Urban Computing Skills Lab
Optimization
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Optimization example

wall H-?

surge h: Unif(0,50)

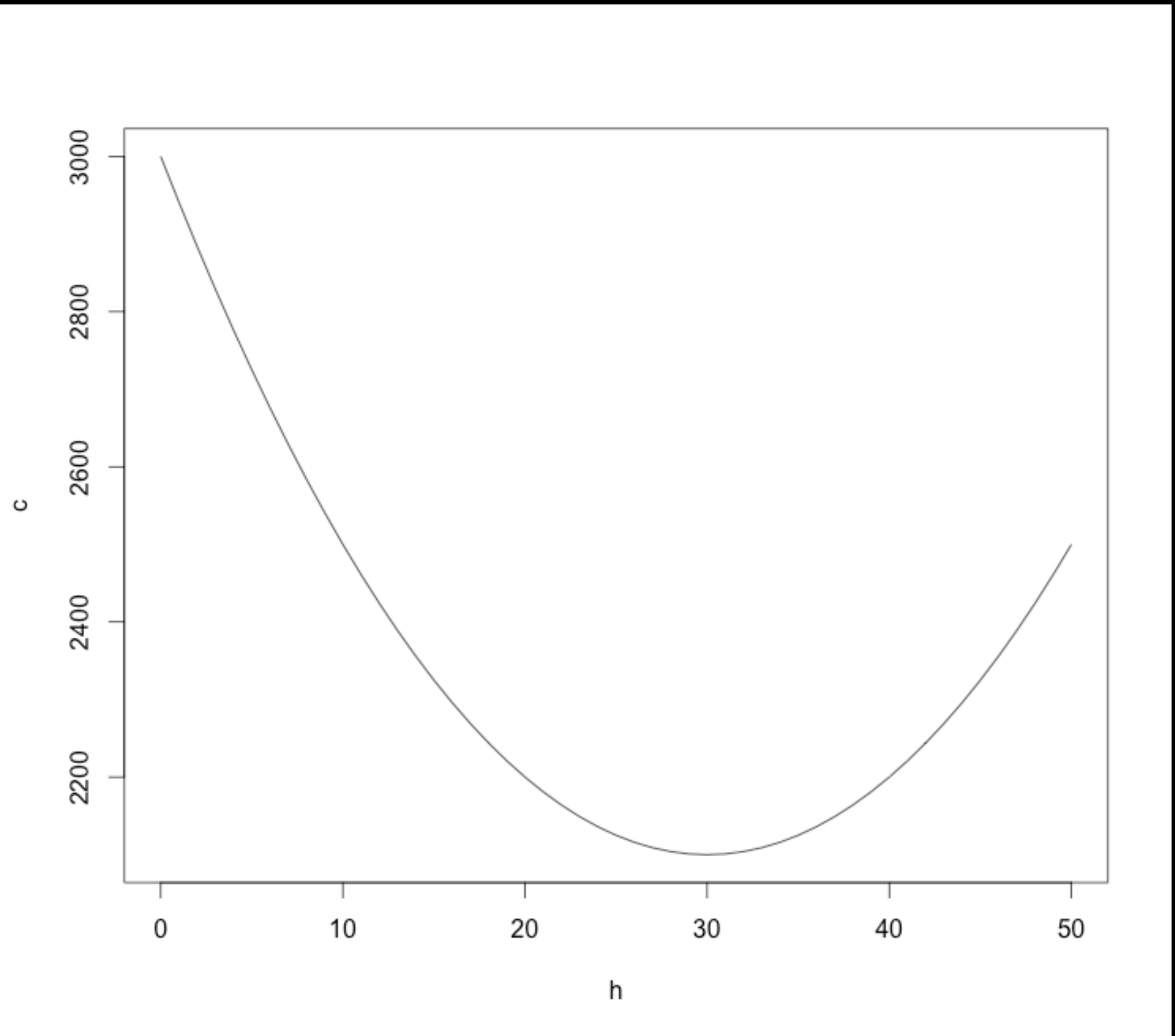
Loss: $3B

Cost: $1M*H^2

Projected losses to minimize (millions $):

