Homework 2

Graphical Models

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Conditional Random Fields - CRF

$$P(\mathbf{y}|\mathbf{x}) = \frac{1}{Z}\left[exp\left(\sum_{i\in S} U(y_i,\mathbf{x}) + \theta_P \sum_{i\in S}\sum_{j\in N_i} P(y_i,y_j,\mathbf{x}_i,\mathbf{x}_j)\right)\right]$$

Where:

- $\mathbf{y} = [y_1, ..., y_n]$: set of labels, where $y_i \in \{1, ..., M\}$ and M is the number of classes and n is the number of sites
- $\mathbf{x}$: observed data
- U and P are the *Unary (Association)* and *Pairwise (Interaction)* potentials
- S: sites in the image indexed by i
- N_i: Neighborhood of a site i, where $j \in N_i$ is a neighbor site of i
- Z: partition function or normalizing constant
- θ_P: *Pairwise* potential weight
- $\theta_P \in [1, 2, \ldots]$

Conditional Random Fields - CRF

$$P(\mathbf{y}|\mathbf{x}) = \frac{1}{Z}\left[exp\left(\boxed{\sum_{i\in S} U(y_i,\mathbf{x})} + \theta_P \sum_{i\in S}\sum_{j\in N_i} P(y_i,y_j,\mathbf{x}_i,\mathbf{x}_j)\right)\right]$$

Unary **or Association Potential:**

How likely an image site i will take a label given its y_i feature vector $\mathbf{f}_i(\mathbf{x})$

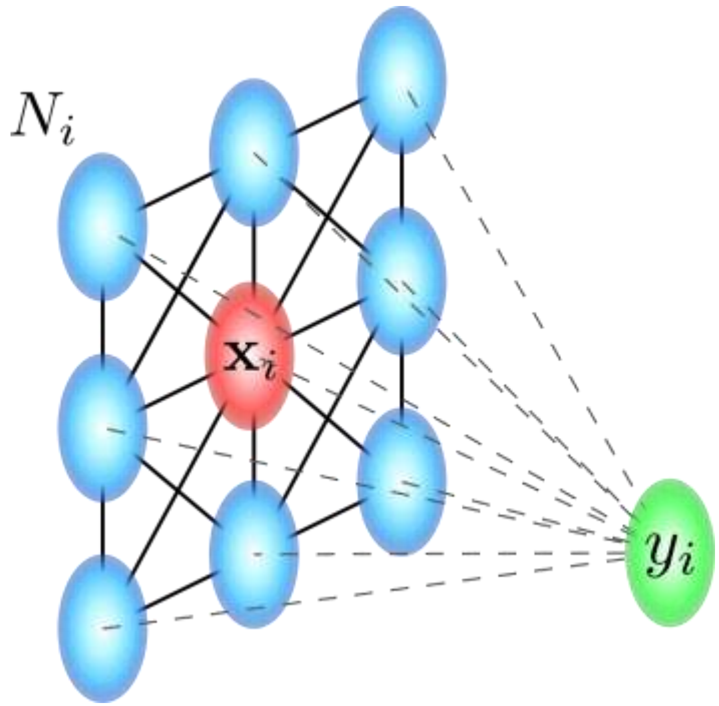
$$U(y_i,\mathbf{x}) = logP(y_i|\mathbf{f}_i(\mathbf{x}))$$

Any discriminative classifier can be used.

Random Forest (RF):

$$logP(y_i|\mathbf{f}_i(\mathbf{x})) = \left(\frac{V_y}{N_T}\right)$$

N_T is the number of trees and V_y is the number of votes for a class y



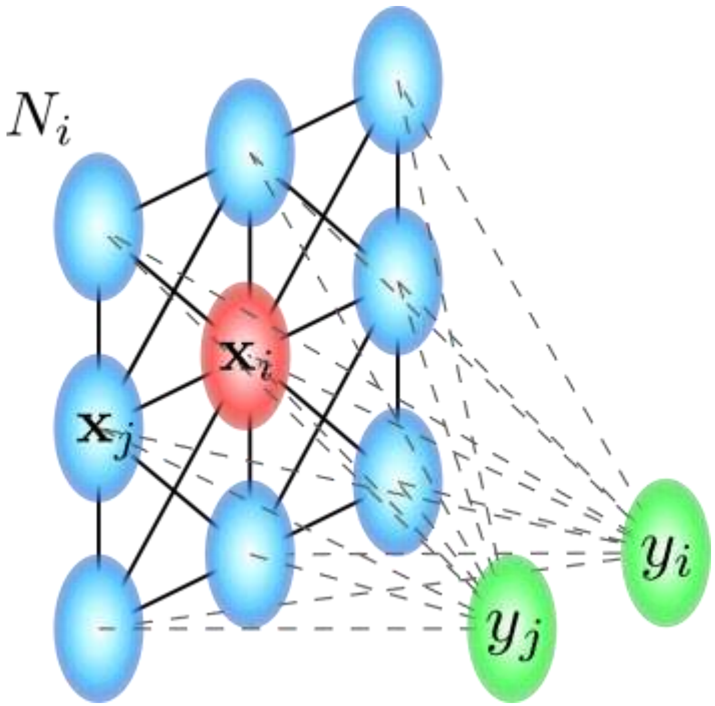
Conditional Random Fields - CRF

$$P(\mathbf{y}|\mathbf{x}) = \frac{1}{Z}\left[exp\left(\sum_{i\in S} U(y_i,\mathbf{x}) + \theta_P \boxed{\sum_{i\in S}\sum_{j\in N_i} P(y_i,y_j,\mathbf{x}_i,\mathbf{x}_j)}\right)\right]$$

Pairwise **or Interaction Potential:**

Potts Model

$$P(y_i,y_j) = \begin{cases} \beta & ,\text{if} \;\; y_i = y_j \\ 0 & ,\text{if} \;\; y_i \neq y_j \end{cases}$$



Conditional Random Fields - CRF

$$P(\mathbf{y}|\mathbf{x}) = \frac{1}{Z}\left[exp\left(\sum_{i\in S} U(y_i,\mathbf{x}) + \theta_P \boxed{\sum_{i\in S}\sum_{j\in N_i} P(y_i,y_j,\mathbf{x}_i,\mathbf{x}_j)}\right)\right]$$

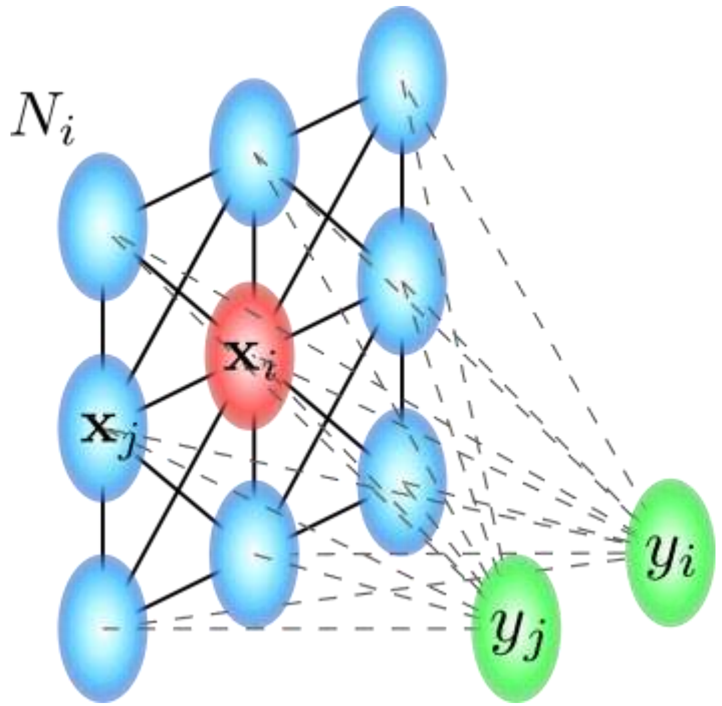
Pairwise **or Interaction Potential:**

Contrast-sensitive Potts Model

$$P(y_i,y_j,\mathbf{x}_i,\mathbf{x}_j) = \begin{cases} p + (1-p)e^{-\frac{d_{ij}^2}{2\sigma^2}} & ,\text{if } \; y_i = y_j \\ 0 & ,\text{if } \; y_i \neq y_j \end{cases}$$

where: $d_{ij} = \|\mathbf{f}_i(\mathbf{x}) - \mathbf{f}_j(\mathbf{x})\|$ and σ^2 is the mean value of d_{ij}^2 determined during training.

