and the "equilibrium" price,

$$\hat{u}_t = Y_t - \hat{\alpha} - \hat{\beta} X_t \quad (1)$$

where $\{Y_t\}$ and $\{X_t\}$ are two non-stationary processes, $\hat{\alpha}$ and $\hat{\beta}$ are the vectors of coefficients from the cointegrating regression between the three variables of interest. In other words, the spread is a "residual" and it captures the difference between the current exchange rate and the estimated long-run exchange rate. That is also the intuitive definition of cointegration, which is that two or more non-stationary series are cointegrated if we can find a linear combination among them that is stationary. In the above example, $\{\hat{u}_t\}$ will be stationary. See coverage in [Alexander \(2008\)](#) for a more detailed explanation or [Appendix B](#) for a distilled version.

The trading strategy will thus open a position when the spread of the pair deviates by more than one historical standard deviation and closes the position if the mean has been crossed or when a new position needs to be opened. Deviations from "equilibrium" values can arise due to a multitude of reasons and there is no consensus in the literature, but there is evidence to support the view that speculation can be a destabilising force. See [Eichengreen and Wypolysz \(1993\)](#) and [Eichengreen et al. \(1994\)](#) for a discussion on the speculative attacks on the European Exchange Rate System (ERM) and how these attacks drove prices rather than economic fundamentals. That is to say that relative-value strategies, like the one implemented, fall under the umbrella of statistical arbitrage and because of that are inherently risky.

[Figure 2A](#) and [Figure 2B](#) illustrate the historical crossings process for a 6-month period for the GBP/CHF and EUR/CHF pairs. A long position is opened when the current spread is below one historical standard deviation and a short one when the current spread is above it. What is meant by long position is the act of buying units of the base currency and selling the equivalent unit amount in the reference currency, i.e. for EUR/CHF a long position means that we buy units of euro and sell equal units of Swiss francs; similarly when we short EUR/CHF we sell euros and buy Swiss francs.

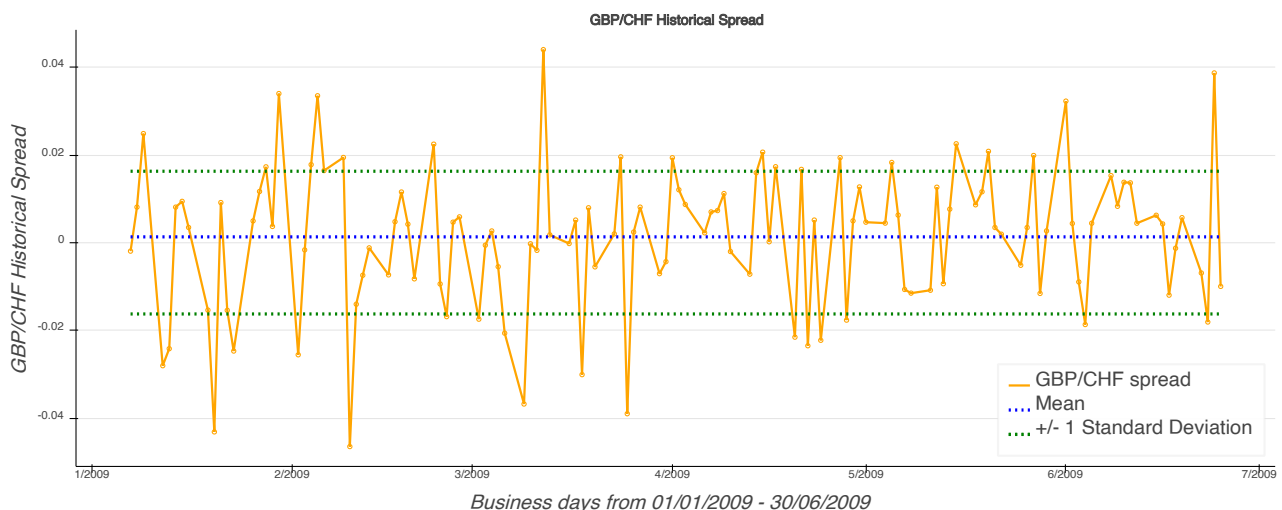


Figure 2A. Historical crossings of GBP/CHF.

