19/09/2016 es. 5

$$A = \begin{pmatrix} -2 & 6 & 0 \\ a & 3 & 1+a \\ 0 & -3 & -a \end{pmatrix}$$

$$G_1 = \begin{pmatrix} 1 & 0 & 0 \\ a/2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad G_1 A = \begin{pmatrix} -2 & 6 & 0 \\ 0 & 3(a+1) & 1+a \\ 0 & -3 & -a \end{pmatrix}$$

$$G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{a+1} & 1 \end{pmatrix} \qquad U = G_2 G_1 A = \begin{pmatrix} -2 & 6 & 0 \\ 0 & 3(a+1) & 1+a \\ 0 & 0 & 1-a \end{pmatrix}$$

$a+1 \neq 0$

$\boxed{a \neq -1}$

$$L = G_1^{-1} G_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -a/2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{a+1} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -a/2 & 1 & 0 \\ 0 & -\frac{1}{a+1} & 1 \end{pmatrix} \qquad A = LU$$

$$\det(A) = \det(LU) = \det(L)\det(U) = 1 \cdot (-2) \cdot 3(a+1)(1-a)$$
$$= -6(a+1)(1-a)$$

A è singolare $\Longleftrightarrow$ $\det(A) = 0$ $\Leftrightarrow$ $\underline{a = -1 \vee a = 1}$

---

per quali a si ottiene $PA = LU$ con $P = I$?
pivot parziale

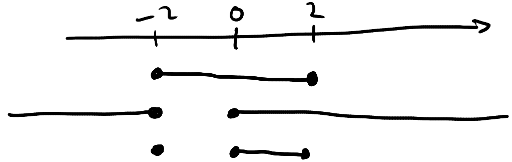
$$\begin{cases} |a| \leq 2 \\ |3(a+1)| \geq 3 \end{cases} \iff \begin{cases} -2 \leq a \leq 2 \\ 3|a+1| \geq 3 \end{cases} \iff |a+1| \geq 1 \iff a+1 \leq -1 \ \vee \ a+1 \geq 1$$

$$\iff a \leq -2 \ \vee \ a \geq 0$$

$$\Updownarrow$$

$$-2 \leq a \leq 2 \ \wedge \ (a \leq -2 \ \vee \ a \geq 0)$$

$$\iff \boxed{a = -2 \ \vee \ a \in [0, 2]}$$



$$a = -\frac{1}{3} \quad , \quad a = 3$$

$$a = -\frac{1}{3} \qquad A = \begin{pmatrix} -2 & 6 & 0 \\ -1/3 & 3 & 2/3 \\ 0 & -3 & 1/3 \end{pmatrix} \qquad P_1 = I \qquad P_1 A = A$$

$$G_1 = \begin{pmatrix} 1 & 0 & 0 \\ -1/6 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad G_1 P_1 A = \begin{pmatrix} -2 & 6 & 0 \\ 0 & 2 & 2/3 \\ 0 & -3 & 1/3 \end{pmatrix} \qquad P_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = P_2^{-1}$$

$$P_2 G_1 P_1 A = \begin{pmatrix} -2 & 6 & 0 \\ 0 & -3 & 1/3 \\ 0 & 2 & 2/3 \end{pmatrix} \qquad G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2/3 & 1 \end{pmatrix}$$

$$\frac{2}{3} \cdot \frac{1}{3} + \frac{2}{3} = \frac{2+6}{9} = \frac{8}{9}$$

$$U = \underbrace{G_2 P_2 G_1 P_1}_{L^{-1}P} A = \begin{pmatrix} -2 & 6 & 0 \\ 0 & -3 & 1/3 \\ 0 & 0 & 8/9 \end{pmatrix}$$

$$P = P_2 P_1 = P_2$$

$$PA = LU$$

$$U = L^{-1} P A$$

$$L^{-1} P = G_2 P_2 G_1 P_1$$

$$L^{-1} = G_2 P_2 G_1 P_1 P^{-1}$$

$$L = P P_1^{-1} G_1^{-1} P_2^{-1} G_2^{-1} = P_2 \cancel{P_1} \cancel{P_1^{-1}} G_1^{-1} P_2^{-1} G_2^{-1} = P_2 G_1^{-1} P_2 G_2^{-1}$$

$$= \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1/6 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2/3 & 1 \end{pmatrix}}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1/6 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2/3 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/6 & -2/3 & 1 \end{pmatrix}$$

