



Political influence on international climate agreements with border carbon adjustment[☆]

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ARTICLE INFO

JEL classification:

D72
F13
F18
H23
Q54
Q56
Q58

Keywords:

Carbon leakage
Climate change
Environmental policy
Lobbying

ABSTRACT

We study the influence of industrial lobbying on national climate policies and the formation of an international environmental agreement if the coalition countries use border carbon adjustments to protect domestic producers. We find that the effects of this political influence crucially depend on the distribution of carbon tax revenues. If these are transferred to the households, lobbying distorts carbon taxes downwards to reduce the tax burden and does not affect coalition sizes. This leads to higher emissions and lower welfare. By contrast, if tax revenues are given back to the firms, lobbies in the outsider countries favor carbon taxes, whereas lobbies in the coalition countries favor carbon subsidies to raise the international commodity price. This reduces the tax difference and the welfare difference between the countries, which reduces the free-rider incentives. Then, lobbying stabilizes the grand coalition and reduces global emissions compared to a “perfect” world without lobbying if the political influence is sufficiently strong.

1. Introduction

Border carbon adjustments (BCA) protect domestic industries in countries with stringent climate policies from negative competitiveness effects (Fischer and Fox, 2012; OECD, 2020). They have been suggested to address two important challenges of international climate policy making: carbon leakage and the formation of international environmental agreements (IEA). Concerning the formation of climate coalitions, it has been shown that trade sanctions against outsiders can have the potential to increase participation (Lessmann et al., 2009; Nordhaus, 2015), a result that has been discussed to potentially also hold with BCA as a specific trade measure (Helm and Schmidt, 2015; Al Khouardjia and Finus, 2020). The fact that BCA can be regarded as some form of “green protectionism” (OECD, 2020) is important when considering the influence of lobby groups on international climate policy making. As protectionist trade policies are subject to lobby influence (Grossman and Helpman, 1994; Goldberg and Maggi, 1999; Gawande and Bandyopadhyay, 2000), we analyze how polluting trade-exposed industries that are organized in lobby groups

[☆] Achim Hagen gratefully acknowledges funding by the German Federal Ministry of Education and Research (BMBF) under grant number 01LA1811C and grant number 01UU2205A. We would like to thank participants of the EAERE 2021, the IIPF 2021, the EEA 2021, the workshop “Instrumentalizing Economics for Political Goals, Instrumentalizing Politics for Economic Goals” at WZB Berlin Social Science Center 2021, the SURED 2022, and the EPSF 2022 for helpful comments. Moreover, the comments from an anonymous reviewer and the editor Andreas Lange significantly improved this article.

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influence climate policy making in the presence of BCA. We study how such lobbying affects carbon taxes and the formation of climate coalitions.

Our model includes perfectly competitive international trade of a commodity. The production of this commodity is associated with carbon dioxide emissions, and countries regulate these emissions with carbon taxes. The full implementation of BCA implies that producers in countries with ambitious climate policies, i.e. the members of the climate coalition, effectively face the same carbon tax as producers in outsider countries with less stringent climate policies, whereas consumption in coalition countries is effectively taxed. BCA thus equalizes producer prices and drives a wedge between consumer prices, such that producer-based emission pricing becomes consumer-based emission pricing (Jakob et al., 2013; Eichner and Pethig, 2015c). With ex ante symmetric countries, this implies that the coalition is a net importer of emissions without BCA (producer-based emission pricing) and a net exporter of emissions with BCA (or with consumer-based emission pricing).¹ We find that if tax revenues are given back to the firms, industrial lobbying aims to raise the international commodity price. Then, lobbying can reduce the tax difference between coalition members and outsiders in the presence of BCA, which reduces the wedge between the consumer prices and, thus, the free-rider incentives. As a result, larger coalitions can be stable and lead to lower global emissions and higher welfare. By contrast, if the tax revenues are transferred to the households, industrial lobbying only aims to reduce the tax burden, which leads to higher global emissions without affecting free-rider incentives.

Our study mainly contributes to two streams of literature. On the one hand, our results enhance the knowledge on IEA in a literature that builds on seminal papers by Hoel (1992), Carraro and Siniscalco (1993) and Barrett (1994), and includes recent contributions by Battaglini and Harstad (2016), Karp and Sakamoto (2021) and Kováč and Schmidt (2021). The papers mentioned above have not considered trade effects. Eichner and Pethig (2013) extended the basic model to include international trade. Eichner and Pethig (2014), Eichner and Pethig (2015a,b) consider different variants concerning the timing of the game and the choice of supply- or demand-side policies, but none of these contributions analyzes the introduction of tariffs. To the best of our knowledge, the first study that considers trade sanctions in an IEA model is (Barrett, 1997). However, this only includes a complete trade ban together with a minimum participation clause, which leads to the stabilization of the grand coalition. Using numerical simulations, Lessmann et al. (2009) and Nordhaus (2015) study trade sanctions as an effective means to stabilize climate coalitions, and Hagen and Schneider (2021) show that the effect of trade sanctions on coalition stability is ambiguous if outsiders retaliate. The results of previous studies concerning the effect of BCA on coalition formation are in general mixed (Felbermayr et al., 2022, pp. 23). In CGE studies with nine world regions, Weitzel et al. (2012) find that BCA cannot stabilize the grand coalition, while (Böhringer et al., 2016) find that BCA stabilizes a coalition of three world regions. Al Khourdajie and Finus (2020) and Baksi and Chaudhuri (2020) study the effect of BCA on coalition stability in models with Cournot competition in segmented markets. Considering multiple firms per country with perfect substitution in consumption, Baksi and Chaudhuri (2020) find that BCA can destabilize the grand coalition. By contrast, considering one firm per country and imperfect substitution in consumption (taste for variety), Al Khourdajie and Finus (2020) find that BCA can stabilize large coalitions up to the grand coalition. Similarly, Helm and Schmidt (2015), who focus on the role of R&D investments, find that BCA can stabilize the grand coalition with monopolistic supply and inelastic demand. These results suggest that BCA stabilizes large coalitions if there is strong market power on the supply side and low substitutability on the demand side. In contrast to these studies, we use a model in which one homogeneous commodity is traded on a perfectly competitive international market and show how lobbying can help to stabilize large coalitions in the presence of BCA.²

On the other hand, we contribute to the literature on the influence of lobbying on environmental policy making. Here, a number of studies have built on the common-agency approach by Grossman and Helpman (1994), including (Aidt, 1998) and Fredriksson and Svensson (2003). Schopf and Voss (2019) and Voss and Schopf (2021) study the influence of lobbying on environmental policies in a dynamic setting. Voss and Schopf (2021) find that industrial lobbying can lead to a reduction of environmental damages. However, they analyze a closed economy and therefore neither consider trade effects and carbon leakage nor coalition formation. Habla and Winkler (2013) study a two-country model of an international permit market under the influence of lobby groups, and Marchiori et al. (2017) and Hagen et al. (2021) show that the influence of lobby groups can have either a stabilizing or a destabilizing effect on climate coalitions, depending on the distribution of lobby groups and the timing of lobbying. Marchiori et al. (2017) and Hagen et al. (2021) neither explicitly account for production and consumption nor for international trade, and none of these studies consider the strategic use of BCA. Closest to our paper, Conconi (2003) analyzes the interaction of environmental and trade policies in the presence of lobby groups and carbon leakage under different trade regimes. However, since the analysis is limited to two countries, the study does not provide insights into the formation of climate coalitions.

To the best of our knowledge, we are the first to provide an integrated analysis of lobbying influence on environmental and trade policies in a multi-country setting with coalition formation. We extend the partial equilibrium framework of Hoel (1994) to include both coalition formation and BCA.³ We apply the common-agency approach of Grossman and Helpman (1994) to study the influence of producer lobbies on carbon taxes and the formation of climate coalitions. We consider different schemes of distributing

¹ In CGE studies with ex ante asymmetric countries, Fischer and Fox (2012, p. 209) and Lanzi et al. (2012, p. S246) confirm that BCA increases the coalition's net exports of emissions compared to the no-policy scenario, and Monjon and Quirion (2011, p. 1964) and Lanzi et al. (2012, p. S246) find that BCA induces negative leakage.

² The papers mentioned above analyze a complete compensation of the carbon tax difference through BCA. For an analysis of the optimal degree of BCA, see Hecht and Peters (2019) and Schopf (2020).

³ We consider symmetric countries and linear-quadratic functions to enable analytical results in the stability analyses.

the carbon tax revenues to firms and households, as these have been shown to matter for the efficiency of environmental policies⁴ and vary strongly between existing carbon tax systems.⁵

In the absence of lobbying, we find that BCA *ceteris paribus* induces the fringe countries to increase their carbon taxes, because this reduces emissions from the coalition countries. Furthermore, it induces the coalition countries to increase their carbon tax, because this effectively reduces demand and therefore emissions from the fringe countries. *Ceteris paribus*, BCA increases the tax difference between the coalition countries and the fringe countries, which increases the welfare difference and thereby the free-rider incentives. Consequently, the stable coalition includes only three countries (Proposition 1).

The ability of competitive firms to organize themselves into a producer lobby that acts on behalf of the entire industry implies that both tax burden and market power motives influence carbon taxes. In the presence of such lobbying, the results crucially depend on the tax revenue distribution. If the tax revenues are lump-sum transferred to the households, all lobbies prefer carbon subsidies to reduce the tax burden. *Ceteris paribus*, the tax difference and, thus, the free-rider incentives remain unaffected. By contrast, if the tax revenues are lump-sum transferred to the firms, the tax burden motive vanishes and a market power motive dominates. In particular, lobbies in fringe countries prefer a positive carbon tax to reduce supply and thereby increase the price. By contrast, lobbies in coalition countries prefer a carbon subsidy, which effectively increases demand and raises the price. *Ceteris paribus*, the tax difference and, thus, the wedge between the consumer prices is reduced.

In particular, lobbying without tax revenue distribution to the firms does not affect the tax difference and the welfare difference. Consequently, the stable coalition includes only three countries and global emissions are greater than without lobbying (Proposition 2). By contrast, lobbying with tax revenue distribution to the firms reduces the tax difference and the welfare difference. Furthermore, a sufficiently strong lobby influence leads to a positive correlation between the coalition sizes and the fringe countries' carbon taxes. Both effects reduce the free-rider incentives and stabilize international climate agreements. In particular, a sufficiently strong lobby influence stabilizes the grand coalition and reduces global emissions compared to a "perfect" world without lobbying (Proposition 3).

To verify that these positive results of lobbying are indeed driven by BCA, we also conduct an analysis with carbon taxes only (Online Appendix C). In this case, a coalition of three countries is stable and a coalition of all countries can be stable, whereby lobbying destabilizes the grand coalition. Furthermore, we conduct an analysis with industrial lobbying only on the membership decision and not on the policy decision (Online Appendix D). In this case, lobbying destabilizes small coalitions but can stabilize the grand coalition if and only if tax revenues are lump-sum transferred to the firms. Finally, we conduct an analysis with environmental lobbying (Online Appendix E). Such environmental lobbying does neither affect the results with nor without industrial lobbying.

Our results suggest that the positive effects of BCA on coalition size, emission reductions and welfare improvements can be underestimated when abstracting from industrial lobbying. However, there is clear evidence for industrial lobbying over national environmental policy (The Guardian, 2021; InfluenceMap, 2022b) and international climate cooperation (The Guardian, 2020; Transparency International, 2020). The effects of this political influence depend on the distribution of carbon tax revenues. If these are transferred to the households, industrial lobbying aims to reduce the national carbon tax (tax burden motive), which leads to higher global emissions but does not influence the free-rider incentives. If, however, the tax revenues are given back to the firms, industrial lobbying aims to influence the international commodity price (market power motive), which reduces the wedge between the consumer prices and welfare levels and, thus, the free-rider incentives. Then, a sufficiently strong political influence stabilizes the grand coalition and makes BCA preferable to IEA with carbon taxes only. This is in line with Helm and Schmidt (2015) and Al Khourdajie and Finus (2020), who find positive effects of BCA with strong market power motives.

In the remainder of the paper we describe the basic model in Section 2 and analyze coalition formation without lobbying in Section 3 and with lobbying in Section 4. Section 5 discusses our extensions, i.e. lobbying without BCA, lobbying only on the membership decision, and environmental lobbying. In Section 6 we address policy implications and model limitations. Section 7 concludes.

2. The model

Consider a trade model with $n \geq 10$ symmetric countries. In each country $i \in N$, the respective representative household derives benefits $B(y_i) = ay_i - \frac{b}{2}y_i^2$ from commodity consumption y_i , where $B' > 0$ and $B'' < 0$, and the respective representative firm faces costs $C(x_i) = \frac{1}{2}x_i^2$ of commodity production x_i , where $C' > 0$ and $C'' > 0$. Emissions are proportional to commodity production, and each country faces climate costs $H(x) = hx$ from global emissions $x := \sum_{j \in N} x_j$, where $H' > 0$ and $H'' = 0$. The commodity is traded on an international market at the commodity price p . The market clearing condition is $\sum_{j \in N} y_j = \sum_{j \in N} x_j$.⁶

⁴ Gersbach and Requate (2004) analyze optimal schemes of refunding emission taxes to the firms with Cournot competition. They find that the first-best can be implemented if the marginal damage from pollution exceeds the market distortion from imperfect competition. Cato (2010) endogenizes the number of firms and adds an entry-license tax. He finds that the first-best can always be implemented with a binding government's budget constraint. Fredriksson and Sterner (2005) analyze industrial lobbying over emission taxes with and without revenue recycling to the firms. They find that both low-pollution-intensity and high-pollution-intensity firms lobby for higher taxes with refunding schemes than without, which results in higher taxes and lower emissions. Sterner and Isaksson (2006) conclude that this industrial lobbying can explain the success of the Swedish experience of NO_x abatement.

⁵ Carl and Fedor (2016) and Klenert et al. (2018) investigate 13 carbon tax systems and find that 44% of revenues are returned to taxpayers. Corporate tax cuts would be the most popular form of tax revenue recycling for reasons of economic efficiency and political economy. However, the distribution of revenues varies strongly between the carbon tax systems (e.g. 25% [75%] of revenue recycling to the firms [households] in Alberta, and 100% of revenue recycling to the firms in British Columbia).

⁶ To the best of our knowledge, Hoel (1994) was the first to use this framework. Harstad (2012, Section 2) shows that this model can be extended to a general equilibrium model without changing the results.

Suppose that m countries represent the environmental coalition E , whereas the remaining $n - m$ countries represent the fringe F . Since the countries are ex ante symmetric, we use the subscript e for coalition countries and the subscript f for fringe countries. The coalition uses a common carbon tax $\chi_e := \chi_i$ for $i \in E$, whereas each fringe country uses an individual carbon tax χ_i for $i \in F$. Then, $\chi_f := \frac{\sum_{i \in F} \chi_i}{n-m}$ is the average carbon tax of the fringe countries and the common carbon tax of the fringe countries in equilibrium. Furthermore, the coalition implements full BCA in the form of a common trade tax $\tau_e = \chi_e - \chi_f$.⁷ This levels the playing field for competition between firms inside and outside the climate coalition.⁸

Note that τ_e is an implicit consumption tax and an implicit production subsidy in the coalition countries, and $\chi_e - \tau_e = \chi_f$ is the implicit carbon tax of the coalition countries. In other words, full BCA yields a unique global producer price $p - \chi_f$, but different national consumer prices $p + \tau_e$ and p . Consequently, national production is identical ($x_e = x_f$), but national consumption is different ($y_e < y_f$), such that the coalition countries export the commodity in equilibrium.

The revenues from the implicit consumption tax ($\tau_e y_e$) are lump-sum transferred to the domestic household, whereas the share $\alpha [1 - \alpha]$ of the costs from the implicit production subsidy ($\tau_e x_e$) is lump-sum collected from the respective firm [household] in the coalition countries. Furthermore, the share $\alpha [1 - \alpha]$ of the revenues from the carbon tax is lump-sum transferred to the respective domestic firm [household] in all countries. Then, the lump-sum transfers to the households T_i^y and to the firms T_i^x are given by

$$\begin{aligned} T_f^y &= (1 - \alpha)\chi_f x_f & \text{and} & & T_f^x &= \alpha\chi_f x_f, \\ T_e^y &= \tau_e y_e + (1 - \alpha)(\chi_e - \tau_e)x_e & \text{and} & & T_e^x &= \alpha(\chi_e - \tau_e)x_e. \end{aligned} \quad (1)$$

Thus, $\alpha = 0$ implies that households receive all tax revenues, whereas $\alpha = 1$ implies that households and firms get back their implicitly paid taxes.⁹ The representative households and firms take the prices, the taxes and the lump-sum transfers as given and do not consider the climate costs. Their consumption and production decisions follow from

$$\begin{aligned} \max_{y_f} B(y_f) - py_f + T_f^y & \quad \text{and} \quad \max_{x_f} (p - \chi_f)x_f - C(x_f) + T_f^x, \\ \max_{y_e} B(y_e) - (p + \tau_e)y_e + T_e^y & \quad \text{and} \quad \max_{x_e} (p - \chi_e + \tau_e)x_e - C(x_e) + T_e^x. \end{aligned} \quad (2)$$

Solving (2) yields the commodity demand and supply in the respective countries

$$\begin{aligned} y_f &= D(p) & \text{and} & & x_f &= S(p - \chi_f), \\ y_e &= D(p + \tau_e) & \text{and} & & x_e &= S(p - \chi_e + \tau_e), \end{aligned} \quad (3)$$

where $D := B'^{-1}$ and $S := C'^{-1}$ with $D' = 1/B'' = -1/b < 0$ and $S' = 1/C'' = 1 > 0$. Using (3) in the market clearing condition yields the functional relationship between the coalition size, the taxes and the international commodity price

$$(n - m)D(p) + mD(p + \tau_e) = \sum_{j \in F} S(p - \chi_j) + mS(p - \chi_e + \tau_e). \quad (4)$$

Totally differentiating (4) for $\tau_e = \chi_e - \frac{\sum_{j \in F} \chi_j}{n-m}$ and rearranging yields the effects of the carbon taxes on the international commodity price

$$\begin{aligned} \frac{\partial p}{\partial \chi_f} &= \frac{S'_f + m(S'_e - D'_e)/(n - m)}{(n - m)(S'_f - D'_f) + m(S'_e - D'_e)} \in (0, 1), \\ \frac{\partial p}{\partial \chi_e} &= \frac{mD'_e}{(n - m)(S'_f - D'_f) + m(S'_e - D'_e)} \in (-1, 0), \end{aligned} \quad (5)$$

where $D'_f := D'(p) = -1/b$, $D'_e := D'(p + \tau_e) = -1/b$, $S'_f := S'(p - \chi_f) = 1$ and $S'_e := S'(p - \chi_e + \tau_e) = 1$. On the one hand, a fringe country's carbon tax reduces its supply, which increases the price, and it reduces the BCA, which reduces supply and raises demand in the coalition countries and, thus, further increases the price. On the other hand, the coalition's carbon tax raises the BCA, which reduces demand in the coalition countries and, thus, reduces the price. Thus, BCA leads to qualitatively different effects of the carbon taxes on the international commodity price.

In the further course of the paper, we analyze a three-stage game of coalition formation.¹⁰ In the first stage of the game, countries decide on their membership in the coalition. Thereby, internal [external] stability implies that no country will leave [join] the coalition if this reduces its government's utility (D'Aspremont et al., 1983). This utility corresponds to a weighted sum of the country's welfare and the political contributions. In the second stage of the game, there is a coalition of m countries, and countries decide on their carbon taxes. Thereby, a fringe country's government maximizes its utility, and the coalition countries' governments maximize the sum of their utility. Finally, production, consumption and trade take place in the third stage of the game.

Throughout the paper, we assume that all countries take the carbon taxes in all other countries as given, i.e. we solve for the Nash equilibrium. Note that the BCA depends on all carbon taxes, such that all countries account for their influence on the coalition's

⁷ In Online Appendix F, we show that all qualitative and quantitative results remain valid with coalition country-specific policies.

⁸ Full BCA is typically assumed in the literature, e.g. Weitzel et al. (2012), Helm and Schmidt (2015), Böhringer et al. (2016), Al Khouradje and Finus (2020) and Baksi and Chaudhuri (2020), and proposed by policy makers, e.g. European Commission (2021) and Deutch (2021).

⁹ Pezzey (1992) and Farrow (1995) reveal the equivalence of tradable permits with grandfathering share α and taxes with revenue parameter α .

¹⁰ For a review of the literature on coalition formation, see Benckroun and Long (2012).

trade tax via their carbon taxes. Furthermore, we assume that all countries account for their influence on the commodity price. Finally, we restrict the parameter space such that the coalition's carbon tax is positive and exceeds the fringe countries' carbon taxes.

3. Border carbon adjustment

In this section, we solve the model without lobbying by backward induction. In the third stage of the game, (4) determines the international commodity price for a given tax policy and a given coalition size, and (3) then determines production, trade and consumption.

In the second stage of the game, each fringe country's optimal policy follows from maximizing its welfare, i.e. its benefits from commodity consumption minus its costs of commodity production minus its climate costs minus its import costs,

$$W_f = B[D(p)] - C[S(p - \chi_i)] - H \left[\sum_{j \in F} S(p - \chi_j) + mS(p - \chi_e + \tau_e) \right] - p[D(p) - S(p - \chi_i)], \quad (6)$$

with respect to its carbon tax subject to $\tau_e = \chi_e - \frac{\sum_{j \in F} \chi_j}{n-m}$ and (5). In Appendix A.1, we solve the maximization problem and find

$$\chi_f = H' - \underbrace{\frac{\frac{\partial p}{\partial \chi_f}}{1 - \frac{\partial p}{\partial \chi_f}}(n-1)H'}_{CL_f < 0} - \underbrace{\frac{\frac{\partial p}{\partial \chi_f}}{1 - \frac{\partial p}{\partial \chi_f}} \frac{1}{S'_f}(y_f - x_f)}_{TOT_f < 0} + \underbrace{\frac{1}{1 - \frac{\partial p}{\partial \chi_f}} \frac{m}{n-m} H'}_{BCA_f > 0} < mH' \text{ for } m \leq n-2. \quad (7)$$

Thus, the carbon tax accounts for the fringe country's climate externality on itself (H'), but also for the increase in foreign emissions via the carbon-leakage effect (CL_f) and for the increase in its import costs via the terms-of-trade effect (TOT_f) induced by the decrease in its commodity supply and, thus, the increase in the commodity price. Furthermore, the carbon tax induces the coalition countries to partially account for their climate externality on the fringe country via the BCA effect (BCA_f), which speaks in favor of a higher carbon tax. However, this effect is not strong enough to bring the carbon tax to the efficient level (nH') when there is more than one fringe country. Note that the fringe's carbon tax is independent of the revenue distribution parameter α , because the representative firm takes the lump-sum transfer as given and revenue distribution has no effect on welfare.

The coalition countries' optimal policy follows from maximizing the sum of their members' welfare, i.e. their benefits from commodity consumption minus their costs of commodity production minus their climate costs plus their export revenues,

$$mW_e = mB[D(p + \tau_e)] - mC[S(p - \chi_e + \tau_e)] - mH \left[\sum_{j \in F} S(p - \chi_j) + mS(p - \chi_e + \tau_e) \right] + mp[S(p - \chi_e + \tau_e) - D(p + \tau_e)], \quad (8)$$

with respect to their carbon tax subject to $\tau_e = \chi_e - \frac{\sum_{j \in F} \chi_j}{n-m}$ and (5). In Appendix A.1, we solve the maximization problem and find

$$\chi_e = \chi_f + \underbrace{\frac{\frac{\partial p}{\partial \chi_e}}{1 + \frac{\partial p}{\partial \chi_e}} \frac{S'_f}{D'_e}(n-m)H'}_{CL_e > 0} - \underbrace{\frac{\frac{\partial p}{\partial \chi_e}}{1 + \frac{\partial p}{\partial \chi_e}} \frac{1}{D'_e}(x_e - y_e)}_{TOT_e < 0} + \underbrace{\frac{\frac{\partial p}{\partial \chi_e}}{1 + \frac{\partial p}{\partial \chi_e}} \frac{S'_e}{D'_e}(mH' - \chi_f)}_{BCA_e > 0} < mH' \text{ for } m \leq n-2. \quad (9)$$

Since the coalition countries take the carbon taxes in the fringe countries and, thus, their own effective carbon tax χ_f as given, they chose their nominal carbon tax χ_e to regulate their commodity demand via their effective consumption tax $\tau_e = \chi_e - \chi_f$. This effective consumption tax accounts for the decrease in foreign emissions via the carbon-leakage effect (CL_e) and for the decrease in the coalition countries' export revenues via the terms-of-trade effect (TOT_e) induced by the decrease in their commodity demand and, thus, in the commodity price. Without these effects, the coalition countries' effective consumption tax would just internalize their climate externality on themselves via the BCA effect (BCA_e): Since their effective carbon tax (χ_f) falls short of this climate externality (mH'), the coalition countries increase their effective consumption tax to reduce the commodity price and, thus, their commodity supply in order to partially offset the underinternalization of their climate externality on themselves. Ceteris paribus, this effect increases the tax difference between a coalition country and a fringe country, but it is not strong enough to bring the carbon tax to the efficient level (nH') when there is more than one fringe country. Consequently, global emissions are then inefficiently high. For the same reasons as for the fringe's carbon tax, the revenue distribution parameter α has no effect on the coalition's carbon tax.

Substituting (7) into (9) and accounting for $S'_f = S'_e$, the difference between the carbon taxes is given by

$$\chi_e - \chi_f = \underbrace{\frac{\frac{\partial p}{\partial \chi_e}}{1 + \frac{\partial p}{\partial \chi_e}} \frac{S'_e}{D'_e} nH'}_{CC_e > 0} - \underbrace{\frac{\frac{\partial p}{\partial \chi_e}}{1 + \frac{\partial p}{\partial \chi_e}} \frac{1}{D'_e}(x_e - y_e)}_{TOT_e < 0} - \underbrace{\frac{\frac{\partial p}{\partial \chi_e}}{1 + \frac{\partial p}{\partial \chi_e}} \frac{S'_e}{D'_e} \left[\frac{\frac{1}{n-m} - \frac{\partial p}{\partial \chi_f}}{1 - \frac{\partial p}{\partial \chi_f}} nH' - \frac{\frac{\partial p}{\partial \chi_f}}{1 - \frac{\partial p}{\partial \chi_f}} \frac{1}{S'_f}(y_f - x_f) \right]}_{\substack{CC_f > 0 \\ TOT_f < 0}}, \quad (10)$$

where the term in square brackets reflects the fringe's carbon tax. The tax difference consists of the climate-costs effects (CC_i) and the terms-of-trade effects. Without terms-of-trade effects, both carbon taxes would be positive to account for the climate costs

($CC_i > 0$), but these climate costs are more important for the coalition than for the fringe ($CC_e > CC_f$). The terms-of-trade effects depress both carbon taxes, such that their effect on the tax difference is ambiguous. If the coalition is small, we show in [Appendix A.1](#) that the terms-of-trade effects reduce the tax difference. Then, each fringe country's terms-of-trade effect is small, because many fringe countries import from few coalition countries, and the coalition's terms-of-trade effect dominates. By contrast, if the coalition is large, the fringe's terms-of-trade effect dominates and the terms-of-trade effects increase the tax difference.