$H^2 + (50-H)/50*3000 = H^2 - 60*H + 3000$

Optimization example: simple solution



$$H^2 - 60H + 3000 \to min$$

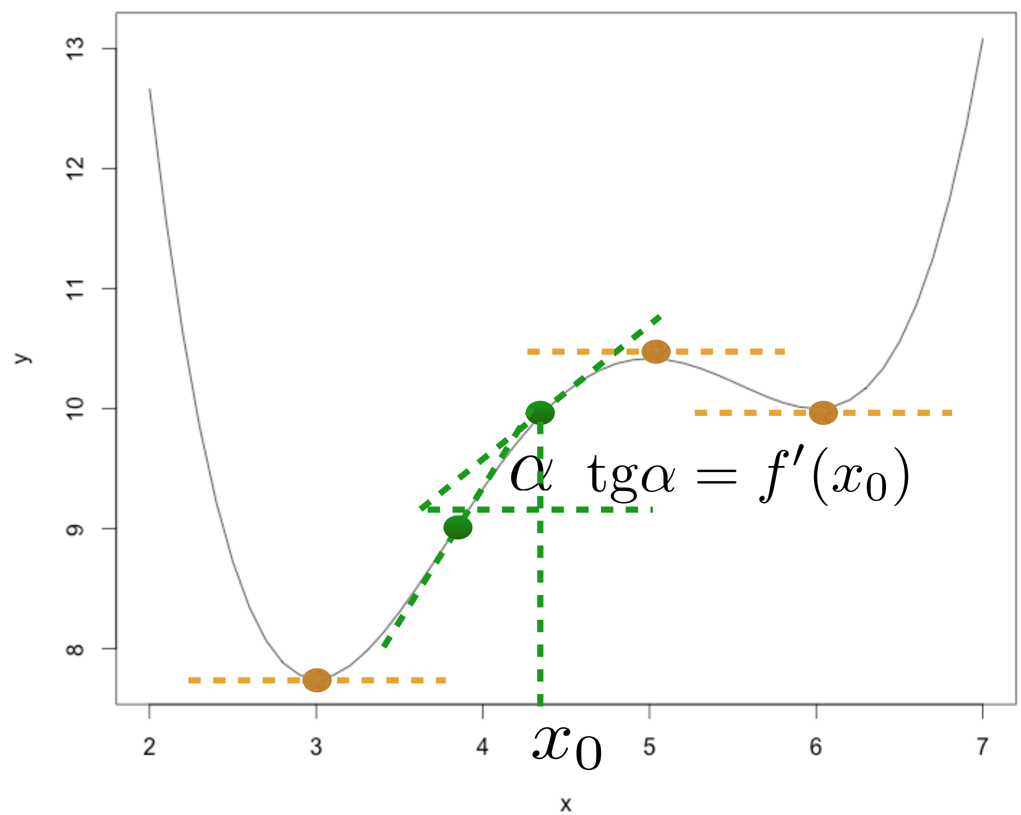
$$(H-30)^2 + 2100 \to min$$

$$H = 30$$

$$e^H - 60H + 3000 \to min?$$

Single-variable smooth function optimization

$$f(x) \to min, max \qquad f'(x_0) \leftarrow \frac{f(x_1) - f(x_0)}{x_1 - x_0} \quad x_1 \to x_0$$



local extrema:

$$f'(x) = 0$$

Optimization example revisited

$$F(H) = H^2 - 60H + 3000 \to min$$

$$F'(H) = 2H - 60 = 0$$

$$H = 30$$

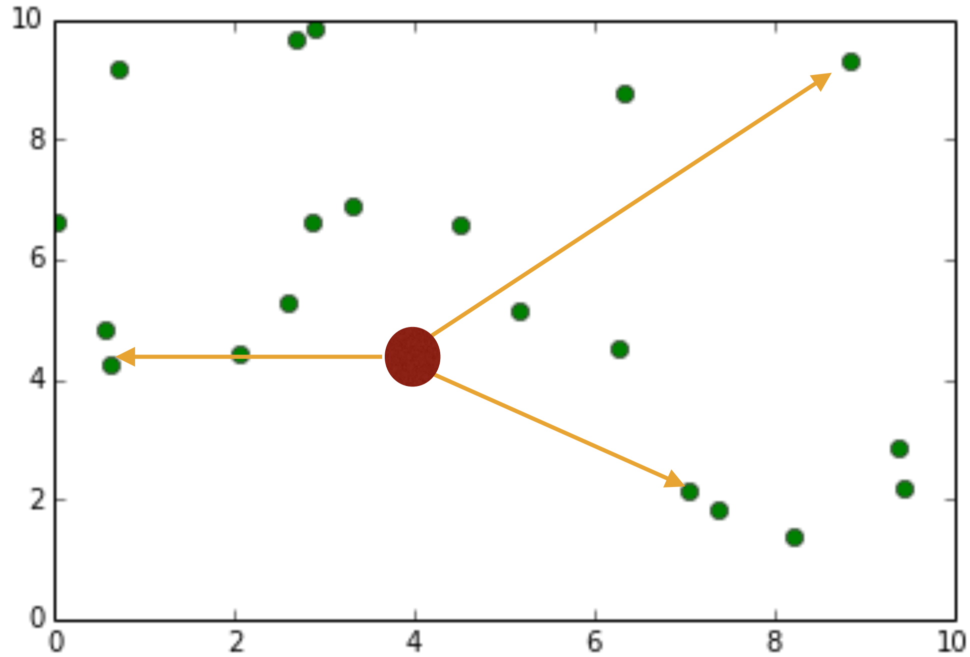
$$F(0) = 3000, F(50) = 2500, F(30) = 2100$$

Multi-variable case

$$F(x_1, x_2, ..., x_n) \to min/max$$

$$\frac{\partial F}{\partial x_i}(x_1^*, x_2^*, ..., x_n^*) = 0$$

Multi-variable example



$$D = \sum_i \left[(x_i - x^*)^2 + (y_i - y^*)^2 \right] \to min$$

Exact solution

$$D = \sum_i \left[(x_i - x^*)^2 + (y_i - y^*)^2\right] \rightarrow min$$

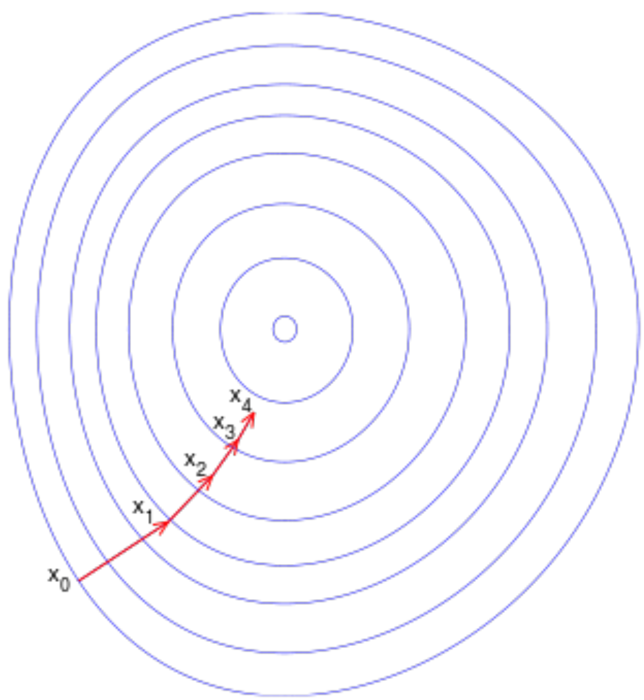
$$\frac{\partial D}{\partial x^*} = \sum_i 2(x_i - x^*) = 0$$

$$\frac{\partial D}{\partial y^*} = \sum_i 2(y_i - y^*) = 0$$

$$\sum_i x_i - nx^* = 0 \qquad \sum_i y_i - ny^* = 0$$

$$x^* = \frac{\sum_i x_i}{n} \qquad y^* = \frac{\sum_i y_i}{n}$$

Gradient methods



$$F(x) \to \min / \max$$

$$x_{k+1} = x_k - \gamma_k \nabla F(x_k)$$

$$\nabla F(x) = \left(\frac{\partial F}{\partial x^1}(x), \frac{\partial F}{\partial x^2}(x), \ldots, \frac{\partial F}{\partial x^n}(x)\right)$$

$$F(x_0) > F(x_1) > F(x_2) > \ldots .$$