Mathematics for Robotics and Control - Assignment 3: Pole-Zero Plots / System Properties

In [3]:
```
import sys
# Windows and Linux so far have no problem with the inline backend...
if not "win" in sys.platform and not "linux" in sys.platform:
    %pylab
# ...on other platforms we did run into problems in the past, so let's
else:
    %pylab inline
```

```
Welcome to pylab, a matplotlib-based Python environment [backend:
module://IPython.kernel.zmq.pylab.backend_inline].
For more information, type 'help(pylab)'.
```

Assignment 3.1

Answer the following questions:

I. How is a PZ plot related to the system's transfer function?
The PZ plot is a graphical representation of a rational transfer functions. Poles are zeros in the factored denominator and Zeros are zeros in the factored numerator. The plot is done in the complex plane.

II. Does a PZ plot completely characterize a system?
If the region of convergence that accords with the intial parameters of the system and multiple pole zero at the same location are specified.

III. Is is possible to recover the differential equation from nothing but a PZ-Plot? If not, why not?
If multiple poles or multiple zeros and these are specified then yes, but if a factor occured in the numerator and denominator and it was cancelled out then this information has been lost. The region of convergence that accords with the original system must also have been included in the PZ plot.

IV. Can you judge a system's stability by it's PZ plot? If so, how? Does the order of a system have any impact on it's stability? Are there different kinds of stability?
The system is stable in the region of convergence. Where this ROC occurs can determine if the system is causal, anti-causal or bounded input, bounded output stable. The order of the system can affect the number of poles and zeros, but it is the location of these poles and zeros that effect stablility.

V. Given a PZ plot, can you judge how long it will take the system to reach equilibrium?

VI. How does the Hurwitz polynomial relate to the analysis of dynamic systems?
Hurwitz polynomials are stable, and a dynamic system is stable if its control matrix is a Hurwitz.

VII. Assuming a PZ plot shows complex poles and/or zeros. What does immediately follow w.r.t.

the complex poles/zeros? Why?
The complex poles and zeros will always come in pairs.

VIII. What is the homogeneous response?
The homogeneous response is the response when the system is set to zero.

IX. What is the time constant τ?
τ is a constant measured in time units which relates to a physical property of the initial system. It is the parameter which is used to establish time scale of the system response for first order system.
$\tau \frac{dy}{dt} + y = 0$

X. What is the canonical form of the homogeneous response of a first-order system?

Assignment 3.1 took me **65** *minutes.*

Assignment 3.2

Plot the PZ plot of the following systems and discuss their properties (i.e. stability, transient response etc.). Give as much info about each system as you can and *specify proof/reasons for each of your claims!*

I. $G(s) = \frac{1}{s^2+\sqrt{(2)}\cdot s+1}$
P= None, Z= $-\frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2} i$
ROC = all: no poles, stable: contains imaginary axis.

II. $G(s) = \frac{4}{s^2+s-2}$
P= None, Z=-2, 1
ROC = all: no poles, stable: contains imaginary axis.

III. $G(s) = \frac{30}{s^2+\frac{3}{10}\cdot s+10}$
P = None, Z= $-0.1500 \pm 3.1587i$
ROC = all:no poles, stable: contains imaginary axis.

IV. $A = \begin{bmatrix} 3 & -3 & 2 \\ 1 & 1 & 1 \\ -4 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \backslash 1 \backslash 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, D = 2$

Assignment 3.2 took me **25** *minutes.*

Use this button to create a .txt file containing the time in minutes you spent working on the assignments. Make sure to include your name in the textbox below. The file will be created in the current directory.

Student's name:

Save timings