Now we turn to the first stage of the game, in which a coalition of a given size m is internally stable if no coalition country has an incentive to leave the coalition $\Phi(m) = W_e(m) - W_f(m-1) \geq 0$, and it is externally stable if no fringe country has an incentive to join the coalition $\Phi(m+1) = W_e(m+1) - W_f(m) \leq 0$. In [Appendix A.3](#), we prove

Proposition 1. *For $m \leq n-1$ each coalition country's carbon tax is smaller than mh , and each fringe country's carbon tax is smaller than h and positive [negative] for $m < [>]0.62n$.*

- Only $m = 3$ is internally and externally stable.
- Compared to the social optimum, global emissions are greater and the welfare of each country is smaller in this equilibrium.

[Proposition 1](#) confirms that the coalition's carbon tax underinternalizes its climate externality on itself ($\chi_e < mh$). Furthermore, it reveals that the carbon-leakage effect (CL_f) and the terms-of-trade effect (TOT_f) outweigh the border-carbon-adjustment effect (BCA_f), such that each fringe country's carbon tax underinternalizes its climate externality on itself ($\chi_f < h$) and is even negative when the coalition is sufficiently large. [Proposition 1a](#) reveals that BCA does not have the potential to stabilize coalitions with more than three countries in our model without lobby influence. Furthermore, the underinternalization of the climate externality implies that global emissions are greater and global welfare is smaller at this coalition–fringe equilibrium than at the social optimum ([Proposition 1b](#)).

To explain why BCA does not stabilize large coalitions, we analyze the effects of an increase in the coalition size in [Lemma 1](#) of [Appendix A.2](#). We show that each fringe country's carbon tax decreases with the coalition size ($\chi_f \downarrow$): On the one hand, each fringe country has an incentive to increase its carbon tax when the coalition becomes larger because it thereby increases the effective carbon tax of more coalition countries ($BCA_f \uparrow$). On the other hand, it has also an incentive to decrease this tax because its imports increase ($TOT_f \downarrow$). The terms-of-trade effect outweighs the border-carbon-adjustment effect, such that each fringe country's carbon tax decreases with the coalition size and eventually becomes negative. Furthermore, we show that the coalition's effective consumption tax increases with the coalition size ($\tau_e = \chi_e - \chi_f \uparrow$): On the one hand, the coalition has an incentive to increase its effective consumption tax when the coalition becomes larger because the difference between the coalition's climate externality and its effective carbon tax increases ($BCA_e \uparrow$). On the other hand, it has also an incentive to decrease this tax because it thereby increases the emissions of less fringe countries ($CL_e \downarrow$). The border-carbon-adjustment effect outweighs the carbon-leakage effect, such that the coalition's effective consumption tax increases with the coalition size. This tax increase typically outweighs the decrease in the effective carbon tax, such that the coalition's nominal carbon tax increases ($\chi_e \uparrow$) when the coalition is not too large ($m \leq n-4$).

The decreasing χ_f and the increasing χ_e cause the international commodity price p to fall, so that consumption in fringe countries increases, and the commodity price in the coalition $p + \tau_e$ to increase, so that consumption in coalition countries decreases. Furthermore, the fall in the international commodity price improves [worsens] the fringe [coalition] countries' terms of trade. Consequently, the welfare difference between a fringe country and a coalition country and, thus, the free-rider incentives increase with the coalition size.¹¹ This explains why BCA does not stabilize large coalitions without lobby influence, even though this would be beneficial for climate and welfare: For producers, the decreasing international commodity price outweighs the decreasing effective carbon tax, such that the producer price $p - \chi_f$ and, thus, production and global emissions decrease with the coalition size. This reduces climate damages, which has a positive effect on each country's welfare that outweighs the decreasing consumption utility in each coalition country ($\partial y_e / \partial m < 0$) and the decreasing producer utility ($\partial x_i / \partial m < 0$), such that each country's welfare and global welfare increase with the coalition size.

So BCA could reduce emissions and increase welfare if it stabilized large coalitions, but this is not the case with welfare maximizing governments. However, BCA has important implications for producers in both coalition and fringe countries. If producers are organized in lobby groups and can thereby influence policies, the effects of BCA on producers alter their political influence via industrial lobbying on government decisions in non-trivial ways. We examine these effects in the following section.

4. Lobbying

In this section, we assume that in each country the representative firm, consisting of many price-taking companies, is represented by a producer lobby that acts on behalf of the entire industry. Therefore, it accounts for the influence of the carbon taxes on the commodity price p and the lump-sum transfer T_i^x . Consequently, its gross utility is given by

$$\begin{aligned} U_f(\chi_i) &= pS(p - \chi_i) - C[S(p - \chi_i)] - (1 - \alpha)\chi_i S(p - \chi_i), \\ U_e(\chi_e) &= pS(p - \chi_e + \tau_e) - C[S(p - \chi_e + \tau_e)] - (1 - \alpha)(\chi_e - \tau_e)S(p - \chi_e + \tau_e). \end{aligned} \quad (11)$$

¹¹ Note that the welfare of each fringe country exceeds that of each coalition country because the countries' producer surpluses and the climate damages coincide, but the commodity consumption and, thus, the consumer surplus of each fringe country exceeds that of each coalition country.

where $pS - C$ is the profit before taxes and $(1 - \alpha)\chi_i S$ is the net tax payment. Note that $\chi_i = \chi_e - \tau_e = \chi_f$ in equilibrium, such that every producer lobby has the same gross utility.

The producer lobby tries to influence both the government's participation decision in the first stage of the game and the government's policy for a given participation decision in the second stage of the game via political contributions $L_i^1(m)$ and $L_i^2(\chi_i)$ (cf. [Habla and Winkler, 2013](#)). Consequently, its net utility is given by

$$U_i(\chi_i) - L_i^1(m) - L_i^2(\chi_i). \quad (12)$$

The government's net utility is given by a linear combination of domestic welfare W_i and political contributions from its producer lobby $L_i^1(m) + L_i^2(\chi_i)$:

$$G_i = W_i(\chi_i) + \mu[L_i^1(m) + L_i^2(\chi_i)], \quad (13)$$

where μ is the lobby parameter, i.e. the relative weight attached to political contributions compared to domestic welfare.

We assume truthful contribution schedules, such that each producer lobby transfers its complete gross utility minus a constant to the government. At the policy stage, this implies

$$L_i^2(\chi_i) = \max[0, U_i(\chi_i) - \bar{U}_i], \quad (14)$$

with the constant \bar{U}_i denoting the producer lobby's net utility with lobbying. From (13) and (14), and since $L_i^1(m)$ is already paid at the participation stage, a fringe government's policy maximizes a weighted sum of its domestic welfare and its producer lobby's gross utility

$$\max_{\chi_i} G_f = \max_{\chi_i} [W_f(\chi_i) + \mu U_f(\chi_i)], \quad (15)$$

and the coalition's policy maximizes a weighted sum of the coalition countries' welfare and their producer lobbies' gross utility (cf. [Marchiori et al., 2017](#))

$$\max_{\chi_e} mG_e = \max_{\chi_e} m[W_e(\chi_e) + \mu U_e(\chi_e)]. \quad (16)$$

On the participation stage, we follow [Habla and Winkler \(2013\)](#) and [Marchiori et al. \(2017\)](#) in assuming that each producer lobby "expects the worst regime to be adopted should it give up lobbying altogether" ([Marchiori et al., 2017](#), p. 126). This implies that a government gets nothing at this stage if it chooses the producer lobby's worse regime, and that it receives the whole producer lobby's net utility gain if it switches to the other regime, i.e.

$$L_e^1(m) = \max[0, \bar{U}_e(m) - \bar{U}_f(m-1)] \quad \text{and} \quad L_f^1(m-1) = \max[0, \bar{U}_f(m-1) - \bar{U}_e(m)]. \quad (17)$$

The government participates if and only if

$$\begin{aligned} G_e(m) - G_f(m-1) &> 0 \\ \Leftrightarrow W_e(m) - W_f(m-1) + \mu[L_e^1(m) - L_f^1(m-1) + L_e^2(m) - L_f^2(m-1)] &> 0 \\ \Leftrightarrow W_e(m) - W_f(m-1) + \mu[U_e(m) - U_f(m-1)] &> 0, \end{aligned} \quad (18)$$

i.e. if the weighted sum of its domestic welfare and its producer lobby's gross utility is greater inside than outside the coalition. Defining this joint welfare by $V_i := W_i + \mu U_i$, the internal stability condition with lobbying becomes $\Phi(m) = V_e(m) - V_f(m-1) \geq 0$, and the external stability condition with lobbying becomes $\Phi(m+1) = V_e(m+1) - V_f(m) \leq 0$. Note that every producer lobby having the same gross utility implies that the welfare difference and the joint welfare difference between a fringe country and a coalition country coincide in equilibrium ($W_f - W_e = V_f - V_e$).

In what follows, we solve the model with lobbying by backward induction. In the third stage of the game, (4) determines the international commodity price for a given tax policy and a given coalition size, and (3) then determines production, trade and consumption.

In the second stage of the game, each fringe country's optimal policy follows from maximizing its joint welfare

$$\begin{aligned} V_f = W_f + \mu U_f &= B[D(p)] - C[S(p - \chi_i)] - H \left[\sum_{j \in F} S(p - \chi_j) + mS(p - \chi_e + \tau_e) \right] - p[D(p) - S(p - \chi_i)] \\ &\quad + \mu \left[pS(p - \chi_i) - C[S(p - \chi_i)] - (1 - \alpha)\chi_i S(p - \chi_i) \right], \end{aligned} \quad (19)$$

with respect to its carbon tax subject to $\tau_e = \chi_e - \frac{\sum_{j \in F} \chi_j}{n-m}$ and (5). In [Appendix B.1](#), we solve the maximization problem and find

$$\chi_f = \frac{1}{1 + \alpha\mu} \chi_f|_{\mu=0} - \frac{\mu}{1 + \alpha\mu} \frac{1 - \alpha - \frac{\partial p}{\partial \chi_f} \chi_f}{1 - \frac{\partial p}{\partial \chi_f}} \frac{\chi_f}{S'_f}. \quad (20)$$

The lobby group has two reasons for influencing the carbon tax, namely reducing the tax burden and increasing the commodity price (market power motive). First consider $\alpha = 0$, such that the households receive all tax revenues. Then, the lobby group prefers a negative carbon tax to reduce the tax burden. In this case, lobbying *ceteris paribus* decreases the carbon tax ($\chi_f = \chi_f|_{\mu=0} - \mu\chi_f/S'_f$).

Now consider $\alpha = 1$, such that the firms get back their paid taxes. Then, the lobby group prefers a positive carbon tax ($\mu \rightarrow \infty \Rightarrow \chi_f > 0$) to reduce the commodity supply and, thus, to raise the commodity price. By contrast, it does not account for the climate externality or for the consumer surplus. The climate part of $\chi_f|_{\mu=0}$ is positive, whereas the consumer part of $\chi_f|_{\mu=0}$, i.e. the terms-of-trade effect for $x_f = 0$, is definitely negative to raise the commodity supply and, thus, to reduce the commodity price. To sum up, we cannot say whether lobbying increases or decreases the carbon tax in this case, but we can say that an otherwise negative carbon tax can become positive due to lobbying. Finally, for $\alpha \in (0, 1)$ the lobby group prefers a negative carbon tax if α is small and $\partial p / \partial \chi_f$ is small, which is ceteris paribus the case if the coalition is small, such that a positive carbon tax would only reduce the commodity supply of a few countries via the BCA.

The coalition countries' optimal policy follows from maximizing the sum of their members' joint welfare

$$\begin{aligned} mV_e = m[W_e + \mu U_e] &= mB[D(p + \tau_e)] - mC[S(p - \chi_e + \tau_e)] - mH\left[\sum_{j \in F} S(p - \chi_j) + mS(p - \chi_e + \tau_e)\right] \\ &+ mp[S(p - \chi_e + \tau_e) - D(p + \tau_e)] \\ &+ m\mu\left[pS(p - \chi_e + \tau_e) - C[S(p - \chi_e + \tau_e)] - (1 - \alpha)(\chi_e - \tau_e)S(p - \chi_e + \tau_e)\right], \end{aligned} \quad (21)$$

with respect to their carbon tax subject to $\tau_e = \chi_e - \frac{\sum_{j \in F} \chi_j}{n-m}$ and (5). In [Appendix B.1](#), we solve the maximization problem and find

$$\begin{aligned} \chi_e = \chi_f + \frac{\frac{\partial p}{\partial \chi_e} S'_f}{1 + \frac{\partial p}{\partial \chi_e} D'_e} (n-m)H' - \frac{\frac{\partial p}{\partial \chi_e}}{1 + \frac{\partial p}{\partial \chi_e} D'_e} \frac{1}{D'_e} (x_e - y_e) + \frac{\frac{\partial p}{\partial \chi_e} S'_e}{1 + \frac{\partial p}{\partial \chi_e} D'_e} (mH' - \chi_f) \\ - \mu \frac{\frac{\partial p}{\partial \chi_e} S'_e}{1 + \frac{\partial p}{\partial \chi_e} D'_e} \left(\frac{x_e}{S'_e} + \alpha \chi_f \right), \end{aligned} \quad (22)$$

where the first line coincides with (9). Just like the lobby group in each fringe country, the lobby groups in the coalition countries want to reduce the tax burden and increase the commodity price (market power motive). However, in contrast to the fringe countries' carbon taxes, the coalition's carbon tax reduces the commodity demand and, thus, the commodity price. Consequently, the lobby groups definitely prefer a negative carbon tax if the households receive all tax revenues ($\alpha = 0$) or if the fringe countries' carbon taxes are positive ($\chi_f \geq 0$), such that the firms get back additional paid taxes through increased production. By contrast, if $\alpha > 0$ and $\chi_f < 0$, the lobby groups could in principle prefer a positive carbon tax to avoid lump-sum transfers to the government. However, with linear-quadratic costs the lobby groups in the coalition countries always prefer a negative carbon tax.¹²

Substituting (20) into (22) and accounting for $x_f/S'_f = x_e/S'_e$, the difference between the carbon taxes is given by

$$\chi_e - \chi_f = (\chi_e - \chi_f)|_{\mu=0} - \alpha \mu \frac{1}{1 - \frac{\partial p}{\partial \chi_f}} \frac{\frac{\partial p}{\partial \chi_e}}{1 + \frac{\partial p}{\partial \chi_e} D'_e} \frac{x_e}{D'_e}. \quad (23)$$

Thus, ceteris paribus, lobbying reduces the difference between the carbon taxes if the firms get back some of their paid taxes ($\alpha > 0$). First consider $\alpha = 0$, such that the households receive all tax revenues. Then, the tax burden motives outweigh the market power motives, each lobby group prefers a carbon subsidy and the effects exactly cancel out each other,¹³ such that the wedge between the carbon taxes stays unchanged. Now consider $\alpha = 1$, such that only market power motives (and not tax burden motives) are relevant. Then, the lobby groups in the coalition countries prefer a carbon subsidy, because this raises the commodity demand in the coalition countries and thereby the commodity price. By contrast, the lobby group in each fringe country prefers a positive carbon tax, because this reduces the commodity supply in its country and, via the BCA, in the coalition countries and thereby increases the commodity price. Both effects narrow the wedge between the carbon taxes. Note that these market power motives are so strong that lobbying ceteris paribus reduces the difference between the carbon taxes for any $\alpha > 0$.

In what follows, we derive the coalition–fringe equilibrium with lobbying for the two polar cases $\alpha = 0$ and $\alpha = 1$. Starting with the case where the households receive all tax revenues ($\alpha = 0$), we prove in [Appendix B.3](#)

Proposition 2. Consider $\alpha = 0$ and $\mu > 0$. For $m \leq n-1$ each coalition country's carbon tax is smaller than mh , and each fringe country's carbon tax smaller than h and negative for $m \geq 0.62n$.

- Only $m = 3$ is internally and externally stable.
- Compared to the unique equilibrium without lobbying, global emissions are greater and the welfare of each country is smaller in this equilibrium with industrial lobbying.

[Propositions 1](#) and [2](#) reveal that the climate coalition and each fringe country underinternalizes its climate externality, i.e. $\chi_e < mh$ and $\chi_f < h$, both without lobbying and with lobbying and tax revenue distribution to the households. [Proposition 2a](#) shows that BCA remains ineffective in stabilizing large climate coalitions with lobbying and tax revenue distribution to the households.

¹² Note that $\partial[x_e/S'_e + \chi_f]/\partial \chi_f = x_e S''_e / (S'_e)^2$, such that $S''_e = -C'''_e / (C''_e)^3 \leq 0 \Leftrightarrow C'''_e \geq 0$ is sufficient for $x_e/S'_e + \chi_f > 0$ and, thus, for $x_e/S'_e + \alpha \chi_f > 0$.

¹³ For $\alpha = 0$, the carbon taxes are given by $\chi_e = x_e|_{\mu=0} - \mu x_e/S'_e$ and $\chi_f = \chi_f|_{\mu=0} - \mu x_f/S'_f$.

Furthermore, the negative distortional effects of lobbying worsen the climate problem and reduce global welfare compared to a “perfect” world without lobbying (Proposition 2b).

To explain why lobbying does not have a stabilizing effect for $\alpha = 0$, we analyze the effects of lobbying on taxes, quantities and (joint) welfare levels in Lemma 2 of Appendix B.2. We show that the tax burden motives outweigh the market power motives if the households receive all tax revenues, so that lobbying then reduces both carbon taxes. However, it does not affect the difference between the carbon taxes. Consequently, production and consumption increase, but the difference between the consumption levels stays unchanged. Even without lobbying, the carbon taxes are too small to maximize global welfare and since lobbying decreases the carbon taxes further, the welfare of each country decreases. However, lobbying does not affect the difference between the welfare levels. These results imply that lobbying without tax revenue distribution to the firms worsens the climate problem and reduces global welfare for a given coalition size. Furthermore, lobbying has no qualitative effect on the free-rider incentives, as the differences between the carbon taxes, the consumption levels and the welfare levels do not change.¹⁴

Continuing with the case where firms get back their paid taxes ($\alpha = 1$), the influence of lobbying on the effects of an increase in the coalition size and on the stable coalition size are quite involved. Clearly, the results coincide with those from Proposition 1 (and Lemma 1 of Appendix A.2) if the lobby influence is sufficiently weak ($\mu \rightarrow 0$). In the remainder of this section, we thus focus on a lobby influence that is sufficiently strong to change some results. Defining¹⁵

$$\underline{\mu} := \frac{n(n-2)b + (n-1)^2}{2nb + 2n - 1} \frac{b+1}{b} > \frac{n-2}{2}, \quad (24)$$

we prove in Appendix B.5

Proposition 3. Consider $\alpha = 1$ and $\mu > \underline{\mu}$. For $m < [=]n - 1$ each coalition country's carbon tax is smaller [greater] than mh , and each fringe country's carbon tax is positive [negative].

- $m = n$ is internally stable.
- Compared to the unique equilibrium without lobbying, global emissions are smaller, the welfare of each coalition country is greater if $a \leq 4nh$, and the welfare of each fringe country and global welfare are greater if $a \leq 3nh$ in this equilibrium with industrial lobbying.
- Compared to the social optimum, global emissions are smaller [greater] if $a \geq 4nh$ [$a \leq 2nh$] in this equilibrium with industrial lobbying.

Propositions 1 and 3 show that the climate coalition underinternalizes its climate externality for $m < n - 1$, i.e. $\chi_e < mh$, both without lobbying and with a sufficiently strong lobby influence. Furthermore, since the lobby groups in the fringe countries prefer carbon taxes to raise the commodity price, a sufficiently strong lobby influence prevents carbon subsidies in the fringe countries for $m < n - 1$. While BCA turned out to be ineffective in stabilizing large climate coalitions without lobbying (Proposition 1a) and with lobbying and tax revenue distribution to the households (Proposition 2a), Proposition 3a reveals that a sufficiently strong lobby influence stabilizes the grand coalition if tax revenues are given back to the firms. If there is only one fringe country ($m = n - 1$), the fringe country chooses a negative carbon tax as in case of large coalitions without lobbying (Proposition 1), and the coalition overinternalizes its climate externality.¹⁶ As a result, commodity price and gross profits are lower than in the grand coalition, which is therefore stabilized if lobbying is sufficiently strong. Then, lobbying reduces global emissions, and it increases the welfare of each country for sufficiently large marginal climate costs (Proposition 3b).¹⁷ Large marginal climate costs imply that the positive environmental effects outweigh the negative distortional effects of lobbying. By contrast, if tax revenues are transferred to the households, lobbying increases global emissions and reduces the welfare of each country compared to a “perfect” world without lobbying (Proposition 2b). Finally, global emissions can even be smaller than at the social optimum for sufficiently small marginal climate costs (Proposition 3c). In this case, the grand coalition overinternalizes the climate externality, i.e. $\chi_e > nh$, to reduce the commodity supply and, thus, to raise the commodity price.

To explain why lobbying has a stabilizing effect for $\alpha = 1$, we analyze the effects of lobbying on taxes, quantities and (joint) welfare levels in Lemma 4 of Appendix B.4. We show that the market power motives outweigh the tax burden motives if the firms get back their paid taxes, so that lobbying then reduces the difference between the carbon taxes. Furthermore, it reduces the coalition's carbon tax with a sufficiently strong lobby influence.¹⁸ Consequently, consumption of each coalition country increases, and this larger consumption outweighs the potentially smaller consumption of each fringe country, such that production and, thus, global

¹⁴ In Lemma 3 of Appendix B.2, we show that most results of Lemma 1 remain valid for $\alpha = 0$ and $\mu > 0$. Furthermore, the joint welfare of each country, global joint welfare, and the joint welfare difference between a fringe country and a coalition country are increasing in the coalition size.

¹⁵ At first glance, $\underline{\mu}$ seems to be very large, e.g. $\underline{\mu} > 9$ for $n = 20$, which means the government values lobby contributions nine times more than social welfare. However, Mitra et al. (2006, p. 208) empirically find that the US government and the Turkish government would be pure lobby contribution maximizers ($\mu \rightarrow \infty$) if the proportion of the politically organized population were greater than about 94%–98% and 55%–66%, respectively. Since Goldberg and Maggi (1999, p. 1147) and Gawande and Bandyopadhyay (2000, p. 146) cannot reject the null hypothesis that the entire US population is politically organized, and Mitra et al. (2002, p. 504) find that 61%–91% of the Turkish population is politically organized, $\mu > \underline{\mu}$ seems not to be unrealistically large. (In addition, Gawande and Bandyopadhyay (2000, p. 147) report $\mu = 550$ and $\mu = 485$ for the US sugar quota of 1983 and the US dairy subsidy of 1984, respectively.)

¹⁶ The reason for the overinternalisation in the coalition is that χ_f becomes a substitute for χ_e if lobbying is sufficiently strong, see (22). The negative carbon tax in the fringe country then induces the coalition to choose a high carbon tax.

¹⁷ For $n \geq 17$, the welfare of each country and global welfare are greater with lobbying than without if $a \leq 4nh$.

¹⁸ Then, the negative direct effect of lobbying on χ_e in the second line of (22) is strong and outweighs the potential positive indirect effect of lobbying via χ_f on χ_e in the first line of (22).

Table 1

Values for stable coalitions $m = 3$, $m = 4$, $m = 18$ and $m = 20$ for $n = 20$, $a = 80$, $b = 286$ and $\mu = 13.5$. x denotes global production, W global welfare, and V joint global welfare.

Coalition size	m	3		4		18		20	
		$\mu = 0$	$\mu > 0$	$\mu = 0$	$\mu > 0$	$\mu = 0$	$\mu > 0$	$\mu = 0$	$\mu > 0$
Stable		Yes	No	No	Yes	No	Yes	No	Yes
Taxes	χ_f	0.004	0.016	0.004	0.017	-0.050	0.221	–	–
	χ_e	2.993	2.411	3.989	3.210	17.98	12.42	20.00	39.22
	τ_e	2.989	2.395	3.985	3.193	17.99	12.20	–	–
Prices	p	0.281	0.294	0.279	0.294	0.173	0.461	20.21	39.36
	$p + \tau_e$	3.270	2.688	4.265	3.486	18.21	12.66	–	–
	$p - \chi_f$	0.277	0.277	0.276	0.276	0.222	0.240	–	–
Quantities	y_f	0.279	0.279	0.279	0.279	0.279	0.278	–	–
	y_e	0.268	0.270	0.265	0.268	0.216	0.235	0.209	0.142
	x	5.543	5.549	5.519	5.529	4.447	4.795	4.181	2.842
Welfare	W_f	5.606	5.601	5.631	5.621	6.707	6.347	–	–
	W_e	5.591	5.591	5.603	5.603	6.138	6.087	6.272	5.628
	W	112.1	112.0	112.5	112.3	123.9	122.3	125.4	112.6
Joint welfare	V_f	5.606	6.182	5.631	6.201	6.707	7.450	–	–
	V_e	5.591	6.172	5.603	6.183	6.138	7.190	6.272	81.00
	V	112.1	123.6	112.5	124.0	123.9	144.3	125.4	1620
Gross profits	$U_e = U_f$	0.039	0.043	0.039	0.043	0.014	0.082	4.203	5.583

emissions increase for a given coalition size. Finally, the consumption difference and the (joint) welfare difference between a fringe country and a coalition country are proportional to the tax difference, so that lobbying reduces these differences and, thus, the free-rider incentives.

Furthermore, we analyze the effects of an increase in the coalition size with a sufficiently strong lobby influence in [Lemma 5](#) of [Appendix B.4](#). Without lobbying, the fringe countries' carbon taxes always decrease in m and become negative for $m > 0.62n$ ([Propositions 1](#) and [1](#) of [Appendix A.2](#)), whereas they are always positive and increase in m for $m < n - 1$ with lobbying if $\mu > \mu$. The reason for this increase is that the positive effect of the fringe countries' carbon taxes on the commodity price increases with the coalition size, since then the commodity supply of more coalition countries is reduced via the BCA, and that the lobby groups in the fringe countries prefer higher carbon taxes when the commodity price reacts more strongly to tax changes (market power motive). Finally, the increase in the fringe's carbon tax induced by an increase in the coalition size outweighs the potential decrease in the difference between the coalition's and the fringe's carbon tax, such that each fringe country's consumption decreases with m . This reduces the free-rider incentives in case of large climate coalitions.

Summing up, if tax revenues are given back to the firms, a sufficiently strong lobby influence reduces the free-rider incentives because it reduces the (joint) welfare difference for a given coalition size and because it reduces each fringe country's consumption when the coalition becomes larger. Note that there is no BCA in case of the grand coalition. Thus, the implicit consumption tax τ_e becomes zero, which raises demand, and the implicit production tax χ_e becomes large, because the carbon-leakage and the terms-of-trade effects vanish and because all lobbies prefer positive taxes if they get back the corresponding expenses, which reduces supply. As a result, commodity price and gross profits are higher than for any other coalition size, which stabilizes the grand coalition if the lobby influence is sufficiently strong. In other words, the presence of BCA beyond the grand coalition stabilizes it.

