Space filling as volume filling in LiDAR scans of trees

The original derivation of the 3/4 scaling result hinges on two properties of vascular networks: area-preservation and space-filling. Mathematically, these properties can be expressed in terms of predicted optimal values of the radius and length scaling ratios beta and gamma. By measuring beta at the node level (individual pairs of parent and child branches), we have been able to demonstrate that these branching networks exhibit area-preservation. However, measurements of gamma in this and previous studies have not supported the space-filling prediction. This indicates either that trees are not even approximately space-filling, or that relying on measurements of branch length scaling is not sufficient to reveal the space-filling properties of diverse botanical networks.

Here, we show that evaluating volume preservation at multiple levels within a given branching hierarchy reveals space-filling properties of diverse tropical and temperate trees. This approach is valuable because it allows us to effectively average over extensive asymmetry in branch lengths that previously obscured space-filling properties of the trees. Despite the fact that space-filling and length scaling as expressed by gamma are usually evaluated at the node level, the original definition of space-filling was written in terms of branching generations k = 0...N, where N is the 'depth' or total number of bifurcation levels within a symmetrical network.

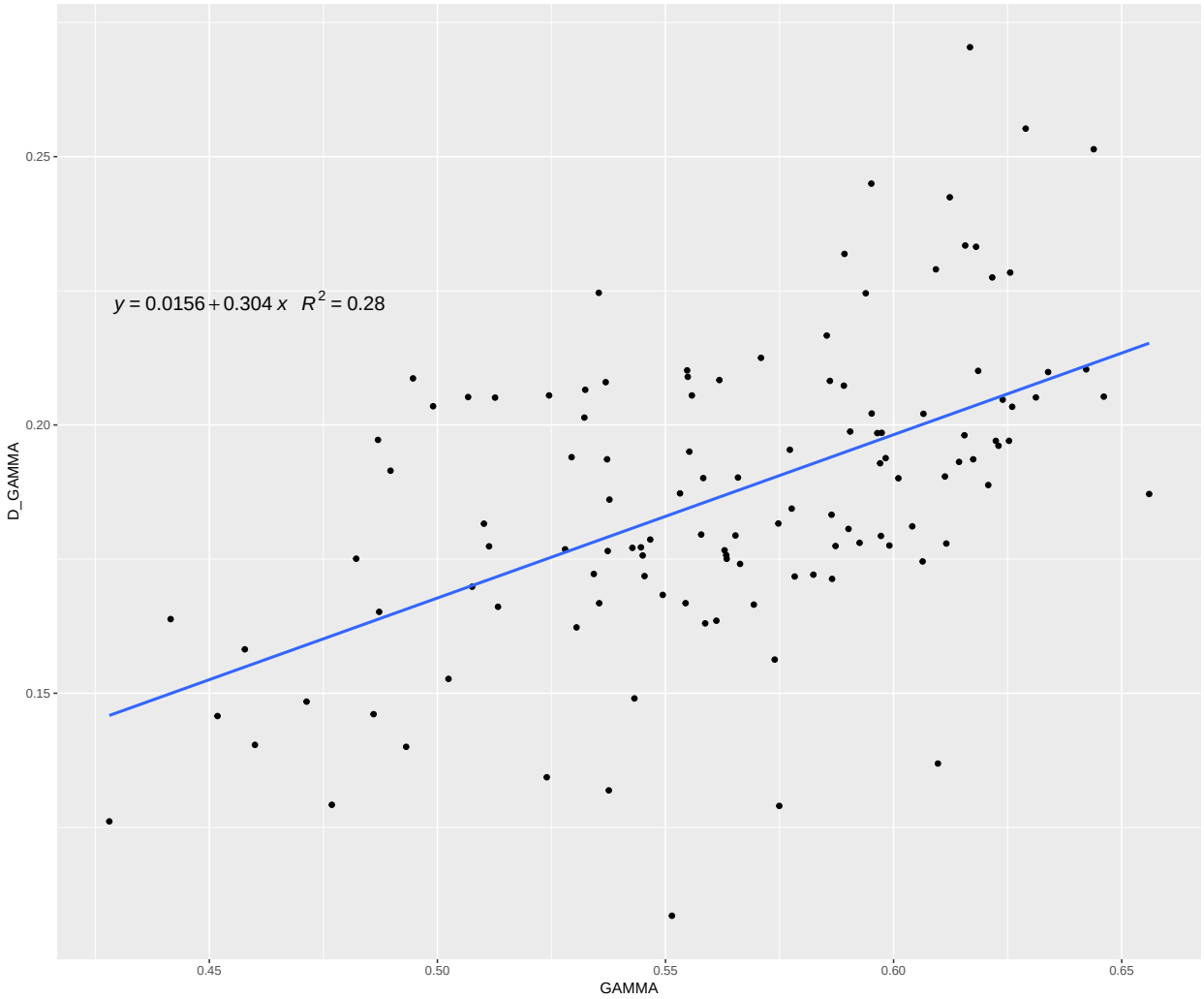
$$n_k l_k^3 = n_{k+1} l_{k+1}^3 \tag{1}$$

