COMPITO DEL 9/9/2020

1) $\mathbb{F} = \mathbb{F}(2, t, e_{max}, e_{min})$, con arrotondamento.

- $\begin{cases} realmin = \frac{1}{16} \\ e_{max} = t \leftarrow \\ realmax/u = 56 \end{cases}$

$realmin = B^{-e_{min}-1} = 2^{-e_{min}-1}$

$u = \frac{B^{1-t}}{2} = 2^{-t}$

$realmax = B^{e_{max}}(1 - B^{-t}) =$
$= 2^t(1-2^{-t}) = 2^t - 1$

$\Longleftrightarrow \begin{cases} 2^{-e_{min}-1} = 2^{-4} \Longleftrightarrow e_{min} = 3 \\ e_{max} = t = 3 \\ (2^t - 1)/2^{-t} = 56 \Longleftrightarrow (2^t - 1)\cdot 2^t = 56 \Longleftrightarrow 2^{2t} - 2^t - 56 = 0 \end{cases}$

$\boxed{y := 2^t}$ $y > 0$

$y^2 - y - 56 = 0 \Longrightarrow \underline{y = 8} \vee \cancel{y = -7}$

$2^t = 8 = 2^3 \Longrightarrow t = 3$

$\mathbb{F} = \mathbb{F}(2, 3, 3, 3)$

- $x = 1.\overline{101}_2$ $\qquad \tilde{x} = fl(x) = fl(1.\overline{101}_2) = fl(0.1\overline{101}_2 \cdot 2) = 0.111_2 \cdot 2$

  $y = 10.\overline{101}_2$ $\qquad \tilde{y} = fl(y) = fl(0.10\overline{101}_2 \cdot 2^2) = 0.101_2 \cdot 2^2$

  $$\tilde{z} = \underline{\tilde{x}\, fl(+)\, \tilde{y} \;=\; fl(\tilde{x} + \tilde{y})} = fl\left(\begin{array}{r} 0.0111_2 \cdot 2^2 + \\ 0.1010_2 \cdot 2^2 \\ \hline 1.0001_2 \cdot 2^2 \end{array}\right) = fl(1.0001_2 \cdot 2^2)$$

  $$= fl(0.10001_2 \cdot 2^3)$$

  $$= 0.100_2 \cdot 2^3$$

- $\tilde{x} = 0.111_2 \cdot 2 = 2 \cdot \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right) = 1 + \frac{1}{2} + \frac{1}{4} = \frac{7}{4}$

  $x = 1.\overline{101}_2 = \dfrac{1101_2 - 1_2}{111_2} = \dfrac{1+4+8-1}{1+2+4} = \dfrac{12}{7}$

  $y, \tilde{y} \ldots$

- $\tilde{z} \cdot 2^e = \text{realmin} = \frac{1}{16} = 2^{-4}$ $\qquad \tilde{z} = 2^{-1} \cdot 2^3 = 2^2$

  $2^2 \cdot 2^e = 2^{-4} \iff 2 + e = -4 \iff e = -6$

  $\text{realmax} - \tilde{z}$?

2) $y = f(x)$, $f(x) = e^{g(x)}$, $g: \mathbb{R} \to \mathbb{R}$

$$f'(x) = e^{g(x)} \cdot g'(x)$$

- $$\mathrm{cond}_f(x) = \frac{|x|\,|f'(x)|}{|f(x)|} = \frac{|x| \cdot \cancel{|e^{g(x)}|}\,|g'(x)|}{\cancel{|e^{g(x)}|}} = |x| \cdot |g'(x)|$$
$$= \mathrm{cond}_g(x) \cdot |g(x)|$$