While [Proposition 2](#) characterizes the unique equilibrium with lobbying for $\alpha = 0$, [Proposition 3](#) characterizes the grand-coalition equilibrium with lobbying for $\alpha = 1$ but does not cover other possible equilibria. In the following, we therefore use a numerical example to show other possible stable coalitions for $\alpha = 1$. In this example, we normalize $h = 1$ and set $n = 20$, $a = 4nh = 80$, $b = 286$ and $\mu = 13.5 > \underline{\mu} = 9.03$. The stable coalitions with lobbying are $m = 4$, $m = 18$ and $m = n$,¹⁹ while the only stable coalition without lobbying is $m = 3$ ([Proposition 1](#)). We present the taxes, prices, quantities, (joint) welfare levels and gross profits at these coalition sizes with and without lobbying in [Table 1](#).

We start with comparing the taxes in coalition and fringe countries. We see that χ_f decreases [increases] with the size of the stable coalition without [with] lobbying. By contrast, χ_e and τ_e increase with the size of the stable coalition both with and without lobbying.²⁰ For a given coalition size, lobbying raises the tax level in the fringe countries and it reduces the tax level in the coalition countries, which narrows the wedge between the carbon taxes for $m \leq 18$. However, we see that the tax level in the grand coalition is almost twice the Pigouvian level: Since there is no BCA in case of the grand coalition, χ_e becomes an implicit production tax, and the lobby groups prefer a high tax level to reduce supply and thereby increase the price.

¹⁹ In [Appendix B.6](#), we derive the parameter restrictions and conduct a sensitivity analysis. Result [1](#) of [Appendix B.6](#) shows that $m = n - 2$ is stable and $m \in [3, 8]$ is large if a/h and μ are sufficiently large and b and n are sufficiently small.

²⁰ [Fig. 1](#) in [Appendix B.6](#) shows that with lobbying χ_e is lower for $m = 18$ than for $m = 17$.

These high tax levels in case of the grand coalition also explain why consumer prices are highest for $m = 20$. For $m \leq 18$, we see that the consumer price in the fringe countries p decreases [increases] with the size of the stable coalition without [with] lobbying. Thus, with lobbying the price-raising effect of an increasing χ_f outweighs the price-reducing effect of an increasing χ_e . However, the consumer price in the coalition countries $p + \tau_e$ increases and the producer price $p - \chi_f$ decreases with the size of the stable coalition both with and without lobbying. For a given coalition size, lobbying raises p and reduces $p + \tau_e$, which narrows the wedge between the consumer prices.

The effects of m and μ on the consumer prices are directly reflected in their effects on the respective consumption levels. In particular, lobbying reduces the consumption difference between fringe and coalition countries. Production decreases with increasing coalition size both with and without lobbying, but for a given coalition size, production (and the amount of global emissions, which is proportional to production) is greater with lobbying than without for $m \leq 18$. In the grand coalition, the overinternalization of the climate externality with lobbying leads to remarkably lower consumption and production levels than without lobbying.²¹

As a result, with lobbying welfare is only slightly higher in the grand coalition than for $m = 3$ and $m = 4$, where the climate externality is strongly underinternalized. With lobbying, joint welfare (and gross profits) are much lower for $m = 18$ than in the grand coalition, but welfare is much higher, because the climate externality is not overinternalized. In other words, the grand coalition is stable with lobbying and it raises welfare compared to the unique equilibrium without lobbying, but coalitions $m \in (3, n)$ can also be stable with lobbying and can even outperform the grand coalition in terms of welfare. Without lobbying, we do not see this welfare detrimental effect of overinternalization and welfare of coalition and fringe countries (and thus also global welfare) is highest in the grand coalition, lowest for $m = 3$ and greater for $m = 18$ than for $m = 4$. Not surprisingly, for a given coalition size welfare is lower and gross profits are higher with lobbying than without. Furthermore, gross profits are much higher in the grand coalition than for any other coalition size. The reason for this is that firms get back their paid taxes and thus benefit from the high price in the grand coalition.

The comparison of key values with and without lobbying reveals why lobbying stabilizes coalitions $m = 4$, $m = 18$ and $m = 20$. For a given coalition size, lobbying raises the tax level in the fringe countries and reduces the tax level in the coalition countries (Lemma 4 of Appendix B.4).²² This reduces the consumption difference and, thus, the (joint) welfare difference, which can be decisive for coalition stability in particular for small coalition sizes with low absolute (joint) welfare differences, and thereby stabilizes the coalition $m = 4$. Furthermore, lobbying leads to a positive correlation between the coalition size and the fringe countries' carbon taxes (Lemma 5 of Appendix B.4) and, even more important, to a negative correlation between the coalition size and the coalition's carbon taxes in the transition from $m = 17$ to $m = 18$. This leads to a sharp increase in the joint welfare of each coalition country, which stabilizes coalition $m = 18$. Firms profit from the high price in the grand coalition. With lobbying, this is relevant for coalition stability, since lobbying induces governments to take gross profits into account when making membership decisions, which helps to stabilize the grand coalition.

Summing up, our numerical example shows that for given coalition sizes, global emissions are greater with lobbying than without for $m \leq 18$ (Lemma 4). Thus, lobbying exacerbates the underinternalization of the climate externality. This effect outweighs the reduced consumption distortion due to the reduced tax difference with lobbying, such that global welfare is smaller with lobbying than without for given coalition sizes. However, the stable coalitions are larger with lobbying than without, and global emissions are smaller and global welfare is greater with lobbying in coalitions $m = 4$, $m = 18$ and $m = 20$ than without lobbying in coalition $m = 3$, although the market power motive leads to an overinternalization of the climate externality in case of the grand coalition with lobbying (Proposition 3c).

5. Extensions

Given these positive results of lobbying on coalition size and even on equilibrium welfare if tax revenues are given back to the firms, we study equilibrium policies and coalition formation with lobbying at both stages of the game but without BCA in Online Appendix C to check whether these effects depend on the implementation of BCA or do hold more generally. The most important results of this analysis are the following: Without BCA lobbying does not have a stabilizing effect and does not lead to larger coalitions than those emerging in the standard case without lobbying, but can even be destabilizing (Proposition C.1). Furthermore, without [with] lobbying the welfare of each country and global welfare are higher [lower] in the equilibrium without BCA than in the $m = 3$ [$m = n$] equilibrium with BCA [if marginal climate costs are above a minimum threshold] (Proposition C.2). Thus, the presence of industrial lobbying speaks for the introduction of BCA if tax revenues are given back to the firms.

Following Habla and Winkler (2013), so far we have considered lobbying at both stages of the game. Thereby, our focus was on the influence of lobbying on coalition stability via its influence on carbon taxes at the second stage of the game. However, lobbying

²¹ We do not report lobbying contributions in Table 1 because they are irrelevant for the stability analysis. At the second stage of the game, contributions reflect the welfare difference with and without lobbying in the respective country. In our example, the policy distortions and, thus, the contributions at the second stage of the game increase with the coalition size in each country. At the first stage of the game, contributions are positive if and only if the respective lobby group prefers the membership decision of its government over the alternative regime, i.e. leaving [joining] the coalition in case of a coalition [fringe] country. In our example, net profits at the second stage of the game increase for $m \in [1, 3]$ and $m \in [11, 18]$, and they decrease for $m \in [3, 11]$, such that contributions at the first stage of the game are positive in each coalition country for $m \in [1, 3]$ and $m \in [12, 18]$, and they are positive in each fringe country for $m \in [3, 10]$. The sum of first and second stage contributions is higher in fringe countries than in coalition countries for $m \in [4, 10]$, and total contributions are highest in the grand coalition.

²² Figs. 1 and 2 in Appendix B.6 depict the tax levels with and without lobbying for all possible coalition sizes.

also affects coalition stability at the first stage of the game: Since a government receives political contributions at the first stage of the game if and only if its lobby group prefers the current regime, industrial lobbying stabilizes a coalition for a given policy if and only if profits increase with the coalition size. In Online Appendix D, we study lobbying only at the first stage of the game to separate the effects of lobbying on the membership decision from the effects of lobbying on the policy decision. It turns out that for almost any coalition size, profits decrease when the coalition becomes larger because both output and price decrease with the coalition size. Consequently, first-stage lobbying does not stabilize medium-sized coalitions $m \in [4, n - 1]$, but can destabilize small-sized coalitions $m \in [1, 3]$. With regard to the grand coalition, things are somewhat different. Without second-stage lobbying, the grand coalition maximizes global welfare by setting the carbon tax equal to global marginal climate costs. When a country leaves the grand coalition, it opts for a carbon subsidy that reduces the coalition's carbon tax to zero. If tax revenues are transferred to the households ($\alpha = 0$), then profits will increase as a result of the exit, and the grand coalition is unstable as in the case without lobbying. By contrast, if tax revenues are given back to the firms ($\alpha = 1$), then profits will fall as a result of the exit, which can stabilize the grand coalition. In particular, the grand coalition is stable if and only if α and μ are sufficiently large. Then, first-stage lobbying leads to global welfare maximization.

Finally, we check whether environmental lobbying alters the positive results of industrial lobbying in Online Appendix E. Most results of the paper remain valid with environmental lobbying. In particular, with BCA only $m = 3$ is stable with environmental lobbying in the absence of industrial lobbying, and $m = n$ is stable if and only if $\mu > \mu$ with environmental lobbying in the presence of industrial lobbying if tax revenues are given back to the firms. Finally, without BCA environmental lobbying has no influence on the size of the stable coalition. In fact, environmental lobbying raises the perceived climate costs in all countries, which affects the size of the carbon taxes, but does not affect the tax difference qualitatively.

6. Discussion

When designing climate policy, the impact and consequences of lobby influence should be taken into account. Our analysis shows that industrial lobbying can promote international cooperation in the presence of BCA, because BCA changes the implications of climate policies for the profits of affected firms. The distribution of carbon tax revenues plays a major role. When these are returned to households, emissions rise and welfare falls due to the industrial lobbying. However, when tax revenues are transferred to firms, the coalition becomes larger, which can reduce emissions and increase welfare. These results suggest that at least some tax revenues should be returned to firms in order to incentivize firms so that lobby influence helps mitigate climate change. Furthermore, our results suggest that countering industrial lobbying on the policy decision can be useful to suppress market power motives, while industrial lobbying only on the participation decision can lead to the implementation of the social optimum.

Our analysis considers competitive firms that cooperate in concentrated lobby groups, which gives them the opportunity to integrate their market power motive into climate policy making. They benefit from higher prices, which can lead to positive effects on climate policy as we have shown above. Nevertheless, we want to stress the fact that lobbying also has negative welfare effects, which can be severe. In our analysis, this is the case through both the tax burden motive and the market power motive (which can for example lead to a strong overinternalization of the climate externality in the grand coalition, see [Proposition 3](#)). It is important to note that we abstract from other regulatory dimensions beyond climate policy. The ability of industrial lobby groups to influence national and international policy can also lead to welfare detrimental policy distortions in other dimensions, both in the realm of environmental regulations (e.g. regulation of local pollutants, prevention of biodiversity loss) and in other fields (e.g. human rights, health).²³ The question of how the distribution of revenues and costs between lobbying firms and households affects such distortions remains a subject for future research.

In order to focus on our main research question in a tractable model, we have made a number of simplifying assumptions, which we discuss in the following. As typically assumed in the literature on self-enforcing environmental agreements, we assume ex-ante identical countries. Some contributions have studied the effect of asymmetry in benefits and costs from emissions ([Fuentes-Albero and Rubio, 2010](#); [Glanemann, 2012](#); [Pavlova and De Zeeuw, 2013](#); [Finus and McGinty, 2019](#)). In these models, asymmetry stabilizes coalitions if and only if benefits from own emissions and climate costs (or costs from own abatement and benefits from global abatement) are negatively correlated. Such stabilizing effects of asymmetric benefits and costs from emissions could potentially also occur with BCA and lobbying and could be studied with the help of calibrated simulation models if the increasing complexity of the model makes it hard to derive clear-cut analytical results.

The linear-quadratic specification with additively separable climate costs is also chosen in line with the literature in order to obtain analytical results. Exceptions in the literature that deviate from this assumption are [Nkuiya \(2020\)](#) with isoelastic benefits from own emissions, [Eckert and Nkuiya \(2022\)](#) with isoelastic climate costs and [Karp and Simon \(2013\)](#) with linear abatement benefits and convex abatement costs. In these models, coalitions can be larger than with the linear-quadratic specification. In integrated assessment models, it is typically assumed that production is a multiplicative function of benefits from global abatement and costs from own abatement. For numerical results on coalition stability in these models see, e.g., [Lessmann et al. \(2015\)](#) and [Marrouch and Chaudhuri \(2016\)](#). Incorporating BCA and lobbying in such numerical models could provide a way to obtain quantitative results if the derivation of analytical results is not feasible. One potential challenge in such an attempt would be the calibration of the lobby influence.

²³ Examples of such lobby activities include the weakening of European air pollution rules ([Myllyvirta, 2015](#)) and the Endangered Species Act ([InfluenceMap, 2022a](#)), targeting health-promoting proposals and policy recommendations from the WHO ([Russ et al., 2022](#)) and opposing international legally binding rules to prevent human rights violations related to activities of transnational corporations ([Martens, 2014](#)).

Concerning the timing of the game, we assume that countries simultaneously decide about their climate policies. An alternative to this Nash assumption is that coalition countries are Stackelberg leaders while fringe countries are Stackelberg followers. In most models, coalitions are small when countries play Nash, and they are always larger when coalition countries are Stackelberg leaders and fringe countries are Stackelberg followers (Finus et al., 2024). From these results we infer that our Nash assumption leads to conservative results concerning the effectiveness of BCA in the presence of lobbying.

We follow Eichner and Pethig (2013, 2014, 2015a,b) in assuming that there is only one polluting commodity to focus on the effects of lobbying and allow for analytical results. Harstad (2012) and Eichner and Pethig (2013) show that this model can be extended to a general equilibrium model with a second non-polluting commodity without changing the results. Extending the analysis to more than one commodity could allow to analyze settings in which countries import and export polluting commodities at the same time. Then, one could distinguish between import and export BCA. Such an expanded framework would come at the expense of analytical tractability of the model, which could be addressed by assuming inelastic demand Helm and Schmidt (2015), conducting numerical stability analysis (Al Khourdajie and Finus, 2020) or restricting the stability analysis to the grand coalition only (Baksi and Chaudhuri, 2020). This task remains for future research.

We also make the simplifying assumption that production and emissions are proportional. This means that the only way to curb emissions is to reduce production. We thus abstract from alternative ways of emission reductions such as carbon capture and storage or clean technologies. In a model with international trade, one could distinguish between production and emissions. In particular, the cost function could be a function of production x_i and emissions z_i , i.e. $C(x_i, z_i)$ with $\partial C/\partial x_i > 0$, $\partial^2 C/\partial x_i^2 > 0$, $\partial C/\partial z_i < 0$, $\partial^2 C/\partial z_i^2 > 0$ and $(\partial^2 C/\partial x_i^2)(\partial^2 C/\partial z_i^2) - [\partial^2 C/(\partial x_i \partial z_i)]^2 > 0$. For a given commodity price, a higher carbon tax would then reduce emissions, and it would reduce production if and only if production and emissions are substitutes, i.e. $\partial^2 C/(\partial x_i \partial z_i) < 0$. In this case, we would expect that the results will not change qualitatively.

7. Conclusion

In this paper we examine the influence of producer lobbies on climate policies and coalition formation with BCA in a partial equilibrium model with international trade. Countries use carbon taxes to regulate carbon emissions and the climate coalition implements BCA to reduce carbon leakage. Our findings highlight a novel mechanism that has the potential to stabilize large climate coalitions. If tax revenues are given back to the firms, lobbies in fringe countries prefer positive taxes to increase the price by reducing supply in their countries and, through the effect of BCA, in the coalition countries. In coalition countries, lobbies prefer a carbon subsidy to effectively increase demand, which again increases the price. Thus, lobbying in all countries *ceteris paribus* reduces the tax difference and the welfare difference between members and outsiders and thereby the free-rider incentives. This helps to stabilize larger coalitions with lower global emissions. If climate damages are severe, this reduction in emissions outweighs the distortional effects of lobbying and leads to higher global welfare than in the equilibrium without lobbying. However, if tax revenues are transferred to the households, lobbies in all countries prefer carbon subsidies to reduce the tax burden, which leads to higher global emissions without affecting free-rider incentives. Thus, whether or not lobbying stabilizes larger coalitions in the presence of BCA crucially depends on the distribution of carbon tax revenues: If these are given back to the firms, market power motives dominate and coalitions can become large and effective. By contrast, if the carbon tax revenues are transferred to the households, tax burden motives dominate and coalitions remain small and become even less effective in the fight against climate change than without lobbying.

As the introduction of BCA raises on the political agenda, our findings are highly relevant for policymakers considering the pros and cons of BCA. We reveal a channel through which the political distortion of nationally welfare maximizing governments in case of protection of trade-exposed industries through BCA can lead to a globally improved situation, which might seem counterintuitive at first sight. In this case, political influence turns “green protection” into environmental protection.

Funding

Achim Hagen has received funding by the German Federal Ministry of Education and Research (BMBF) under grant number 01LA1811C and grant number 01UU2205A. We have not received any further financial support for the conduct of the research and/or preparation of the article from any third party.

CRedit authorship contribution statement

Achim Hagen: Writing – original draft, Writing – review & editing, Conceptualization, Formal analysis, Methodology, Visualization. **Mark Schopf:** Conceptualization, Formal analysis, Methodology, Visualization, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Appendix to Section 3

A.1. Derivation of Eqs. (7) and (9)

Accounting for $\tau_e = \chi_e - \frac{\sum_{j \in F} \chi_j}{n-m}$, the first-order condition of (6) with respect to χ_i reads

$$0 = -[p - C'_i - H'_i]S'_i + \frac{H'_i \sum_{j \in E} S'_j}{n-m} + \left\{ [B'_i - p]D'_i + [p - C'_i]S'_i + x_i - y_i - H'_i \sum_{j \in N} S'_j \right\} \frac{\partial p}{\partial \chi_i}. \quad (\text{A.1})$$

Substituting $p = B'_i = C'_i + \chi_i$ for $i \in F$ from (2) and rearranging yields

$$0 = -[\chi_i - H'_i]S'_i + \frac{H'_i \sum_{j \in E} S'_j}{n-m} + \left[\chi_i S'_i + x_i - y_i - H'_i \sum_{j \in N} S'_j \right] \frac{\partial p}{\partial \chi_i} \quad (\text{A.2})$$

$$\Leftrightarrow \chi_i = H'_i - \frac{\frac{\partial p}{\partial \chi_i}}{1 - \frac{\partial p}{\partial \chi_i}} \frac{\sum_{j \in N \setminus i} S'_j}{S'_i} H'_i - \frac{\frac{\partial p}{\partial \chi_i}}{1 - \frac{\partial p}{\partial \chi_i}} \frac{1}{S'_i} (y_i - x_i) + \frac{1}{1 - \frac{\partial p}{\partial \chi_i}} \frac{\sum_{j \in E} S'_j}{(n-m)S'_i} H'_i. \quad (\text{A.3})$$

By making use of symmetry, i.e. $\chi_i = \chi_f$, $D_i = D_f$ for all $i \in F$ and $S_i = S_f = S_e$, $H_i = H$ for all $i \in N$, (A.3) is equivalent to (7). Furthermore, rearranging (7) yields

$$\begin{aligned} \chi_f = & \frac{m+2}{2} H' - \frac{\{(m+2)(n-m-1)[nS'_i - mD'_e] - m(n-m)(n-m-2)D'_f\} H'}{2\{(n-m-1)[nS'_i - mD'_e] - (n-m)^2 D'_f\}} \\ & - \frac{[nS'_i - mD'_e](y_f - x_f)}{S'_i \{(n-m-1)[nS'_i - mD'_e] - (n-m)^2 D'_f\}} < \frac{m+2}{2} H' \Leftarrow m \leq n-2. \end{aligned} \quad (\text{A.4})$$

Accounting for $\tau_e = \chi_e - \frac{\sum_{j \in F} \chi_j}{n-m}$, the first-order condition of (8) with respect to χ_e reads

$$0 = \sum_{j \in E} [B'_j - p]D'_j + \sum_{j \in E} \left\{ [B'_j - p]D'_j + [p - C'_j]S'_j + x_j - y_j - H'_j \sum_{j \in N} S'_j \right\} \frac{\partial p}{\partial \chi_e}. \quad (\text{A.5})$$

Substituting $p = B'_i = \frac{\sum_{j \in F} \chi_j}{n-m} = C'_i + \chi_e - \frac{\sum_{j \in F} \chi_j}{n-m}$ for $i \in E$ from (2) and rearranging yields

$$\begin{aligned} 0 = & \left[\chi_e - \frac{\sum_{j \in F} \chi_j}{n-m} \right] \sum_{j \in E} D'_j \left[1 + \frac{\partial p}{\partial \chi_e} \right] + \sum_{j \in E} \left[\frac{\sum_{j \in F} \chi_j}{n-m} S'_j + x_j - y_j - H'_j \sum_{j \in N} S'_j \right] \frac{\partial p}{\partial \chi_e} \\ \Leftrightarrow \chi_e = & \frac{\sum_{j \in F} \chi_j}{n-m} + \frac{\frac{\partial p}{\partial \chi_e}}{1 + \frac{\partial p}{\partial \chi_e}} \frac{\sum_{j \in F} S'_j}{\sum_{j \in E} D'_j} \sum_{j \in E} H'_j - \frac{\frac{\partial p}{\partial \chi_e}}{1 + \frac{\partial p}{\partial \chi_e}} \frac{1}{\sum_{j \in E} D'_j} \sum_{j \in E} (x_j - y_j) \\ & + \frac{\frac{\partial p}{\partial \chi_e}}{1 + \frac{\partial p}{\partial \chi_e}} \frac{\sum_{j \in E} S'_j}{\sum_{j \in E} D'_j} \left[\sum_{j \in E} H'_j - \frac{\sum_{j \in F} \chi_j}{n-m} \right]. \end{aligned} \quad (\text{A.7})$$

By making use of symmetry, i.e. $\chi_i = \chi_f$ for all $i \in F$, $D_i = D_e$ for all $i \in E$ and $S_i = S_f = S_e$, $H_i = H$ for all $i \in N$, (A.7) is equivalent to (9). Furthermore, substituting (7) into (9) and rearranging yields

$$\begin{aligned} \chi_e = mH' + & \frac{(n-m)\{[(m-1)(n-m-2) + m-2][nS'_i - (n-m)D'_f] - (n-m-1)m^2 D'_e\} D'_f H'}{[nS'_i - (n-m)D'_f]\{(n-m-1)[nS'_i - mD'_e] - (n-m)^2 D'_f\}} \\ & - \frac{m\{S'_i[nS'_i - mD'_e - (n-m)D'_f] - D'_f[nS'_i - mD'_e]\}(x_e - y_e)}{(n-m)S'_i[nS'_i - mD'_e]\{(n-m-1)[nS'_i - mD'_e] - (n-m)^2 D'_f\}} < mH' \Leftarrow m \leq n-2. \end{aligned} \quad (\text{A.8})$$

Finally, substituting (7) and (9) into $\chi_e - \chi_f$ and rearranging yields

$$\begin{aligned} \chi_e - \chi_f = mH' + & \frac{m(n-m)[n(n-m)S'_i - m(n-m-1)D'_e - (n-m)^2 D'_f] D'_f}{[nS'_i - (n-m)D'_f]\{(n-m-1)[nS'_i - mD'_e] - (n-m)^2 D'_f\}} H' \\ & - \frac{[(n-m)^2 - n][nS'_i - mD'_e] - (n-m)^3 D'_f}{[nS'_i - (n-m)D'_f]\{(n-m-1)[nS'_i - mD'_e] - (n-m)^2 D'_f\}} (y_f - x_f) \end{aligned} \quad (\text{A.9})$$

$$\begin{aligned} = mH' + & \frac{m(n-m)[n(n-m)S'_i - m(n-m-1)D'_e - (n-m)^2 D'_f] D'_f}{[nS'_i - (n-m)D'_f]\{(n-m-1)[nS'_i - mD'_e] - (n-m)^2 D'_f\}} H' \\ & + \frac{[n - (n-m)^2]nS'_i - [m - (n-m)^2]nD'_e - (n-m)^3 (D'_e - D'_f)}{[nS'_i - (n-m)D'_f]\{(n-m-1)[nS'_i - mD'_e] - (n-m)^2 D'_f\}} (y_f - x_f), \end{aligned} \quad (\text{A.10})$$

where the second line of (A.9) is negative for $m \leq n - \sqrt{n}$, and the second line of (A.10) is positive for $m \geq n - 0.5(\sqrt{4n+1} - 1)$ and $D'' = -B'''/(B'')^3 \geq 0 \Leftrightarrow B''' \geq 0$.

The equilibrium with arbitrary carbon taxes is characterized by

$$B'(y_f) = B'(y_e) - (\chi_e - \chi_f), \quad (\text{A.11a})$$

$$B'(y_f) = C'(x_i) + \chi_f, \quad (\text{A.11b})$$

$$B'(y_e) = C'(x_i) + \chi_e, \quad (\text{A.11c})$$

$$(n-m)y_f + my_e = (n-m)x_f + mx_e, \quad (\text{A.11d})$$

where $\chi_f \leq \chi_e < mH'$ for $m \leq n-2$ in the coalition–fringe equilibrium. In Online Appendix C, we find $\chi_f = \chi_e = nH'$ in the fully cooperative equilibrium. Suppose x_i is weakly smaller in the coalition–fringe equilibrium than in the fully cooperative equilibrium. From (A.11b) and (A.11c), $B(y_f)$ and $B(y_e)$ are then smaller and, thus, y_f and y_e are then greater in the coalition–fringe equilibrium than in the fully cooperative equilibrium, which contradicts that x_i is weakly smaller in the coalition–fringe equilibrium than in the fully cooperative equilibrium. From (A.11a), $B(y_f)$ is weakly smaller than $B(y_e)$, such that y_f is weakly greater than y_e and, thus, greater than $y_f = y_e = x_i$ in the fully cooperative equilibrium.