This expression allowed the derivation of γ:

$$n_k l_k^3 = n_{k+1} l_{k+1}^3 1 = \frac{n_{k+1}}{n_k} \frac{l_{k+1}}{l_k}^3 1/n = \frac{l_{k+1}}{l_k}^3 n^{-1/3} = \frac{l_{k+1}}{l_k} \gamma = \frac{l_{k+1}}{l_k} = n^{-1/3} \tag{2}$$

As mentioned before, measuring β and γ at the node level in LiDAR scans of trees reveals the theoretical expectation of area-preservation, but space-filling reperesented by length scaling falls short of the expected value.

A common hypothesis for why space-filling is apparently not obeyed in these networks is the presence of pervasive asymmetry, especially in branch lengths. We measured Brummer's asymmetrical length-scaling ratio $\delta\gamma$ to confirm that asymmetry was indeed affecting measurements of γ. This correlation at least indicates that this is the case:



In symmetrical networks, space-filling remains as much a property of intermediate branching levels as it does of individual nodes in the hierarchy. Measuring space-filling at the level of branching generations in asymmetrical networks poses its own problems - asymmetrical networks can't directly rely on a simple branch-ordering scheme to map branching generations within a hierarchy. Symmetrical networks assume the existence of branching depth N, and posit the following relationship between branching generation k and distal tips n_N:

$$n_N = 2^k \tag{3}$$

Include a better map of how branching suctions look across path lengths to directly answer Van's question -tips/Strahler more robust to asymmetry than naive branch indexing.

An intuitive and metabolically relevant ordering scheme would be to assign a branch order to branches based on the distal number of terminal tips. This approach turns out to be more-or-less similar to the more well-known Strahler ordering scheme:

	FILENAME	NUM_BRANCHES	MAX_BRANCH_ORDER	BRANCH_VOLUME	TREE_HEIGHT
1	1_sans_feuilles	956	886	69.900	22.560
2	100_sans_feuilles	23510	18726	4784.000	42.270
3	101_sans_feuilles	2463	1323	1140.000	27.550
4	102_sans_feuilles	241	228	12.870	21.600
5	103_sans_feuilles	10065	8933	1133.000	41.550
6	104_sans_feuilles	9607	6815	2792.000	38.590
7	105_sans_feuilles	519	439	79.850	25.110
8	106_sans_feuilles	1093	850	242.100	27.660
9	11_sans_feuilles	186	176	10.220	14.920
10	18_sans_feuilles	9181	8374	806.900	42.620
11	2-16--3	84	47	36.720	9.649
12	2-16-1	38	21	16.840	3.299
13	2-16-3	11	4	6.778	3.406
14	2-16-4	208	97	110.800	10.840
15	2-16-7	87	41	46.500	8.974
16	28_sans_feuilles	598	590	7.657	20.040
17	29_sans_feuilles	1047	667	379.500	15.040
18	3-19--1	245	90	155.500	10.590
19	3-19--3	200	109	91.570	11.660
20	3-19-1	192	91	101.600	11.380