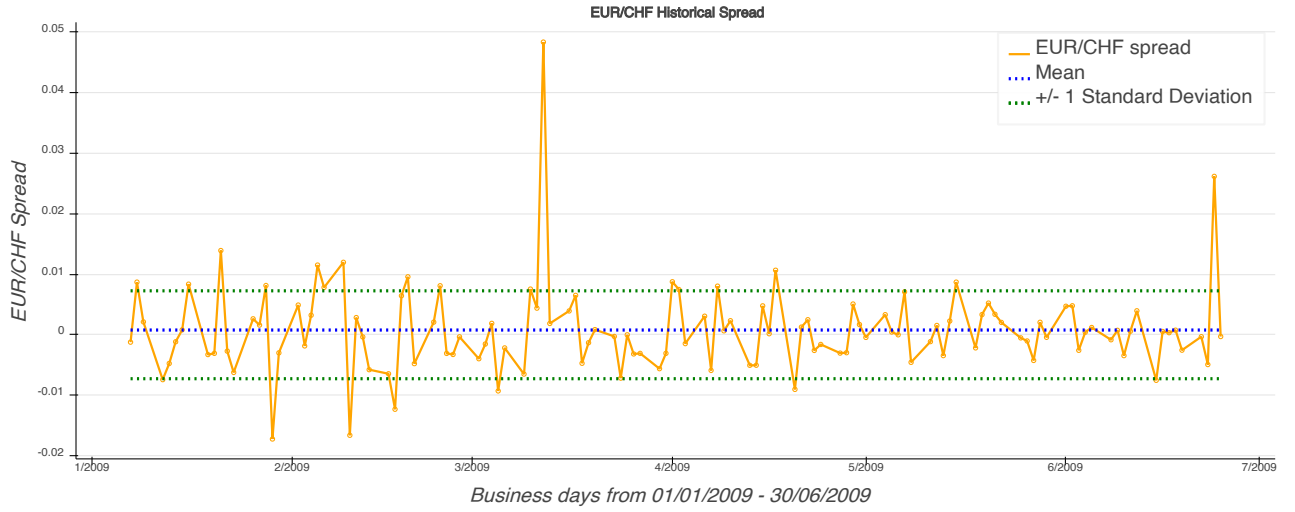


Fig. 2B. Historical crossings of EUR/CHF

3.1. Returns computation

For a combination of cointegrated pairs indexed k , the total value-weighted return at day t is defined as,

$$r_t^k = \sum_{i=1}^n \left(I_t^i \frac{P_t^i - P_{t-1}^i}{P_{t-1}^i} \frac{P_{t-1}^i \beta_i}{\sum_{j=1}^n P_{t-1}^j \beta_j} \right) \quad (2)$$

where P_t^i is the price of the i -th pair at time t expressed in Swiss Francs; I_t^i is a tri-state variable that is either 0 (not invested), 1 (long), -1 (short); β_i is the absolute cointegration coefficient for i -th pair; and $n = 3$ because we have three cointegrated pairs. I chose to express pairs in Swiss Francs because two of the three pairs are already measured in this currency, namely EUR/CHF and GBP/CHF, which leaves only the pair EUR/GBP to be converted to CHF, which is just another conversion of GBP/CHF. This distinction is important because there are cases where a profitable position in EUR/GBP becomes

unprofitable after conversion to GBP/CHF and vice versa. "Exchange risk on exchange risk" in essence. See [Appendix A](#). for additional information on profit calculations.

