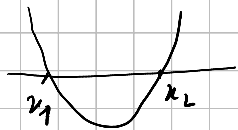
23/07/2020 es. 3 $\qquad f(x) = x^3 - x^2 - 8x + 12 \qquad \alpha < \beta$

$$\lim_{x \to +\infty} f(x) = +\infty$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

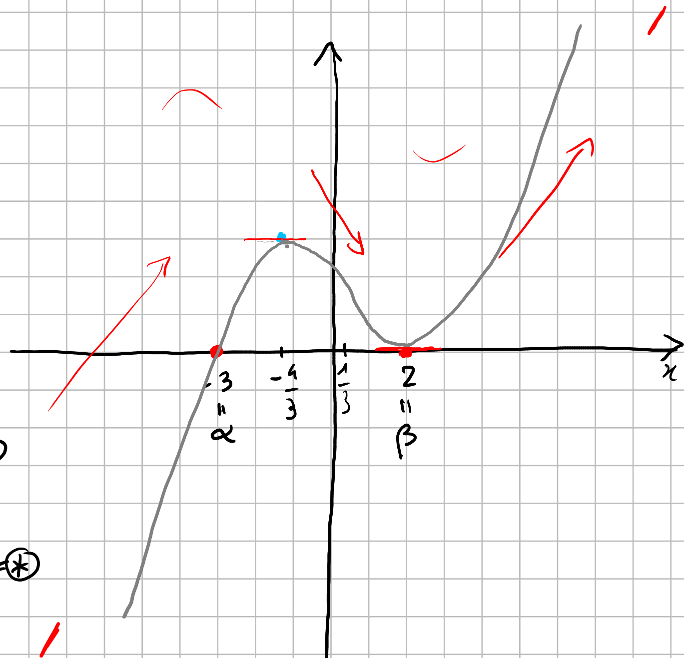
$$f'(x) = 3x^2 - 2x - 8$$



$$\Delta = 4 + 4 \cdot 3 \cdot 8 = 100$$

$$x_2 = \frac{2+10}{6} = \frac{12}{6} = 2 \qquad f(x_2) = 8 - 4 - 16 + 12 = 0$$

$$x_1 = \frac{2-10}{6} = -\frac{8}{6} = -\frac{4}{3} \qquad f(x_1) = -\frac{64}{27} - \frac{16}{9} + \frac{32}{3} + 12 = (*)$$



$f'(x) > 0 \iff x < x_1 \lor x > x_2$

$\quad = 0 \iff x = x_1 \lor x = x_2$

$\quad < 0 \iff x_1 < x < x_2$

$$f''(x) = 6x - 2$$

$f''(x) > 0 \iff x > \frac{1}{3}$

$\quad = 0 \qquad =$

$\quad < 0 \qquad <$

$$(*) = \frac{-64 - 48 + 288 +}{27} \qquad \boxed{f(x_1) > 0}$$

$f(2) = f'(2) = 0 \Rightarrow 2$ è radice doppia

$\beta = 2$

$\alpha < -4/3$

$\alpha < -\frac{4}{3}$

$f(x) = x^3 - x^2 - 8x + 12$

$f(-2) = -8 - 4 + 16 + 12 = 16 > 0$

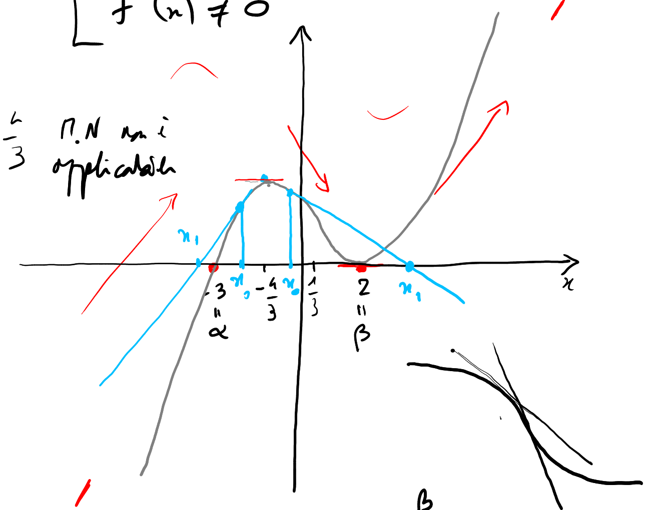
$f(-3) = -27 - 9 + 24 + 12 = 0 \qquad \alpha = -3$

