

Model Matematika Transformasi Koordinat ...

❑ Model Matematika

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} v_X \\ v_Y \\ v_Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & -z & y & x \\ 0 & 1 & 0 & z & 0 & -x & y \\ 0 & 0 & 1 & -y & x & 0 & z \end{bmatrix} \begin{bmatrix} t_x \\ t_y \\ t_z \\ \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ ds \end{bmatrix} + \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

Memperhatikan tingkat ketelitian koordinat titik sekutu pada sistem (x,y,z) & (X,Y,Z)

❑ Linearisasi Model Matematika & Perataan Kombinasi

$$\alpha = 1 + ds^o$$

$$\underbrace{\begin{bmatrix} \alpha & \varepsilon_z^o & -\varepsilon_y^o & -1 & 0 & 0 \\ -\varepsilon_z^o & \alpha & \varepsilon_z^o & 0 & -1 & 0 \\ \varepsilon_y^o & -\varepsilon_z^o & \alpha & 0 & 0 & -1 \end{bmatrix}}_A \underbrace{\begin{bmatrix} v_x \\ v_y \\ v_z \\ v_X \\ v_Y \\ v_Z \end{bmatrix}}_v + \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & -z & y & x \\ 0 & 1 & 0 & z & 0 & -x & y \\ 0 & 0 & 1 & -y & x & 0 & z \end{bmatrix}}_B \underbrace{\begin{bmatrix} t_x \\ t_y \\ t_z \\ \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ ds \end{bmatrix}}_x = \underbrace{\begin{bmatrix} F_x^o \\ F_y^o \\ F_z^o \end{bmatrix}}_F$$

Bentuk Matrik untuk Kasus 2 Titik Sekutu ...

❑ Matrix A & B

$$A = \begin{bmatrix} \alpha_1 & \varepsilon_{z1}^o & -\varepsilon_{y1}^o & -1 & 0 & 0 \\ -\varepsilon_{z1}^o & \alpha_1 & \varepsilon_{z1}^o & 0 & -1 & 0 \\ \varepsilon_{y1}^o & -\varepsilon_{z1}^o & \alpha_1 & 0 & 0 & -1 \\ 0 & 0 & 0 & \alpha_2 & \varepsilon_{z2}^o & -\varepsilon_{y2}^o \\ 0 & 0 & 0 & -\varepsilon_{z2}^o & \alpha_1 & \varepsilon_{z1}^o \\ 0 & 0 & 0 & \varepsilon_{y2}^o & -\varepsilon_{z2}^o & \alpha_2 \end{bmatrix}$$


$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & -z_1 & y_1 & x_1 \\ 0 & 1 & 0 & z_1 & 0 & -x_1 & y_1 \\ 0 & 0 & 1 & -y_1 & x_1 & 0 & z_1 \\ 1 & 0 & 0 & 0 & -z_2 & y_2 & x_2 \\ 0 & 1 & 0 & z_2 & 0 & -x_2 & y_2 \\ 0 & 0 & 1 & -y_2 & x_2 & 0 & z_2 \end{bmatrix}$$

Bentuk Matrik untuk Kasus 2 Titik Sekutu ...

❑ Matrix v & Q

$$v^T = \begin{bmatrix} v_{x1} & v_{y1} & v_{z1} & v_{X1} & v_{Y1} & v_{Z1} & v_{x2} & v_{y2} & v_{z2} & v_{X2} & v_{Y2} & v_{Z2} \end{bmatrix}$$

$$Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}$$



$$Q_k = \begin{bmatrix} \sigma_{xk}^2 & & & & \\ & \sigma_{yk}^2 & & & \\ & & \sigma_{zk}^2 & & \\ & & & \sigma_{Xk}^2 & \\ & & & & \sigma_{Yk}^2 \\ & & & & & \sigma_{Zk}^2 \end{bmatrix}$$

Solusi Perataan Kombinasi ...

1. Matrix x

$$W = (AQA^T)^{-1}$$

$$N = (B^T W B)$$

$$x = N^{-1}(B^T W F)$$

2. Matrix v

$$v = QA^T W (F - Bx)$$

3. Variansi a posteriori

$$\bar{\sigma}^2 = \frac{v^T W v}{r}, \quad r = \text{degree of freedom}$$

4. Ketelitian matrik x

$$\sum_{xx} = \bar{\sigma}^2 (B^T W B)^{-1}$$

5. Kofaktor residual

$$Q_{vv} = QA^T W (I - BN^{-1}B^T W)AQ$$

Tes Outlier Titik Sekutu ...

1. Nilai kritis Tau

$$\tau_{\alpha/2} = \frac{(t_{\alpha/2,r})\sqrt{r}}{\sqrt{r-1 + (t_{\alpha/2,r})^2}}$$

$t_{\alpha/2,dof}$ = t-student value

α = significant level (1 – confidence level)

2. Loop sebanyak m (isi matrik v)

for $i = 1, m$

$$\bar{v}(i) = \frac{v(i)}{Q_{vv}(i,i)}$$

if $\frac{|\bar{v}(i)|}{\bar{\sigma}} > \tau_{\alpha/2}$ then $l(i)$ is outlier

end