

# ECON106V LAB #7

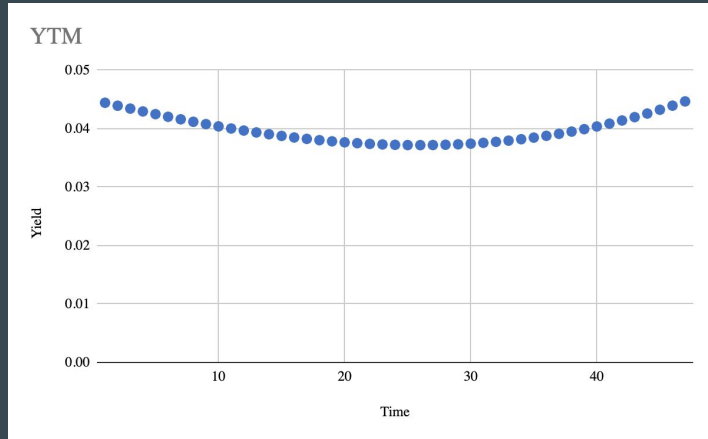


Aaron Chien, Ashley Lu, Yingyin Tan

## Are most bonds in the data premium bonds or discount bonds? Do bonds with high coupon tend to sell at higher or lower prices? Why?

Most bonds in the data are *discount bonds*. There are 37 discount bonds and 18 premium bonds. Bonds with high coupon tend to sell at higher prices because they offer a higher yield compared to similar bonds with lower coupons. However, the price of a bond is also influenced by many other factors, including changes in prevailing interest rates, the creditworthiness of the issuer, and the remaining term to maturity. When interest rates in the market decrease, investors are willing to pay more for bonds with higher coupons to capture the higher yields offered. In the dataset, bonds with high coupon (i.e., 6.75%) tend to sell above \$100, whereas bonds with a low coupon (i.e., 1.5%) tend to sell below \$100.

# How accurate is the model of the yield curve? What is the percentage error if you calculate price using the model?



The model of the yield curve is not perfectly accurate, but it is reliable. Since the yield curve is upward sloping, it is a reliable predictor of future economic conditions, because an upward sloping curve implies that short-term interest rates are lower than long-term interest rates, which suggest that investors are optimistic about the future economic outlook and expect inflation to remain under control.

Percentage Error: 11.4594592

**According to the expectations hypothesis, do investors expect interest rate to go up over time? Did their expectations prove correct from over the last year?**

According to the expectations hypothesis, investors expect that long-term interest rates will be the average of current short-term interest rates expected to prevail over the life of the bond; meaning if short-term interest rates are expected to rise in the future, investors would expect long-term interest rates to rise as well. In general, interest rates in the US have been low for several years, and have remained relatively stable over the last year, with some fluctuations.

Whether their expectations have proved correct over the last year depends on the specific time frame and the interest rate in question. In our scenario, the expectations was not fully accurate, but it is a good source of information, so it could be partially reliable.

According to the expectations hypothesis, as of Feb. 17, 2023, what is the market expectation about the 1 year short-term interest rate on Feb. 15, 2024? Is it lower or higher than the YTM on a ZCB paying off on Feb. 15, 2025? Why?

Expected 1-year short-term interest rate

$$= (0.0434271209436095 - 0.04294339045) / (1 + 0.04294339045) = 0.0463812\%$$

It is higher than the YTM (0.0429433904%) on a ZCB paying off on Feb. 15, 2025.

YTM is higher to compensate for the higher expected future short-term interest rates.

Suppose you invest in a ZCB paying off on Feb. 15, 2036, and hold it to maturity, and assume that inflation is 2% per year over this time period. What is your real return?

Assuming: 15 years until maturity...

$$\text{Total nominal return} = \$100 - \$107.33 = -\$7.33$$

$$\text{Average annual nominal return} = (-\$7.33 / \$107.33) / 15 = -0.00455293642 = -0.455293642\%$$

$$\text{Real Return} = -0.455293642\% - 2\% = -2.455293642\%$$

Suppose you run a corporation and issue a bond on Feb. 15, 2022 with 15 years to maturity, and a coupon rate of 10%. Coupons are paid semi-annually. If your bonds were default free, what would be its price on Feb. 17, 2023.

Assuming...

Coupon Payments Remaining:  $(15 \text{ years} \times 2) = 30$

Coupon Rate: 10%

Face Value: \$100

Coupon Payment: \$10

Since the bond is default-free, we can use a market interest rate as a proxy for the discount rate

Market Interest Rate: 8%

PV of remaining coupon payments:  $\$9.61 + \$9.24 + \$8.88 + \dots + = \$172.92$

PV of Face Value:  $\$100 / (1 + 0.04)^{30} = \$30.83$

Bond Price:  $\$172.92 + \$30.831 = \$203.75$

Price of the Bond on Feb. 17, 2023 (Default-Free): **\$203.75**

Suppose that investors' expect you to default with probability of 3% per year, and expect no recovery in case of default (i.e., if you default, you would not be able to pay them anything). If investors were risk neutral, what would be the price of your bond on Feb. 17, 2023?

Assuming...

Coupon Payments Remaining:  $(15 \text{ years} \times 2) = 30$

Coupon Rate: 10%

Face Value: \$100

Coupon Payment: \$100

Since the bond is default-free, we can use a market interest rate as a proxy for the discount rate

Market Interest Rate: 8%

PV of remaining coupon payments:  $\$9.61 + \$9.24 + \$8.88 + \dots + = \$172.92$

Expected Face Value:  $\$100 * [ (1 - \exp(-0.03 \times 30)) ] = \$59.34$

PV of Expected Face Value:  $\$59.34 / (1 + 0.04)^{30} = \$18.29$

Bond Price:  $\$172.92 + \$18.29 = \$191.21$

Price of the Bond on Feb. 17, 2023 (Risk-Neutral): **\$191.21**