3.2 Transaction costs

I take the view of an institutional trader and assume that transaction costs will be low. In the case of Forex, the main costs are two: operational costs and spread costs. Operational costs are what the broker or dealer (which is very often a bank) charges as commission for the operation, while the spread cost is the additional markup imposed by the broker or dealer in the bid-ask spread quoted. Additionally, a lot of brokers offer a "0% operational cost" policy, which in reality ends up carrying over as a hidden cost in the advertised bid-ask spread. But taking into account how liquid the spot Forex market is, bid-ask spreads quoted need to be competitive, otherwise traders would move to a more convenient dealer/broker. This fact makes it such that quoted bid-ask spreads will be narrow, and therefore the spread cost will be ignored in this strategy. A more sophisticated strategy on other hand will have to include said costs, and generally, a conservative half-spread cost per transaction would be sufficient in that case.

4. Data

The data is publicly available from dukascopy.com and the set is daily close prices of EUR/CHF, EUR/GBP and GBP/CHF pairs. See [Appendix C](#). for supplementary data. It should be noted that due to the decentralised structure of the Forex market a single broker or dealer will not be representative of the entire market, however, keeping in mind the arguments made in the previous section, the quoted prices will be a good approximation of market-wide values. The range was chosen because it is of particular interest to observe the long-run behaviour of the three pairs in the decade following the Global Financial Crisis (GFC) of 2007-2008.

5. Empirical Results

5.1 Cointegration results

Before performing cointegration tests, I ensure that the pairs are integrated of order 1, or $I(1)$, by running Augmented Dickey-Fuller (ADF) tests for unit roots on the data in levels,

and in first differences to confirm stationarity, or $I(0)$. This is more of a sanity check since visual inspection is enough to show that the series are non-stationary. Afterwards, an appropriate lag order for the vector autoregressive (VAR) model is chosen with the help of information criteria: Akaike (AIC), Bayesian (BIC), and Hannan-Quinn (HQC). The lag order suggested by these criteria is not always optimal in terms of correcting for serial correlation in the residuals, therefore in some instances I chose higher order lags. Then, cointegration tests, with the lag order selected in the previous step, are performed sequentially starting from 2009 up to 2019, and if there is cointegration in a given year, in the following a trading cycle begins, otherwise, another formation period is started in the following year. [Figure 3](#), illustrates this process. The results from the cointegration tests are presented in [Table 1](#).

Notably, in 2011, 2012 and 2019 I found no evidence of cointegration. If we view currencies as a proxy of economic and political stability of a region, then the absence of cointegration in 2011 and 2012 could be justified by reasoning that those years were the peak of the European debt crisis which threatened the very existence of the bloc. Partial stability and faith in the monetary union was restored in the iconic speech from Mario Draghi in which he vowed, as president of the European Central Bank (ECB) at the time, to do "whatever it takes" to contain the crisis and prevent the collapse of the euro system ([Eichengreen, 2015](#)). His commitment was signalled by announcing Outright Monetary Transactions (OMT), which allows the ECB to purchase unlimited, in the literal sense, amounts of sovereign debt to suppress the effects of speculation and prevent defaults.

The announcement was to assure investors that their euro-denominated government bonds would be fulfilled by the ECB in case of default. [Altavilla et al. \(2014\)](#) find that the OMT announcement caused a 2% reduction in the 2-year government bond rate in Italy and Spain and an increase in real economic activity, credit, and prices; which are posited as evidence of the effectiveness of the announcement. Along with OMT, in 2012 the European Stability Mechanism (ESM) was created to assist distressed member countries and banks due to the crucial role they play, they are after all the main dealers in the currency markets. These stabilisation mechanisms are perhaps why cointegration is identified in the following years.