$\boxed{x_0 \quad -2,\ -0.5,\ -\frac{4}{3},\ \frac{1}{3},\ 3,\ 0}$

<u>Teo</u> $J_\alpha = [\alpha, \alpha + \rho]$ e $J_\alpha = [\alpha - \rho, \alpha]$

$\begin{cases} f(x) f''(x) > 0 \quad \forall x \in J_\alpha \setminus \{\alpha\} \\ f'(x) \neq 0 \end{cases} \Rightarrow \forall x_0 \in J_\alpha$ il Metodo di Newton converge in modo monotono

$x_i = -\frac{4}{3}$ M.N non è applicabile



$]-\infty, -3[\ni x_0$ M.N. converge a α in modo monotono quadraticamente

$x_0 = -3 = \alpha$ è la radice e M.N converge in 1 passo

$x_0 \in]2, +\infty[$ M.N converge a β in modo monotono linearmente

$x_0 = 2 = \beta$ è già la radice e M.N non è applicabile

$x_0 \in]\frac{1}{3}, 2[$ MN converge a β monotono lineare

$x_0 \in]-\frac{4}{3}, \frac{1}{3}]$ M.N. converge a β linearmente in modo monotono a partire da x_1

$x_0 \in]-3, -\frac{4}{3}[$ MN converge a α quadraticamente in modo monotono a partire da x_1

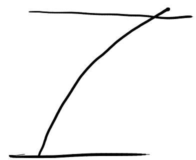
$$f(x) = x^3 - x^2 - 8x + 12 \qquad g(x) = x - \frac{f(x)}{m} \qquad x_{k+1} = g(x_k)$$

$$g(\alpha) = \alpha - \frac{f(\alpha)}{m} = \alpha \qquad g(\beta) = \beta - \frac{f(\beta)}{m} = \beta$$

- conv. locale monotona a α con fattore asintotico $\frac{1}{4}$

$$g'(\alpha) = \frac{1}{4} \qquad g'(x) = 1 - \frac{f'(x)}{m} \qquad f'(x) = 3x^2 - 2x - 8$$

$$f'(\alpha) = f'(-3) = 27 + 6 - 8 = 25 \qquad g'(\alpha) = 1 - \frac{25}{m} = \frac{1}{4}$$



$$\frac{25}{m} = \frac{3}{4} \qquad \frac{m}{25} = \frac{4}{3} \qquad m = \frac{25 \cdot 4}{3} = \frac{100}{3}$$

$$x_0 = -2 \qquad \left(e_{k+1} = g(x_k) - g(\alpha) = \underline{g'(\xi_k)} e_k \;, \; \xi_k \in \,]\alpha, x_k[\right)$$

$x \in [-3, -2]$

$g'(x) \in [\frac{1}{4}, \frac{76}{100}]$

$$g'(-2) = 1 - \frac{f'(-2) \cdot 3}{100} = 1 - \frac{24}{100} = \frac{76}{100} < 1$$

$$f'(-2) = 12 + 4 - 8 = 8$$

$$g''(x) = -\frac{f''(x)}{m} = -\frac{(6x - 2) \cdot 3}{100} = -\frac{6}{100}(3x - 1)$$

$$g''(x) \underset{<}{\overset{>}{=}} 0 \iff x \underset{>}{\overset{<}{=}} \frac{1}{3}$$