A.2. Characterization of the equilibrium for $\mu = 0$

Consider $S'_i = 1$, $D'_i = -1/b$ and $H'_i = h$, such that $\frac{\partial p}{\partial \chi_i} = \frac{nb+m}{n(n-m)(b+1)}$, $\frac{\partial p}{\partial \chi_e} = -\frac{m}{n(b+1)}$ and $S''_i = D''_i = \frac{\partial^2 p}{\partial \chi_i^2} = \frac{\partial^2 p}{\partial \chi_e^2} = 0$. Then, the first-order conditions (A.2) and (A.6) become

$$0 = -\chi_i S'_i \left[1 - \frac{\partial p}{\partial \chi_i} \right] + H'_i S'_i \left[1 + \frac{m}{n-m} - n \frac{\partial p}{\partial \chi_i} \right] + [S_i(p - \chi_i) - D_i(p)] \frac{\partial p}{\partial \chi_i}, \quad (\text{A.12})$$

$$0 = (\chi_e - \chi_f) m D'_i \left[1 + \frac{\partial p}{\partial \chi_e} \right] + m \left[\chi_f S'_i - n H'_i S'_i + S_i(p - \chi_f) - D_i(p + \chi_e - \chi_f) \right] \frac{\partial p}{\partial \chi_e}, \quad (\text{A.13})$$

and the second-order conditions read

$$0 > -S'_i \left[1 - \left[\frac{\partial p}{\partial \chi_i} \right]^2 \right] - D'_i \left[\frac{\partial p}{\partial \chi_i} \right]^2 = -\frac{n(n-m-1)[n(n-m+1)(b+1)+1]b - (m-1)b - m^2}{n^2(n-m)^2b(b+1)}, \quad (\text{A.14})$$

$$0 > m S'_i \left[\frac{\partial p}{\partial \chi_e} \right]^2 + m D'_i \left[1 - \left[\frac{\partial p}{\partial \chi_e} \right]^2 \right] = -\frac{m(n^2b + n^2 - m^2)}{n^2b(b+1)}. \quad (\text{A.15})$$

From (A.14) [(A.15)], the second-order condition of each fringe [coalition] country is violated for $m = n-1$ [satisfied for $m \leq n$]. Differentiating the second-order condition of each fringe country with respect to m yields $\frac{2(nb+m)}{n(n-m)^3b} > 0$, and substituting $m = n-2$ into the second-order condition of each fringe country yields

$$0 > -\frac{3n^2b^2 + 2n(n+2)b - (n-2)^2}{4n^2b(b+1)}, \quad (\text{A.16})$$

which is satisfied for $b > b_1 := \frac{2\sqrt{n^2-2n+4}-(n+2)}{3n}$. Thus, the second-order condition of each fringe country is satisfied for $m \leq n-2$ if and only if $b > b_1$.

Next we analyze the interior solution for $m \leq n-2$. Then, the equilibrium with arbitrary carbon taxes is characterized by

$$a - by_e - (\chi_e - \chi_f) = a - by_f, \quad (\text{A.17a})$$

$$a - by_e = x_e + \chi_e, \quad (\text{A.17b})$$

$$a - by_f = x_f + \chi_f, \quad (\text{A.17c})$$

$$my_e + (n-m)y_f = mx_e + (n-m)x_f. \quad (\text{A.17d})$$

Solving for x_i and y_i yields

$$x_i = \frac{a - \chi_i}{b+1} - \frac{b}{b+1} \cdot (y_i - x_i), \quad (\text{A.18a})$$

$$y_i = \frac{a - \chi_i}{b+1} + \frac{1}{b+1} \cdot (y_i - x_i), \quad (\text{A.18b})$$

$$x = \frac{na - m\chi_e - (n-m)\chi_f}{b+1}, \quad (\text{A.18c})$$

where

$$y_e - x_e = -\frac{(n-m)(\chi_e - \chi_f)}{nb}, \quad (\text{A.19})$$

$$y_f - x_f = \frac{m(\chi_e - \chi_f)}{nb}. \quad (\text{A.20})$$

Substituting $S'_i = 1$, $D'_i = -1/b$ and $H'_i = h$ into (7) and (9) yields

$$\chi_f = \frac{n(n-m)h - (bn+m)(y_f - x_f)}{n(n-m-1)(b+1) + n-m}, \quad (\text{A.21})$$

$$\chi_e = \chi_f + \frac{mb(nh - \chi_f + y_e - x_e)}{nb + n - m}. \quad (\text{A.22})$$

Using (A.19) and (A.20), solving (A.21) and (A.22) for χ_f and χ_e yields

$$\begin{aligned} \chi_f &= \frac{n(n^2 - nm - m^2)h}{\Gamma} = h - \frac{n^2(n-m-1)\left[b - \frac{n^2-6n+4}{n(n-2)}\right]h}{\Gamma} \\ &\quad - \frac{h}{(n-2)^4\Gamma} [n(n-1)(N_7^2 + 8N_7 + 11)NM_2^3 + nm(2N_7^3 + 28N_7^2 + 116N_7 + 118)NM_2^2 \\ &\quad + nm^2(N_7^3 + 14N_7^2 + 55N_7 + 38)NM_2 + m^3(N_7^4 + 20N_7^3 + 148N_7^2 + 472N_7 + 527)], \end{aligned} \quad (\text{A.23})$$

$$\begin{aligned} \chi_e &= \frac{nm[n(n-m-1)b + (n^2 - nm - m^2)/m]h}{\Gamma} \\ &= mh - \frac{(n-m)[(m-1)NM_2^2 + 2(m^2 + m - 2)NM_2 + 3m^2 - 4]h}{\Gamma}, \end{aligned} \quad (\text{A.24})$$

where $N_i := n - i$, $NM_i := n - m - i$, and

$$\chi_e - \chi_f = \frac{n^2m(n-m-1)bh}{\Gamma}, \quad (\text{A.25})$$

where

$$\Gamma := n^2(n-m-1)b + (n+m)(n-m)^2 - nm, \quad (\text{A.26})$$

$$\frac{\partial \Gamma}{\partial m} = -n^2b - (n+3m+1)(n-m) - m < 0. \quad (\text{A.27})$$

From (A.25), $\chi_e - \chi_f$ is positive if and only if Γ is positive. Since Γ decreases with m , $\chi_e - \chi_f$ is positive for all $m \leq n-2$ if and only if Γ is positive for $m = n-2$. This is the case if and only if $b > \underline{b}_2 := \frac{n^2-10n+8}{n^2}$. From (A.24), χ_e is positive if and only if $n(n-m-1)b + (n^2 - nm - m^2)/m$ is positive. Since this term decreases with m , χ_e is positive for all $m \leq n-2$ if and only if this term is positive for $m = n-2$. This is the case if and only if $b > \underline{b}_3 := \frac{n^2-6n+4}{n(n-2)}$. Furthermore,

$$\underline{b}_3 - \underline{b}_2 = \frac{6N_4^2 + 24N_4 + 16}{n^2(n-2)}, \quad (\text{A.28})$$

$$\underline{b}_3 - \underline{b}_1 = \frac{2N_6^3 + 16N_6^2 + 30N_6 + 4}{(n-2)[(n-4)(2n-1) + (n-2)\sqrt{n^2-2n+4}]}, \quad (\text{A.29})$$

Thus, $\chi_e > 0$ implies $\chi_e - \chi_f > 0$ and $\frac{\partial^2 W_f}{\partial \chi_f^2} < 0$ for $m \leq n-2$ and $n \geq 6$.

Lemma 1 summarizes the effects of an increase in the coalition size:

Lemma 1. Consider $\alpha \in [0, 1]$, $\mu = 0$ and $m \leq n-2$. Then, an increase in the coalition size has the following effects:

- Each coalition country's carbon tax is increasing in m for $m \leq n-4$, each fringe country's carbon tax is decreasing in m , and the tax difference between a coalition country and a fringe country is increasing in m .
- Each country's production is decreasing in m for $m \leq n-4$.
- Each coalition country's consumption is decreasing in m for $m \leq n-4$, each fringe country's consumption is increasing in m , and the consumption difference between a fringe country and a coalition country is increasing in m .
- Each fringe country's welfare is increasing in m , and each coalition country's welfare and global welfare are increasing in m for $m \leq n-5$.
- The welfare difference between a fringe country and a coalition country is increasing in m .

Proof. Differentiating (A.23), (A.24) and (A.25) with respect to m yields

$$\begin{aligned} \frac{\partial \chi_f}{\partial m} &= -\frac{nh}{(n-2)\Gamma^2} \{n^2(n-2)[(2m-1)NM_2 + m^2 + m - 2](b - \underline{b}_3) + (2m-2)NM_2^4 + (10m^2 - 6m - 8) \\ &\quad \cdot NM_2^3 + (16m^3 + 5m^2 - 40m - 4)NM_2^2 + (9m^4 + 16m^3 - 38m^2 - 40m + 16)NM_2 + (m-2)(m+1) \\ &\quad \cdot (m^3 + 8m^2 + 4m - 8)\} < 0, \end{aligned} \quad (\text{A.30a})$$

$$\begin{aligned} \frac{\partial \chi_e}{\partial m} &= \frac{nh}{(n-2)^2(n-4)^4\Gamma^2} \{n^3(n-2)^2(n-4)^4(n-m-1)^2(b - \underline{b}_3)^2 + n(n-2)[3NM_4^5 + (9m+41)NM_4^4 \\ &\quad + (10m^2 + 98m + 213)NM_4^3 + (4m^3 + 81m^2 + 389m + 510)NM_4^2 + (22m^3 + 217m^2 + 664m + 520) \\ &\quad \cdot NM_4 + 29m^3 + 194m^2 + 408m + 128](b - \underline{b}_3) + n(2N_6^6 + 49N_6^5 + 485N_6^4 + 2456N_6^3 + 6592N_6^2 \end{aligned}$$

$$\begin{aligned}
& + 8600N_6 + 3952)NM_4^4 + n^2m(4N_6^5 + 84N_6^4 + 670N_6^3 + 2476N_6^2 + 4056N_6 + 2144)NM_4^3 + nm^2 \\
& \cdot (3N_6^6 + 84N_6^5 + 981N_6^4 + 5936N_6^3 + 18932N_6^2 + 28688N_6 + 14688)NM_4^2 + m^3(12N_6^6 + 300N_6^5 \\
& + 3184N_6^4 + 17912N_6^3 + 54272N_6^2 + 79520N_6 + 39808)NM_4 + nm^4(6N_6^4 + 116N_6^3 + 716N_6^2 \\
& + 1592N_6 + 944) \} > 0 \Leftarrow m \leq n-4,
\end{aligned} \tag{A.30b}$$

$$\begin{aligned}
\frac{\partial(\chi_e - \chi_f)}{\partial m} &= \frac{n^2 h}{\Gamma^2} [n^2(n-m-1)^2 b + NM_2^4 + (2m+7)NM_2^3 + (2m^2+9m+18)NM_2^2 + (5m^2+12m+20)NM_2 \\
& + (m+2)(m^2+4)]b > 0,
\end{aligned} \tag{A.30c}$$

which proves Lemma 1a.

Indicating the fully cooperative equilibrium by an asterisk and the fully uncooperative equilibrium by a circle, we find in Online Appendix C

$$x_i^* = y_i^* = \frac{a-nh}{b+1} < x_i^\circ = y_i^\circ = \frac{a-nh}{b+1} + \frac{n(n-1)h}{(n-1)b+n}. \tag{A.31}$$

Substituting (A.23) and (A.24) into (A.18), (A.19) and (A.20) yields

$$\begin{aligned}
x_i &= x_i^\circ - \frac{m[n^2(n-1)(n-2)(n-m-1)(b-\underline{b}_3) + (2NM_2 + 5m+5)NM_2^3]h}{(n-2)[(n-1)b+n]\Gamma} < x_i^\circ \\
&= x_i^* + \frac{n(n-m)(n-m-1)(n+m)h}{\Gamma} > x_i^*,
\end{aligned} \tag{A.32a}$$

$$\begin{aligned}
y_e &= y_i^\circ - \frac{nm[n(n-1)(n-2)^3(n-m-1)(b-\underline{b}_3) + (2N_4^4 + 21N_4^3 + 80N_4^2 + 128N_4 + 68)NM_2^2]h}{(n-2)^3[(n-1)b+n]\Gamma} \\
&\quad - \frac{nm[nm(N_4^3 + 8N_4^2 + 20N_4 + 12)NM_2 + m^2(2N_4^2 + 8N_4 + 4)]h}{(n-2)^3[(n-1)b+n]\Gamma} < y_i^\circ \\
&= y_i^* + \frac{n^2(n-m)(n-m-1)h}{\Gamma} > y_i^*,
\end{aligned} \tag{A.32b}$$

$$y_f = y_i^\circ + \frac{nm[mNM_2^2 + (m^2+3m-1)NM_2 + 2m^2+m-2]h}{[(n-1)b+n]\Gamma} > y_i^\circ > y_i^*. \tag{A.32c}$$

Differentiating with respect to m yields

$$\begin{aligned}
\frac{\partial x_i}{\partial m} &= -\frac{nh}{(n-2)(n-4)^4\Gamma^2} [2mn^2(n-2)(n-4)^4(n-m-1)^2(b-\underline{b}_3) + n^3(n-2)NM_4^4 + nm(2N_5^5 + 26N_5^4 \\
&\quad + 122N_5^3 + 258N_5^2 + 300N_5 + 268)NM_4^3 + nm^2(2N_5^5 + 35N_5^4 + 203N_5^3 + 469N_5^2 + 435N_5 + 328) \\
&\quad \cdot NM_4^2 + nm^3(8N_5^4 + 84N_5^3 + 270N_5^2 + 212N_5 + 156)NM_4 + m^4(4N_5^4 + 72N_5^3 + 224N_5^2 + 144N_5 \\
&\quad + 108)] < 0 \Leftarrow m \leq n-4,
\end{aligned} \tag{A.33a}$$

$$\begin{aligned}
\frac{\partial y_e}{\partial m} &= -\frac{n^2 h}{(n-2)\Gamma^2} [n^2(n-2)(n-m-1)^2(b-\underline{b}_3) + 2NM_4^5 + (4m+29)NM_4^4 + (3m^2+44m+164)NM_4^3 \\
&\quad + (m^3+21m^2+180m+448)NM_4^2 + (4m^3+46m^2+328m+580)NM_4 + 2m^3+32m^2+228m \\
&\quad + 272] < 0 \Leftarrow m \leq n-4,
\end{aligned} \tag{A.33b}$$

$$\frac{\partial y_f}{\partial m} = \frac{n^3 h}{\Gamma^2} [2mNM_2^2 + (6m-1)NM_2 + m^2+3m-2] > 0, \tag{A.33c}$$

$$\begin{aligned}
\frac{\partial(y_e - y_f)}{\partial m} &= -\frac{n^2 h}{\Gamma^2} [n^2(n-m-1)^2 b + NM_2^4 + (2m+7)NM_2^3 + (2m^2+9m+18)NM_2^2 + (5m^2+12m+20)NM_2 \\
&\quad + (m+2)(m^2+4)] < 0,
\end{aligned} \tag{A.33d}$$

which proves Lemma 1b and Lemma 1c. Note that

$$\begin{aligned}
\frac{\partial \left(\frac{\Gamma^2 \frac{\partial \chi_e}{\partial m}}{n^4(n-m-1)h} \right)}{\partial m} &= -b^2 - \frac{NM_2^4 + 9NM_2^3 + (2m^2+5m+25)NM_2^2 + (4m^2+12m+26)NM_2 + 6m+3m^2+8}{n^2(n-m-1)^2} b \\
&\quad - \frac{(6m-1)NM_2^3 + (9m^2+25m-6)NM_2^2 + 6(4m^2+5m-2)NM_2 + 2m^3+14m^2+8m-8}{n^3(n-m-1)^2},
\end{aligned} \tag{A.34a}$$

$$\frac{\Gamma^2 \frac{\partial \chi_e}{\partial m}}{n^4(n-m-1)h} \Big|_{m=n-2} = b^2 - \frac{n-8}{n} b - \frac{2(3n-4)(n^2-4n+2)}{n^3}, \tag{A.34b}$$

$$\frac{\partial \left(\frac{\Gamma^2 \frac{\partial x_i}{\partial m}}{2n^3 m(n-m-1)^2 h} \right)}{\partial m} = \frac{(n^2 - m^2)[N M_2^2 + (3m^2 - m + 3)N M_2 + 2m^3 + m^2 - 3m + 2]}{2n^2 m^2 (n - m - 1)^3}, \quad (\text{A.34c})$$

$$\frac{\Gamma^2 \frac{\partial x_i}{\partial m}}{n^3 m(n-m-1)^2 h} \Big|_{m=n-2} = -b + \frac{(3n-4)(n^2-4n+2)}{n^2(n-2)}, \quad (\text{A.34d})$$

$$\frac{\partial \left(\frac{\Gamma^2 \frac{\partial y_e}{\partial m}}{2n^4 (n-m-1)^2 h} \right)}{\partial m} = \frac{2(n-m)[N M_2^3 + 4N M_2^2 + (2m+4)N M_2 + m^2 + m]}{n^2 (n-m-1)^3}, \quad (\text{A.34e})$$

$$\frac{\Gamma^2 \frac{\partial y_e}{\partial m}}{n^4 (n-m-1)^2 h} \Big|_{m=n-2} = -b + \frac{3(n-4)}{n}, \quad (\text{A.34f})$$

such that $\frac{\partial x_e}{\partial m} > 0$, $\frac{\partial x_i}{\partial m}, \frac{\partial y_e}{\partial m} < 0$ holds for $m \leq n-2$ if b is sufficiently large; otherwise, $\frac{\partial x_e}{\partial m}, \frac{\partial x_i}{\partial m}, \frac{\partial y_e}{\partial m}$ change their signs once in $m \in (n-4, n-2)$.

The welfare of each country is given by $W_i = ay_i - \frac{b}{2}y_i^2 - \frac{1}{2}x_i^2 - p(y_i - x_i) - hx$, where $p = a - by_f$ from (2). Substituting (A.31) yields $W_i^* = \frac{(a-nh)^2}{2(b+1)}$, and substituting (A.32) yields

$$W_e = W_i^* - \frac{n^2(n^2 - m^2)(n-m-1)^2(n^2b + n^2 - m^2)h^2}{2\Gamma^2} < W_i^*, \quad (\text{A.35a})$$

$$W_f = W_i^* - \frac{n^2(n-m-1)^2[n^2(n^2 - 2m^2)b + (n-m)^2(n+m)^2]h^2}{2\Gamma^2} < W_i^* \Leftrightarrow m \leq 0.71n, \quad (\text{A.35b})$$

$$W = W^* - \frac{n^3(n-m)(n-m-1)^2[n(n^2 - m^2 + nm)b + (n-m)(n+m)^2]h^2}{2\Gamma^2} < W^*, \quad (\text{A.35c})$$

$$W_f - W_e = \frac{n^4 m^2 (n-m-1)^2 b h^2}{2\Gamma^2} > 0, \quad (\text{A.35d})$$

Differentiating with respect to m yields

$$\begin{aligned} \frac{\partial W_e}{\partial m} &= \frac{n^2(n-m-1)h^2}{(n-2)^2 \Gamma^3} \{n^4 m(n-2)^2(n-m-1)^2(b - \underline{b}_3)^2 + n^2(n-2)[3m^5 m + (9m^2 + 39m + 1)N M_4^4 \\ &\quad + (8m^3 + 93m^2 + 190m + 14)N M_4^3 + (2m^4 + 61m^3 + 342m^2 + 418m + 72)N M_4^2 + (9m^4 + 148m^3 \\ &\quad + 522m^2 + 376m + 160)N M_4 + 7m^4 + 114m^3 + 272m^2 + 64m + 128](b - \underline{b}_3) + 2N M_5^8 m + (12m^2 \\ &\quad + 55m + 2)N M_5^7 + (29m^3 + 285m^2 + 650m + 58)N M_5^6 + (36m^4 + 579m^3 + 2818m^2 + 4333m \\ &\quad + 710)N M_5^5 + (24m^5 + 581m^4 + 4637m^3 + 14999m^2 + 18012m + 4742)N M_5^4 + (8m^6 + 295m^5 \\ &\quad + 3549m^4 + 18906m^3 + 46392m^2 + 48737m + 18590)N M_5^3 + (m^7 + 68m^6 + 1229m^5 + 10021m^4 \\ &\quad + 40931m^3 + 83851m^2 + 86146m + 42550)N M_5^2 + (5m^7 + 149m^6 + 1873m^5 + 12491m^4 + 43995 \\ &\quad \cdot m^3 + 83642m^2 + 93435m + 52250)N M_5 + 2m^7 + 25m^6 + 575m^5 + 4874m^4 + 18091m^3 + 37185 \\ &\quad \cdot m^2 + 48150m + 26250\} > 0 \Leftrightarrow m \leq n-5, \end{aligned} \quad (\text{A.36a})$$

$$\begin{aligned} \frac{\partial W_f}{\partial m} &= \frac{n^2(n-m-1)h^2}{(n-2)^7 \Gamma^3} \{2n^4 m(n-2)^7(n-m-1)^2(b - \underline{b}_3)^2 + n^2(n-2)^4[n^3 N M_2^3 + nm(6N_9^4 + 176N_9^3 \\ &\quad + 1917N_9^2 + 9174N_9 + 16243)N M_2^2 + nm^2(n-1)(n-4)(9n-8)N M_2 + m^3(N_9^4 + 29N_9^3 + 317 \\ &\quad \cdot N_9^2 + 1535N_9 + 2734)](b - \underline{b}_3) + 2n^4(n^2 - 4n + 2)N M_2^6 + 4n^2 m(n^2 - 4n + 2)(N_9^4 + 28N_9^3 \\ &\quad + 290N_9^2 + 1318N_9 + 2227)N M_2^5 + 2n^2 m^2(6N_9^6 + 253N_9^5 + 4396N_9^4 + 40252N_9^3 + 204662N_9^2 \\ &\quad + 547371N_9 + 601036)N M_2^4 + 2nm^3(n-2)(6N_9^6 + 267N_9^5 + 4870N_9^4 + 46488N_9^3 + 244176N_9^2 \\ &\quad + 666421N_9 + 734636)N M_2^3 + nm^4(4N_9^7 + 214N_9^6 + 4783N_9^5 + 57831N_9^4 + 407422N_9^3 \\ &\quad + 1664240N_9^2 + 3618991N_9 + 3184019)N M_2^2 + 4nm^5(2N_9^6 + 76N_9^5 + 1161N_9^4 + 8982N_9^3 \\ &\quad + 35966N_9^2 + 65406N_9 + 30839)N M_2 + m^6(N_9^7 + 48N_9^6 + 957N_9^5 + 10184N_9^4 + 61483N_9^3 \\ &\quad + 203816N_9^2 + 316023N_9 + 122816)\} > 0, \end{aligned} \quad (\text{A.36b})$$

$$\begin{aligned} \frac{\partial W}{\partial m} &= \frac{n^3(n-m-1)h^2}{2(n-2)^2 \Gamma^3} \{n^3 m(4n-3m)(n-2)^2(n-m-1)^2(b - \underline{b}_3)^2 + n(n-2)[12N M_4^6 m + (39m^2 \\ &\quad + 208m + 2)N M_4^5 + (45m^3 + 567m^2 + 1446m + 36)N M_4^4 + (20m^4 + 524m^3 + 3204m^2 + 5080m \\ &\quad + 256)N M_4^3 + (2m^5 + 172m^4 + 2239m^3 + 8740m^2 + 9216m + 896)N M_4^2 + (9m^5 + 476m^4 + 4162 \\ &\quad \cdot m^3 + 11400m^2 + 7552m + 1536)N M_4 + 7m^5 + 418m^4 + 2848m^3 + 5600m^2 + 1536m + 1024](b \\ &\quad - \underline{b}_3) + 8N M_5^8 m + (42m^2 + 224m + 4)N M_5^7 + (90m^3 + 1031m^2 + 2676m + 116)N M_5^6 + (99m^4 \end{aligned}$$

$$\begin{aligned}
& + 1888m^3 + 10565m^2 + 17752m + 1420)NM_5^5 + (57m^5 + 1714m^4 + 16062m^3 + 58294m^2 \\
& + 71252m + 9484)NM_5^4 + (15m^6 + 773m^5 + 11494m^4 + 70576m^3 + 185768m^2 + 176624m \\
& + 37180)NM_5^3 + (m^7 + 145m^6 + 3731m^5 + 37068m^4 + 167746m^3 + 338511m^2 + 264124m \\
& + 85100)NM_5^2 + (5m^7 + 408m^6 + 7435m^5 + 56971m^4 + 202320m^3 + 321465m^2 + 220440m \\
& + 104500)NM_5 + 2m^7 + 270m^6 + 4960m^5 + 32974m^4 + 94966m^3 + 119300m^2 + 82100m \\
& + 52500\} > 0 \Leftrightarrow m \leq n-5,
\end{aligned} \tag{A.36c}$$

$$\begin{aligned}
\frac{\partial(W_f - W_e)}{\partial m} &= \frac{n^4 m(n-m-1)h^2}{\Gamma^3} \{n^2(n-m-1)^2b + NM_2^4 + (2m+7)NM_2^3 + (2m^2+9m+18)NM_2^2 \\
&+ (5m^2+12m+20)NM_2 + (m+2)(m^2+4)\}b > 0,
\end{aligned} \tag{A.36d}$$

which proves Lemma 1d and Lemma 1e. Note that

$$\begin{aligned}
\frac{\partial \left(\frac{\Gamma^3 \frac{\partial W_e}{\partial m}}{n^6 m(n-m-1)^3 h^2} \right)}{\partial m} &= -\frac{1}{n^4 m^2(n-m-1)^3} \{n^2[NM_2^4 + (2m^3+3m^2+m+7)NM_2^3 + (12m^3+12m^2+3m+18)NM_2^2 \\
&+ (3m^4+25m^3+9m^2+20)NM_2 + 7m^4 + 13m^3 - 6m^2 - 4m + 8]b + NM_2^6 + (3m^2+3m+11)NM_2^5 \\
&+ (14m^3+27m^2+25m+50)NM_2^4 + (14m^4+94m^3+86m^2+80m+120)NM_2^3 + (78m^4+218m^3+108m^2 \\
&+ 120m+160)NM_2^2 + (12m^5+136m^4+192m^3+24m^2+80m+112)NM_2 + 20m^5+72m^4+40m^3 \\
&- 32m^2+16m+32\},
\end{aligned} \tag{A.37a}$$

$$\frac{\Gamma^3 \frac{\partial W_e}{\partial m}}{n^6 m(n-m-1)^3 h^2} \Big|_{m=n-2} = b^2 - \frac{5N_5^3 + 47N_5^2 + 135N_5 + 109}{n^2(n-2)}b - \frac{8(n-1)(3n-4)(n^2-4n+2)}{n^4(n-2)}, \tag{A.37b}$$

$$\begin{aligned}
\frac{\partial \left(\frac{2\Gamma^3 \frac{\partial W}{\partial m}}{n^6 m(4n-3-m)(n-m-1)^3 h^2} \right)}{\partial m} &= -\frac{1}{n^3 m^2(4n-3-m)^2(n-m-1)^3} \{n[8NM_2^6 + (36m^2+12m+88)NM_2^5 + (12m^4+104m^3 \\
&+ 276m^2+92m+400)NM_2^4 + (6m^5+152m^4+676m^3+748m^2+256m+960)NM_2^3 + (48m^5+597m^4 \\
&+ 1548m^3+744m^2+288m+1280)NM_2^2 + (3m^6+135m^5+927m^4+1408m^3-48m^2+64m+896)NM_2 \\
&+ 7m^6+117m^5+490m^4+368m^3-352m^2-64m+256]b + 4[2NM_2^7 + (6m^2+5m+26)NM_2^6 + (34m^3 \\
&+ 60m^2+51m+144)NM_2^5 + (50m^4+283m^3+225m^2+210m+440)NM_2^4 + (19m^5+349m^4+894m^3 \\
&+ 360m^2+440m+800)NM_2^3 + (117m^5+888m^4+1292m^3+120m^2+480m+864)NM_2^2 + (6m^6+234m^5 \\
&+ 964m^4+776m^3-288m^2+240m+512)NM_2 + 10m^6+152m^5+368m^4+96m^3-240m^2+32m \\
&+ 128]\},
\end{aligned} \tag{A.37c}$$

$$\begin{aligned}
\frac{2\Gamma^3 \frac{\partial W}{\partial m}}{n^6 m(4n-3-m)(n-m-1)^3 h^2} \Big|_{m=n-2} &= b^2 - \frac{5N_5^4 + 82N_5^3 + 444N_5^2 + 894N_5 + 463}{n^2(n-2)(n+6)}b \\
&- \frac{16(n-1)(3n-4)(n^2-4n+2)}{n^3(n-2)(n+6)},
\end{aligned} \tag{A.37d}$$

such that $\frac{\partial W_e}{\partial m}, \frac{\partial W}{\partial m} > 0$ holds for $m \leq n-2$ if b is sufficiently large; otherwise, $\frac{\partial W_e}{\partial m}, \frac{\partial W}{\partial m}$ change their signs once in $m \in (n-5, n-2)$. \square

Finally we analyze the corner solution for $m = n-1$. Substituting $m = n-1$ into (A.23) and (A.24) yields $\chi_f = \chi_e = nh$, which represents a minimum for the fringe country and a maximum for each coalition country. Substituting $m = n-1$ and (A.19) into (A.22) and rearranging yields

$$\chi_e(\chi_f) = \chi_f + \frac{n(n-1)b}{n^2b+2n-1}(nh - \chi_f) = \frac{nb+2n-1}{n^2b+2n-1} \left[\frac{n(n-1)b}{nb+2n-1}nh + \chi_f \right]. \tag{A.38}$$

Consequently, $\chi_f \leq nh$ to ensure $\chi_e \geq \chi_f$, and $\chi_f \geq \underline{\chi}_f := \frac{n(n-1)b}{nb+2n-1}nh$ to ensure $\chi_e \geq 0$. Substituting (A.19), (A.20) and (A.38) into (A.18) yields

$$x_i(\chi_f) = \frac{a-nh}{b+1} + \frac{2n-1}{n^2b+2n-1}(nh - \chi_f) > x_i^*, \tag{A.39a}$$

$$y_e(\chi_f) = \frac{a-nh}{b+1} + \frac{n}{n^2b+2n-1}(nh - \chi_f) > y_i^*, \tag{A.39b}$$

$$y_f(\chi_f) = \frac{a-nh}{b+1} + \frac{n^2}{n^2b+2n-1}(nh - \chi_f) > y_i^*. \tag{A.39c}$$

Thus, $\chi_f = \underline{\chi}_f$ and $\chi_e = 0$ is the corner solution for $m = n-1$.