$p \in [0;1]$ controls the weights of the data-dependent and data-independent smoothing terms.



Conditional Random Fields - CRF

The following approaches are proposed to be tested:

1. $\boldsymbol{RF_{Pixel}}$**:** Using only the unary potential. Association Potential with *Random Forest* (RF) classifier.

2. $\boldsymbol{CRF_{RF+Potts}}$**:** Using the unary and pairwise potentials. Association Potential with *Random Forest* (RF) and Spatial Interaction potential with *Potts* Model.

3. $\boldsymbol{CRF_{RF+ContrastPotts}}$**:** Using the unary and pairwise potentials. Association Potential with *Random Forest* (RF) and Spatial Interaction potential with *Contrast-Sensitive Potts* Model.

Perform the following experiments:

1. Run RF_{Pixel}, $CRF_{RF+Potts}$ and $CRF_{RF+ContrastPotts}$.
2. Change the θ_p to find the one that produce the best result.
3. Show maps for different approaches and values of θ_P, as well as accuracy metrics (OA, AA, Precision, Recall, F1-score)

Markov Random Fields - CRF

The following approaches are proposed to be tested:

1. $\boldsymbol{NB_{Pixel}}$**:** Using only the unary potential. Association Potential with *Naïve Bayes* (NB) classifier.

2. $\boldsymbol{MRF_{NB+Potts}}$**:** Using the unary and pairwise potentials. Association Potential with *Naïve Bayes* (NB) and Spatial Interaction potential with *Potts* Model.

3. $\boldsymbol{MRF_{NB+ContrastPotts}}$**:** Using the unary and pairwise potentials. Association Potential with *Naïve Bayes* (NB) and Spatial Interaction potential with *Contrast-Sensitive Potts* Model.

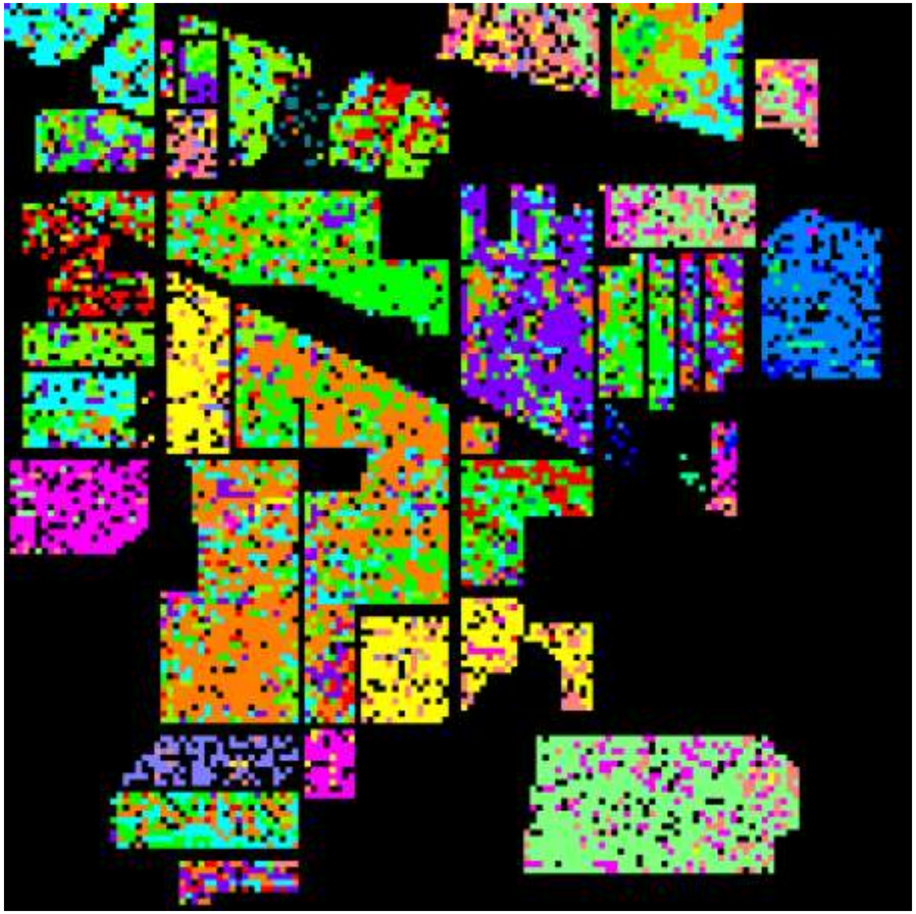
Perform the following experiments:

1. Run NB_{Pixel}, $MRF_{NB+Potts}$ and $MRF_{NB+ContrastPotts}$.
2. Change the θ_p to find the one that produce the best result.
3. Show maps for different approaches and values of θ_P, as well as accuracy metrics (OA, AA, Precision, Recall, F1-score)