- conv locale a α quadratica $\qquad f'(x) = 3x^2 - 2x - 8$

$$g'(\alpha) = \boxed{0 = 1 - \frac{25}{m}} \qquad \frac{25}{m} = 1 \qquad \boxed{m = 25}$$

$$g''(\alpha) = \frac{-f''(\alpha)}{m} = -\frac{(6\alpha - 2)}{25} = -\frac{(-18-2)}{25} = \frac{20}{25} = \frac{4}{5} \neq 0$$

$x_0 = -2$

$x \quad x \in [-3, -2] \qquad g'(x) \in \left[0, \frac{17}{25}\right]$

$$g'(x) = 1 - \frac{f'(x)}{25} \qquad g''(x) = -\frac{f''(x)}{25} = -\frac{(6x-2)}{25} = -\frac{2}{25}(3x-1)$$

$$g'(-2) = 1 - \frac{12+4-8}{25} = 1 - \frac{8}{25} = \frac{17}{25} < 1$$

$g''(x) \underset{<}{\overset{>}{=}} 0 \iff x \underset{>}{\overset{<}{=}} \frac{1}{3}$

- $m = -8$ conv locale a β

$$g'(x) = 1 - \frac{f'(x)}{8} \qquad g'(\beta) = 1 - \frac{f'(\beta)}{8} = 1 - \frac{f'(2)}{8} = 1 - \frac{12-4-8}{8} = 1$$

$x_0 = 1$ $\qquad$ conv. sublineare monotona

$$g'(1) = 1 - \frac{f'(1)}{8} = 1 - \frac{3-2-8}{8} = 1 + \frac{7}{8} = \frac{15}{8} > 1$$

$[1, \beta]$

$$x_1 = g(x_0) = x_0 - \frac{f(x_0)}{8} = 1 - \frac{1-1-8+12}{8} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$f(x) = x^3 - x^2 - 8x + 12$$

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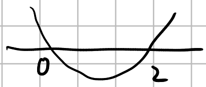
$f(x) = x^3 - 3x^2 + 1 \qquad \alpha < \beta < \gamma$