$a = 3$

$$A = \begin{pmatrix} -2 & 6 & 0 \\ 3 & 3 & 4 \\ 0 & -3 & -3 \end{pmatrix}$$

$$P_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad P_1 A = \begin{pmatrix} 3 & 3 & 4 \\ -2 & 6 & 0 \\ 0 & -3 & -3 \end{pmatrix}$$

$$P_1 = P_1^{-1}$$

$$G_1 = \begin{pmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad G_1 P_1 A = \begin{pmatrix} 3 & 3 & 4 \\ 0 & 8 & 8/3 \\ 0 & -3 & -3 \end{pmatrix} \qquad P_2 = I$$

$$P_2 G_1 P_1 A = G_1 P_1 A$$

$$G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3/8 & 1 \end{pmatrix} \qquad U = G_2 P_2 G_1 P_1 A = \begin{pmatrix} 3 & 3 & 4 \\ 0 & 8 & 8/3 \\ 0 & 0 & -2 \end{pmatrix} \qquad P = P_2 P_1 = P_1$$

$$L = P P_1^{-1} G_1^{-1} P_2^{-1} G_2^{-1} = \cancel{P_2} \cancel{P_1} \cancel{P_1^{-1}} G_1^{-1} \cancel{P_2^{-1}} G_2^{-1} = G_1^{-1} G_2^{-1} =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ -2/3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3/8 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2/3 & 1 & 0 \\ 0 & -3/8 & 1 \end{pmatrix}$$

<u>24/9/2018 es. 4</u>

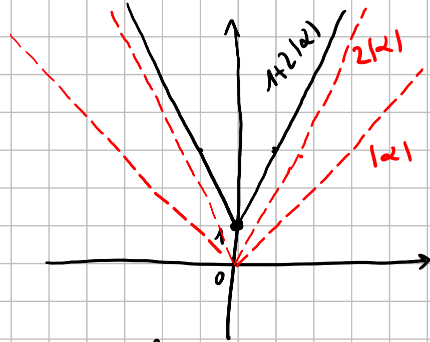
$$A = \begin{pmatrix} 1 & \alpha & 0 \\ \alpha & 1 & \alpha \\ 0 & \alpha & 1 \end{pmatrix}$$

$$\alpha \longmapsto \|A\|_\infty = \max_{i \in \{1,2,3\}} \sum_{j=1}^{3} |a_{ij}| = \max\{1+|\alpha|, 1+2|\alpha|\} = 1+2|\alpha|$$

$$G_1 = \begin{pmatrix} 1 & 0 & 0 \\ -\alpha & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad G_1 A = \begin{pmatrix} 1 & \alpha & 0 \\ 0 & 1-\alpha^2 & \alpha \\ 0 & \alpha & 1 \end{pmatrix}$$

$$G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{-\alpha}{1-\alpha^2} & 1 \end{pmatrix}$$

$$1-\alpha^2 \neq 0$$
$$\alpha^2 \neq 1$$
$$\boxed{\alpha \neq -1 \wedge \alpha \neq 1}$$



$$U = G_2 G_1 A = \begin{pmatrix} 1 & \alpha & 0 \\ 0 & 1-\alpha^2 & \alpha \\ 0 & 0 & \frac{1-2\alpha^2}{1-\alpha^2} \end{pmatrix}$$

$$-\frac{\alpha^2}{1-\alpha^2} + 1 = \frac{-\alpha^2 + 1 - \alpha^2}{1-\alpha^2} = \frac{1-2\alpha^2}{1-\alpha^2}$$

$$L = G_1^{-1} G_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ \alpha & 1 & 0 \\ 0 & \frac{\alpha}{1-\alpha^2} & 1 \end{pmatrix}$$

$$A = LU$$

$$\frac{\alpha}{1-\alpha^2} = \frac{1/2}{3/4} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}$$

$$1 - \alpha \cdot \frac{\alpha}{1-\alpha^2} = 1 - \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$1-\alpha^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\begin{cases} \alpha > 0 \\ \|A\|_\infty = 2 \end{cases} \Longleftrightarrow \begin{cases} \alpha > 0 \\ 1 + 2|\alpha| = 2 \end{cases} \Longleftrightarrow \alpha = \frac{1}{2}$$

$$2|\alpha| = 1$$
$$|\alpha| = 1/2$$

$$A = \begin{pmatrix} 1 & 1/2 & 0 \\ 1/2 & 1 & 1/2 \\ 0 & 1/2 & 1 \end{pmatrix} \quad U = \begin{pmatrix} 1 & 1/2 & 0 \\ 0 & 3/4 & 1/2 \\ 0 & 0 & 2/3 \end{pmatrix} \quad L = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 2/3 & 1 \end{pmatrix}$$

calcolando $PA = LU$ con pivot parziale si scambiano righe al primo passo (in particolare si scambiano 1^a e 2^a riga) se $|\alpha| > 1$, cioè se $\alpha < -1 \vee \alpha > 1$.