As for 2019, the lack of cointegration could be explained by reasoning that it was due to political uncertainty surrounding Brexit and its execution rather than the referendum result in itself, since that had already been priced in from 2016. It is likely that it wasn't so much the initial shock of the referendum but political uncertainty during the negotiations that prevented the markets from properly assessing and pricing the euro and the sterling in the long-term. Counterintuitively, in this period the sterling appreciated against the euro ([Figure 4](#)), so not sure if that argument is valid. However, for the entire range of observations (from 2009 - 2019), the cointegration test strongly indicates existence of long-term price dependence between the pairs EUR/CHF, GBP/CHF, and EUR/GBP.

Left branches are what happens when cointegration is found, right branches when there's no cointegration

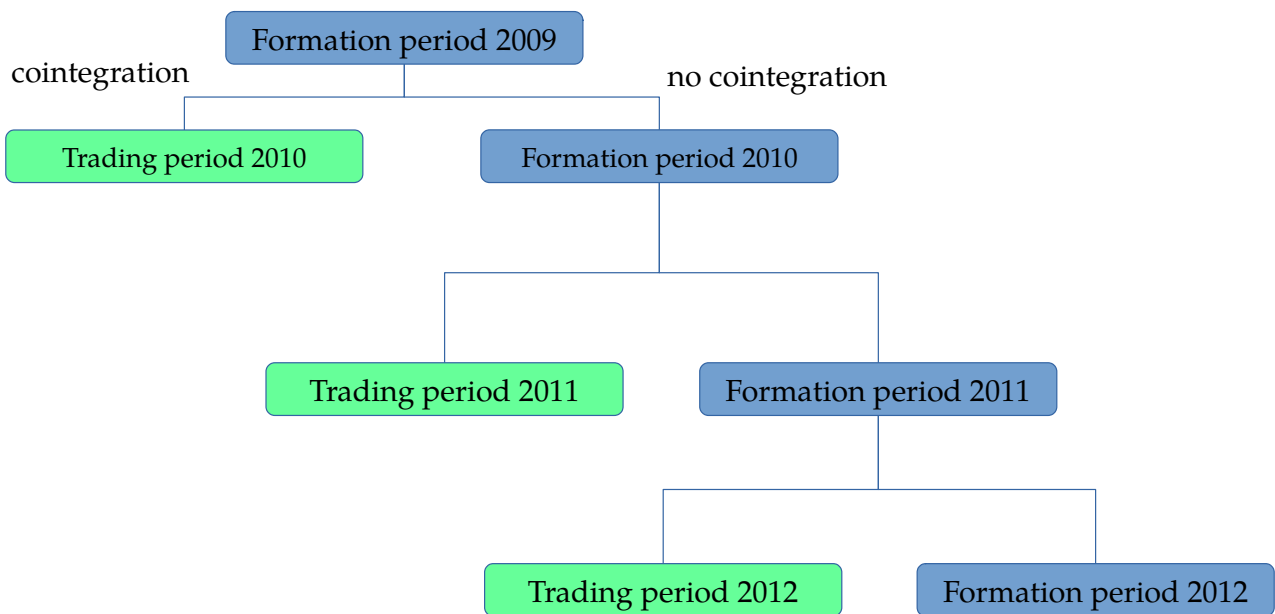


Figure 3. Formation and trading cycles.



Figure 4. EUR/GBP in 2019

5.1 Trading results

After cointegration has been identified in the formation period, a VECM is estimated with the rank chosen from Johansen's test. [Table 2.](#) presents the key results. Looking at the table it might be tempting to conclude that due to error correction terms (ECT) of positive sign, there is no convergence and the model is explosive but that is not necessarily the case. In a VECM it is possible to have some variables with positive and significant ECT, what is important is that their linear combination is stationary. The cointegration β enter [\(2\)](#). Daily returns have been annualised assuming a 252 business day calendar, since the trading period lasts a year. Volatility of returns, as expressed in terms of standard deviation, has also been annualised. These statistics can be observed in [Table 3.](#)