$$\mathrm{cond}_g(x) = \frac{|x|\,|g'(x)|}{|g(x)|}$$

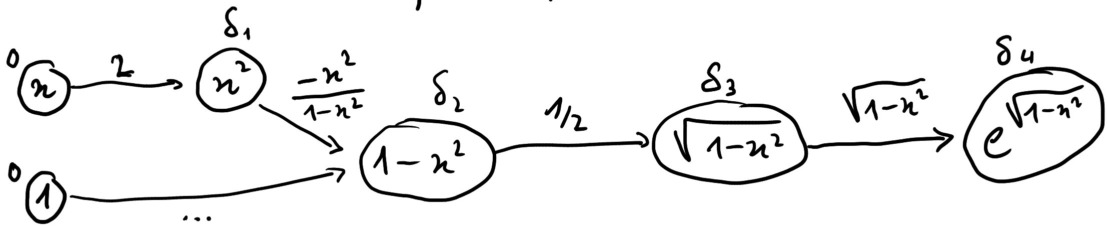
dominio di g
$1 - x^2 \geq 0$
$x^2 \leq 1$
$-1 \leq x \leq 1$

$$g(x) = \sqrt{1-x^2}$$

$$\mathrm{cond}_g(x) = \frac{|x|}{|\sqrt{1-x^2}|} \cdot \left| \frac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}} \right| = \left| \frac{x^2}{1-x^2} \right| = \frac{x^2}{|1-x^2|}$$

$$\mathrm{cond}_f(x) = \frac{x^2}{|1-x^2|} \cdot |\sqrt{1-x^2}|$$
$$= \frac{x^2}{|\sqrt{1-x^2}|}$$

problema malcondizionato per $x \approx 1$ e $x \approx -1$

- 

$$c_{\exp}(x) = \frac{x\,e^x}{e^x} = x$$

$$\varepsilon_{alg} = \delta_4 + \sqrt{1-x^2}\left(\delta_3 + \frac{1}{2}\left(\delta_2 - \frac{x^2}{1-x^2}\delta_1\right)\right)$$

$$|\varepsilon_{alg}| \leq u + \sqrt{1-x^2}\left(u + \frac{1}{2}u + \frac{x^2}{2|1-x^2|}u\right)$$

$$= u + \sqrt{1-x^2}\cdot\frac{3}{2}u + \frac{x^2}{2|\sqrt{1-x^2}|}\cdot u$$

alg. instabile per $x \approx 1$ e $x \approx -1$

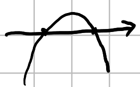
- $g(x) = \sqrt{(1+x)(1-x)}$

$\ldots \quad |\varepsilon_{alg}| \leq u + \sqrt{1-x^2}\cdot\frac{5}{2}u$ (forse)

3) $f(x) = -2x^3 + 2x^2 + 10x + 6$

radici $\alpha < \beta$

$$\lim_{x\to\pm\infty} f(x) = \mp\infty$$



$f'(x) = \underline{-6}x^2 + 4x + 10 \qquad \Delta = 16 + 240 = 256$

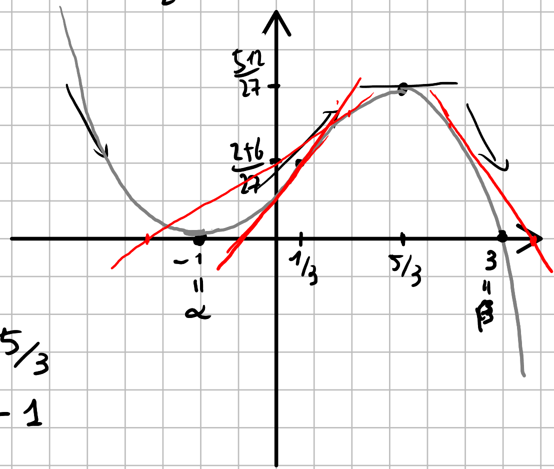
$$f'(x) = 0 \iff x = \frac{-4 \pm \sqrt{256}}{-12} = \frac{-4 \pm 16}{-12} = \begin{cases} 5/3 \\ -1 \end{cases}$$