$$\begin{cases} \alpha < 0 \\ \|A\|_\infty = 5 \end{cases} \Longleftrightarrow \begin{matrix} 1 + 2|\alpha| = 5 \\ 2|\alpha| = 4 \end{matrix} \Longleftrightarrow \begin{cases} \alpha < 0 \\ |\alpha| = 2 \end{cases} \Longleftrightarrow \alpha = -2$$

$$A = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 1 \end{pmatrix} \qquad P_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad P_1 A = \begin{pmatrix} -2 & 1 & -2 \\ 1 & -2 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

$$G_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad G_1 P_1 A = \begin{pmatrix} -2 & 1 & -2 \\ 0 & -3/2 & -1 \\ 0 & -2 & 1 \end{pmatrix} \qquad \begin{matrix} P_2 \\ \shortparallel \\ P_2^{-1} \end{matrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$P_2 G_1 P_1 A = \begin{pmatrix} -2 & 1 & -2 \\ 0 & -2 & 1 \\ 0 & -3/2 & -1 \end{pmatrix} \qquad G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3/4 & 1 \end{pmatrix}$$

$$U = G_2 P_2 G_1 P_1 A = \begin{pmatrix} -2 & 1 & -2 \\ 0 & -2 & 1 \\ 0 & 0 & -7/4 \end{pmatrix}$$

$$PA = LU \qquad P = P_2 P_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$L = P_2 \cancel{P_1} \cancel{P_1}^{-1} G_1^{-1} P_2^{-1} G_2^{-1}$$

$$= P_2 G_1^{-1} P_2 G_2^{-1}$$

$$= \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \; \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3/4 & 1 \end{pmatrix}}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1/2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3/4 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/2 & 3/4 & 1 \end{pmatrix}$$

12/02/2018 es. 5 $\qquad f(x) = 2\log_2(x)$

$$P_0 = \left(\tfrac{1}{2}, f\left(\tfrac{1}{2}\right)\right) = \left(\tfrac{1}{2}, -2\right)$$
$$P_1 = (1, f(1)) = (1, 0)$$
$$P_2 = (2, f(2)) = (2, 2)$$

polinomio p interpolante P_0, P_1, P_2 nella forma di Newton

$$\begin{array}{cccc}
1/2 & -2 & & \\
 & & \dfrac{0+2}{1-1/2} = \dfrac{2}{1/2} = 4 & \\
1 & 0 & & \dfrac{+2-4}{2-1/2} = \dfrac{\overset{-2}{\cancel{-6}}}{3/2} = \cancel{-\dfrac{12}{3} = -4} = -\dfrac{4}{3} \\
 & & \dfrac{+2-0}{2-1} = +2 & \\
2 & +2 & &
\end{array}$$

$$p(x) = -2 + 4\left(x-\tfrac{1}{2}\right) - \tfrac{4}{3}\left(x-\tfrac{1}{2}\right)(x-1)$$

$$p(2) = -2 + 4\left(2-\tfrac{1}{2}\right) - \tfrac{4}{3}\left(2-\tfrac{1}{2}\right)(2-1) = -2 + 6 - 2 = 2$$

$x \in \left[\tfrac{1}{2}, 2\right]$

$$f(x) - p(x) = \frac{f^{(3)}(\xi)}{3!}\left(x-\tfrac{1}{2}\right)(x-1)(x-2) \qquad \xi \in \left[\tfrac{1}{2}, 2\right]$$

$$\max_{x\in[\frac{1}{2},2]} |f(x)-p(x)| \leq \frac{\max_{x\in[1/2,2]} |f^{(3)}(x)|}{3!} \left(2-\frac{1}{2}\right)^3$$

