Urban Computing Skills Lab
Conditional probabilities and Bayes theorem
Summer, 2016

Prof. Stanislav Sobolevsky

Conditional probability - example

Snowfall in NYC happens 11 days a year

And 4 of those days happen in January

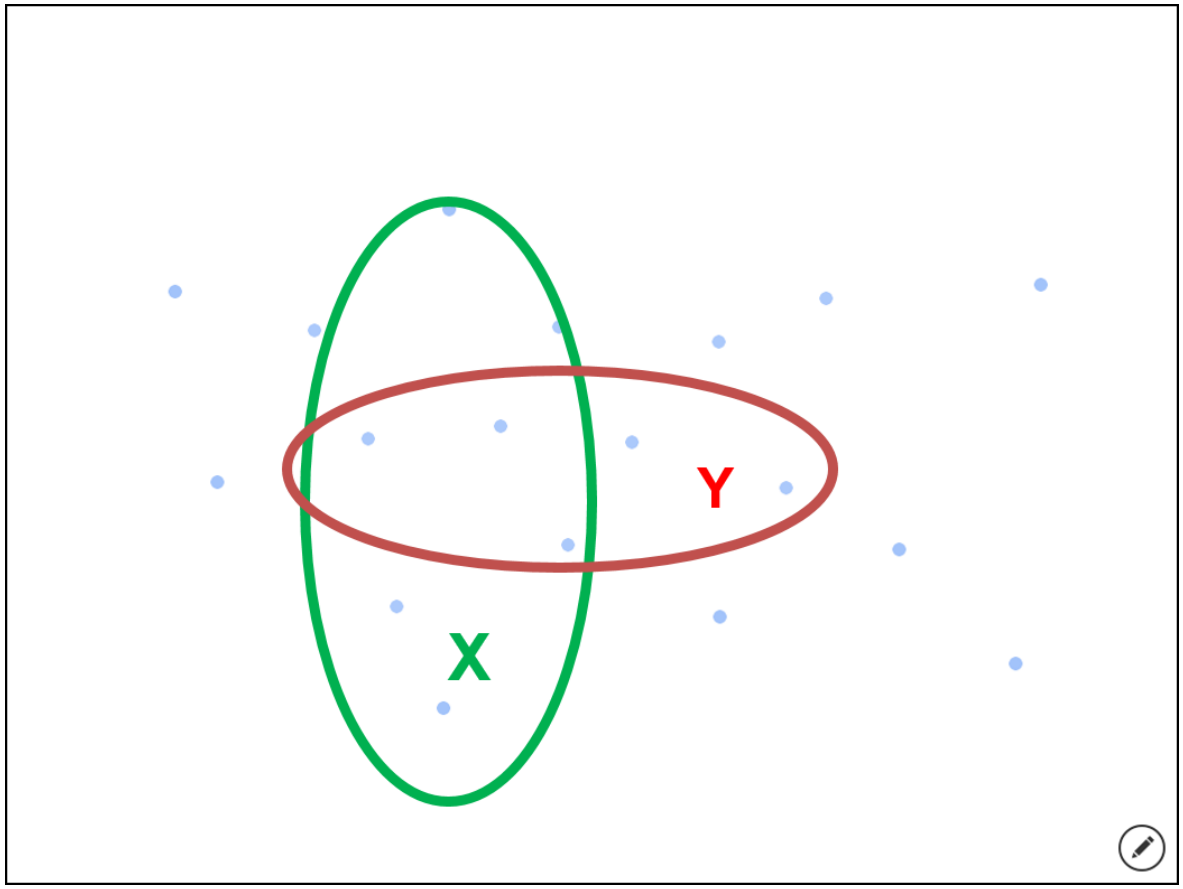
P(snowfall)=11/365

P(snowfall | January)=4/31

Two dices - x and y P(x+y>6) =0.5

P(x+y>6|x=1) =1/6

Conditional probability



$P(X|Y)$

$$P(X \cap Y) = P(X|Y)P(Y)$$

Independent events revisited

$$P(X \cap Y) = P(X)P(Y)$$

$$P(X|Y) = P(X)$$

$$P(X \cap Y) = P(X|Y)P(Y) = P(X)P(Y)$$

Summation rule

$$P(Y) = P(Y \cap X) + P(Y \cap \bar{X}) =$$

$$= P(Y|X)P(X) + P(Y|\bar{X})P(\bar{X})$$

Conditional probability - Bayes theorem

$$P(X \cap Y) = P(X|Y)P(Y)$$

$$P(X \cap Y) = P(Y|X)P(X)$$

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)}$$

Example

peterpanbus.com

30 men, 20 women

us.megabus.com

10 men, 20 women

Probability to see a lady coming out?

If a lady comes out what is the probability that it was Mega Bus?

Summation rule-2

$$P(Y) = \sum_k P(Y|X = k)P(X = k)$$

Example - water quality test

99% accurate

10% false alarms

prior: 1% of spots are polluted

Positive test - probability of pollution?

$$P(Polluted = 1|Test = 1) = \frac{P(Test = 1|Polluted = 1)P(Polluted = 1)}{P(Test = 1)} =$$

$$= \frac{0.99 * 0.01}{P(Test = 1)}$$

$$P(Test = 1) = P(Test = 1|Polluted = 1)P(Polluted = 1) + P(Test = 1|Polluted = 0)P(Polluted = 0) =$$

$$= 0.99 * 0.01 + 0.1 * 0.99 = 0.99 * 0.11$$

$$P(Polluted = 1|Test = 1) = \frac{1}{11} = 9.09\%$$