Results

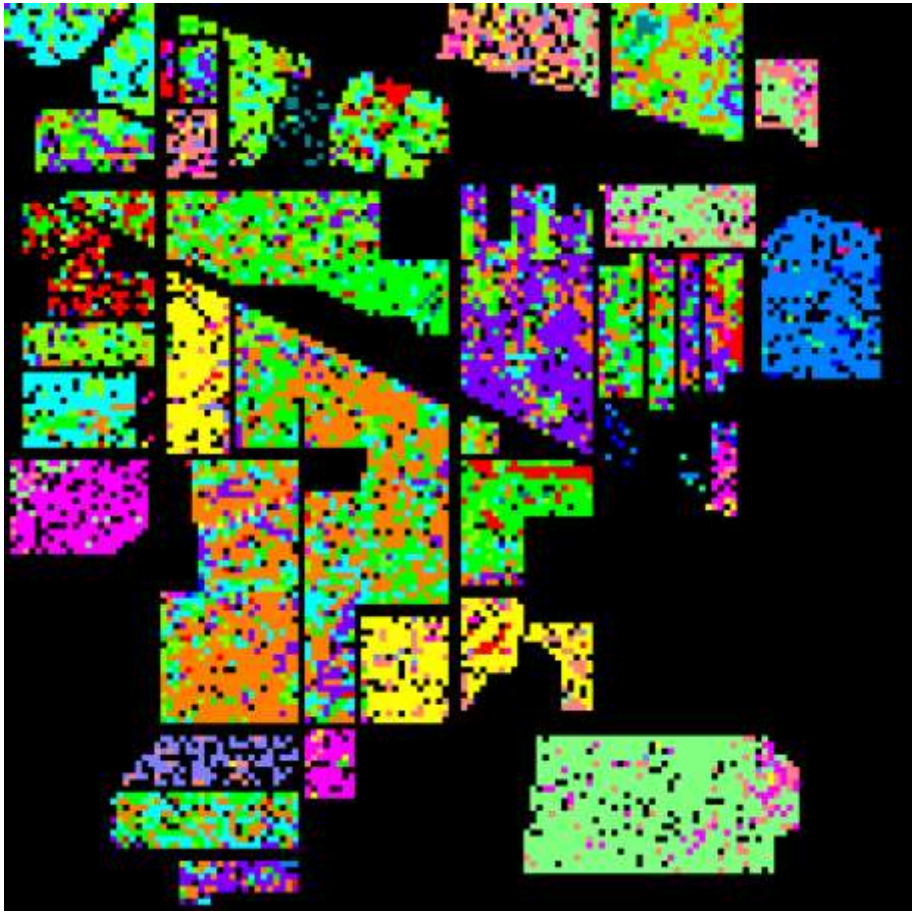
$CRF_{RF+Potts}$

$\theta_{SP} = 1$



$CRF_{RF+ContrastPotts}$

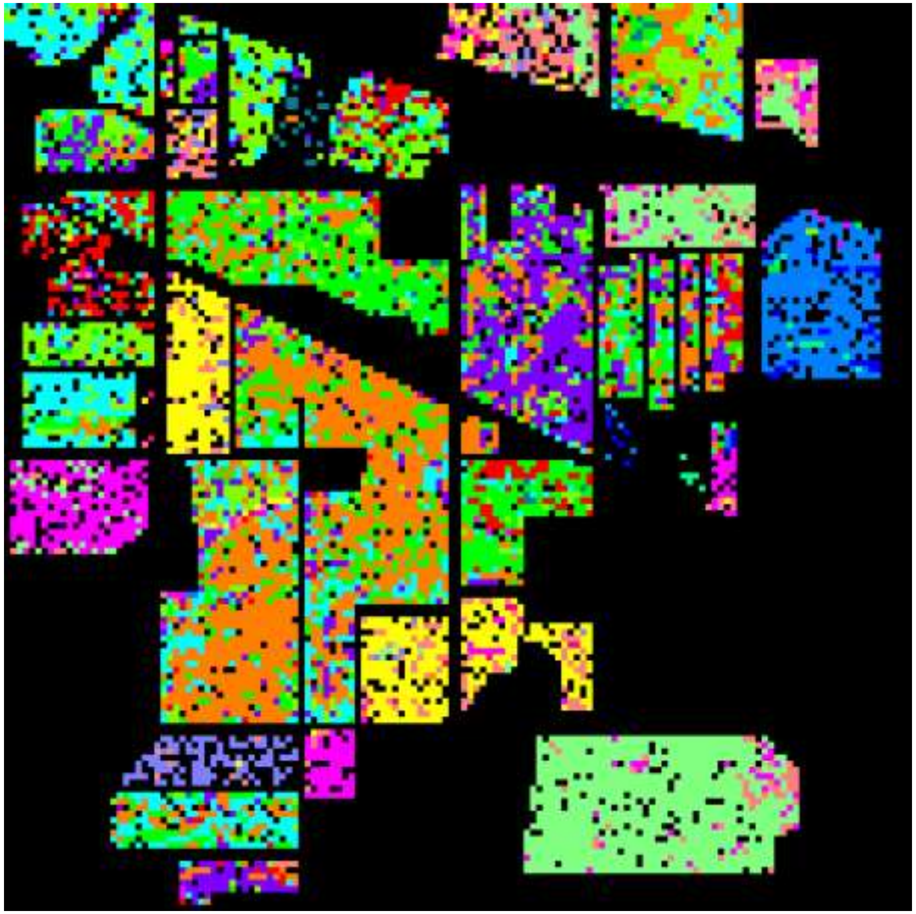
$\theta_{SP} = 1$



Results

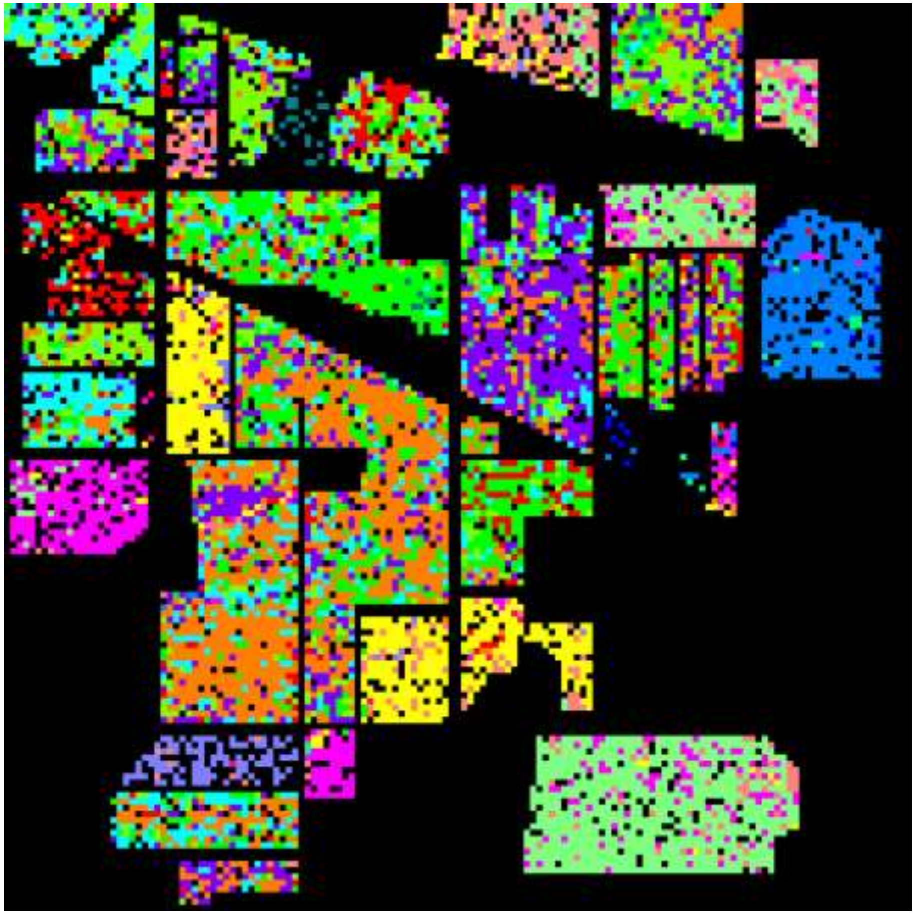
$CRF_{RF+Potts}$

$\theta_{SP} = 2$



$CRF_{RF+ContrastPotts}$