$$f(x) = 2\log_2(x) \qquad f'(x) = 2\cdot\frac{1}{\ln 2}\cdot\frac{1}{x}$$

$$\left(\begin{array}{l} D(\ln x) = \frac{1}{x} \\ D(\log_a x) = \frac{1}{\ln a}\cdot\frac{1}{x} \end{array} \right)$$

$$f''(x) = \frac{2}{\ln 2}\cdot\left(\frac{-1}{x^2}\right) = -\frac{2}{\ln 2}\cdot\frac{1}{x^2}$$

$$f'''(x) = -\frac{2}{\ln 2}\cdot\left(\frac{-2}{x^3}\right) = \frac{4}{\ln 2}\cdot\frac{1}{x^3}$$

$$f^{(4)}(x) = \frac{4}{\ln 2}\cdot\left(\frac{-3}{x^4}\right) = -\frac{12}{\ln 2}\cdot\frac{1}{x^4} < 0$$ dove esiste

x $x>0$ $f'''(x)>0$

e $f'''(x)$ è decrescente

$$\Downarrow$$

$$\max_{x\in[\frac{1}{2},2]} |f^{(3)}(x)| = f^{(3)}\left(\frac{1}{2}\right) =$$

$$= \frac{4}{\ln 2}\cdot\frac{1}{1/8} = \frac{8\cdot 4}{\ln 2} = \frac{32}{\ln 2}$$

$$\max_{x\in[\frac{1}{2},2]} |f(x)-p(x)| \leq \frac{32}{\ln 2}\cdot\frac{1}{3!}\cdot\frac{3^3}{2^3}$$

$$= \frac{1}{\ln 2}\cdot\frac{\overset{2^5}{\overset{\parallel}{32}}\cdot 3^{\cancel{3}\,2}}{\cancel{3}\cdot 2\cdot 2^3} = \frac{3^2\cdot 2}{\ln 2} = \frac{18}{\ln 2}$$

$P_3 = (4, f(4)) = (4, 4)$ $\qquad$ $\tilde{p}$ interpolante P_0, P_1, P_2, P_3

$1/2$	-2			
		$\frac{0+2}{1-1/2} = \frac{2}{1/2} = 4$		
1	0		$\frac{+2-4}{2-1/2} = \frac{\cancel{-6}\ -2}{3/2} = \cancel{-\frac{12}{3}} = \cancel{-4} = -\frac{4}{3}$	
		$\frac{+2-0}{2-1} = +2$		$\frac{-1/3+4/3}{4-\frac{1}{2}} = \frac{1}{7/2} = \frac{2}{7}$
2	$+2$			
		$\frac{4-2}{4-2} = 1$	$\frac{1-2}{4-1} = -\frac{1}{3}$	
4	4			

$$p(x) = -2 + 4\left(x - \tfrac{1}{2}\right) - \frac{4}{3}\left(x - \tfrac{1}{2}\right)(x-1)$$

$$\tilde{p}(x) = p(x) + \frac{2}{7}\left(x - \tfrac{1}{2}\right)(x-1)(x-2)$$

$$\tilde{p}(4) = -2 + \cancel{4}^{\,2} \cdot \frac{7}{\cancel{2}} - \frac{\cancel{4}^{\,2}}{\cancel{3}} \cdot \frac{7}{\cancel{2}} \cdot \cancel{3} + \frac{\cancel{2}}{\cancel{7}} \cdot \frac{\cancel{7}}{\cancel{2}} \cdot 3 \cdot 2 = -2 + \cancel{14} - \cancel{14} + 6 = 4$$

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q di miglior approssimazione dei punti P_0, P_1, P_2

$$q(x) = q_0 + q_1 x$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1/2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1/2 \\ 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1/2 & 1 & 2 \end{pmatrix} \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 7/2 \\ 7/2 & 21/4 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \iff \begin{cases} 3a_0 + \frac{7}{2} a_1 = 0 \\ \frac{7}{2} a_0 + \frac{21}{4} a_1 = 3 \end{cases}$$

$$\iff \begin{cases} a_0 = -\frac{7}{6} a_1 \\ \frac{7}{2}\left(-\frac{7}{6} a_1\right) + \frac{21}{4} a_1 = 3 \end{cases}$$

$$\iff a_1 \left(-\frac{\overset{49}{\cancel{21}}}{12} + \frac{21}{4} \right) = 3$$

$$\iff a_1 \frac{-\overset{49}{\cancel{21}} + 63}{12} = 3$$

$$\iff a_1 = 3 \cdot \frac{12}{\underset{14}{\cancel{42}}} = 3 \cdot \frac{6}{7} = \frac{18}{7}$$