$\lim_{x \to \pm\infty} f(x) = \pm\infty$



$f'(x) = 3x^2 - 6x = 3x(x-2)$

$f'(x) = 0 \iff x = 0 \lor x = 2$

$f(0) = 1 \quad , \quad f(2) = 8 - 12 + 1 = -3$

$f'(x) > 0 \iff x < 0 \lor x > 2$

$ < 0 \iff 0 < x < 2$

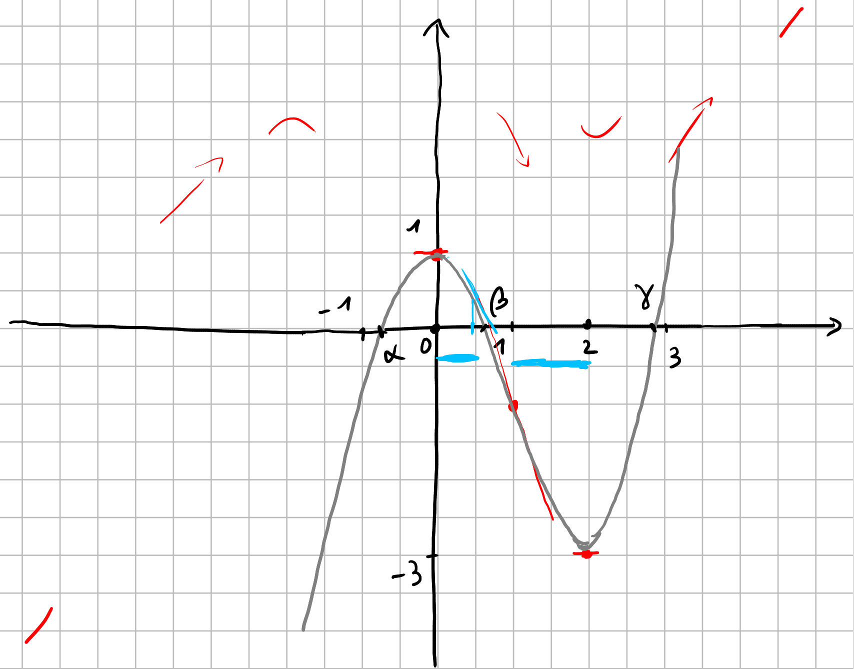
$f''(x) = 6x - 6 = 6(x-1)$

$f''(x) = 0 \iff x = 1$

$f''(x) \underset{<}{>} 0 \iff x \underset{<}{>} 1$

$f(1) = 1 - 3 + 1 = -1$

$f'(1) = 3 \cdot 1 \cdot (1-2) = -3$



$\alpha < 0 \quad , \quad 0 < \beta < 1 \quad , \quad \gamma > 2$

$f(-1) = -1 - 3 + 1 = -3 < 0 \qquad -1 < \alpha < 0$

$f(3) = 27 - 27 + 1 = 1 > 0 \qquad 2 < \gamma < 3$

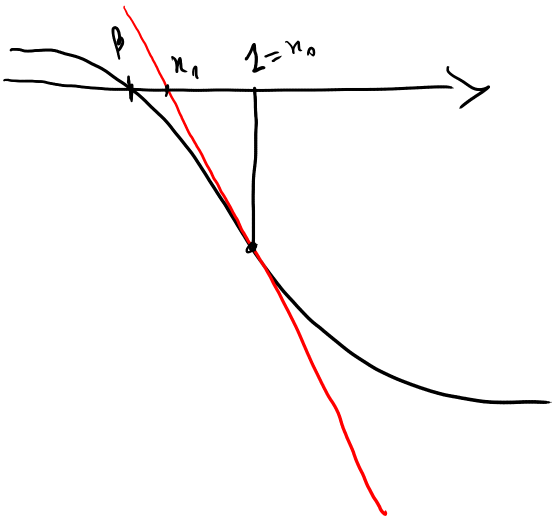
ordine di conv. quadratico per α, β, γ (tutte radici semplici)

$]\beta, 1[\ \ni x \quad f(x)\, f''(x) > 0$ conv. monotona a β

$x_0 = -0,5$; la succ converge a α? conv. quadratica a α, monotona a partire da x_1

$x_0 = 3$; la succ. converge a γ? conv. quad. monotona a γ

$x_0 = 1$; la succ. converge a β? conv. quad. monotona a β



$$g(x) = 3 - \frac{1}{x^2}$$

$$f(x) = x^3 - 3x^2 + 1 \qquad \alpha < \beta < \gamma$$

$$\boxed{x = g(x)} = 3 - \frac{1}{x^2}$$

$$x_{k+1} = g(x_k)$$

$$g'(\alpha)$$

$$x^3 = 3x^2 - 1$$

$$g'(x) = -(-2x^{-3}) = \frac{2}{x^3}$$

$$x^3 - 3x^2 + 1 = 0 \qquad \boxed{f(x) = 0}$$

$$g'(x) \neq 0 \qquad \forall x \in \mathbb{R} \setminus \{0\}$$

$$|g'(x)| = 1$$

$$\| \quad \frac{2}{|x^3|}$$

$$|x^3| = 2$$

$$x^3 = 2 \quad \vee \quad x^3 = -2$$

$$x = \sqrt[3]{2} \quad \vee \quad x = -\sqrt[3]{2}$$

se una delle radici è $\pm\sqrt[3]{2}$ si ha conv. locale sublinear

negli altri casi, se si ha la convergenza, questa è lineare

se c'è la convergenza locale, è monotona se la radice è positiva
alternata se la radice è negativa

perché il segno di $g'(x)$ è il segno di x.