Table 1. Results from Johansen tests

Period	rank	Trace	p-value	Max eigenvalue	p-value
2009 - 2010	0	31.648	0.0297	23.945	0.0173
	1	7.7026	0.5048	7.6728	0.4217
	2	0.029810	0.8629	0.029810	0.8629
2011 - 2012	0	19.937	0.4381	11.439	0.6140
	1	8.4974	0.4210	6.5028	0.5572
	2	1.9945	0.1579	1.9945	0.1579
2013 - 2014	0	50.408	0.0000	31.333	0.0008
	1	19.075	0.0124	18.624	0.0081
	2	0.45070	0.5020	0.45070	0.5020
2015 - 2016	0	43.668	0.0005	35.189	0.0001
	1	8.4793	0.4229	7.8963	0.3979
	2	0.58291	0.4452	0.58291	0.4452
2017 - 2018	0	53.339	0.0000	42.888	0.0000
	1	10.451	0.2520	6.9191	0.5072
	2	3.5322	0.0602	3.5322	0.0602

2019 - 2019	0	19.600	0.4613	15.519	0.2648
	1	4.0811	0.8908	2.5108	0.9642
	2	1.5702	0.2102	1.5702	0.2102
2009 - 2019	0	51.261	0.0000	38.760	0.0000
	1	12.501	0.1352	7.7987	0.4082
	2	4.7024	0.0301	4.7024	0.0301

Trace test, H_0 : rank = r vs H_1 : rank = $r + 1$

Max eigenvalue test, H_0 : rank = r vs H_1 : rank $\leq r + 1$

Table 2. VECM summary results.

Period	Variable	ECT	p-value ECT	Cointegration beta
2009 - 2010	Δ EUR/CHF	-0.281482 (0.233610)	0.2288	1.0 (0.0)
	Δ GBP/CHF	-1.38464 (0.428135)	0.0013***	-0.85868 (0.0038861)
	Δ EUR/GBP	0.617162 (0.200363)	0.0022***	-1.6546 (0.0092380)
2013 - 2014	Δ EUR/CHF	0.189912 (0.0866118)	0.0288**	1.0 (0.0)
	Δ GBP/CHF	0.615337 (0.215214)	0.0044***	-0.95383 (0.029520)
	Δ EUR/GBP	-0.0784701 (0.117151)	0.5033	-1.6570 (0.041001)
2015 - 2016	Δ EUR/CHF	-0.0439625 (0.00753193)	0.0000***	1.0 (0.0)
	Δ GBP/CHF	-0.0616351 (0.0111721)	0.0000***	1.1317 (0.32007)
	Δ EUR/GBP	0.00273225 (0.00379657)	0.4721	1.6573 (0.52444)
2017 - 2018	Δ EUR/CHF	-0.662918 (0.413738)	0.1097	1.0 (0.0)
	Δ GBP/CHF	0.834192 (0.734245)	0.2564	-0.87899 (0.0015687)
	Δ EUR/GBP	-0.833990 (0.456654)	0.0684*	-1.2598 (0.0035118)

2009 - 2019	$\Delta\text{EUR/CHF}$	0.0746676 (0.0307298)	0.0152**	1.0 (0.0)
	$\Delta\text{GBP/CHF}$	0.163738 (0.0480724)	0.0007***	-0.87643 (0.0048310)
	$\Delta\text{EUR/GBP}$	-0.0223879 (0.0205316)	0.2756	-1.4656 (0.013107)

Cointegrating coefficients have been renormalised for EUR/CHF, hence all $\beta_{\text{EUR/CHF}}$ are 1.0. The cointegrating equation is,

$$\text{EUR/CHF}_t = \text{Constant} + \beta_{\text{GBP/CHF}} \text{GBP/CHF}_t + \beta_{\text{EUR/GBP}} \text{EUR/GBP}_t \quad (\text{standard errors in parenthesis})$$

*** indicates significance at the 1% level, ** at 5%, * at 10%

Table 3. Return statistics

Period	R (%)	σ (%)	Risk-free rate	Sharpe Ratio	Trades with negative returns (%)
2010	3.76	4.42	0.335	0.7741	47.11
2014	1.43	1.54	0.062	0.8899	45.45
2016	1.59	1.62	-0.737	1.4427	46.34
2018	-1.95	2.39	-0.649	-0.5434	50

R is the annualised portfolio return and σ the annualised volatility. The reference risk-free rate is the average of daily yields of 1-year CHF-denominated government bonds for the relevant trading year.