$$a_0 = -\frac{\cancel{7}}{\cancel{6}} \cdot \frac{\overset{3}{\cancel{18}}}{\cancel{7}} = -3$$

$$a(x) = -3 + \frac{18}{7} x$$

r di primo grado miglior approssimaz. di P_1, P_2, P_3 $\qquad r(x) = r_0 + r_1 x$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} r_0 \\ r_1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 7 \\ 7 & 21 \end{pmatrix} \begin{pmatrix} r_0 \\ r_1 \end{pmatrix} = \begin{pmatrix} 6 \\ 20 \end{pmatrix} \iff \begin{cases} 3r_0 + 7r_1 = 6 \\ 7r_0 + 21r_1 = 20 \end{cases}$$

$$r_0 = \frac{6 - 7r_1}{3}$$

$$\frac{7}{3}(6 - 7r_1) + 21r_1 = 20 \iff 14 - \frac{49}{3}r_1 + 21r_1 = 20$$

$$\iff r_1\left(-\frac{49}{3} + 21\right) = 6 \iff r_1\left(\frac{-49+63}{3}\right) = 6 \iff r_1 = 6 \cdot \frac{3}{14}$$

$$= \frac{18}{14} = \frac{9}{7}$$

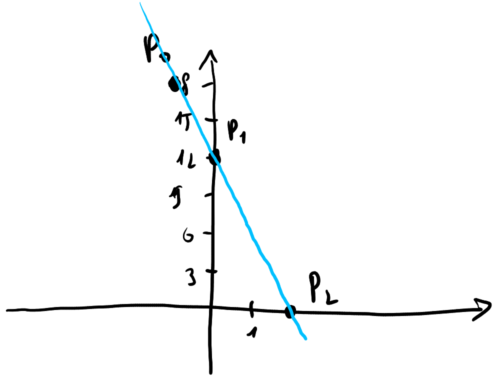
$$r_0 = \frac{6 - 7 \cdot 9/7}{3} = \frac{6-9}{3} = \frac{-3}{3} = -1$$

$$r(x) = -1 + \frac{9}{7}x.$$

8/2/2021 n. 5

$$P_0 = (-1, 18) \qquad P_1 = (0, 12) \qquad P_2 = (2, 0)$$

$$\frac{12-18}{0+1} = -6 \qquad \frac{0-12}{2-0} = -6 \qquad 0$$



$$p(x) = 18 - 6(x+1)$$

$\tilde{p}(x)$ che interpola P_0, P_1, P_2

e tale che $\tilde{p}'(0) = -8$

-1	18			
		-6		
0	12		$\frac{-8+6}{0+1} = -2$	
		-8		$\frac{1+2}{2+1} = 1$
0	12		$\frac{-6+8}{2-0} = 1$	
		-6		
2	0			

$$\tilde{p}(x) = 18 - 6(x+1) - 2(x+1)x + 1(x+1)x^2$$

q di primo grado di migl. appr. dei punti P_0, P_1, P_2 e $P_3(3,6)$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} q_0 \\ q_1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 2 & 3 \end{pmatrix} \begin{pmatrix} 18 \\ 12 \\ 0 \\ 6 \end{pmatrix}$$

$q(x) = q_0 + q_1 x$

$$\Longleftrightarrow \begin{pmatrix} 4 & 4 \\ 4 & 14 \end{pmatrix} \begin{pmatrix} q_0 \\ q_1 \end{pmatrix} = \begin{pmatrix} 36 \\ 0 \end{pmatrix} \Longleftrightarrow \begin{cases} 4q_0 + 4q_1 = 36 \\ 4q_0 + 14q_1 = 0 \end{cases}$$

$$\Longleftrightarrow \begin{cases} q_0 + q_1 = 9 \\ 2q_0 + 7q_1 = 0 \end{cases} \Longleftrightarrow \begin{cases} q_0 = 9 - q_1 \\ 18 - 2q_1 + 7q_1 = 0 \end{cases} \Longleftrightarrow \begin{cases} q_0 = 9 + \frac{18}{5} = \frac{45+18}{5} = \frac{63}{5} \\ 5q_1 = -18 \end{cases} \quad q_1 = -\frac{18}{5}$$

$$q(x) = \frac{63}{5} - \frac{18}{5}x.$$