$f'(x) > 0 \iff -1 < x < 5/3$

$f'(x) < 0 \iff x < -1 \vee x > \frac{5}{3}$

$f(-1) = 2 + 2 - 10 + 6 = 0$

$f(5/3) = 512/27 > 0$



$f''(x) = -12x + 4$

$f''(x) = 0 \iff x = 1/3$

$> 0 \iff x < 1/3$

$< 0 \iff x > 1/3$

$f\left(\frac{1}{3}\right) = \frac{256}{27}$

$f'\left(\frac{1}{3}\right)$

$\boxed{\alpha = -1}$ $\beta > \frac{5}{3}$ $f(2) = 18 > 0$ $\beta > 2$

$f'(\alpha) = f'(-1) = -6 - 4 + 10 = 0$ $f(3) = 0$ $\boxed{\beta = 3}$

$x_0 \in]-\infty, \underset{-1}{\overset{\shortparallel}{\alpha}}[$ convergenza monotona per il Teorema perché $f \cdot f'' > 0, f' \neq 0$, lineare perché α è una radice doppia

$x_0 = \alpha = -1$

$x_0 \in]-1, \frac{1}{3}[$ per il T. convergenza monotona lineare

$x_0 \in [\frac{1}{3}, \frac{5}{3}[$ convergenza lineare, monotona (almeno) a partire dalla 2^a iter.

$x_0 = \frac{5}{3}$ il metodo non si può applicare

$x_0 \in]3, +\infty[$, $f < 0, f'' < 0 \Rightarrow f f'' > 0$

$f' \neq 0$ $\Rightarrow$ per il T. convergenza monotona

$x_0 = 3$

dato che β è radice semplice, è quadratica

$x_0 \in]\frac{5}{3}, 3[$ convergenza quadratica, monotona dalla 2^a iterazione

- $x_0 \in \{-2, 0, 1/3, 5/3, 2, 3\}$ ...

- $g(x) = x - \frac{f(x)}{m}$ $\qquad g(\alpha) = \alpha - \frac{\overbrace{f(\alpha)}^{\overset{0}{\|}}}{m} = \alpha$, lo stesso per β

$x_k = g(x_{k-1})$

- localmente convergente a $\beta = 3$ in modo monotono con fattore asintotico $\frac{1}{5}$

$g'(\beta) = g'(3) = \frac{1}{5}$ $\qquad g'(x) = 1 - \frac{f'(x)}{m} = 1 - \frac{-6x^2 + 4x + 10}{m}$

$g'(3) = 1 + \frac{32}{m} = \frac{1}{5}$ $\qquad f'(3) = -6 \cdot 9 + 4 \cdot 3 + 10 = -54 + 12 + 10 = -32$

$\frac{32}{m} = \frac{1}{5} - 1 = -\frac{4}{5}$ $\qquad m = -\frac{5}{\cancel{4}_1} \cdot \cancel{32}^{8} = \underline{\underline{-40}}$

$x_0 = 2$

$g''(x) < 0 \iff x > \frac{1}{3}$ quindi $g'(x)$ è decrescente se $x > \frac{1}{3}$

in $[2, 3]$ g' è decrescente

$\max_{x \in [2,3]} g'(x) = g'(2) = 1 - \frac{-24 + 8 + 10}{-40} = 1 + \frac{-6}{40} \in]0, 1[$

se $x \in [2,3]$ $g'(x) \in]0,1[$, la succ. che parte da $x_0=2$ è convergente.

- convergenza locale a β con ordine quadratico

$$g'(\beta) = g'(3) = 0 = 1 + \frac{32}{m} \quad \dots$$

la succ. $x_0 = 2$ converge? ...

- $m = 20$, studiamo la convergenza locale a $\alpha = -1$ e dire se la succ. $x_0 = 0$ è convergente e con che ordine.

$$g'(\alpha) = 1 - \frac{f'(\alpha)}{20} = 1$$

se si ha convergenza, la convergenza è sublineare

studiamo g' in $[-1,0]$

$$g''(x) = -\frac{f''(x)}{m} = \frac{12x-4}{20} = \frac{3x-1}{5}$$

$g'(x)$ è decrescente in $[-1,0]$

$$g'(0) = 1 - \frac{f'(0)}{20} = 1 - \frac{10}{20} = \frac{1}{2}$$

$g''(x) < 0 \Leftrightarrow x < \frac{1}{3}$

$\rightarrow$ se $x \in]-1,0]$ $g'(x) \in [1/2, 1[$

la succ $x_0=0$ converge a α in modo monotono e sublineare.