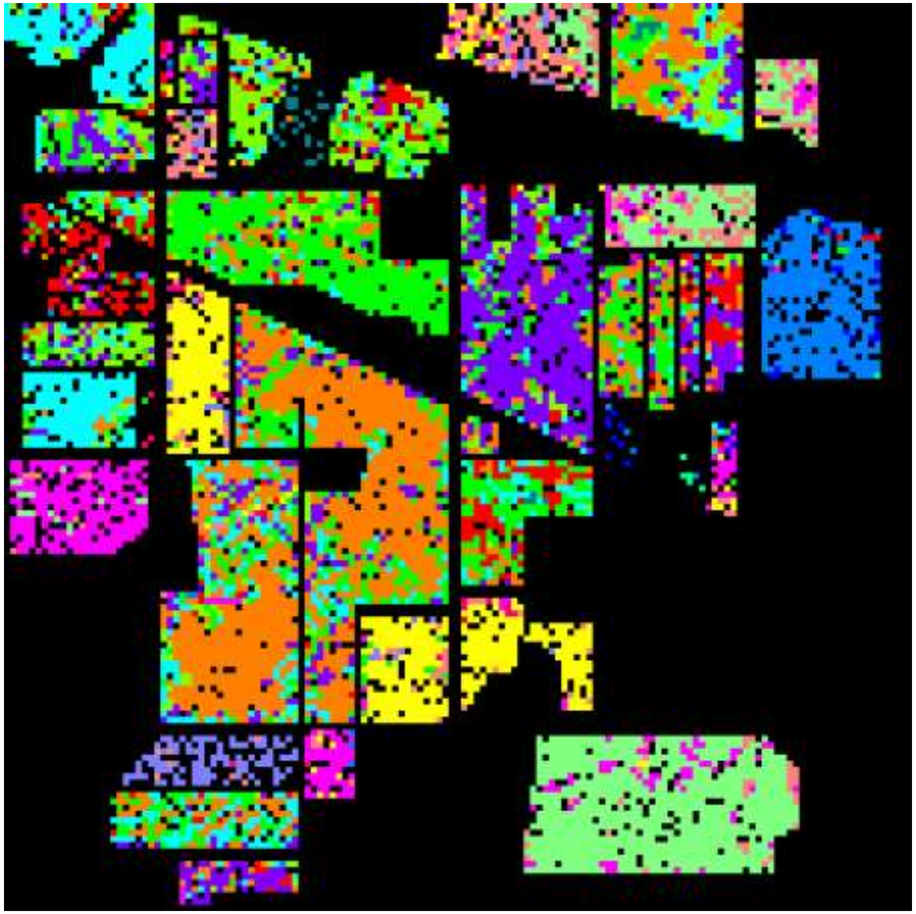
$\theta_{SP} = 2$



Results

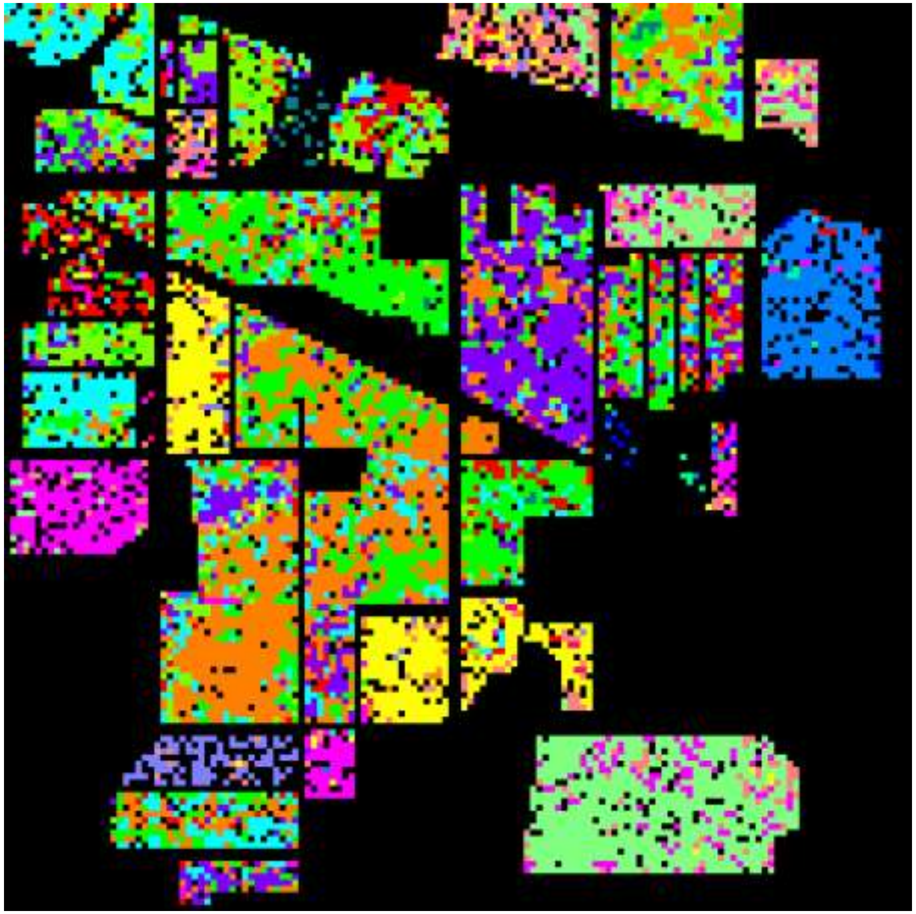
$CRF_{RF+Potts}$

$\theta_{SP} = 3$



$CRF_{RF+ContrastPotts}$

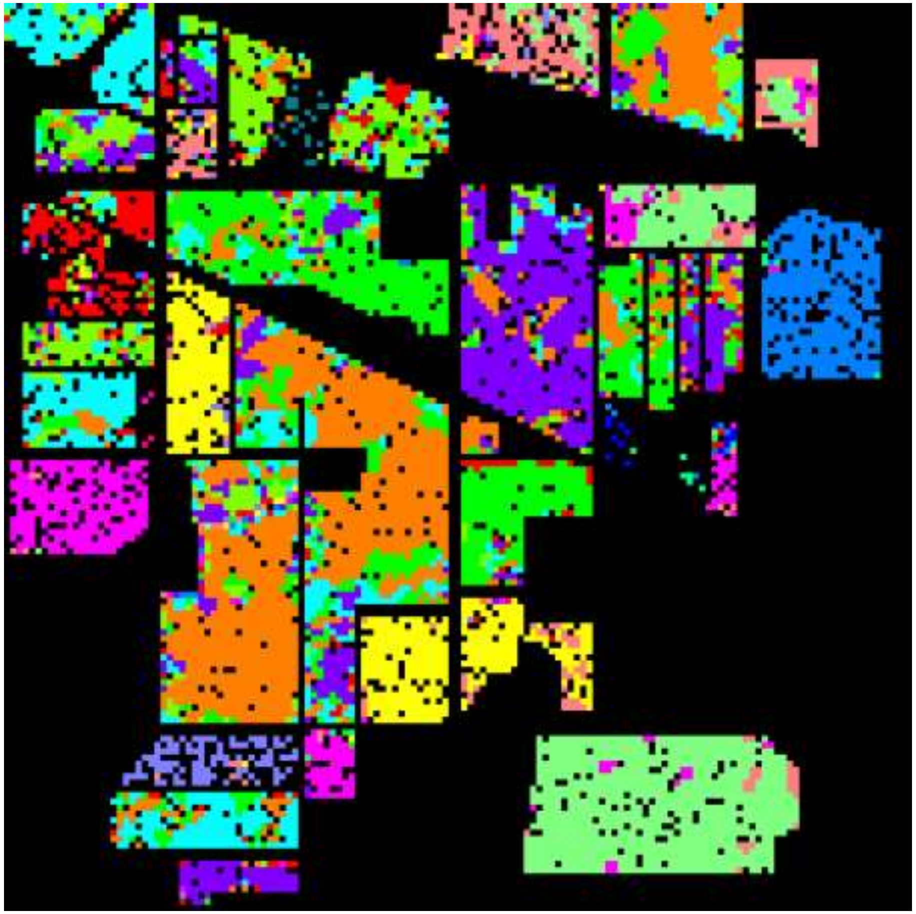
$\theta_{SP} = 3$



Results

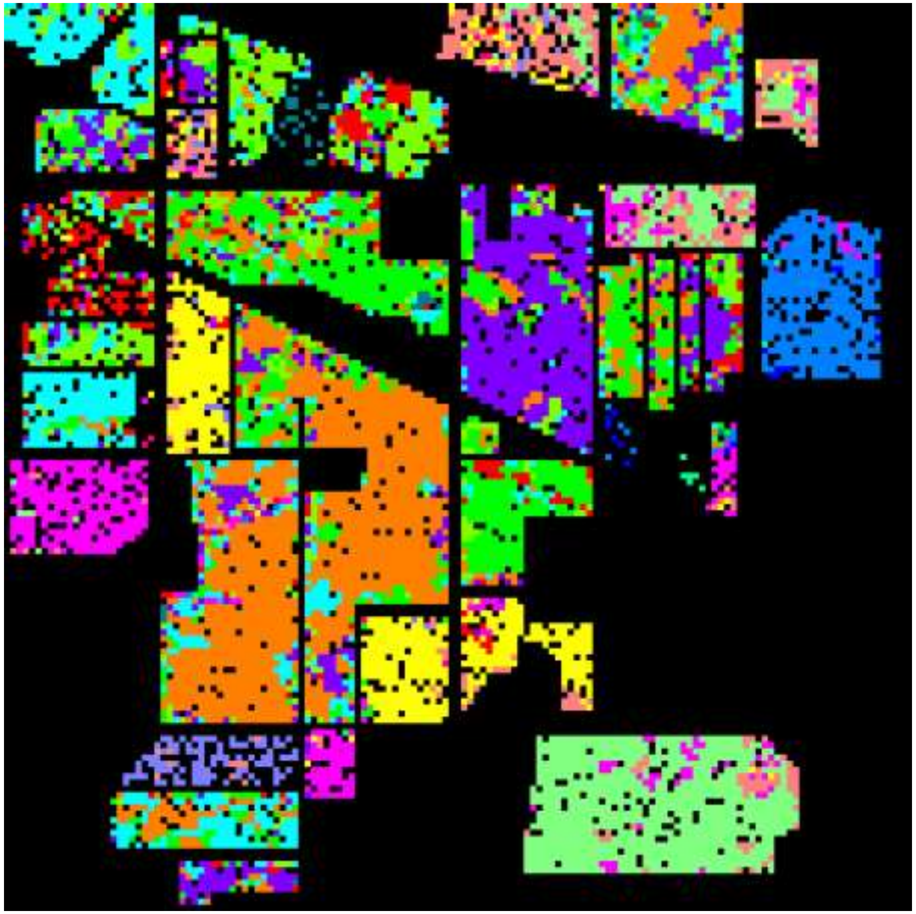