A.3. Proof of Proposition 1

From (A.23), χ_f smaller than h for $m \leq n-2$, and it is negative if and only if $n^2 - nm - m^2$ is negative, which is the case if and only if $m/n > \frac{2}{1+\sqrt{5}} \approx 0.62$. From (A.24), χ_e is smaller than mh for $m \leq n-2$. Furthermore, $\chi_f = \frac{\chi_f}{\chi_f} < 0$ and $\chi_e = 0$ for $m = n-1$.

Substituting (A.35a) and (A.35b) into $\Phi(m) := W_e(m) - W_f(m-1)$ yields

$$\begin{aligned} \Phi(m) = & -\frac{n^2(n-m)h^2}{2[\Gamma(m)\Gamma(m-1)]^2} \{n^6(n-m)(n-m-1)^2(m^2-4m+2)b^3 + n^4(n-m)(n-m-1)[(2m^2-8m+4)n^3 \\ & - (2m^3-8m^2+4)n^2 - (2m^4-6m^3-2m^2+6m-1)n + 2m^5-8m^4+6m^3-2m^2-m+1]b^2 \\ & + n^2[(m^2-4m+2)n^7 - (3m^3-12m^2+2m+6)n^6 + (m^4-6m^3+6m^2-2m+5)n^5 + (5m^5-16m^4 \\ & - 10m^3+16m^2-3m+2)n^4 - (5m^6-20m^5+9m^4-6m^3+10m^2-2m+1)n^3 - (m^7-17m^5+26m^4 \\ & - 12m^3+4m^2-5m+2)n^2 + (m-1)(3m^7-7m^6-5m^5+3m^4+3m^3-9m^2+5m-1)n \\ & - m(m-1)^2(m^6-2m^5+m^4+m^2+2m-1)]b - (n-m)[n^2 - (3m^2-4m+1)n - m^3+2m^2-m] \\ & \cdot [2n^5-2mn^4 - (4m^2-2m+2)n^3 + (4m^3-4m^2+2m-1)n^2 + (2m^4-2m^3+m^2-2m+1)n - 2m^5 \\ & + 4m^4-m^3-2m^2+m]\}. \end{aligned} \quad (\text{A.40})$$

Substituting $b = \underline{b}_3 + \beta$, $m = \frac{4\eta}{1+\eta} + \frac{n-2}{1+\eta}$ and $n = N_6 + 6$ with $\beta, \eta \geq 0$ into (A.40), it can be shown that $\Phi(m)$ is negative for $m \in [4, n-2]$ and $n \geq 6$. The expression is available upon request. This proves $m \in [4, n-2]$ being internally unstable.

Substituting $m = 2$ and $m = 3$ into (A.40) yields

$$\begin{aligned} \Phi(2) = & \frac{n^2h^2}{2(n-2)[\Gamma(2)\Gamma(1)]^2} \{2n^6(n-2)^3(n-3)^2(b-\underline{b}_3)^3 + n^4(n-2)^2(n-3)(10N_6^4 + 166N_6^3 + 971N_6^2 \\ & + 2343N_6 + 1948)(b-\underline{b}_3)^2 + n^2(n-2)(16N_6^8 + 530N_6^7 + 7475N_6^6 + 58466N_6^5 + 276487N_6^4 \\ & + 806422N_6^3 + 1409751N_6^2 + 1341060N_6 + 525472)(b-\underline{b}_3) + 8N_6^{11} + 372N_6^{10} + 7710N_6^9 \\ & + 93832N_6^8 + 743331N_6^7 + 4013416N_6^6 + 15017258N_6^5 + 38762564N_6^4 + 67209560N_6^3 \\ & + 73838920N_6^2 + 45526048N_6 + 11571712\} > 0, \end{aligned} \quad (\text{A.41})$$

$$\begin{aligned} \Phi(3) = & \frac{n^2(n-3)h^2}{2(n-2)^3[\Gamma(3)\Gamma(2)]^2} \{2n^6(n-2)^3(n-3)(n-4)^2(b-\underline{b}_3)^3 + n^4(n-2)^2(n-3)(n-4)(5N_{10}^4 + 152N_{10}^3 \\ & + 1671N_{10}^2 + 7794N_{10} + 12800)(b-\underline{b}_3)^2 + n^2(n-2)(8N_{10}^9 + 544N_{10}^8 + 16214N_{10}^7 + 277496N_{10}^6 \\ & + 2997965N_{10}^5 + 21131059N_{10}^4 + 96697048N_{10}^3 + 274941348N_{10}^2 + 435274496N_{10} + 285584384) \\ & \cdot (b-\underline{b}_3) + 4N_{10}^{12} + 372N_{10}^{11} + 15681N_{10}^{10} + 395497N_{10}^9 + 6632298N_{10}^8 + 77669250N_{10}^7 \\ & + 648515864N_{10}^6 + 3865190452N_{10}^5 + 16152917640N_{10}^4 + 45327176224N_{10}^3 + 78090355680N_{10}^2 \\ & + 67110609664N_{10} + 12951611392\} > 0. \end{aligned} \quad (\text{A.42})$$

m being internally stable implies $m-1$ being externally unstable. This proves $m = 1$ and $m = 2$ being externally unstable and $m = 3$ being internally and externally stable.

The welfare of each country is given by $W_i = ay_i - \frac{b}{2}y_i^2 - \frac{1}{2}x_i^2 - p(y_i - x_i) - hx$, where $p = a - by_f$ from (2). Substituting (A.31) for $m = n$, (A.32) for $m = n-2$ and (A.39) for $m = n-1$ into $\Phi(m) = W_e(m) - W_f(m-1)$ yields

$$\begin{aligned} \Phi(n) = & -\frac{(nh - \underline{\chi}_f)^2}{2(n-2)(n^2b + 2n-1)^2} [n^2(n-2)(n^2-4n+2)(b-\underline{b}_3) \\ & + N_7^5 + 25N_7^4 + 236N_7^3 + 1020N_7^2 + 1882N_7 + 926] < 0, \end{aligned} \quad (\text{A.43})$$

$$\begin{aligned} \Phi(n-1) = & -\frac{(2n-1)(nh - \underline{\chi}_f)^2}{2(n^2b + 2n-1)} - \frac{n^2h^2}{2(n-2)[\Gamma(n-2)]^2} [(n-2)(n^2-8n+8)(b-\underline{b}_3) \\ & + N_{10}^5 + 36N_{10}^4 + 484N_{10}^3 + 2904N_{10}^2 + 6832N_{10} + 1952] < 0. \end{aligned} \quad (\text{A.44})$$

This proves $m = n$ and $m = n-1$ being internally unstable. Consequently, only $m = 3$ is internally and externally stable. This proves Proposition 1a.

(A.32) and (A.39) reveal $y_f > x_i > y_e > x_i^* = y_i^*$ for $m < n$. $W_e < W_f$ and $W < W^*$ imply $W_e < W_i^*$ for $m < n$, and (A.35b) reveals $W_f < W_i^*$ for $m < 0.71n$. This proves Proposition 1b. \square

Appendix B. Appendix to Section 4

B.1. Derivation of Eqs. (20) and (22)

Accounting for $\tau_e = \chi_e - \frac{\sum_{j \in F} \chi_j}{n-m}$, the first-order condition of (19) with respect to χ_i reads

$$0 = -[p - C'_i - H'_i]S'_i + \frac{H'_i \sum_{j \in E} S'_j}{n-m} + \left\{ [B'_i - p]D'_i + [p - C'_i]S'_i + x_i - y_i - H'_i \sum_{j \in N} S'_j \right\} \frac{\partial p}{\partial \chi_i} + \mu_i \left\{ -[p - C'_i]S'_i + \{[p - C'_i]S'_i + x_i\} \frac{\partial p}{\partial \chi_i} + (1 - \alpha_i)[\chi_i S'_i - \chi_i S'_i \frac{\partial p}{\partial \chi_i} - x_i] \right\}. \quad (\text{B.1})$$

Substituting $p = B'_i = C'_i + \chi_i$ for $i \in F$ from (2) and rearranging yields

$$0 = -[(1 + \alpha_i \mu_i)\chi_i - H'_i]S'_i + \frac{H'_i \sum_{j \in E} S'_j}{n-m} + \left[(1 + \alpha_i \mu_i)\chi_i S'_i + (1 + \mu_i)x_i - y_i - H'_i \sum_{j \in N} S'_j \right] \frac{\partial p}{\partial \chi_i} - (1 - \alpha_i)\mu_i x_i \quad (\text{B.2})$$

$$\Leftrightarrow \chi_i = \frac{1}{1 + \alpha_i \mu_i} \left\{ H'_i - \frac{\frac{\partial p}{\partial \chi_i}}{1 - \frac{\partial p}{\partial \chi_i}} \frac{\sum_{j \in N \setminus i} S'_j}{S'_i} H'_i - \frac{\frac{\partial p}{\partial \chi_i}}{1 - \frac{\partial p}{\partial \chi_i}} \frac{1}{S'_i} (y_i - x_i) + \frac{1}{1 - \frac{\partial p}{\partial \chi_i}} \frac{\sum_{j \in E} S'_j}{(n-m)S'_i} H'_i \right\} - \frac{\mu_i}{1 + \alpha_i \mu_i} \frac{1 - \alpha_i - \frac{\partial p}{\partial \chi_i}}{1 - \frac{\partial p}{\partial \chi_i}} \frac{x_i}{S'_i}. \quad (\text{B.3})$$

By making use of symmetry, i.e. $\alpha_i = \alpha$, $\mu_i = \mu$, $\chi_i = \chi_f$, $D_i = D_f$ for all $i \in F$ and $S_i = S_f = S_e$, $H_i = H$ for all $i \in N$, (B.3) is equivalent to (20).

Accounting for $\tau_e = \chi_e - \frac{\sum_{j \in F} \chi_j}{n-m}$, the first-order condition of (21) with respect to χ_e reads

$$0 = \sum_{j \in E} [B'_j - p]D'_j + \sum_{j \in E} \left\{ [B'_j - p]D'_j + [p - C'_j]S'_j + x_j - y_j - H'_j \sum_{j \in N} S'_j \right\} \frac{\partial p}{\partial \chi_e} + \sum_{j \in E} \mu_j \left\{ [p - C'_j]S'_j + x_j - (1 - \alpha_j) \frac{\sum_{j \in F} \chi_j}{n-m} S'_j \right\} \frac{\partial p}{\partial \chi_e}. \quad (\text{B.4})$$

Substituting $p = B'_i = \frac{\sum_{j \in F} \chi_j}{n-m} = C'_i + \chi_e - \frac{\sum_{j \in F} \chi_j}{n-m}$ for $i \in E$ from (2) and rearranging yields

$$0 = \left[\chi_e - \frac{\sum_{j \in F} \chi_j}{n-m} \right] \sum_{j \in E} D'_j \left[1 + \frac{\partial p}{\partial \chi_e} \right] + \sum_{j \in E} \left[(1 + \alpha_j \mu_j) \frac{\sum_{j \in F} \chi_j}{n-m} S'_j + (1 + \mu_j)x_j - y_j - H'_j \sum_{j \in N} S'_j \right] \frac{\partial p}{\partial \chi_e} \quad (\text{B.5})$$

$$\Leftrightarrow \chi_e = \frac{\sum_{j \in F} \chi_j}{n-m} - \frac{\frac{\partial p}{\partial \chi_e}}{1 + \frac{\partial p}{\partial \chi_e}} \frac{1}{\sum_{j \in E} D'_j} \sum_{j \in E} \left[(1 + \alpha_j \mu_j) \frac{\sum_{j \in F} \chi_j}{n-m} S'_j + (1 + \mu_j)x_j - y_j - H'_j \sum_{j \in N} S'_j \right]. \quad (\text{B.6})$$

By making use of symmetry, i.e. $\chi_i = \chi_f$ for all $i \in F$, $\alpha_i = \alpha$, $\mu_i = \mu$, $D_i = D_e$ for all $i \in E$ and $S_i = S_f = S_e$, $H_i = H$ for all $i \in N$, (B.6) is equivalent to (22).

B.2. Characterization of the equilibrium for $\alpha = 0$ and $\mu \geq 0$

Consider $S'_i = 1$, $D'_i = -1/b$ and $H'_i = h$, such that $\frac{\partial p}{\partial \chi_i} = \frac{nb+m}{n(n-m)(b+1)}$, $\frac{\partial p}{\partial \chi_e} = -\frac{m}{n(b+1)}$ and $S''_i = D''_i = \frac{\partial^2 p}{\partial \chi_i^2} = \frac{\partial^2 p}{\partial \chi_e^2} = 0$. Then, for $\alpha = 0$ the first-order conditions (B.2) and (B.5) become

$$0 = -\chi_i S'_i \left[1 - \frac{\partial p}{\partial \chi_i} \right] + H'_i S'_i \left[1 + \frac{m}{n-m} - n \frac{\partial p}{\partial \chi_i} \right] + [(1 + \mu)S_i(p - \chi_i) - D_i(p)] \frac{\partial p}{\partial \chi_i} - \mu S_i(p - \chi_i), \quad (\text{B.7})$$

$$0 = (\chi_e - \chi_f)mD'_i \left[1 + \frac{\partial p}{\partial \chi_e} \right] + m \left[\chi_f S'_i - nH'_i S'_i + (1 + \mu)S_i(p - \chi_f) - D_i(p + \chi_e - \chi_f) \right] \frac{\partial p}{\partial \chi_e}, \quad (\text{B.8})$$

and the second-order conditions read

$$0 > -S'_i \left[1 - \left[\frac{\partial p}{\partial \chi_i} \right]^2 \right] - D'_i \left[\frac{\partial p}{\partial \chi_i} \right]^2 + \mu S'_i \left[1 - \frac{\partial p}{\partial \chi_i} \right]^2 \quad (\text{B.9})$$

$$0 > (1 + \mu)mS'_i \left[\frac{\partial p}{\partial \chi_e} \right]^2 + mD'_i \left[1 - \left[\frac{\partial p}{\partial \chi_e} \right]^2 \right] = -\frac{m[(n^2 b + n^2 - m^2)(b+1) - \mu m^2 b]}{n^2 b(b+1)^2}. \quad (\text{B.10})$$

From (B.9) and (B.10), the second-order conditions are increasing in μ . Thus, the second-order condition of each fringe country is violated for $m = n - 1$ (Appendix A.2), and it is satisfied for $m \leq n - 2$ if and only if

$$\mu < \frac{\{n^2[(n-m)^2 - 1]b^2 + n[n(n-m)^2 - 2m]b - m^2\}(b+1)}{[n(n-m-1)b + n^2 - nm - m]^2 b}, \quad (\text{B.11a})$$

$$\mu|_{m=1} = 1 + b - \frac{[n(n-2)b-1]^2(b-1)^2}{[n(n-2)b+(n^2-n-1)]^2b}, \quad (\text{B.11b})$$

$$\begin{aligned} \mu - \mu|_{m=1} = & \frac{n^2(m-1)(b+1)^3}{(n-2)^2b[n(n-2)b+(n^2-n-1)]^2[n(n-m-1)b+n^2-nm-m]^2} \\ & \cdot \{2n^2(n-2)^3(n-m-1)(b-\underline{b}_3)^2 + n(n-2)[(6N_8^3 + 112N_8^2 + 679N_8 + 1322)NM_2 + m(5N_8^2 \\ & + 59N_8 + 158)](b-\underline{b}_3) + 2[(2N_8^5 + 59N_8^4 + 679N_8^3 + 3785N_8^2 + 10120N_8 + 10228)NM_2 + m \\ & \cdot (N_8^4 + 21N_8^3 + 148N_8^2 + 372N_8 + 160)]\}, \end{aligned} \quad (\text{B.11c})$$

and, thus, if and only if $\mu < \tilde{\mu}_1 := \mu|_{m=1}$. Furthermore, the second-order condition of each coalition country is satisfied for $m \leq n-1$ if and only if it is satisfied for $m = n-1$ and, thus, if and only if $\mu < \frac{(n^2b+2n-1)(b+1)}{(n-1)^2b} (> 1+b)$.

Next we analyze the interior solution for $m \leq n-2$. Substituting $S'_i = 1$, $D'_i = -1/b$ and $H'_i = h$ into (20) and (22) for $\alpha = 0$ yields

$$\chi_f = \frac{n(n-m)h - (bn+m)(y_f - x_f)}{n(n-m-1)(b+1) + n-m} - \mu x_f, \quad (\text{B.12})$$

$$\chi_e = \chi_f + \frac{m[nh - \chi_f + y_e - x_e]b}{nb + n - m} - \mu x_e. \quad (\text{B.13})$$

Using (A.18), (A.19) and (A.20), solving (B.12) and (B.13) for χ_f and χ_e yields

$$\chi_f = \frac{n(n^2 - nm - m^2)h}{\Gamma} - \frac{\mu[(a-nh)\Gamma + n(n-m-1)(n^2 - m^2)(b+1)h]}{(b+1-\mu)\Gamma}, \quad (\text{B.14})$$

$$\chi_e = \frac{nm[n(n-m-1)b + (n^2 - nm - m^2)/m]h}{\Gamma} - \frac{\mu[(a-nh)\Gamma + n(n-m-1)(n^2 - m^2)(b+1)h]}{(b+1-\mu)\Gamma}, \quad (\text{B.15})$$

and

$$\chi_e - \chi_f = \frac{n^2m(n-m-1)bh}{\Gamma}. \quad (\text{B.16})$$

From (B.15), χ_e is positive for $m \leq n-2$ if and only if

$$\mu < \frac{n[nm(n-m-1)b + n^2 - nm - m^2](b+1)h}{a\Gamma + nm(n-m)(n-m-1)bh}, \quad (\text{B.17a})$$

$$\mu|_{m=1} = \frac{n[n(n-2)b + n^2 - n - 1](b+1)h}{a\Gamma(1) + n(n-1)(n-2)bh}, \quad (\text{B.17b})$$

$$\begin{aligned} \mu - \mu|_{m=1} = & \frac{n(m-1)(b+1)h}{(n-2)(n-3)^2[a\Gamma(m) + nm(n-m)(n-m-1)bh][a\Gamma(1) + 2n(n-2)(n-3)bh]} \\ & \cdot \{ \{n^3(n-2)^2(n-3)^3(n-m-1)(b-\underline{b}_3)^2 + n(n-2)[n(n-1)(3N_6^2 + 23N_6 + 37)NM_3 + m \\ & \cdot (3N_6^4 + 62N_6^3 + 462N_6^2 + 1448N_6 + 1557)NM_3 + m^2(5N_6^3 + 72N_6^2 + 318N_6 + 411)](b-\underline{b}_3) \\ & + n(n-1)(2N_6^4 + 29N_6^3 + 144N_6^2 + 276N_6 + 156)NM_3 + m(2N_6^6 + 55N_6^5 + 605N_6^4 + 3369N_6^3 \\ & + 9827N_6^2 + 13750N_6 + 6708)NM_3 + m^2(n-1)(2N_6^4 + 35N_6^3 + 195N_6^2 + 377N_6 + 168)\}(a \\ & - nh) + n(n-2)\{n(n-2)(n-3)^2(n^2 + m)(n-m-1)(b-\underline{b}_3) + n^2(n-1)^2(n-5)NM_3 + m(N_6^5 \\ & + 26N_6^4 + 253N_6^3 + 1136N_6^2 + 2252N_6 + 1356)NM_3 + m^2(N_6^4 + 21N_6^3 + 142N_6^2 + 341N_6 \\ & + 159)\}(b+1)h\}, \end{aligned} \quad (\text{B.17c})$$

and, thus, if and only if $\mu < \tilde{\mu}_2 := \mu|_{m=1}$.

Lemma 2 summarizes the effects of an increase in the lobby parameter:

Lemma 2. Consider $\alpha = 0$, $\mu \geq 0$ and $m \leq n-2$. Then, an increase in the lobby parameter has the following effects:

- Each country's carbon tax is decreasing in μ , and the tax difference between a coalition country and a fringe country is independent of μ .
- Each country's production is increasing in μ .
- Each country's consumption is increasing in μ , and the consumption difference between a fringe country and a coalition country is independent of μ .
- Each country's welfare is decreasing in μ , and the (joint) welfare difference between a fringe country and a coalition country is independent of μ .

Proof. Differentiating (B.14) and (B.15) with respect to μ yields

$$\frac{\partial \chi_f}{\partial \mu} = \frac{\partial \chi_e}{\partial \mu} = - \frac{[\Gamma(a-nh) + n(n^2 - m^2)(n-m-1)(b+1)h](b+1)}{\Gamma(b+1-\mu)^2} < 0, \quad (\text{B.18})$$

such that $\frac{\partial(\chi_e - \chi_f)}{\partial \mu} = 0$. This proves Lemma 2a.

Indicating the fully cooperative equilibrium by an asterisk, we find in Online Appendix C

$$x_i^* = y_i^* = \frac{a - \chi_i^*}{b + 1} = \frac{a - nh}{b + 1 - \mu}. \quad (\text{B.19})$$

Substituting (B.14) and (B.15) into (A.18), (A.19) and (A.20) yields

$$x_i = x_i|_{\mu=0} + \frac{\mu[\Gamma(a - nh) + n(n^2 - m^2)(n - m - 1)(b + 1)h]}{\Gamma(b + 1 - \mu)} > x_i|_{\mu=0}, \quad (\text{B.20})$$

$$y_i = y_i|_{\mu=0} + \frac{\mu[\Gamma(a - nh) + n(n^2 - m^2)(n - m - 1)(b + 1)h]}{\Gamma(b + 1 - \mu)} > y_i|_{\mu=0}, \quad (\text{B.21})$$

such that $y_e - y_f = (y_e - y_f)|_{\mu=0} < 0$, $y_e - x_e = (y_e - x_e)|_{\mu=0} < 0$ and $y_f - x_f = (y_f - x_f)|_{\mu=0} > 0$. Differentiating with respect to μ yields

$$\frac{\partial x_i}{\partial \mu} = \frac{\partial y_i}{\partial \mu} = \frac{\Gamma(a - nh) + n(n^2 - m^2)(n - m - 1)(b + 1)h}{\Gamma(b + 1 - \mu)^2} > 0, \quad (\text{B.22})$$

such that $\frac{\partial(y_e - y_f)}{\partial \mu} = 0$, $\frac{\partial(y_e - x_e)}{\partial \mu} = 0$ and $\frac{\partial(y_f - x_f)}{\partial \mu} = 0$. This proves Lemma 2b and Lemma 2c.

The welfare of each country is given by $W_i = ay_i - \frac{b}{2}y_i^2 - \frac{1}{2}x_i^2 - p(y_i - x_i) - hx$, where $p = a - by_f$ from (2). Substituting (B.20) and (B.21) yields

$$W_i = W_i|_{\mu=0} - \frac{\mu}{2\Gamma^2(b + 1)(b + 1 - \mu)^2} \{ \mu\Gamma^2(a - nh)^2 + 2(n^2 - m^2)(n - m - 1)\Gamma(b + 1)^2(a - nh)nh + (n^2 - m^2)^2(n - m - 1)^2(b + 1)^2[2(b + 1 - \mu)(nh)^2] \} < W_i|_{\mu=0}, \quad (\text{B.23})$$

such that $W_f - W_e = V_f - V_e = (W_f - W_e)|_{\mu=0} = (V_f - V_e)|_{\mu=0} > 0$. Differentiating with respect to μ yields

$$\frac{\partial W_i}{\partial \mu} = -\frac{1}{\Gamma^2(b + 1 - \mu)^3} \{ \mu\Gamma^2(a - nh)^2 + (n^2 - m^2)(n - m - 1)\Gamma(b + 1)(b + 1 - \mu)(a - nh)nh + (n^2 - m^2)^2(n - m - 1)^2(b + 1)^3(nh)^2 \} < 0, \quad (\text{B.24})$$

such that $\frac{\partial(W_f - W_e)}{\partial \mu} = \frac{\partial(V_f - V_e)}{\partial \mu} = 0$. This proves Lemma 2d. \square

Lemma 3 summarizes the effects of an increase in the coalition size:

Lemma 3. Consider $\alpha = 0$, $\mu \geq 0$ and $m \leq n - 2$. Then, an increase in the coalition size has the following effects:

- Each coalition country's carbon tax is increasing in m for $m \leq n - 4$, and the tax difference between a coalition country and a fringe country is increasing in m .
- Each country's production is decreasing in m for $m \leq n - 4$.
- Each coalition country's consumption is decreasing in m for $m \leq n - 4$, and the consumption difference between a fringe country and a coalition country is increasing in m .
- Each fringe country's (joint) welfare is increasing in m , and each coalition country's (joint) welfare and global (joint) welfare are increasing in m for $m \leq n - 5$.
- The (joint) welfare difference between a fringe country and a coalition country is increasing in m .