4) $$A = \begin{pmatrix} 2-\alpha & 2 & \alpha+1 \\ 2 & 1 & -2 \\ \alpha+1 & -2 & 5 \end{pmatrix}$$

$$\frac{4+\alpha-2}{\alpha-2} = \frac{\alpha+2}{\alpha-2}$$

$$\frac{2(\alpha+1)-2\alpha+4}{\alpha-2} = \frac{6}{\alpha-2}$$

- $\boxed{\alpha \neq 2}$

$$G_1 = \begin{pmatrix} 1 & 0 & 0 \\ \frac{2}{\alpha-2} & 1 & 0 \\ \frac{\alpha+1}{\alpha-2} & 0 & 1 \end{pmatrix}$$

$$G_1 A = \begin{pmatrix} 2-\alpha & 2 & \alpha+1 \\ 0 & \frac{4}{\alpha-2}+1 & \frac{2(\alpha+1)}{\alpha-2}-2 \\ 0 & \frac{2(\alpha+1)}{\alpha-2}-2 & \frac{(\alpha+1)^2}{\alpha-2}+5 \end{pmatrix}$$

$$= \begin{pmatrix} 2-\alpha & 2 & \alpha+1 \\ 0 & \frac{\alpha+2}{\alpha-2} & \frac{6}{\alpha-2} \\ 0 & \frac{6}{\alpha-2} & \frac{(\alpha+1)^2}{\alpha-2}+5 \end{pmatrix}$$

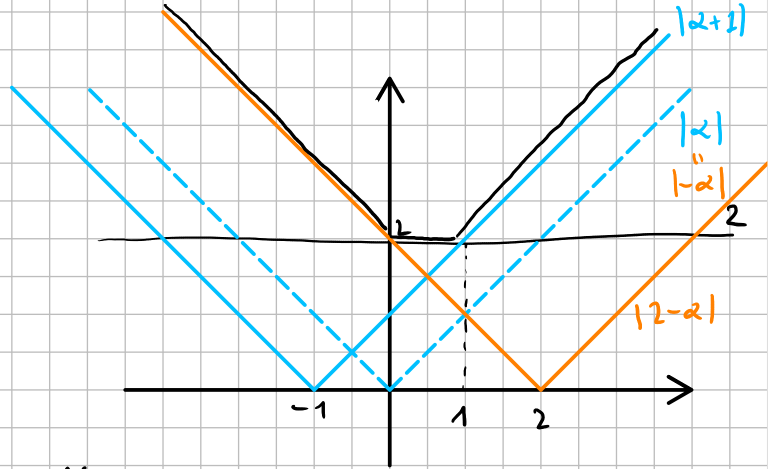
$\boxed{\alpha \neq -2}$

$$G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{-6}{\alpha+2} & 1 \end{pmatrix}$$

$$U = G_2 G_1 A = \begin{pmatrix} 2-\alpha & 2 & \alpha+1 \\ 0 & \frac{\alpha+2}{\alpha-2} & \frac{6}{\alpha-2} \\ 0 & 0 & -\frac{36}{(\alpha+2)^2}+\frac{(\alpha+1)^2}{\alpha-2}+5 \end{pmatrix}$$

$$L = G_1^{-1} G_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{2}{2-\alpha} & 1 & 0 \\ \frac{\alpha+1}{2-\alpha} & \frac{6}{\alpha+2} & 1 \end{pmatrix}$$

esiste se $\alpha \neq 2$ e $\alpha \neq -2$



- $\max\{|2-\alpha|, 2, |\alpha+1|\}$

$$= \begin{cases} |2-\alpha| & \text{se } \alpha \le 0 \\ 2 & \text{se } 0 \le \alpha \le 1 \\ |\alpha+1| & \text{se } \alpha \geq 1 \end{cases}$$

Se $\alpha \le 0$ non si fanno scambi;

se $0 < \alpha \le 1$ si scambiano 1^a e 2^a riga;

se $\alpha > 1$ si scambiano 1^a e 3^a riga.