$CRF_{RF+Potts}$

$\theta_{SP} = 4$



$CRF_{RF+ContrastPotts}$

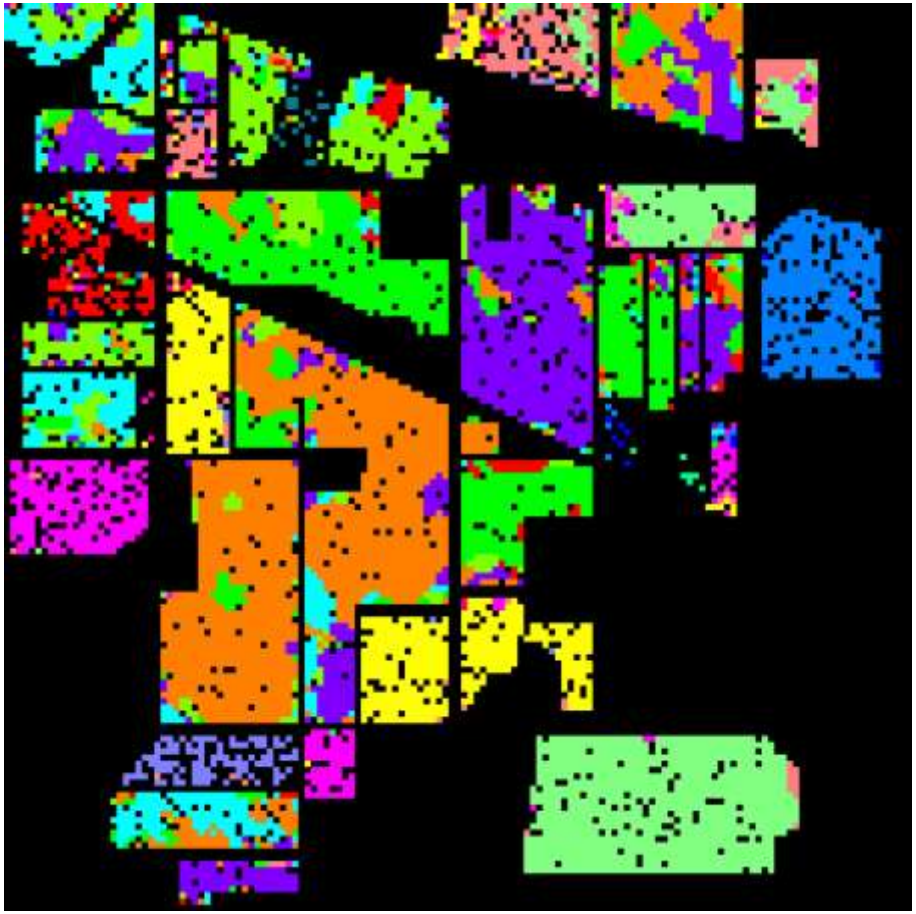
$\theta_{SP} = 4$



Results

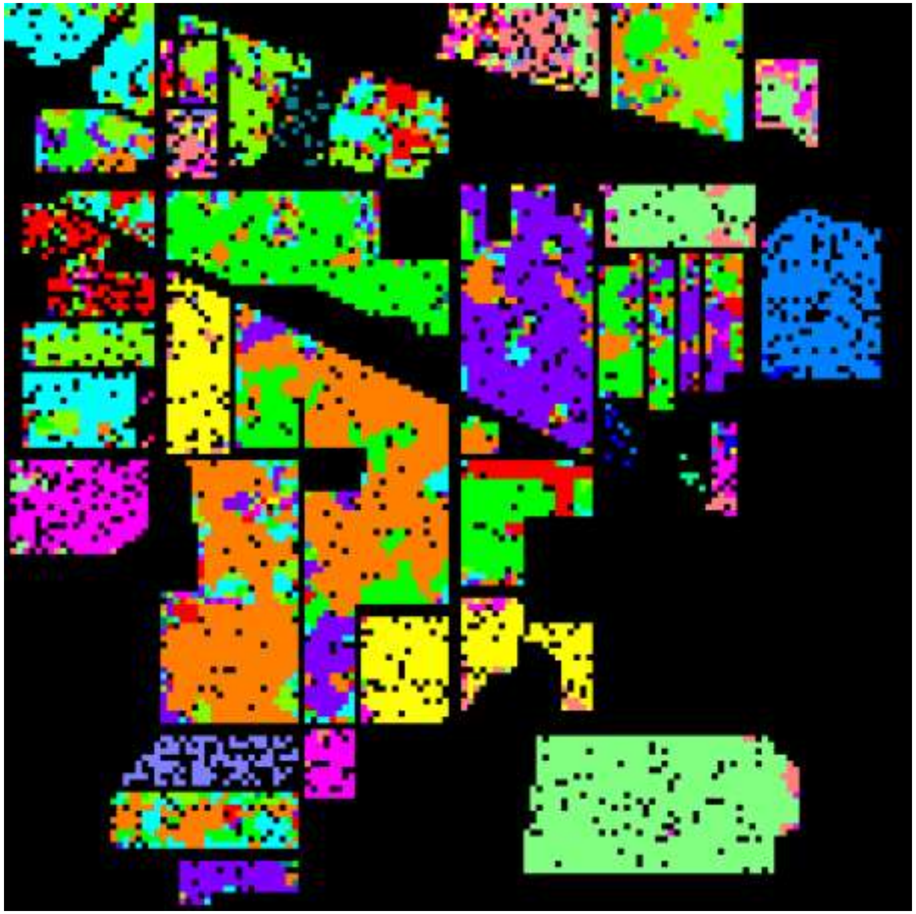
$CRF_{RF+Potts}$

$\theta_{SP} = 5$



$CRF_{RF+ContrastPotts}$