Proof. Defining

$$\begin{aligned} \Omega := & 2n^2m(n - 2)(n - m - 1)^2(b - b_3) + [n^3(n - 2)NM_4^4 + nm(2N_5^5 + 26N_5^4 + 122N_5^3 + 258N_5^2 + 300N_5 \\ & + 268)NM_4^3 + nm^2(2N_5^5 + 35N_5^4 + 203N_5^3 + 469N_5^2 + 435N_5 + 328)NM_4^2 + nm^3(8N_5^4 + 84N_5^3 + 260N_5^2 \\ & + 212N_5 + 156)NM_4 + m^4(4N_5^4 + 64N_5^3 + 224N_5^2 + 144N_5 + 108)]/(n - 4)^4 > 0 \Leftrightarrow m \leq n - 4, \end{aligned} \quad (\text{B.25})$$

differentiating (B.14) and (B.15) with respect to m yields

$$\frac{\partial \chi_f}{\partial m} = \frac{\partial \chi_f}{\partial m}|_{\mu=0} + \frac{\mu n \Omega (b + 1)h}{(n - 2)\Gamma^2(b + 1 - \mu)}, \quad (\text{B.26})$$

$$\frac{\partial \chi_e}{\partial m} = \frac{\partial \chi_e}{\partial m}|_{\mu=0} + \frac{\mu n \Omega (b + 1)h}{(n - 2)\Gamma^2(b + 1 - \mu)} > 0 \Leftrightarrow m \leq n - 4, \quad (\text{B.27})$$

such that $\frac{\partial(\chi_e - \chi_f)}{\partial m} = \frac{\partial(\chi_e - \chi_f)}{\partial m}|_{\mu=0} > 0 \Leftrightarrow m \leq n - 2$. This proves Lemma 3a. Note that $\frac{\partial \chi_f}{\partial m}|_{\mu=0} < 0 \Leftrightarrow m \leq n - 2$, but $\frac{\partial^2 \chi_f}{\partial m \partial \mu} > 0 \Leftrightarrow m \leq n - 4$,

such that $\frac{\partial \chi_f}{\partial m}|_{\mu>0} > 0$ cannot be excluded.

Differentiating (B.20) and (B.21) with respect to m yields

$$\frac{\partial x_i}{\partial m} = \frac{\partial x_i}{\partial m}|_{\mu=0} - \frac{\mu n^2 \Omega h}{(n - 2)\Gamma^2(b + 1 - \mu)} < 0 \Leftrightarrow m \leq n - 4, \quad (\text{B.28a})$$

$$\frac{\partial y_e}{\partial m} = \frac{\partial y_e}{\partial m}|_{\mu=0} - \frac{\mu n^2 \Omega h}{(n - 2)\Gamma^2(b + 1 - \mu)} < 0 \Leftrightarrow m \leq n - 4, \quad (\text{B.28b})$$

$$\frac{\partial y_f}{\partial m} = \frac{\partial y_f}{\partial m} \Big|_{\mu=0} - \frac{\mu n^2 \Omega h}{(n-2)\Gamma^2(b+1-\mu)}, \quad (\text{B.28c})$$

such that $\frac{\partial(y_e - y_f)}{\partial m} = \frac{\partial(y_e - y_f)}{\partial m} \Big|_{\mu=0} < 0 \Leftarrow m \leq n-2$. This proves Lemma 3b and Lemma 3c. Note that $\frac{\partial y_f}{\partial m} \Big|_{\mu=0} > 0 \Leftarrow m \leq n-2$, but $\frac{\partial^2 y_f}{\partial m \partial \mu} < 0 \Leftarrow m \leq n-4$, such that $\frac{\partial y_f}{\partial m} \Big|_{\mu>0} < 0$ cannot be excluded.

Differentiating (B.23) with respect to m yields

$$\frac{\partial W_e}{\partial m} = \frac{\partial W_e}{\partial m} \Big|_{\mu=0} + \frac{\mu n \Omega \{ \Gamma(a-nh) + (n^2 - m^2)(n-m-1)[2(b+1) - \mu]nh \} h}{(n-2)\Gamma^3(b+1-\mu)^2/(b+1)} > 0 \Leftarrow m \leq n-5, \quad (\text{B.29a})$$

$$\frac{\partial W_f}{\partial m} = \frac{\partial W_f}{\partial m} \Big|_{\mu=0} + \frac{\mu n \Omega \{ \Gamma(a-nh) + (n^2 - m^2)(n-m-1)[2(b+1) - \mu]nh \} h}{(n-2)\Gamma^3(b+1-\mu)^2/(b+1)} > 0 \Leftarrow m \leq n-4, \quad (\text{B.29b})$$

$$\frac{\partial W}{\partial m} = \frac{\partial W}{\partial m} \Big|_{\mu=0} + n \frac{\mu n \Omega \{ \Gamma(a-nh) + (n^2 - m^2)(n-m-1)[2(b+1) - \mu]nh \} h}{(n-2)\Gamma^3(b+1-\mu)^2/(b+1)} > 0 \Leftarrow m \leq n-5, \quad (\text{B.29c})$$

such that $\frac{\partial(W_f - W_e)}{\partial m} = \frac{\partial(W_f - W_e)}{\partial m} \Big|_{\mu=0} > 0 \Leftarrow m \leq n-2$. Using $\mu \leq \tilde{\mu}_2$, it can be shown that $\frac{\partial W_f}{\partial m} > 0 \Leftarrow m \leq n-2$. The expression is available upon request.

The joint welfare of each country is given by $V_i = ay_i - \frac{b}{2}y_i^2 - \frac{1}{2}x_i^2 - p(y_i - x_i) - hx + \mu(px_i - \frac{1}{2}x_i^2 - \chi_f x_i)$, where $p = a - by_f$ from (2). Substituting (B.20) and (B.21) yields

$$V_i = W_i \Big|_{\mu=0} - \frac{\mu \{ \Gamma^2(a-nh)^2 - (n^2 - m^2)^2(n-m-1)^2(b+1)^2[2(b+1) - \mu](nh)^2 \}}{2\Gamma^2(b+1)(b+1-\mu)}. \quad (\text{B.30})$$

Differentiating with respect to m yields

$$\frac{\partial V_e}{\partial m} = \frac{\partial W_e}{\partial m} \Big|_{\mu=0} + \frac{\mu(n^2 - m^2)(n-m-1)\Omega(b+1)(nh)^2}{(n-2)\Gamma^3(b+1-\mu)} > 0 \Leftarrow m \leq n-5, \quad (\text{B.31a})$$

$$\frac{\partial V_f}{\partial m} = \frac{\partial W_f}{\partial m} \Big|_{\mu=0} + \frac{\mu(n^2 - m^2)(n-m-1)\Omega(b+1)(nh)^2}{(n-2)\Gamma^3(b+1-\mu)} > 0 \Leftarrow m \leq n-4, \quad (\text{B.31b})$$

$$\frac{\partial V}{\partial m} = \frac{\partial W}{\partial m} \Big|_{\mu=0} + n \frac{\mu(n^2 - m^2)(n-m-1)\Omega(b+1)(nh)^2}{(n-2)\Gamma^3(b+1-\mu)} > 0 \Leftarrow m \leq n-5. \quad (\text{B.31c})$$

such that $\frac{\partial(V_f - V_e)}{\partial m} = \frac{\partial(V_f - V_e)}{\partial m} \Big|_{\mu=0} > 0 \Leftarrow m \leq n-2$. Using $\mu \leq \tilde{\mu}_2$, it can be shown that $\frac{\partial V_f}{\partial m} > 0 \Leftarrow m \leq n-2$. The expression is available upon request. This proves Lemma 3d and Lemma 3e. \square

Finally we analyze the corner solution for $m = n-1$. Substituting $m = n-1$ into (B.14) and (B.15) yields $\chi_f = \chi_e = \frac{(b+1)nh - \mu a}{b+1-\mu}$, which represents a minimum for the fringe country and a maximum for each coalition country. Substituting $m = n-1$, (A.18) and (A.19) into (B.13) and rearranging yields

$$\begin{aligned} \chi_e(\chi_f) &= \chi_f + \frac{n(n-1)b(b+1-\mu)}{(2n-1)(b+1)^2 + (n-1)^2b(b+1-\mu)} \left[\frac{(b+1)nh - \mu a}{b+1-\mu} - \chi_f \right] \\ &= \frac{(b+1)(nb + 2n-1) + \mu(n-1)b}{(2n-1)(b+1)^2 + (n-1)^2b(b+1-\mu)} \left[\chi_f + \frac{n(n-1)b[(b+1)nh - \mu a]}{(b+1)(nb + 2n-1) + \mu(n-1)b} \right]. \end{aligned} \quad (\text{B.32})$$

Consequently, $\chi_f \leq \frac{(b+1)nh - \mu a}{b+1-\mu}$ to ensure $\chi_e \geq \chi_f$, and $\chi_f \geq \underline{\chi}_f := -\frac{n(n-1)b[(b+1)nh - \mu a]}{(b+1)(nb + 2n-1) + \mu(n-1)b}$ to ensure $\chi_e \geq 0$. Substituting (A.18), (A.20) and (B.32) into (A.18) yields

$$x_i(\chi_f) = \frac{a-nh}{b+1-\mu} + \frac{(2n-1)(b+1)}{(2n-1)(b+1)^2 + (n-1)^2b(b+1-\mu)} \left[\frac{(b+1)nh - \mu a}{b+1-\mu} - \chi_f \right] > x_i^*, \quad (\text{B.33a})$$

$$y_e(\chi_f) = \frac{a-nh}{b+1-\mu} + \frac{n(b+1-\mu) + \mu(2n-1)}{(2n-1)(b+1)^2 + (n-1)^2b(b+1-\mu)} \left[\frac{(b+1)nh - \mu a}{b+1-\mu} - \chi_f \right] > y_i^*, \quad (\text{B.33b})$$

$$y_f(\chi_f) = \frac{a-nh}{b+1-\mu} + \frac{n^2(b+1-\mu) + \mu(2n-1)}{(2n-1)(b+1)^2 + (n-1)^2b(b+1-\mu)} \left[\frac{(b+1)nh - \mu a}{b+1-\mu} - \chi_f \right] > y_i^*. \quad (\text{B.33c})$$

Thus, $\chi_f = \underline{\chi}_f$ and $\chi_e = 0$ is the corner solution for $m = n-1$.

B.3. Proof of Proposition 2

From (B.14), χ_f smaller than h for $m \leq n-2$, and it is negative if $n^2 - nm - m^2$ is non-positive, which is the case if and only if $m/n \geq \frac{2}{1+\sqrt{5}} \approx 0.62$. From (B.15), χ_e is smaller than mh for $m \leq n-2$. Furthermore, $\chi_f = \underline{\chi}_f < 0$ and $\chi_e = 0$ for $m = n-1$.

Substituting (B.30) into $\Phi(m) := V_e(m) - V_f(m-1)$ yields

$$\begin{aligned} \Phi(m) &= \Phi(m) \Big|_{\mu=0} + \frac{\mu n^2(n-m)^2(b+1)h^2}{2(n-2)^7(n-4)^2[\Gamma(m)\Gamma(m-1)]^2(b+1-\mu)} \{ n^2(n-2)^6(n-m-1)(2n^2-2m^2+2m \\ &\quad - 1)(b-b_3) + n(n-1)(4N_{10}^4 + 146N_{10}^3 + 1983N_{10}^2 + 11865N_{10} + 26348NM_2^5 + 2m(n-1)NM_2^4 \\ &\quad \cdot (8N_{10}^5 + 377N_{10}^4 + 7068N_{10}^3 + 65860N_{10}^2 + 304795N_{10} + 559948) + m^2NM_2^3(18N_{10}^6 + 1052N_{10}^5 \end{aligned}$$

$$\begin{aligned}
& + 25431N_{10}^4 + 325532N_{10}^3 + 2327254N_{10}^2 + 8809452N_{10} + 13791152) + m^3NM_2^2(4N_{10}^6 + 290N_{10}^5 \\
& + 8246N_{10}^4 + 119976N_{10}^3 + 951888N_{10}^2 + 3928796N_{10} + 6615216) + m^4NM_2(10N_{10}^5 + 542N_{10}^4 \\
& + 11322N_{10}^3 + 114869N_{10}^2 + 568806N_{10} + 1103064) + m^5(2N_{10}^4 + 136N_{10}^3 + 2538N_{10}^2 + 18502N_{10} \\
& + 46968)\{n^2(n-2)(n-4)^2(n-m-1)(2m-1)(b-\underline{b}_3) + n(n-1)(n-2)NM_3^3 + n(m-1)(2N_{10}^3 \\
& + 46N_{10}^2 + 347N_{10} + 860)NM_3^2 + (m-1)^2(2N_{10}^4 + 66N_{10}^3 + 804N_{10}^2 + 4268N_{10} + 8272)NM_3 \\
& + (m-1)^3(4N_{10}^2 + 48N_{10} + 104)\},
\end{aligned} \tag{B.34}$$

such that $\frac{\partial \Phi(m)}{\partial \mu} > 0 \Leftrightarrow m \leq n-3$. Substituting $\mu = \tilde{\mu}_2$, $b = \underline{b}_3 + \beta$, $m = \frac{4\eta}{1+\eta} + \frac{n-2}{1+\eta}$ and $n = N_6 + 6$ with $\beta, \eta \geq 0$ into (B.34), it can be shown that $\Phi(m)$ is negative for $m \in [4, n-2]$ and $n \geq 6$. The expression is available upon request. This proves $m \in [4, n-2]$ being internally unstable. Furthermore, $\Phi(m)|_{\mu=0} > 0$ and $\frac{\partial \Phi(m)}{\partial \mu} > 0$ for $m \in \{2, 3\}$ implies $\Phi(m)|_{\mu>0} > 0$ for $m \in \{2, 3\}$, such that $m = 1$ and $m = 2$ are externally unstable and $m = 3$ is internally and externally stable.

The joint welfare of each country is given by $V_i = ay_i - \frac{b}{2}y_i^2 - \frac{1}{2}x_i^2 - p(y_i - x_i) - hx + \mu(px_i - \frac{1}{2}x_i^2 - \chi_f x_i)$, where $p = a - by_f$ from (2). Substituting (B.19) for $m = n$, (B.20), (B.21) for $m = n-2$ and (B.33) for $m = n-1$ into $\Phi(m) = V_e(m) - V_f(m-1)$ yields

$$\begin{aligned}
\Phi(n) = & - \frac{[(\chi_f - nh)(b+1-\mu) + \mu(a-nh)]^2/[n(n-2)b+n^2-n-1]^2}{2[(2n-1)(b+1)^2 + (n-1)^2b(b+1-\mu)]^2(b+1-\mu)} \\
& \cdot \{n[n^2(n-2)b+2n^2-4n+1][n(n-2)(n^2-4n+2)(b-\underline{b}_3) + N_9^4 + 24N_9^3 + 194N_9^2 \\
& + 551N_9 + 186](b+1)^2 + (\tilde{\mu}_1 - \mu)(n-1)^4[n(n-2)b+n^2-n-1]^2b\} < 0,
\end{aligned} \tag{B.35}$$

$$\begin{aligned}
\Phi(n-1) = & - \frac{(2n-1)[(\chi_f - nh)(b+1-\mu) + \mu(a-nh)]^2}{2[(2n-1)(b+1)^2 + (n-1)^2b(b+1-\mu)](b+1-\mu)} \\
& - \frac{n^2h^2}{2[\Gamma(n-2)]^2[(n-2)(n^2+n-1)(b+1) + n-1](b+1-\mu)} \{(n-1)[n(n-2)(N_{10}^4 + 33N_{10}^3 \\
& + 390N_{10}^2 + 1892N_{10} + 2936)(b-\underline{b}_3) + N_{10}^6 + 47N_{10}^5 + 880N_{10}^4 + 8196N_{10}^3 + 37960N_{10}^2 \\
& + 70224N_{10} + 2320](b+1)^2 + (\tilde{\mu}_2 - \mu)(n-2)^4[(n-2)(n^2+n-1)(b+1) + n-1]b\} < 0.
\end{aligned} \tag{B.36}$$

This proves $m = n$ and $m = n-1$ being internally unstable. Consequently, only $m = 3$ is internally and externally stable. This proves Proposition 2a.

Since $m = 3$ is internally and externally stable for $\mu > 0$ and $\mu = 0$, Lemma 2 implies that global emissions are greater and the welfare of each country is smaller for $\mu > 0$ than for $\mu = 0$. This proves Proposition 2b. \square

B.4. Characterization of the equilibrium for $\alpha = 1$ and $\mu \geq 0$

Consider $S'_i = 1$, $D'_i = -1/b$ and $H'_i = h$, such that $\frac{\partial p}{\partial \chi_i} = \frac{nb+m}{n(n-m)(b+1)}$, $\frac{\partial p}{\partial \chi_e} = -\frac{m}{n(b+1)}$ and $S''_i = D''_i = \frac{\partial^2 p}{\partial \chi_i^2} = \frac{\partial^2 p}{\partial \chi_e^2} = 0$. Then, for $\alpha = 1$ the first-order conditions (B.2) and (B.5) become

$$0 = -(1+\mu)\chi_i S'_i \left[1 - \frac{\partial p}{\partial \chi_i}\right] + H'_i S'_i \left[1 + \frac{m}{n-m} - n \frac{\partial p}{\partial \chi_i}\right] + [(1+\mu)S_i(p-\chi_i) - D_i(p)] \frac{\partial p}{\partial \chi_i}, \tag{B.37}$$

$$0 = (\chi_e - \chi_f)mD'_i \left[1 + \frac{\partial p}{\partial \chi_e}\right] + m[(1+\mu)\chi_f S'_i - nH'_i S'_i + (1+\mu)S_i(p-\chi_f) - D_i(p+\chi_e-\chi_f)] \frac{\partial p}{\partial \chi_e}, \tag{B.38}$$

and the second-order conditions read

$$0 > -(1+\mu)S'_i \left[1 - \left[\frac{\partial p}{\partial \chi_i}\right]^2\right] - D'_i \left[\frac{\partial p}{\partial \chi_i}\right]^2, \tag{B.39}$$

$$0 > (1+\mu)mS'_i \left[\frac{\partial p}{\partial \chi_e}\right]^2 + mD'_i \left[1 - \left[\frac{\partial p}{\partial \chi_e}\right]^2\right] = -\frac{m[(n^2b+n^2-m^2)(b+1) - \mu m^2b]}{n^2b(b+1)^2}. \tag{B.40}$$

From (B.39) [(B.40)], the second-order condition of each fringe [coalition] country is decreasing [increasing] in μ . Thus, the second-order condition of each fringe country is satisfied for $m \leq n-2$ if $b > \underline{b}_1$ (Appendix A.2), and it is satisfied for $m = n-1$ if and only if

$$0 > \frac{[n(n-2)b + (n-1)^2](b+1) - \mu(2nb + 2n-1)b}{n^2b(b+1)^2} \tag{B.41}$$

and, thus, if and only if $\mu > \underline{\mu} := \frac{n(n-2)b+(n-1)^2}{2nb+2n-1} \frac{b+1}{b} > (n-2)(b+1)/(2b)$. Furthermore, the second-order condition of each coalition country is satisfied for $m \leq n-1$ if and only if it is satisfied for $m = n-1$ and, thus, if and only if $\mu < \frac{(n^2b+2n-1)(b+1)}{(n-1)^2b} > (1+b)$.

Next we analyze the interior solution for $m \leq n-2$. Substituting $S'_i = 1$, $D'_i = -1/b$ and $H'_i = h$ into (20) and (22) for $\alpha = 1$ yields

$$\chi_f = \frac{1}{1+\mu} \frac{\{n(n-m)h - (bn+m)[y_f - (1+\mu)x_f]\}}{n(n-m-1)(b+1) + n-m}, \tag{B.42}$$

$$\chi_e = \chi_f + \frac{m[nh - (1+\mu)\chi_f + y_e - (1+\mu)x_e]b}{nb + n - m}. \tag{B.43}$$

Using (A.18), (A.19) and (A.20), solving (A.21) and (A.22) for χ_f and χ_e yields

$$\chi_f = \frac{n\{\mu[(na - m^2h)b + ma] + (n^2 - m^2 - nm)(b + 1)h\}}{\Psi} \\ = mh - \frac{[\mu m(n - m) + (m - 1)(n - m - 2) + m - 2]h}{(1 + \mu)(n - m)} - \frac{(nb + m)(\chi_e - \chi_f)}{(1 + \mu)m(n - m)b}, \quad (\text{B.44})$$

$$\chi_e = \frac{-\mu^2 nm(n - m)ab + n[nm(n - m - 1)b + (n^2 - nm - m^2)](b + 1)h}{\Psi} \\ - \frac{\mu n\{[(mn - m^2 - n)b - m[(n - m - 1)nb + n^2 - nm - m]bh\}}{\Psi}, \quad (\text{B.45})$$

and

$$\chi_e - \chi_f = \frac{nm\{(1 + \mu)(n - m - 1)[(b + 1)nh - \mu a] - \mu[(1 + \mu)a - nh]\}b}{\Psi}, \quad (\text{B.46})$$

where

$$\Psi := -\mu^2 m^2(n - m)b + \mu[(m^3 - nm^2 + n^2)b + nm] + (1 + \mu)(b + 1)\Gamma, \quad (\text{B.47})$$

$$\frac{\partial \Psi}{\partial m} = -(1 + \mu)^2 m(2n - 3)m - (1 + \mu)[n^2 b^2 + n(2n + 1)b + (n - m)(n + 3)m] - n < 0 \Leftarrow m \leq 2/3n \\ = -(1 + \mu)(1 + b - \mu)m(3m - 2n)b - (1 + \mu)[(n - m)(n + 3)m(b + 1)^2 + nb] - n \\ < 0 \Leftarrow m \geq 2/3n \text{ and } \mu \leq 1 + b. \quad (\text{B.48})$$

Using (B.46) in (B.45) yields

$$\chi_e = \frac{1}{nm(n - 2)[(1 + \mu)(n - m - 1)(b + 1) + \mu]b} \{ \mu m(n - 2)\{na + [n(n - m - 1)(b + 1) + (n - m)](\chi_e \\ - \chi_f)\}b + \{nm(n - 2)(n - m - 1)(b - \underline{b}_3) + (n - m - 2)[n^2 + m(n^2 - 5n + 2)]\}(b + 1)(\chi_e - \chi_f)\}, \quad (\text{B.49})$$

such that χ_e is positive for $m \leq n - 2$ if $\chi_e - \chi_f \geq 0$.

Substituting (B.44) and (B.45) into (A.18), and differentiating with respect to m yields

$$y_e \Psi = (1 + \mu)[\mu m(n - m)^2 + \Gamma](a - nh) + (1 + \mu)n(n - m)(n - m - 1)(b - \underline{b}_3)h + \{(15625[(2\mu + 2)N_{10}^3 \\ + (51\mu + 50)N_{10}^2 + (430\mu + 412)N_{10} + 1196\mu + 1116)]K_{84}^3 + 1875[(7\mu^2 + 29\mu + 22)N_{10}^3 + (196\mu^2 \\ + 743\mu + 536)N_{10}^2 + (1820\mu^2 + 6326\mu + 4308)N_{10} + 5600\mu^2 + 17888\mu + 11408]mK_{84}^2 + 75[(56\mu^2 \\ + 200\mu + 144)N_{10}^3 + (1568\mu^2 + 4687\mu + 3194)N_{10}^2 + (14560\mu^2 + 36378\mu + 23168)N_{10} + 44800\mu^2 \\ + 93980\mu + 55180]m^2 K_{84} + [(336\mu^2 + 1136\mu + 800)N_{10}^3 + (9408\mu^2 + 14583\mu + 15800)N_{10}^2 \\ + (87360\mu^2 - 6290\mu + 97600)N_{10} + 268800\mu^2 - 395200\mu + 186000]m^3\}h/[9261n(n - 2)] \\ > 0 \Leftarrow m \leq 0.84n, \quad (\text{B.50})$$

$$\frac{\partial y_e \Psi}{\partial m} = -(1 + \mu)\{(1 + b - \mu)[bn^2 + (n - m)(n + 3)m + n] + \mu[m(4n - 3)m + 6nm - 6m^2 + n]\}(a - nh) \\ / (b + 1) - (1 + \mu)\{(1 + b - \mu)n(2n - 2m - 1) + \mu[(n + 3)m(n - m - 1) + 3m]\}nh - 2\mu n^2 h \\ < 0 \Leftarrow \mu \leq 1 + b, \quad (\text{B.51})$$

where $K_i := \frac{i}{100}n - m$. $y_e \Psi$ being positive for $m \leq 0.84n$ implies that Ψ must also be positive for $m \leq 0.84n$ to ensure $y_e > 0$ for $m \leq 0.84n$. Ψ being positive implies that $(\chi_e - \chi_f)\Psi$ must also be positive to ensure $\chi_e - \chi_f \geq 0$. $(\chi_e - \chi_f)\Psi$ being positive implies $\mu < \bar{\mu} := 1 + b$ from (B.46). Note that $\mu < \bar{\mu}$ implies $\mu < (n^2 b + 2n - 1)(b + 1)/[(n - 1)^2 b]$, such that the second-order condition of each coalition country is satisfied for $m \leq n - 1$. Furthermore, note that (B.46), (B.47) and (B.50) imply $[n^2 m(n - m - 1)bh]\Psi = [\mu nm(n - m)b]y_e \Psi + [\mu m(n - m)^2 + \Gamma](\chi_e - \chi_f)\Psi$, such that Ψ is positive [negative] if $y_e \Psi$ and $(\chi_e - \chi_f)\Psi$ are positive [negative]. Finally, note that (B.46), (B.48) and (B.51) imply that Ψ , $y_e \Psi$ and $(\chi_e - \chi_f)\Psi$ are positive for all $m \leq n - 2$ if and only if they are positive for $m = n - 2$.

Lemma 4 summarizes the effects of a sufficiently strong lobby influence:

Lemma 4. Consider $\alpha = 1$, $\mu > \underline{\mu}$ and $m \leq n - 2$. Then, compared to a given coalition size without lobbying, industrial lobbying has the following effects:

- Each coalition country's carbon tax and the tax difference between a coalition country and a fringe country are smaller.
- Each country's production is greater.
- Each coalition country's consumption is greater, and the consumption difference between a coalition country and a fringe country is smaller.
- The (joint) welfare difference between a fringe country and a coalition country is smaller.