- $\alpha = -6$

$$A = \begin{pmatrix} 8 & 2 & -5 \\ 2 & 1 & -2 \\ -5 & -2 & 5 \end{pmatrix} \qquad P_1 = I_3, \; P_1 A = A$$

$$G_1 = \begin{pmatrix} 1 & 0 & 0 \\ -1/4 & 1 & 0 \\ 5/8 & 0 & 1 \end{pmatrix} \qquad G_1 P_1 A = \begin{pmatrix} 8 & 2 & -5 \\ 0 & 1/2 & -3/4 \\ 0 & -3/4 & 15/8 \end{pmatrix}$$

$$-\frac{25}{8} + 5 = \frac{-25+40}{8} = \frac{15}{8}$$

$$P_2^{-1} = P_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad P_2 G_1 P_1 A = \begin{pmatrix} 8 & 2 & -5 \\ 0 & -3/4 & 15/8 \\ 0 & 1/2 & -3/4 \end{pmatrix}$$

$$G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2/3 & 1 \end{pmatrix}$$

$$P = P_2 P_1 = P_2$$

$$\frac{1/2}{3/4} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}$$

$$U = \underbrace{G_2 P_2 G_1 P_1}_{L^{-1} P} A = \begin{pmatrix} 8 & 2 & -5 \\ 0 & -3/4 & 15/8 \\ 0 & 0 & 1/2 \end{pmatrix}$$

$$\frac{\cancel{2}^{\,1}}{\cancel{3}_{\,1}} \cdot \frac{\cancel{15}^{\,5}}{\cancel{8}_{\,4}} - \frac{3}{4} = \frac{5}{4} - \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$$

$$L = P P_1^{-1} G_1^{-1} P_2^{-1} G_2^{-1} = P_2 \cancel{P_1} \cancel{P_1^{-1}} G_1^{-1} P_2^{-1} G_2^{-1} = \underbrace{P_2 G_1^{-1}}\ \underbrace{P_2 G_2^{-1}}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ -5/8 & 0 & 1 \\ 1/4 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2/3 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -5/8 & 1 & 0 \\ 1/4 & -2/3 & 1 \end{pmatrix}$$

- $Ux = d$ ...

5) $f(x) = \log_3 (1 + 2x^2)$

- $P_0 = (0, f(0)) = (0, 0)$
  $P_1 = (1, f(1)) = (1, 1)$
  $P_2 = (2, f(2)) = (2, 2)$
- $P_3 = (-2, f(-2)) = (-2, 2)$

$$\begin{array}{cccc}
0 & & & \\
 & 1 & & \\
1 & & 0 & \\
 & 1 & & \dfrac{1/3}{-2} = -\dfrac{1}{6} \\
2 & & \dfrac{0-1}{-2-1} = \dfrac{1}{3} & \\
 & 0 & & \\
2 & & &
\end{array}$$

$p(x) = 0 + 1 \cdot x = x$

$$\tilde{p}(x) = p(x) - \frac{1}{6} x (x-1)(x-2)$$

- polinomio q di primo grado di migliore approssimaz. di P_0, P_1, P_2, P_3

$q(x) = q_0 + q_1 x$ minimi quadrati

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} q_0 \\ q_1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \\ 2 \end{pmatrix} \qquad \dots$$

6*) $I(t) = e^{rt} I_0$, $t \geq 0$, stimare r, $I_0 > 0$ dai dati rilevati

$k = 1, \dots, N$ $\quad t_k > 0 \quad I_k$ rilevato

$I_0 = e^{l}$ $\qquad I(t) = e^{rt} e^{l} = e^{rt+l}$

$I(t_k) = I_k = e^{rt_k + l} \qquad r t_k + l = \log(I_k)$

$$\begin{pmatrix} 1 & t_1 \\ 1 & t_2 \\ \vdots & \vdots \\ 1 & t_N \end{pmatrix} \begin{pmatrix} l \\ r \end{pmatrix} = \begin{pmatrix} \log(I_1) \\ \log(I_2) \\ \vdots \\ \log(I_N) \end{pmatrix}$$