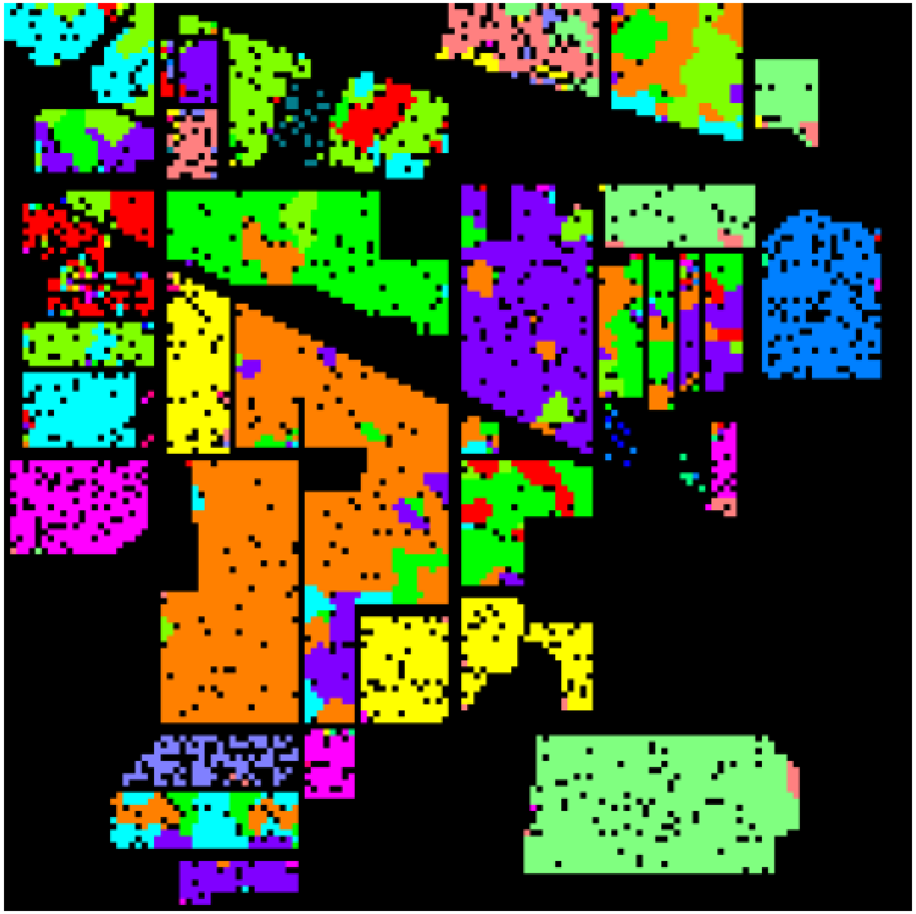
$\theta_{SP} = 5$



Results

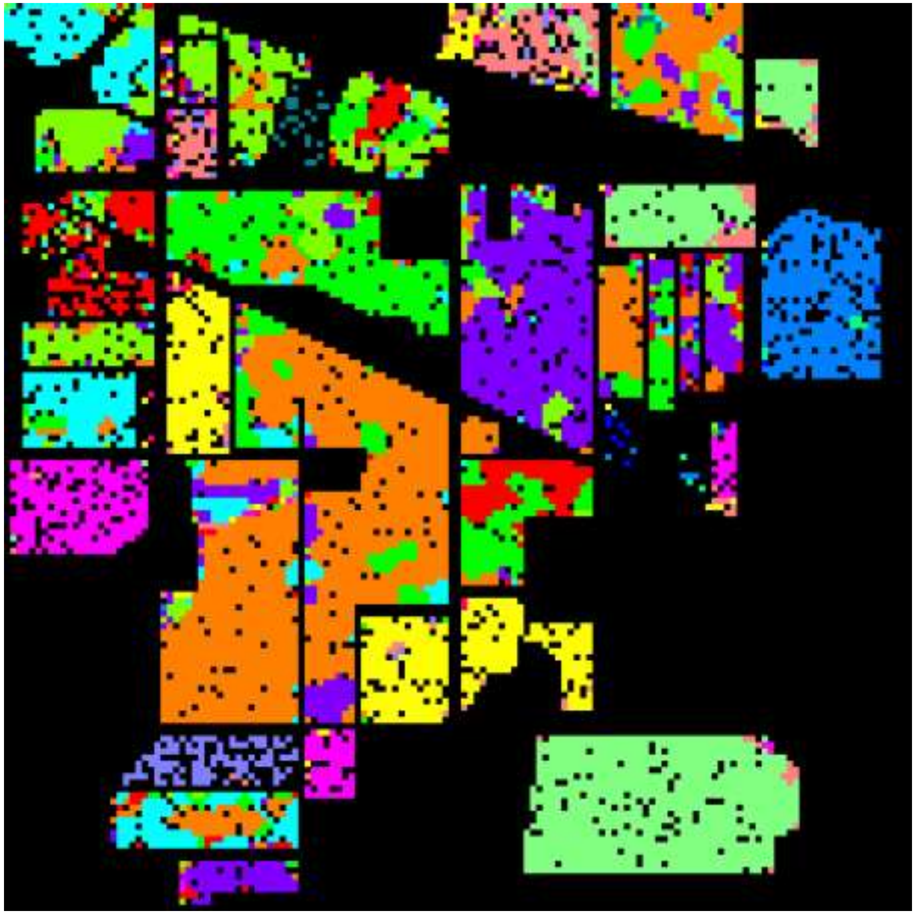
$CRF_{RF+Potts}$

$\theta_{SP} = 6$



$CRF_{RF+ContrastPotts}$

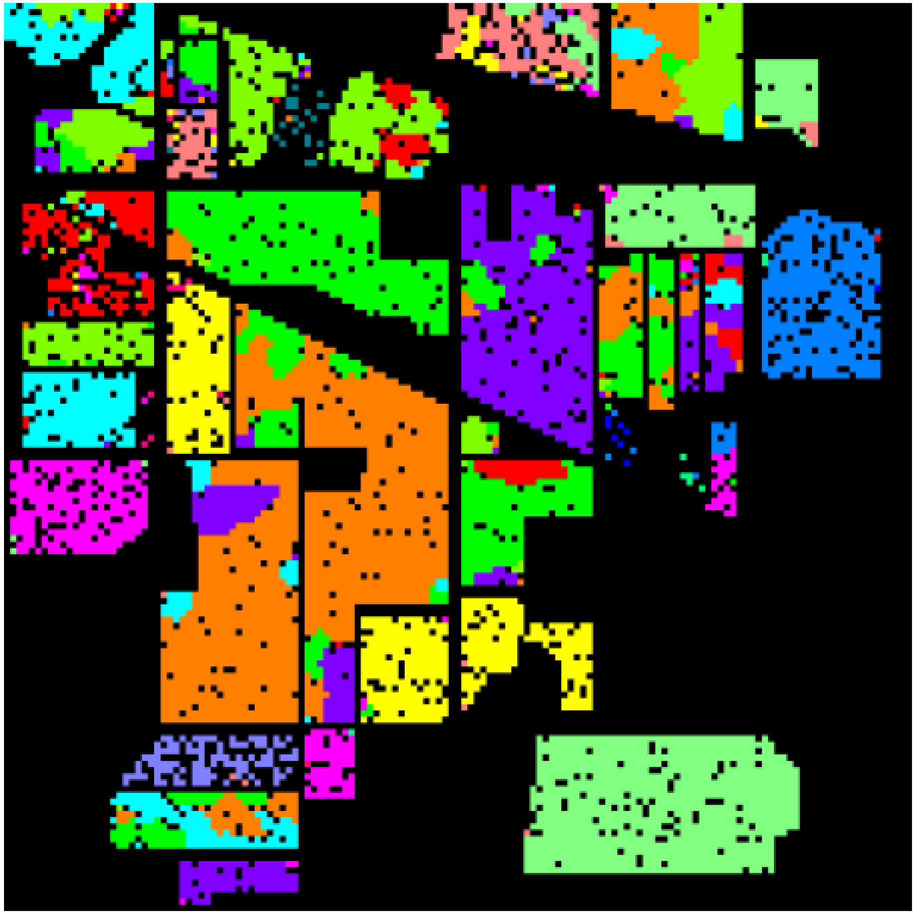
$\theta_{SP} = 6$



Results

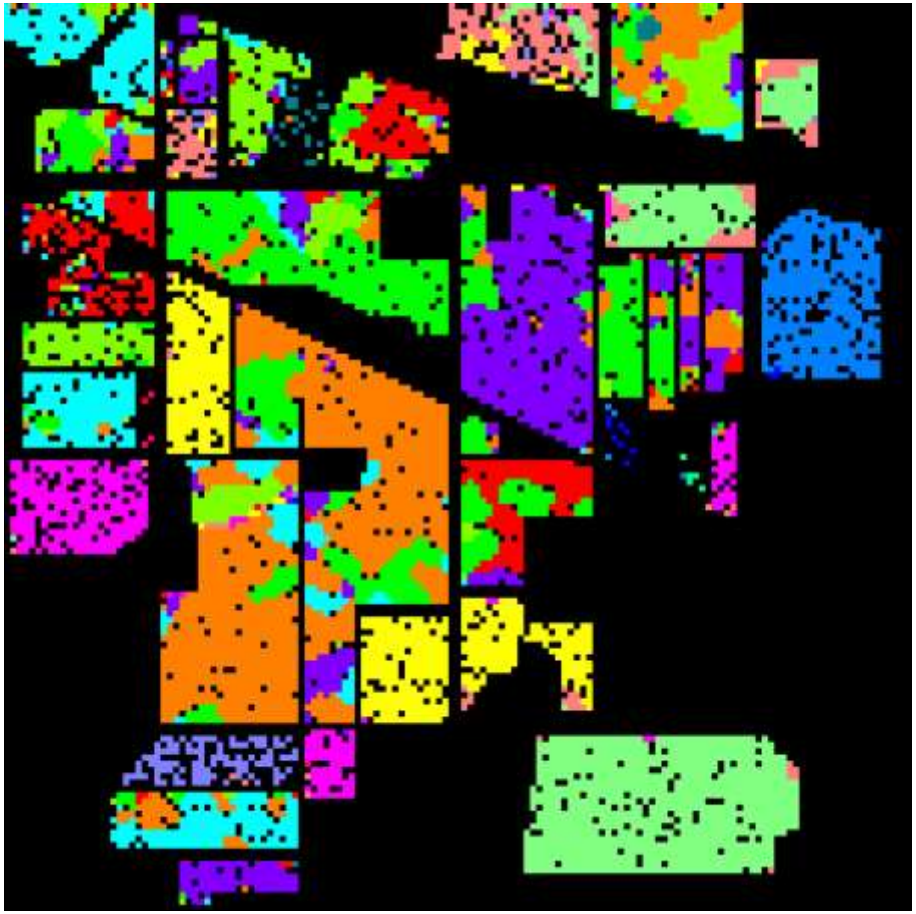
$CRF_{RF+Potts}$