Proof. Indicating the fully cooperative equilibrium by an asterisk, we find in Online Appendix C

$$x_i^* = y_i^* = \frac{a - \chi_i^*}{b + 1} = \frac{(1 + \mu)a - nh}{\mu(2b + 1) + b + 1}. \quad (\text{B.52})$$

Substituting (B.44) and (B.45) into (A.18), (A.19) and (A.20) yields

$$x_i = \frac{\Gamma[(1+\mu)a-nh] - n(n-m-1)[\mu m^2b - (n^2 - m^2)(b+1)]h}{\Psi}, \quad (\text{B.53a})$$

$$y_e = \frac{[\mu m(n-m)^2 + \Gamma][(1+\mu)a-nh] - n^2(n-m-1)[\mu mb - (n-m)(b+1)]h}{\Psi}, \quad (\text{B.53b})$$

$$y_f = \frac{[-\mu m^2(n-m) + \Gamma][(1+\mu)a-nh] + n^3(n-m-1)(b+1)h}{\Psi}. \quad (\text{B.53c})$$

From (B.45), (B.46) and (B.53), we get

$$\begin{aligned} \chi_e - \chi_e|_{\mu=0} = & -\frac{\mu n}{2\Gamma\Psi} \{ \Gamma \{ 2m(n-m)[\mu - (n-2)(b+1)/(2b)]b + n[mNM_2 + 2(m-1)](b-b_3) + 2[NM_2^3m \\ & + (2m^2 + m-1)NM_2^2 + (m^3 + 3m^2 - 7m + 2)NM_2 + 2(m^2 - 1)(m-2)]/(n-2) \} (a-nh) + \{ 2m(n-m)^2 \\ & \cdot [\mu - (n-2)(b+1)/(2b)]b[n(n+m)(n-m-1)b + (n^2 + nm + 2n + 2m + 4)NM_2 + m^2 + 8m + 8] + nm \\ & \cdot (n-m-1)[(n^2 - m^2 + 2n + 2m + 2)NM_2 + 6m + 4](b-b_3) + [2NM_2^6m + (8m^2 + 13m + 2)NM_2^5 + (10m^3 \\ & + 44m^2 + 26m + 16)NM_2^4 + (4m^4 + 50m^3 + 64m^2 + 14m + 48)NM_2^3 + (19m^4 + 70m^3 - 4m^2 + 64)NM_2^2 \\ & + (26m^4 + 22m^3 - 52m^2 + 16m + 32)NM_2 + 4(m-1)(3m^2 - 4m)]/(n-2) \} (b+1)nh \}, \end{aligned} \quad (\text{B.54})$$

$$(\chi_e - \chi_f) - (\chi_e - \chi_f)|_{\mu=0} = -\frac{\mu nm(n-m)b}{\mu m(n-m)^2 + \Gamma} [y_e + nm(n-m)(n-m-1)h/\Gamma] < 0, \quad (\text{B.55})$$

where $\frac{n-2}{2} < \frac{n-2}{2} \frac{b+1}{b} < \underline{\mu}$ is a lower bound for μ . This proves Lemma 4a.

From (B.53), we get

$$\begin{aligned} x_i - x_i|_{\mu=0} = & \frac{\mu}{2\Gamma\Psi(b+1)} \{ \Gamma \{ 2m^2(n-m)[\mu - (n-2)(b+1)/(2b)]b + n[(m^2 - 2)NM_2 + (m-2)(m+1)]b \\ & + m[mNM_2^2 + (m^2 + 2m - 2)NM_2 + 2(m-2)(m+1)] \} (a-nh) + \{ m^2(n+m)(n-m-1)(n-m)^2[\mu - (n-2)(b+1)/(2b)]b \\ & + nm(n-m-1)[mNM_2^3 + (2m^2 + 6m - 2)NM_2^2 + (8m^2 + 8m - 8)NM_2 + 6m^2 - 8]b \\ & + (m^2 + 2)NM_2^5 + (3m^3 + 7m^2 + 4m + 20)NM_2^4 + (2m^4 + 17m^3 + 14m^2 + 32m + 80)NM_2^3 + (10m^4 + 26m^3 \\ & - 2m^2 + 96m + 160)NM_2^2 + (16m^4 - 32m^2 + 128m + 160)NM_2 + 10m^4 - 16m^3 - 24m^2 + 64m + 64 \} \\ & \cdot (b+1)nh \}, \end{aligned} \quad (\text{B.56a})$$

$$\begin{aligned} y_e - y_e|_{\mu=0} = & \frac{\mu}{2\Gamma\Psi(b+1)} \{ \Gamma \{ 2m(n-m)[\mu - (n-2)/2][nb + n-m] + n^2[m(n-m-1) + m-2](b-b_3) \\ & + n[2mNM_2^3 + (3m^2 + 4m - 2)NM_2^2 + (m^3 + 6m^2 - 10m + 4)NM_2 + 2(m-2)(m^2 - 2)]/(n-2) \} (a-nh) \\ & + \{ 2m(n-m)^2[\mu - (n-2)/2][n(n+m)(n-m-1)(b-b_3) + [2NM_2^4 + (7m + 7)NM_2^3 + (7m^2 + 15m + 4) \\ & \cdot NM_2^2 + (2m^3 + 9m^2 + 4m - 8)NM_2 + m^3 + 4m^2 - 4m - 8]/(n-2)] + n^2m(n-m-1)[NM_2^3 + (2m + 6) \\ & \cdot NM_2^2 + (8m + 10)NM_2 + 6m + 4](b-b_3) + n[2NM_2^6m + (7m^2 + 15m + 2)NM_2^5 + (7m^3 + 43m^2 + 40m \\ & + 16)NM_2^4 + (2m^4 + 37m^3 + 80m^2 + 50m + 48)NM_2^3 + (9m^4 + 56m^3 + 44m^2 + 40m + 64)NM_2^2 + (10m^4 \\ & + 26m^3 - 4m^2 + 32m + 32)NM_2 + 4m(m^3 - m^2 + 4)]/(n-2) \} (b+1)nh \}, \end{aligned} \quad (\text{B.56b})$$

$$(y_f - y_e) - (y_f - y_e)|_{\mu=0} = [(\chi_e - \chi_f) - (\chi_e - \chi_f)|_{\mu=0}]/b < 0. \quad (\text{B.56c})$$

This proves Lemma 4b and Lemma 4c.

The joint welfare of each country is given by $V_i = ay_i - \frac{b}{2}y_i^2 - \frac{1}{2}x_i^2 - p(y_i - x_i) - hx + \mu(px_i - \frac{1}{2}x_i^2)$, where $p = a - by_f$ from (2). Substituting (A.18), (A.19) and (A.20) yields

$$(V_f - V_e) - (V_f - V_e)|_{\mu=0} = (W_f - W_e) - (W_f - W_e)|_{\mu=0} = [(\chi_e - \chi_f)^2 - (\chi_e - \chi_f)|_{\mu=0}]/(2b) < 0. \quad (\text{B.57})$$

This proves Lemma 4d. \square

Lemma 5 summarizes the effects of an increase in the coalition size:

Lemma 5. Consider $\alpha = 1$, $\mu > \underline{\mu}$ and $m \leq n-2$. Then, an increase in the coalition size has the following effects:

- Each fringe country's carbon tax is increasing in m .
- Each fringe country's consumption is decreasing in m .

Proof. Differentiating (B.44) and (B.53) with respect to m yields

$$\begin{aligned} \frac{\partial \chi_f}{\partial m} = & \frac{n}{\Psi^2} \{ \mu(1+\mu)\{(n-m)[\mu n(n+3m)b + n^2 + nm + 2m^2](b+1) + (\bar{\mu} - \mu)[n^3b^2 + n(2n^2 + 2nm - 3m^2)b \\ & + n^3 + nm^2 - 2m^3]\} (a-nh) + \{ \mu^3m(2n-1)^2[(n-m)^2(2n+m)b + nm(n-2m)]b + [\mu - (n^2 - 3n + 1) \\ & / (2n-1)]^2(2n-1)^2[n^2(n^2 - 2mn + m^2 + 2m)b^3 + (3n^4 + 2n^3m - 8n^2m^2 + 2nm^3 + 3m^4 + nm^2)b^2 \} \end{aligned}$$

$$\begin{aligned}
& + (3n^4 + 2n^3m - 4n^2m^2 - 2nm^3 + 2m^4)b + n(n-m)(n^2 + nm + 2m^2)] + [\mu - (n^2 - 3n + 1)/(2n - 1)] \\
& \cdot (2n - 1)\{n^3[2(nm^2 + 124m + 26)NM_1^3 + (52m^4 + 170m^3 + 156m^2 + 42m + 6)NM_1^2 - m - 1]^2 + 2n \\
& - 1](n - 1)b^3 + [6NM_1^6 + (40m + 24)NM_1^5 + (94m^2 + 120m + 37)NM_1^4 + 100m^3 + 214 + (16m^5 + 58 \\
& \cdot m^4 + 92m^3 + 32m^2 - 8m - 2)NM_1 + 4m^6 + 6m^5 + 20m^4 + 16m^3 - 4m^2 - 6m - 1]b^2 + n[6NM_1^5 + (34 \\
& \cdot m + 18)NM_1^4 + (68m^2 + 76m + 17)NM_1^3 + (56m^3 + 100m^2 + 43m + 2)NM_1^2 + (18m^4 + 56m^3 + 25m^2 \\
& - 8m - 5)NM_1 + 2m^5 + 14m^4 + 7m^3 - 10m^2 - 9m - 2]b^2 + (n - m)[2NM_1^5 + (12m + 6)NM_1^4 + (32m^2 \\
& + 28m + 5)NM_1^3 + (44m^3 + 46m^2 + 16m - 1)NM_1^2 + (30m^4 + 30m^3 + 6m^2 - 4m - 3)NM_1 + 8m^5 + 6m^4 \\
& - 6m^3 - 8m^2 - 4m - 1]\} + n^4(n - 1)^2(n - m - 1)^2b^3 + [3NM_1^8 + (26m + 12)NM_1^7 + (90m^2 + 78m \\
& + 16)NM_1^6 + (164m^3 + 205m^2 + 60m + 7)NM_1^5 + (173m^4 + 281m^3 + 73m^2 - 13m + 3)NM_1^4 + (110m^5 \\
& + 214m^4 + 18m^3 - 97m^2 - 4m + 11)NM_1^3 + (44m^6 + 88m^5 - 26m^4 - 141m^3 - 26m^2 + 45m + 13)NM_1^2 \\
& + (m^2 - m - 1)(12m^5 + 29m^4 + 23m^3 - 25m^2 - 30m - 6)NM_1 + (m^2 - m - 1)^2(2m^4 + 5m^3 + 9m^2 + 6 \\
& \cdot m + 1)]b^2 + n[3NM_1^7 + (23m + 9)NM_1^6 + (71m^2 + 54m + 5)NM_1^5 + (113m^3 + 119m^2 + 9m - 3)NM_1^4 \\
& + (99m^4 + 128m^3 - 28m^2 - 32m + 6)NM_1^3 + (47m^5 + 77m^4 - 74m^3 - 76m^2 + 16m + 16)NM_1^2 + (m^2 \\
& - m - 1)(11m^4 + 41m^3 - m^2 - 28m - 10)NM_1 + (m^2 - m - 1)^2(m^3 + 9m^2 + 8m + 2)]b^2 + n[NM_1^7 + (7 \\
& \cdot m + 3)NM_1^6 + (22m^2 + 18m + 1)NM_1^5 + (38m^3 + 40m^2 + 5m - 2)NM_1^4 + (37m^4 + 44m^3 - 7m^2 - 8m \\
& + 3)NM_1^3 + (19m^5 + 29m^4 - 31m^3 - 23m^2 + 9m + 8)NM_1^2 + (m^2 - m - 1)(4m^4 + 18m^3 - 3m^2 - 11m \\
& - 5)NM_1 + (m^2 - m - 1)^2(4m^2 + 3m + 1)]h/(2n - 1)^2\} \\
& = \frac{n}{\psi^2} \{ \mu(1 + \mu)\{ (n - m)[\mu n(n + 3m)b + n^2 + nm + 2m^2](b + 1) + (\bar{\mu} - \mu)[n^3b^2 + n(2n^2 + 2nm - 3m^2)b \\
& + n^3 + nm^2 - 2m^3]\}(a - nh) + \{ - \mu^2(\bar{\mu} - \mu)m(2n - 1)^2[(m + 2n)(n - m)^2b + nm(n - 2m)]b + [\mu - (n^2 \\
& - 3n + 1)/(2n - 1)]^2(2n - 1)^2[(3n^4 + 4n^3m - 10n^2m^2 + 4m^4 + nm^2)b^2(b + 1) + (n - m)(3n^3 + 5n^2m \\
& + 2nm^2 - 2m^3)b + n(n - m)(n^2 + nm + 2m^2)] + [\mu - (n^2 - 3n + 1)/(2n - 1)](2n - 1)\{[2NM_1^6 + (12m \\
& + 8)NM_1^5 + (26m^2 + 36m + 13)NM_1^4 + (24m^3 + 64m^2 + 36m + 11)NM_1^3 + (8m^4 + 56m^3 + 48m^2 + 9m \\
& + 5)NM_1^2 + (22m^4 + 40m^3 + 5m^2 - 6m + 1)NM_1 + m(2m^4 + 13m^3 + 7m^2 - 5m - 3)]b^3 + [6NM_1^6 + (44 \\
& \cdot m + 24)NM_1^5 + (110m^2 + 128m + 37)NM_1^4 + (120m^3 + 242m^2 + 120m + 26)NM_1^3 + (58m^4 + 206m^3 \\
& + 152m^2 + 22m + 6)NM_1^2 + (12m^5 + 76m^4 + 104m^3 + 4m^2 - 24m - 2)NM_1 + 2m^6 + 8m^5 + 30m^4 + 12 \\
& \cdot m^3 - 16m^2 - 10m - 1]b^2 + n[6NM_1^5 + (34m + 18)NM_1^4 + (70m^2 + 76m + 17)NM_1^3 + (58m^3 + 100m^2 \\
& + 43m + 2)NM_1^2 + (16m^4 + 60m^3 + 21m^2 - 8m - 5)NM_1 + 18m^4 + 7m^3 - 12m^2 - 9m - 2]b^2 + (n - m) \\
& \cdot [2NM_1^5 + (12m + 6)NM_1^4 + (32m^2 + 28m + 5)NM_1^3 + (44m^3 + 46m^2 + 16m - 1)NM_1^2 + (30m^4 + 30m^3 \\
& + 6m^2 - 4m - 3)NM_1 + 8m^5 + 6m^4 - 6m^3 - 8m^2 - 4m - 1]\} + [NM_1^8 + (8m + 4)NM_1^7 + (26m^2 + 22m \\
& + 6)NM_1^6 + (44m^3 + 52m^2 + 16m + 4)NM_1^5 + (41m^4 + 70m^3 + 8m^2 + 1)NM_1^4 + (20m^5 + 58m^4 - 8m^3 \\
& - 30m^2 + 6m)NM_1^3 + (4m^6 + 28m^5 - 3m^4 - 50m^3 + m^2 + 16m)NM_1^2 + m(m^2 - m - 1)(6m^3 + 12m^2 \\
& - 10m - 10)NM_1 + m(2 + 3m)(m^2 - m - 1)^2]b^3 + [3NM_1^8 + (28m + 12)NM_1^7 + (102m^2 + 80m + 16) \\
& \cdot NM_1^6 + (192m^3 + 217m^2 + 52m + 7)NM_1^5 + (204m^4 + 313m^3 + 41m^2 - 25m + 3)NM_1^4 + (124m^5 \\
& + 260m^4 - 24m^3 - 141m^2 + 11)NM_1^3 + (42m^6 + 124m^5 - 41m^4 - 203m^3 - 22m^2 + 61m + 13)NM_1^2 \\
& + (m^2 - m - 1)(8m^5 + 39m^4 + 37m^3 - 39m^2 - 40m - 6)NM_1 + (m^2 - m - 1)^2(m^4 + 5m^3 + 13m^2 + 8m \\
& + 1)]b^2 + n[3NM_1^7 + (23m + 9)NM_1^6 + (72m^2 + 54m + 5)NM_1^5 + (116m^3 + 118m^2 + 9m - 3)NM_1^4 \\
& + (101m^4 + 128m^3 - 31m^2 - 32m + 6)NM_1^3 + (45m^5 + 83m^4 - 81m^3 - 75m^2 + 16m + 16)NM_1^2 + (m^2 \\
& - m - 1)(8m^4 + 46m^3 - 4m^2 - 28m - 10)NM_1 + 2(m^2 - m - 1)^2(2m^2 + 4m + 1)]b + n[NM_1^7 + (7m \\
& + 3)NM_1^6 + (22m^2 + 18m + 1)NM_1^5 + (38m^3 + 40m^2 + 5m - 2)NM_1^4 + (37m^4 + 44m^3 - 7m^2 - 8m + 3) \\
& NM_1^3 + (19m^5 + 29m^4 - 31m^3 - 23m^2 + 9m + 8)NM_1^2 + (m^2 - m - 1)(4m^4 + 18m^3 - 3m^2 - 11m - 5) \\
& \cdot NM_1 + (m^2 - m - 1)^2(4m^2 + 3m + 1)]h/(2n - 1)^2\}, \tag{B.58a}
\end{aligned}$$

$$\begin{aligned}
& = \frac{n}{\psi^2} \{ \mu(1 + \mu)[2(1 + \mu)m(n - m)^2 + n^2(b + 1)](a - nh) + n\{2m(n - m)^2(\mu - 1 - b)^3 + [4m(n \\
& - m) + n + 3m](n - m)b + 8m(n - m)^2 + n^2](\mu - 1 - b)^2 + 2[m(n - m) + n + 3m](n - m)b^2 + [8m(n \\
& - m)^2 + 5n^2 + 8nm - 12m^2 + n]b + 8m(n - m)^2 + 3n^2 + 2nm - 3m^2(\mu - 1 - b) + [(n - m)(n + 3m)b \\
& + 2(n - m)(n + 3m) + n](b + 1)^2]h\}, \tag{B.58b}
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial y_f}{\partial m} = \frac{n^2(b + 1)}{\psi^2} \{ \mu(1 + \mu)[2(1 + \mu)m(n - m)^2 + n^2(b + 1)](a - nh) + n\{2m(n - m)^2(\mu - 1 - b)^3 + [4m(n \\
& - m) + n + 3m](n - m)b + 8m(n - m)^2 + n^2](\mu - 1 - b)^2 + 2[m(n - m) + n + 3m](n - m)b^2 + [8m(n \\
& - m)^2 + 5n^2 + 8nm - 12m^2 + n]b + 8m(n - m)^2 + 3n^2 + 2nm - 3m^2(\mu - 1 - b) + [(n - m)(n + 3m)b \\
& + 2(n - m)(n + 3m) + n](b + 1)^2]h\}, \tag{B.58c}
\end{aligned}$$

(B.58a) [(B.58b)] is positive if $\mu \geq (n^2 - 3n + 1)/(2n - 1) (< \underline{\mu})$ and $(n - m)^2(2n + m)b + nm(n - 2m) \geq [\leq] 0$, and (B.58c) is positive if $\mu \geq 1 + b (< \underline{\mu})$. This proves Lemma 5a and Lemma 5b. \square

Finally we analyze the interior solution for $m = n - 1$. Substituting $m = n - 1$ into (B.44), (B.46), and (B.50) yields

$$\begin{aligned} \chi_f \Psi|_{m=n-1} &= \mu n(nb + n - 1)(a - nh) + n\{[(2n - 1)b + n(n - 1)][\mu - (n - 2)(b + 1)/(2b)] \\ &\quad + n[b + n^2 - 3n + 2](b + 1)/(2b)\}h > 0, \end{aligned} \quad (\text{B.59a})$$

$$(\chi_e - \chi_f) \Psi|_{m=n-1} = -\mu n(n - 1)[(1 + \mu)a - nh]b < 0, \quad (\text{B.59b})$$

$$y_e \Psi|_{m=n-1} = -[n^2 - 3n + 1 - (n - 1)\mu][(1 + \mu)a - nh], \quad (\text{B.59c})$$

where $(n - 2)(b + 1)/(2b) < \underline{\mu}$ is a lower bound for μ . Consequently, χ_f is negative, $\chi_e - \chi_f$ is positive and y_e is positive for $m = n - 1$ if and only if $\mu < \underline{\mu}_1 := (n^2 - 3n + 1)/(n - 1)$. Substituting $m = n - 2$ into (B.46) and (B.50) yields

$$(\chi_e - \chi_f) \Psi|_{m=n-2} = n^2(n - 2)(1 + \mu)\{b - (2\mu^2 - 1)/(1 + \mu) - 2\mu[a/(nh) - 1]\}b, \quad (\text{B.60})$$

$$\begin{aligned} y_e \Psi|_{m=n-2} &= (1 + \mu)[n^2(b - \underline{b}_2) + 4\mu(n - 2)(a - nh) + \{4(n - 2)\mu[\mu - (n - 2)(b + 1)/(2b)] \\ &\quad + [2nb^2 + n(n + 2)b + 2(n - 2)^2]\mu/b + 2n(b + 1)nh], \end{aligned} \quad (\text{B.61})$$

such that $y_e \Psi|_{m=n-2}$ is positive if $\mu > \underline{\mu}$, which implies that $(\chi_e - \chi_f) \Psi|_{m=n-2}$ must also be positive if $\mu > \underline{\mu}$. $(\chi_e - \chi_f) \Psi|_{m=n-2}$ is positive if b is sufficiently large and μ and $a/(nh)$ are sufficiently small, and it can only be positive if $\mu \leq \underline{\mu}_2 := (\sqrt{b^2 + 8b + 8} + b)/4 < 1 + b/2$.

B.5. Proof of Proposition 3

From (B.59), χ_f is negative for $m = n - 1$. Rearranging (B.44) for $m < n - 1$ yields

$$\begin{aligned} \chi_f &= \frac{n}{\Psi(n - 1)^2 b(2nb + 2n - 1)} \{ \mu(n - 1)^2 b(2nb + 2n - 1)(nb + m)(a - nh) + \{(n - 1)^2 b(2nb + 2n - 1) \\ &\quad \cdot [(n^2 - m^2)b + nm](\mu - \underline{\mu}) + n(b + 1)^2 \{n[n^2(n - m - 1)^2 + 2(n^2 - n + 1)m(n - m - 1) + m^2]b \\ &\quad + m(n - 1)^4\}h \}, \end{aligned} \quad (\text{B.62})$$

which is positive if $\mu \geq \underline{\mu}$. This proves $\chi_f > 0$ for $m < n - 1$. From Lemma 4, χ_e is smaller than $\chi_e|_{\mu=0}$ for $m < n - 1$, and from Lemma 1, $\chi_e|_{\mu=0}$ is smaller than mh for $m \leq n - 1$. This proves $\chi_e < mh$ for $m < n - 1$. Rearranging (B.45) for $m = n - 1$ yields

$$\begin{aligned} \chi_e|_{m=n-1} &= (n - 1)h - \frac{1}{\Psi|_{m=n-1}(2nb + 2n - 1)^2} \{ \mu n(2nb + 2n - 1)\{(n - 1)b(2nb + 2n - 1)(\mu - \underline{\mu}) \\ &\quad + (nb + n - 1)[n(n - 3)b + n^2 - 4n + 2]\}(a - nh) + \{(2n^2 - 3n + 1)b(2nb + 2n - 1)^2(\mu - \underline{\mu})^2 \\ &\quad + [2n^2(n^2 - 3n + 1)b^2 + n(4n^3 - 13n^2 + 10n - 2)b + (n - 1)^3(2n - 1)](2nb + 2n - 1)(\mu - \underline{\mu}) \\ &\quad + n^3(b + 1)^2[n(n - 3)b + n^2 - 4n + 2]\}h \}, \end{aligned} \quad (\text{B.63})$$

which is greater than $(n - 1)h$ if $\mu \geq \underline{\mu}$. This proves $\chi_e > mh$ for $m = n - 1$.

First suppose $\mu > \underline{\mu}$, such that we have an interior solution for $m = n - 1$. The joint welfare of each country is given by $V_i = ay_i - \frac{b}{2}y_i^2 - \frac{1}{2}x_i^2 - p(y_i - x_i) - hx + \mu(px_i - \frac{1}{2}x_i^2)$, where $p = a - by_f$ from (2). Substituting (B.53) for $m = n - 1$ and (B.52) for $m = n$ into $\Phi(n) = V_i^* - V_f(n - 1)$ yields

$$\begin{aligned} \Phi(n)|_{\mu > \underline{\mu}} &= \frac{\mu^2(n - 1)^2[(1 + \mu)a - nh]^2}{2[\Psi(n - 1)]^2[\mu(2b + 1) + b + 1]} \{ (\mu - \underline{\mu})^2(n^2 - 2n + 1)(2nb + 2n - 1)^2b^2 \\ &\quad + (\mu - \underline{\mu})[2n^2(n^2 - 4n - 1)b^2 + 2n^2(2n^2 - 9n + 3)b + 2n^4 - 10n^3 + 9n^2 \\ &\quad - 4n + 1](2nb + 2n - 1)b + n^2[n(n - 3)b + n^2 - 4n + 2]^2(b + 1)^2 \} > 0. \end{aligned} \quad (\text{B.64})$$

This proves $m = n$ being internally stable for $\mu > \underline{\mu}$.

