$$\overline{\chi}y(\overline{\chi}+y)(y+\overline{y})$$
=  $\overline{\chi}y(\overline{\chi}+y) \cdot 1$  (Inverse Law)
=  $\overline{\chi}y(\overline{\chi}+y)$  (Identity Law)
=  $\overline{\chi}y\overline{\chi}+\overline{\chi}yy$  (Distributive Law)
=  $\overline{\chi}+\overline{\chi}y\overline{\chi}+\overline{\chi}yy$  (De Morgan's Law)
=  $(\overline{\chi}+\overline{y}\overline{\chi})+(\overline{\chi}+\overline{y}y)$  (De Morgan's Law)
=  $\overline{\chi}$  (Redundancy Law)
=  $\overline{\chi}$  (Idempotent Law)

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