$\theta_{SP} = 7$



$CRF_{RF+ContrastPotts}$

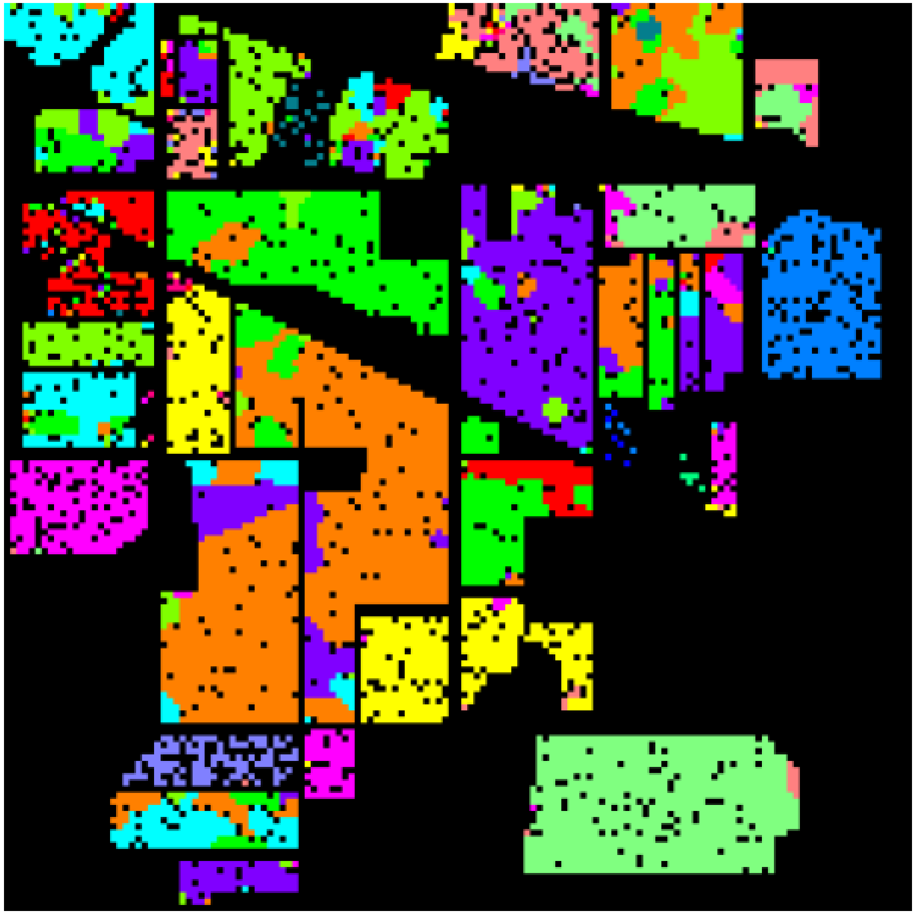
$\theta_{SP} = 7$



Results

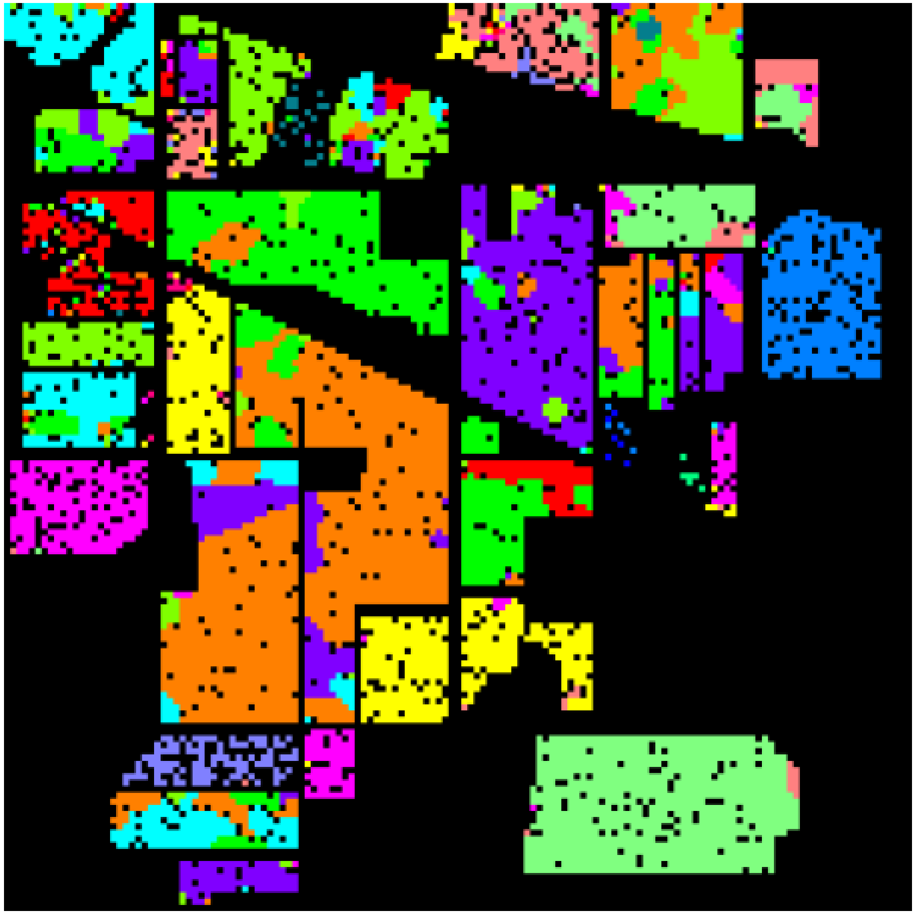
$CRF_{RF+Potts}$

$\theta_{SP} = 8$



$CRF_{RF+ContrastPotts}$

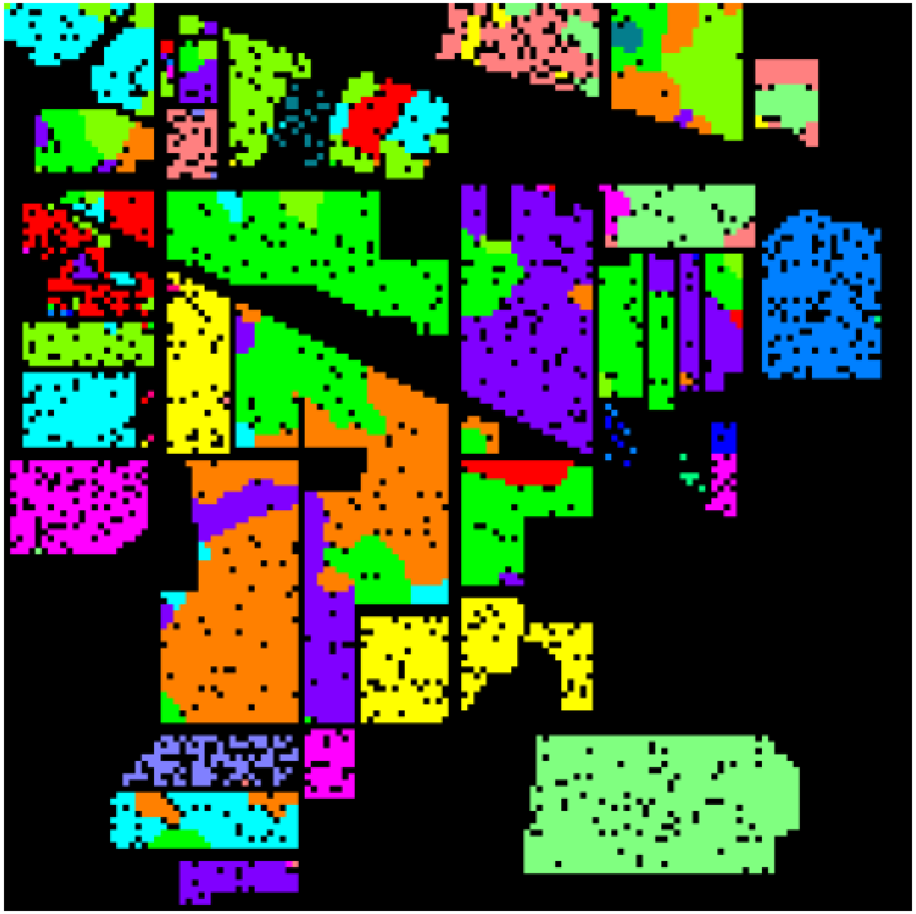