Now suppose $\mu \leq \underline{\mu}$, such that we have a corner solution for $m = n - 1$, which is either $\chi_f \rightarrow -\infty$ or $\chi_f \rightarrow +\infty$. Substituting $m = n - 1$, (A.18) and (A.19) into (B.43) and rearranging yields

$$\chi_e(\chi_f) = \frac{n(n - 1)b[(b + 1)nh - \mu a]}{(2n - 1)(b + 1)^2 + (n - 1)^2b(b + 1 - \mu)} + \left[1 - \frac{n(n - 1)b(b + 1 + \mu b)}{(2n - 1)(b + 1)^2 + (n - 1)^2b(b + 1 - \mu)} \right] \chi_f. \quad (\text{B.65})$$

The joint welfare of each country is given by $V_i = ay_i - \frac{b}{2}y_i^2 - \frac{1}{2}x_i^2 - p(y_i - x_i) - hx + \mu(px_i - \frac{1}{2}x_i^2)$, where $p = a - by_f$ from (2). Substituting (A.18), (A.19), (A.20) and (B.65) for $m = n - 1$ and (B.52) for $m = n$ into $\Phi(n) = V_i^* - V_f(n - 1)$ yields

$$\begin{aligned} \Phi(n)|_{\mu \leq \underline{\mu}} &= \frac{[(1 + \mu)a - nh]^2}{2[\mu(2b + 1) + b + 1]} - \frac{1}{2[\mu(n - 1)^2b - (n^2b + 2n - 1)(b + 1)]^2} \{ [(bn^2 + 2n - 1)^2a(a - 2hn) + n^2 \\ &\quad \cdot (n - 1)^2(2bn^2 + n^2 + 2n - 1)bh^2](b + 1) + \mu[b(n - 1)^4(\mu a^2 - 2nah - n^2bh^2) - (bn^2 + 2n - 1)[(n^2 - 4n \\ &\quad + 2)b - 2n + 1]a^2] - \{2n[n^2(n^2 - 4n + 2)b - (2n - 1)^2](b + 1)^2h - 2\mu n[\mu(n - 1)^4bh + \mu[(b + 1)n^3a - 2(n \end{aligned}$$

$$\begin{aligned}
& -1)^2(bn^2 + 2n - 1)h](b + 1)]b\} \chi_f\} - \frac{1}{8n^2[\mu(n-1)^2b - (n^2b + 2n - 1)(b + 1)]^4(b + 1)} \{\mu^3[n^2(n-1)^2(n^2 \\
& + 2n - 1)(b - \underline{b}_2) + N_{10}^6 + 50N_{10}^5 + 996N_{10}^4 + 9892N_{10}^3 + 49085N_{10}^2 + 98142N_{10} + 4912](b + 1)^2 + \mu^2(\bar{\mu} \\
& - \mu)[2n^4(n-1)^2(b - \underline{b}_2)^2 + n^2(7N_{10}^4 + 224N_{10}^3 + 2628N_{10}^2 + 13268N_{10} + 23909)(b - \underline{b}_2) + 5N_{10}^6 + 218 \\
& \cdot N_{10}^5 + 3790N_{10}^4 + 32824N_{10}^3 + 141528N_{10}^2 + 242910N_{10} + 404](b + 1) + \mu(\bar{\mu} - \mu)^2[2n^4(n^2 - 4n + 2)(b \\
& - \underline{b}_2)^2 + (7N_{10}^6 + 352N_{10}^5 + 7294N_{10}^4 + 79564N_{10}^3 + 480581N_{10}^2 + 1518420N_{10} + 1950100)(b - \underline{b}_2) + 5 \\
& \cdot N_{10}^6 + 210N_{10}^5 + 3518N_{10}^4 + 29384N_{10}^3 + 122417N_{10}^2 + 204114N_{10} + 3672] + (\bar{\mu} - \mu)^3[n^4(n^2 - 4n + 2) \\
& \cdot (b - \underline{b}_2) + n^2(n-1)(N_{10}^3 + 17N_{10}^2 + 73N_{10} + 15)]\} \chi_f^2 < 0.
\end{aligned} \tag{B.66}$$

This proves $m = n$ being internally unstable for $\mu \leq \underline{\mu}$. Consequently, $m = n$ is internally stable if and only if $\mu > \underline{\mu}$. **Proposition 3a.**

The welfare of each country is given by $W_i = ay_i - \frac{b}{2}y_i^2 - \frac{1}{2}x_i^2 - p(y_i - x_i) - hx$. Substituting (B.53) and (B.52) yields

$$\begin{aligned}
x_i^* - x_i(3)|_{\mu=0} = & -\frac{1}{n(n+2)\Gamma(3)[\mu(2b+1) + b + 1](b+1)} \{\mu n(n+2)\Psi(3)b(a - nh) + n\{\mu[n(n+2)(n-4)(n^2 \\
& - 18)(b - \underline{b}_3) + N_7^5 + 24N_7^4 + 197N_7^3 + 678N_7^2 + 934N_7 + 358] + n(n+2)(n-4)(b+1)(n^2 - 9)\}(b \\
& + 1)h\} < 0,
\end{aligned} \tag{B.67a}$$

$$\begin{aligned}
W_i^* - W_e(3)|_{\mu=0} = & \frac{1}{2\Gamma(3)^2(n^2 - 3n + 1)^2[\mu(2b+1) + b + 1]^2(b+1)} \{\mu^2\Gamma(3)^2(n^2 - 3n + 1)^2[n(4b+2)h \\
& - ba]ba + n^2\{\mu^2\{n^2(n-4)^2(4N_8^5 + 109N_8^4 + 1122N_8^3 + 5288N_8^2 + 10586N_8 + 5319)b^4 + (n-4)(6N_8^8 \\
& + 276N_8^7 + 5358N_8^6 + 57005N_8^5 + 360514N_8^4 + 1370826N_8^3 + 3005350N_8^2 + 3387353N_8 + 1476612)b^3 \\
& + (2N_8^9 + 93N_8^8 + 1822N_8^7 + 19688N_8^6 + 130274N_8^5 + 570177N_8^4 + 1849666N_8^3 + 4985629N_8^2 \\
& + 10071392N_8 + 9965932)b^2 + (N_8^8 + 68N_8^7 + 2027N_8^6 + 33854N_8^5 + 342966N_8^4 + 2150454N_8^3 \\
& + 8147172N_8^2 + 17068024N_8 + 15157084)b + (N_8^3 + 18N_8^2 + 117N_8 + 269)(2N_8^5 + 68N_8^4 + 893N_8^3 \\
& + 5674N_8^2 + 17467N_8 + 20851)\} + (n-1)(n-4)^2[n^2b + n^2 - 9](n^2 - 9)(b+1)^2\{\mu[(4n^2 - 11n + 3)b \\
& + 2n^2 - 5n + 1] + (n^2 - 3n + 1)(b+1)\}(\bar{\mu}_1 - \mu)\}h^2\},
\end{aligned} \tag{B.67b}$$

$$\begin{aligned}
W_i^* - W_f(3)|_{\mu=0} = & \frac{1}{2\Gamma(3)^2[\mu(2b+1) + b + 1]^2(b+1)} \{\mu^2\Gamma(3)^2[n(3b+2)h - ba]b(a - nh) + n^2\{\mu^2\{3n^2(n \\
& - 4)^2(n^2 - 24)b^3 + (n-4)(5N_8^5 + 178N_8^4 + 2376N_8^3 + 14794N_8^2 + 42980N_8 + 46992)b^2 + (2N_8^6 + 76 \\
& \cdot N_8^5 + 1125N_8^4 + 8284N_8^3 + 32174N_8^2 + 63048N_8 + 49191)(b - \underline{b}_3)^2 + (2N_8^8 + 94N_8^7 + 1830N_8^6 \\
& + 19174N_8^5 + 117449N_8^4 + 425886N_8^3 + 873654N_8^2 + 881064N_8 + 282972)/[n(n-2)]\} + (n-4)^2[n^2 \\
& \cdot (n^2 - 18)b + (n-3)^2(n+3)^2][\mu(2b+1) + b + 1](b+1)\}h^2\} \\
= & \frac{1}{2\Gamma(3)^2(n^2 - 3n + 1)^2[\mu(2b+1) + b + 1]^2(b+1)} \{\mu^2\Gamma(3)^2(n^2 - 3n + 1)^2\{[n(4b+1)h - ba]a + n(a - nh) \\
& \cdot h\}b + n^2\{\mu^2\{n^2(n-4)^2(4N_{17}^5 + 253N_{17}^4 + 6018N_{17}^3 + 64211N_{17}^2 + 267284N_{17} + 101880)b^4 + (n-4) \\
& \cdot (N_{17}^9 + 149N_{17}^8 + 9715N_{17}^7 + 364303N_{17}^6 + 8665928N_{17}^5 + 135671504N_{17}^4 + 1398054576N_{17}^3 \\
& + 9141545072N_{17}^2 + 34396241973N_{17} + 56681992585)b^3 + (2N_{17}^{10} + 316N_{17}^9 + 22348N_{17}^8 + 931804N_{17}^7 \\
& + 25371240N_{17}^6 + 471452062N_{17}^5 + 6055802035N_{17}^4 + 53101665492N_{17}^3 + 304246675525N_{17}^2 \\
& + 1028631774008N_{17} + 1558516743104)b^2 + N_{17}^{10} + 158N_{17}^9 + 11193N_{17}^8 + 468232N_{17}^7 + (N_{17}^8 + 68 \\
& \cdot N_{17}^7 + 2027N_{17}^6 + 33854N_{17}^5 + 342966N_{17}^4 + 2150454N_{17}^3 + 12810419N_{17}^2 + 239539402N_{17} + 3100607279 \\
& \cdot N_{17}^4 + 27436562886N_{17}^3 + 158856178619N_{17}^2 + 543513238886N_{17} + 834561606746)b + (N_{17}^3 + 45 \\
& \cdot N_{17}^2 + 684N_{17} + 3509)(2N_{17}^5 + 158N_{17}^4 + 4961N_{17}^3 + 77413N_{17}^2 + 600496N_{17} + 1852891)\} + (n-1) \\
& \cdot (n-4)^2[n^2(n^2 - 18)b + (n^2 - 9)^2](b+1)^2\{\mu[(4n^2 - 11n + 3)b + 2n^2 - 5n + 1] + (n^2 - 3n + 1)(b+1)\} \\
& \cdot (\bar{\mu}_1 - \mu)\}h^2\}.
\end{aligned} \tag{B.67c}$$

Thus, global emissions are smaller, the welfare of each coalition country is greater if $n(4b+2)h - ba \geq 0 \Leftrightarrow a \leq 4nh$ and, since $W_f > W/n > W_e$, the welfare of each fringe country and global welfare are greater if $n(3b+2)h - ba \geq 0 \Leftrightarrow a \leq 3nh$ or if $n \geq 17$ and $n(4b+1)h - ba \geq 0 \Leftrightarrow a \leq 4nh$ for $m = n$ with lobbying than for $m = 3$ without lobbying. This proves **Proposition 3b.**

From (B.52), we get

$$x_i^* - x_i|_{\mu=0} = \frac{(2nh - a)b + nh}{[\mu(2b+1) + b + 1](b+1)} = -\frac{nh(b - \underline{b}_3) + \left[a - \left[\frac{42}{11} - \frac{(n-10)(9n-8)}{11(n^2-6n+4)}\right]nh\right]bb_3}{[\mu(2b+1) + b + 1](b+1)b_3}, \tag{B.68}$$

which is positive [negative] if $a \leq 2nh$ [$a \geq 4nh$]. This proves **Proposition 3c.** \square

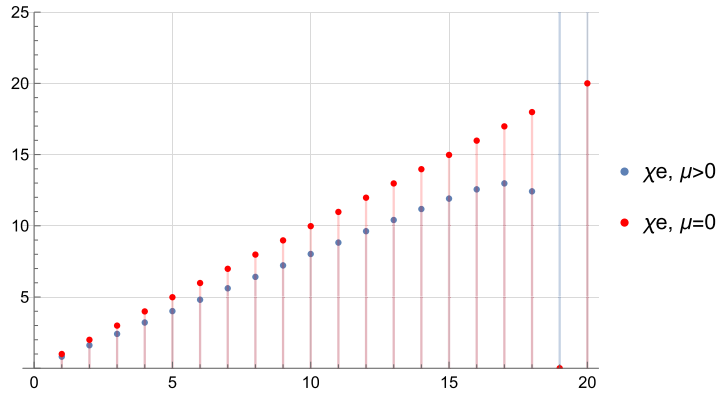


Fig. 1. Comparison of coalition countries' taxes with and without lobbying for $n = 20$, $a = 80$, $b = 286$, $\mu = 13.5$.

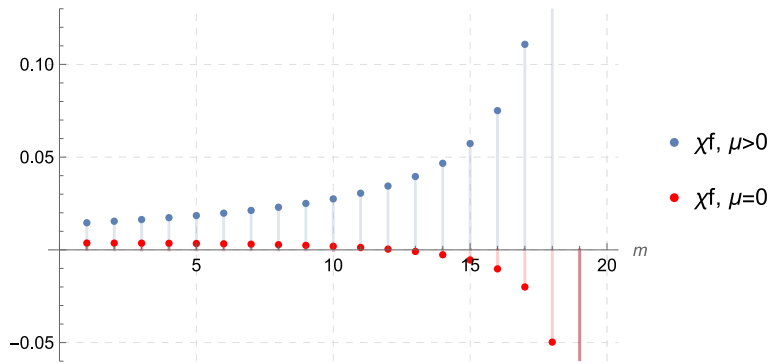


Fig. 2. Comparison of fringe countries' taxes with and without lobbying for $n = 20$, $a = 80$, $b = 286$, $\mu = 13.5$.

B.6. Numerical example

Whether the values of and the differences between the carbon taxes, the production and consumption quantities and the welfare levels increase or decrease with μ and m depends on n, m, μ as well as on a/h and b . Consequently, we normalize $h = 1$ without loss of generality. In choosing μ , a and b , we consider the following parameter restrictions: We focus on equilibria in which $m = n$ is internally stable, i.e. $\mu > \underline{\mu} = \frac{n(n-2)b+(n-1)^2}{2nb+2n-1}$. For positive quantities in case of $m = n$, we require $a/nh \geq 1$. For positive quantities and positive tax differences in case of $m = n - 1$ and $m \leq n - 2$, we require $\mu < \bar{\mu}_1 = \frac{n^2-3n+1}{n-1}$ and $(\chi_e - \chi_f)\Psi|_{m=n-2} > 0$, respectively. The latter condition is satisfied if b is sufficiently large and μ and a/nh are sufficiently small. Using $\mu = \bar{\mu}_1$ and $a/nh = 8$ as upper bounds yields the lower bound $b > \frac{16n^4-82n^3+119n^2-38n+1}{n(n-1)(n-2)}$. Using this threshold in μ yields $\mu > \frac{(n-2)(8n^3-41n^2+60n-19)(8n^4-41n^3+60n^2-20n+1)}{(8n^4-42n^3+63n^2-22n+1)(16n^3-82n^2+119n-37)}$.

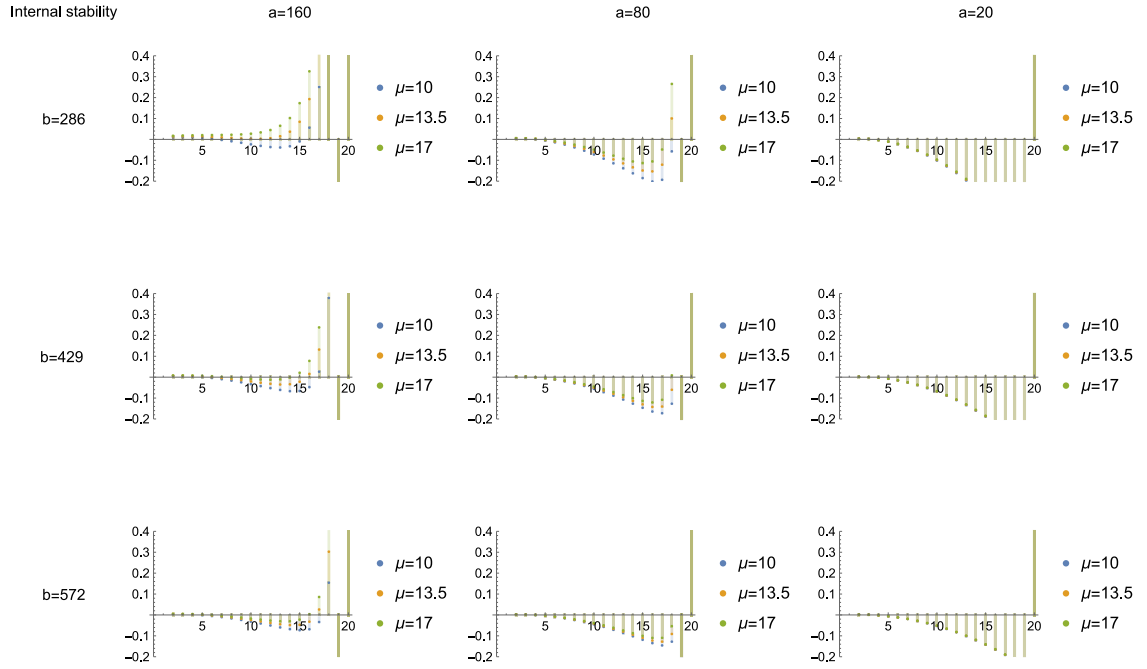
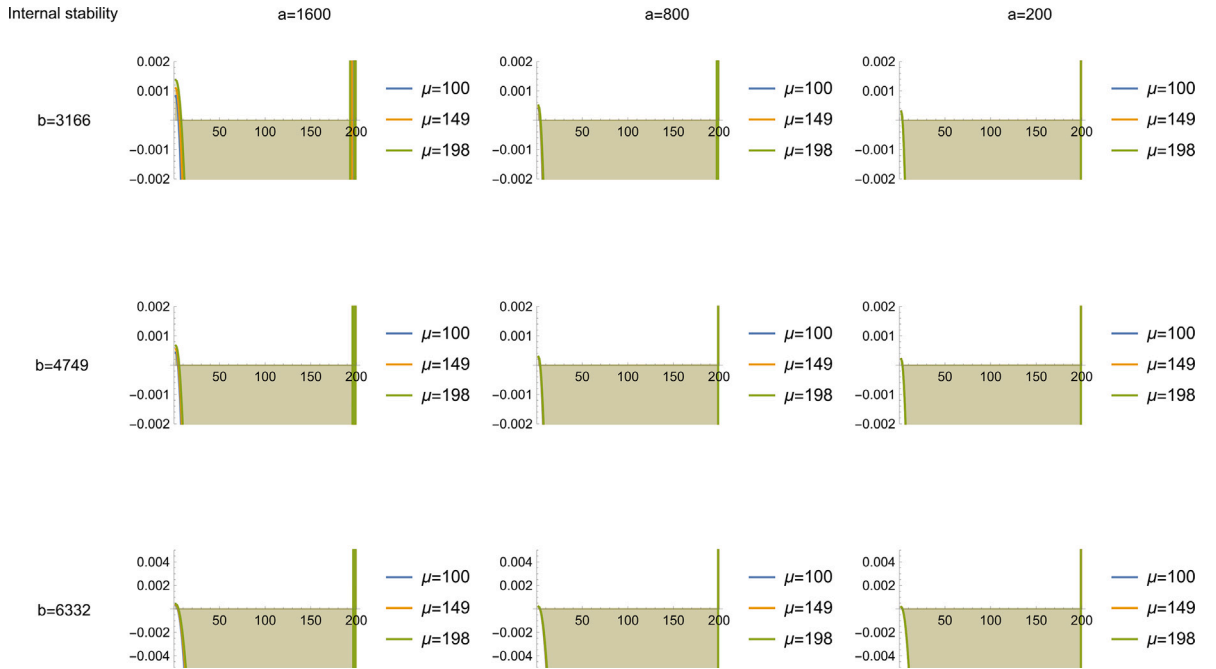
We set $n = 20$ in the body of the paper, and we check the robustness of our results by setting $n = 200$ later in this appendix of the paper. For $n = 20$, we then get the parameter restrictions $a \in [20, 160]$, $\mu \in [9.07, 17.95]$ and $b \geq 285.21$, and for $n = 200$, we then get the parameter restrictions $a \in [200, 1600]$, $\mu \in [99.06, 198.0]$ and $b \geq 3165.9$. Finally, we vary b between its lower bound and twice its lower bound in the sensitivity analysis, i.e. $b \in [286, 572]$ and $b \in [3166, 6332]$, respectively.

Figs. 3 and 4 illustrate the internal stability condition for different values of a , b , μ and n . Note that if some coalition m is internally stable, then the coalition $m - 1$ is externally unstable. Since $\mu > \underline{\mu}$, the grand coalition is always stable. Furthermore, there is either a small stable coalition $m \in [3, 8]$ and/or a large stable coalition $m = n - 2$.

Tables 2 and 3 summarize the stable coalitions in addition to the grand coalition for different values of a , b , μ and n . From these tables, we infer that $m = n - 2$ is stable and $m \in [3, 8]$ is large if a/h and μ are sufficiently large and b and n are sufficiently small.

Tables 4 and 5 compare global welfare at the stable coalitions with lobbying, i.e. $m \in [3, 8]$, $m = n - 2$ and $m = n$, versus global welfare at the stable coalition without lobbying, i.e. $m = 3$. From these tables and Tables 2 and 3, we infer that $m = n - 2$ is welfare superior to $m \in [3, 8]$, $m = n$ and $m = 3$ without lobbying. Furthermore, $m = n$ is welfare superior to $m \in [3, 8]$ and $m = 3$ without lobbying if and only if a/h is sufficiently small, and $m \in [3, 8]$ is welfare superior to $m = 3$ without lobbying if and only if the coalition size is larger with lobbying than without.

Tables 6 and 7 compare global emissions at the stable coalitions with lobbying, i.e. $m \in [3, 8]$, $m = n - 2$ and $m = n$, versus global welfare at the stable coalition without lobbying, i.e. $m = 3$. From these tables and Tables 2 and 3, we infer that global emissions are smaller with lobbying than without if and only if the coalition size is larger with lobbying than without.

Fig. 3. Internal stability condition with $n = 20$ and $h = 1$.Fig. 4. Internal stability condition with $n = 200$ and $h = 1$.

We summarize our results in

Result 1. Consider $\mu > \underline{\mu}$.

- $m = n - 2$ is stable and $m \in [3, 8]$ is large if a/h and μ are sufficiently large and b and n are sufficiently small.
- $m = n - 2$ is welfare superior to $m \in [3, 8]$, $m = n$ and $m = 3$ without lobbying.

Table 2Stable coalitions in addition to $m = n$ with $n = 20$ and $h = 1$.

$\begin{matrix} b & \mu \\ \hline \end{matrix}$	$a = 160$			$a = 80$			$a = 20$		
	10	13.5	17	10	13.5	17	10	13.5	17
286	$6, n - 2$	$n - 2$	$n - 2$	4	$4, n - 2$	$4, n - 2$	3	3	3
429	$5, n - 2$	$6, n - 2$	$7, n - 2$	3	3	$4, n - 2$	3	3	3
572	$4, n - 2$	$5, n - 2$	$5, n - 2$	3	3	3	3	3	3

Table 3Stable coalitions in addition to $m = n$ with $n = 200$ and $h = 1$.

$\begin{matrix} b & \mu \\ \hline \end{matrix}$	$a = 1600$			$a = 800$			$a = 200$		
	100	149	198	100	149	198	100	149	198
3166	$5, n - 2$	$6, n - 2$	$8, n - 2$	3	4	$4, n - 2$	3	3	3
4749	$4, n - 2$	$5, n - 2$	$5, n - 2$	3	3	3	3	3	3
6332	$4, n - 2$	$4, n - 2$	$5, n - 2$	3	3	3	3	3	3

Table 4Global welfare difference between stable coalitions for $\mu > 0$ and $\mu = 0$ with $n = 20$ and $h = 1$.

m	$\begin{matrix} b & \mu \\ \hline \end{matrix}$	$a = 160$			$a = 80$			$a = 20$		
		10	13.5	17	10	13.5	17	10	13.5	17
$\in [3, 7]$	286	1.04	-	-	0.307	0.259	0.211	-0.018	-0.025	-0.032
$n - 2$		8.21	5.32	1.45	-	10.2	9.31	-	-	-
n		-99.9	-103	-104	0.803	0.487	0.295	10.2	10.1	10.1
$\in [3, 7]$	429	0.470	0.723	0.966	-0.028	-0.040	0.189	-0.008	-0.011	-0.014
$n - 2$		6.70	5.83	4.71	-	-	7.06	-	-	-
n		-66.8	-68.7	-69.8	0.517	0.305	0.177	6.80	6.75	6.71
$\in [3, 7]$	572	0.155	0.348	0.311	-0.016	-0.022	-0.029	-0.005	-0.006	-0.008
$n - 2$		5.36	4.98	4.50	-	-	-	-	-	-
n		-50.2	-51.6	-52.5	0.380	0.221	0.125	5.10	5.06	5.04

Table 5Global welfare difference between stable coalitions for $\mu > 0$ and $\mu = 0$ with $n = 200$, and $h = 1$.

m	$\begin{matrix} b & \mu \\ \hline \end{matrix}$	$a = 1600$			$a = 800$			$a = 200$		
		100	149	198	100	149	198	100	149	198
$\in [3, 8]$	3166	0.631	0.865	1.45	-0.059	0.261	0.199	-0.018	-0.026	-0.035
$n - 2$		968	588	5.77	-	-	1014	-	-	-
n		-9987	-10024	-10042	12.8	8.66	6.58	950	949	948
$\in [3, 8]$	4749	0.191	0.416	0.331	-0.026	-0.042	-0.057	-0.008	-0.012	-0.016
$n - 2$		751	647	493	-	-	-	-	-	-
n		-6660	-6684	-6697	8.33	5.60	4.21	633	633	632
$\in [3, 8]$	4749	0.162	0.131	0.331	-0.026	-0.015	-0.023	-0.032	-0.007	-0.009
$n - 2$		590	548	486	-	-	-	-	-	-
n		-4996	-5014	-5023	6.18	4.13	3.09	475	474	474

c. $m = n$ is welfare superior to $m \in [3, 8]$ and $m = 3$ without lobbying iff a/h is sufficiently small.

d. $m \in [3, 8]$ is welfare superior to $m = 3$ without lobbying iff the coalition size is larger with lobbying than without.

e. Global emissions are smaller with lobbying than without iff the coalition size is larger with lobbying than without.

Appendix C. Supplementary material

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.jeem.2024.102979>.

Table 6

Global emissions difference between stable coalitions for $\mu > 0$ and $\mu = 0$ with $n = 20$ and $h = 1$.

m	μ	$a = 160$			$a = 80$			$a = 20$		
		10	13.5	17	10	13.5	17	10	13.5	17
$\in [3, 7]$	286	-0.059	-	-	-0.017	-0.014	-0.011	0.011	0.015	0.019
$n - 2$		-0.534	-0.309	-0.074	-	-0.749	-0.644	-	-	-
n		-5.33	-5.38	-5.41	-2.68	-2.70	-2.71	-0.697	-0.689	-0.684
$\in [3, 7]$	429	-0.027	-0.041	-0.056	0.002	0.002	-0.010	0.000	0.001	0.001
$n - 2$		-0.486	-0.391	-0.292	-	-	-0.538	-	-	-
n		-3.56	-3.60	-3.62	-1.79	-1.80	-1.81	-0.466	-0.460	-0.457
$\in [3, 7]$	572	-0.009	-0.020	-0.017	0.001	0.001	0.002	0.000	0.000	0.000
$n - 2$		-0.412	-0.360	-0.306	-	-	-	-	-	-
n		-2.67	-2.70	-2.71	-1.35	-1.35	-1.36	-0.350	-0.345	-0.343

Table 7

Global emissions difference between stable coalitions for $\mu > 0$ and $\mu = 0$ with $n = 200$ and $h = 1$.

m	μ	$a = 1600$			$a = 800$			$a = 200$		
		100	149	198	100	149	198	100	149	198
$\in [3, 8]$	3166	-0.003	-0.004	-0.007	0.000	-0.001	-0.001	0.000	0.000	0.000
$n - 2$		-6.56	-3.40	-0.029	-	-	-7.07	-	-	-
n		-50.3	-50.4	-50.4	-25.2	-25.2	-25.2	-6.34	-6.33	-6.33
$\in [3, 8]$	4749	-0.001	-0.002	-0.002	0.000	0.000	0.000	0.000	0.000	0.000
$n - 2$		-5.72	-4.39	-3.01	-	-	-	-	-	-
n		-33.6	-33.6	-33.6	-16.8	-16.8	-16.8	-4.23	-4.22	-4.22
$\in [3, 8]$	6332	-0.001	-0.001	-0.002	0.000	0.000	0.000	0.000	0.000	0.000
$n - 2$		-4.78	-4.05	-3.30	-	-	-	-	-	-
n		-25.2	-25.2	-25.2	-12.6	-12.6	-12.6	-3.17	-3.17	-3.16

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