6. Conclusions

In line with initial expectations, the results from Johansen's tests suggest that there's a long-run relationship in the decade following the GFC among the major European currencies: euro, pound sterling, and Swiss franc. Although this relationship seems to brake in some of the periods under consideration, notably at the peak of the European debt crisis in 2011-2012, and in 2019 during Brexit negotiations, overall, the currencies tend to be cointegrated. This is a confirmation that in the long-run economic fundamentals prevail, even if the individual currency pairs might react differently to short-run shocks, as indicated by the mixed signs in the ECT, the deviations are eventually corrected. This is not surprising if we consider that currencies are a reflection of a region's political and economic stability. With a union like the EU and the stabilising effect of the ECB, intra-

regional frictions are reduced. Hence, integration of exchange rates is to be expected. The results from the trading sessions show that in three out of the four years, the strategy was profitable. Transaction costs were muted so these returns are overstated but are nonetheless promising if we take into consideration that the strategy can be further refined by taking into account residual amplitude or even use overlapping, instead of sequential, formation and trading cycles to juice up returns ([Jegadeesh and Titman, 1993](#)). The seemingly low returns could be posited as evidence of market efficiency, or inadequacy of the strategy to effectively capitalise on short-term violations of the law of one price. Although the positive sharpe ratios during profitable trading years are promising, they don't mean a lot by themselves, unless they can be compared to another portfolio in the same year.

Appendix A. Contract and position size

Contract sizes in the Forex market are represented in lots, which is just the number of units of the currency to buy or sell. A standard lot is made of 100,000 units of a currency, a mini lot is made of 10,000, a micro lot 1,000, and a nano lot 100. When we enter a long position of 1 standard lot of EUR/GBP for example, at an exchange rate of EUR/GBP = 0.96775, that means that €100,000 are bought and £96,775. For this simple strategy I mainly focus on returns rather than absolute gains and losses, or profits and loss (PnL).

More importantly, the key assumption when calculating returns is that due to positions being opened at market close, the orders can be filled in fully and that there is no price risk due to slippage. Slippage is the difference between expected price of the trade and the actual price at which the trade is executed. Slippage is particularly relevant for large orders and when volatility is high, which is during the busiest trading hours of the main market of a specific currency. This assumption is only possible because of the 24/7 nature of spot Forex (24 hours/7 days a week). Even if the main market of a currency is closed, i.e. Japanese market is closed, Yen can still be traded from other markets that are open.

Appendix B. Johansen framework

The Johansen test is based on the following VECM,

$$\Delta X_t = c + \Pi X_{t-1} + \sum_{i=1}^{k-1} \delta_i \Delta X_{t-i} + \epsilon_t$$

where X_t is a vector of m exchange rates, c is a vector of constants, δ_i are $m \times m$ matrices of coefficients, ϵ_t is a multivariate white noise process, and Π is a $m \times m$ matrix called long-run impact matrix. Testing for cointegration is essentially testing for the rank of Π . If there's cointegration then its rank will be $1 \leq r < m$, and the matrix can be refactored as,

$$\Pi = \alpha\beta$$

where α and β are $m \times r$ matrices of full rank. The columns of β contain the r linearly independent cointegrating vectors, which represent the number of cointegration relationships in the model.

Appendix C. Supplementary data

Supplementary data and code can be found on <https://github.com/acheronte/disso>.

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