$\theta_{SP} = 8$



Results

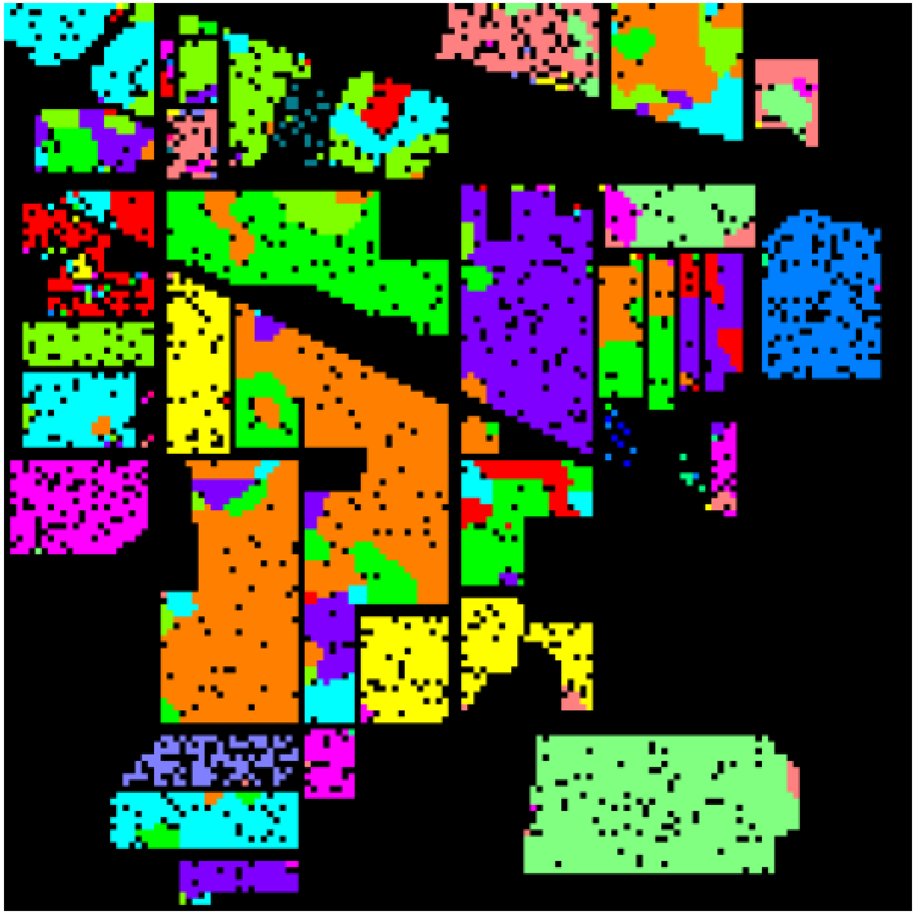
$CRF_{RF+Potts}$

$\theta_{SP} = 9$



$CRF_{RF+ContrastPotts}$

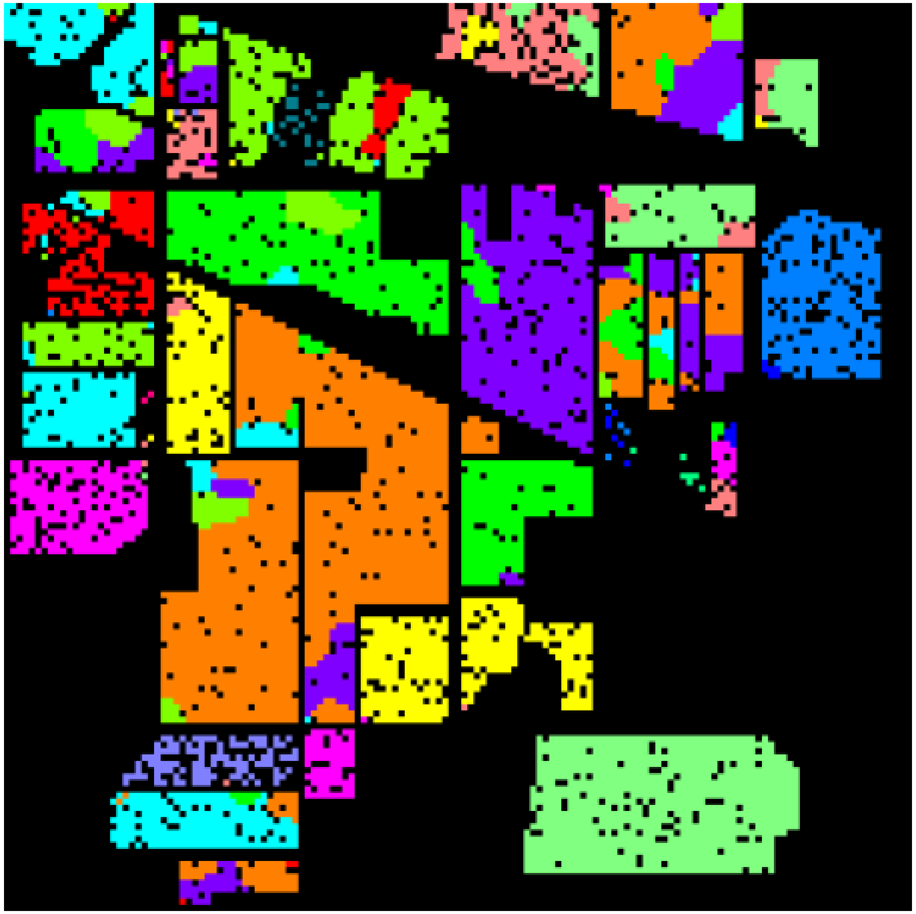
$\theta_{SP} = 9$



Results

$CRF_{RF+Potts}$

$\theta_{SP} = 10$



$CRF_{RF+ContrastPotts}$

$\theta_{SP} = 10$