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## 21              3-19-2          137              68        68.690       9.908
## 22              3-19-3           85              42        42.110      10.010
## 23             3-22--2           66              36        30.920       8.360
## 24              3-22-1           38              18        19.710       8.178
## 25             3-22-10           87              49        37.540       9.717
## 26             3-22-11           34              17        17.260       7.375
## 27             3-22-13           56              29        26.180       7.660
## 28              3-22-2          128              51        77.310       9.138
## 29              3-22-3          137              68        68.740      11.350
## 30              3-22-4           67              40        27.450       9.612
## 31              3-22-5          123              66        56.610      10.580
## 32              3-22-6           26              15        10.500       6.915
## 33              3-22-7           35              19        16.090       7.046
## 34              3-22-8          105              42        63.170       7.823
## 35     32_sans_feuille         3606            3506       100.300      37.050
## 36    52_sans_feuilles         1285            1226        58.730      28.740
## 37    54_sans_feuilles          480             469        11.060      22.090
## 38    59_sans_feuilles          133             131         1.957      14.350
## 39     6_sans_feuilles        50789           40497     10291.000      47.370
## 40    60_sans_feuilles        30277           24864      5413.000      43.710
## 41    61_sans_feuilles          178             136        41.570      17.400
## 42    62_sans_feuilles          901             818        83.320      23.600
## 43    65_sans_feuilles         3203            2646       556.900      33.550
## 44    66_sans_feuilles         6382            5376      1006.000      29.440
## 45    68_sans_feuilles          488             449        39.190      25.240
## 46    69_sans_feuilles          468             321       146.900      14.660
## 47    71_sans_feuilles          493             438        55.510      22.490
## 48    72_sans_feuilles           86              74        11.730      11.430
## 49            74_sans         4036            2912      1124.000      32.370
## 50    75_sans_feuilles         7646            6844       802.100      42.020
## 51    76_sans_feuilles         2161            2151        10.510      32.050
## 52    79_sans_feuilles         3618            3375       242.400      30.750
## 53     8_sans_feuilles        34036           25513      8523.000      47.000
## 54    80_sans_feuilles         4064            3458       606.300      30.830
## 55    83_sans_feuilles          994             954        40.270      34.090
## 56    87_sans_feuilles           89              77        11.850      11.480
## 57            88_sans         1354            1008       346.100      24.670
## 58    92_sans_feuilles         3843            2912       930.400      28.510
## 59    93_sans_feuilles         2432            2333        99.180      34.020
## 60    94_sans_feuilles         4300            4185       115.100      41.920
## 61    95_sans_feuilles           68              60         8.226      10.850
## 62    98_sans_feuilles        10712            9281      1430.000      48.130
## 63             Ery_01          374             305        68.440      15.470
## 64             Ery_02          315             241        74.660      17.110
## 65             Ery_03          324             237        87.710      15.380
## 66             Ery_04          415             319        96.000      17.220
## 67             Ery_05          624             472       152.500      18.260
## 68             Ery_06          188             170        17.540      14.190
## 69             Ery_07          280             232        47.720      15.300
## 70             Ery_08          506             298       208.300      14.980
## 71             Ery_09          542             389       152.300      18.640
## 72             Ery_10          632             446       185.400      19.390
## 73             Ery_11          447             357        90.600      17.830
## 74             Ery_12          375             300        75.570      18.180
```

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## 75  ID63_sans_feuilles        21910            14945      6966.000      42.870
## 76                     L1         3730             1595      2135.000      14.810
## 77                     L2         9027             3123      5904.000      15.560
## 78                     L3         6385             1511      4874.000      15.070
## 79                     L4         8038             1917      6121.000      16.350
## 80                     L5         4970             1297      3673.000      12.090
## 81                     L6         9594             2636      6958.000      17.090
## 82                     L7         7652             1562      6090.000      15.730
## 83                Pinus01          846              484       362.300      16.280
## 84                Pinus02          289              234        55.150      13.180
## 85                Pinus03          498              284       214.400      15.770
## 86                Pinus04          421              274       147.500      16.770
## 87                Pinus05          508              360       148.300      14.800
## 88                Pinus06          622              463       158.800      16.070
## 89                Pinus07          395              306        88.840      12.010
## 90                Pinus08          149               94        54.960      11.540
## 91                Pinus09          466              279       186.700      14.630
## 92                Pinus10          426              348        78.330      16.110
## 93                    Q01          911              615       296.000      24.350
## 94                    Q02          918              621       297.900      26.180
## 95                    Q03         1617             1061       555.300      27.550
## 96                    Q04          965              786       179.100      26.780
## 97                    Q05         1055              735       320.200      23.390
## 98                    Q06         1077              787       289.300      28.930
## 99                    Q07         1247              668       579.300      27.120
## 100                   Q08         1532              872       659.900      26.960
## 101                   Q09         1475             1058       417.800      29.830
## 102                   Q10         1194              663       530.600      21.990
## 103                   Q11         1145              684       460.500      24.790
## 104                   Q12         1433             1108       325.600      25.950
## 105                    S1          455               58       397.000       7.629
## 106                    S2          718               97       621.100       9.230
## 107                    S3          361               44       317.100       6.914
## 108                    S4          605               86       519.800       9.362
## 109                    S6          710               86       623.800       8.931
## 110                    S7          184               31       152.900       5.892
## 111                    S8          841               87       753.900       8.694
## 112 wytham_winter_1551          365              138       227.300      14.270
## 113 wytham_winter_1777          197              109        87.590      15.680
## 114 wytham_winter_2225          398              176       221.500      14.410
## 115  wytham_winter_229         1936             1211       725.700      22.020
## 116  wytham_winter_424         1889              947       941.300      23.790
## 117   wytham_winter_45        12098             5146      6952.000      23.260
## 118  wytham_winter_600         5407             2139      3268.000      23.070
## 119   wytham_winter_81        15490             6027      9463.000      22.630
## 120  wytham_winter_869          656              363       292.700      17.300
## 121  wytham_winter_990          562              264       297.300      17.180
##     TREE_VOLUME TRUNK_VOLUME TRUNK_LENGTH BRANCH_LENGTH TREE_AREA DBH_QSM
## 1        23.310       78.210          63             5    21.840 0.34090
## 2        40.500     1009.000         782             9   259.400 1.02400
## 3        25.920      630.800         534             6    89.410 0.47690
## 4        22.300       36.340          53             3     9.242 0.17050
## 5        41.910      243.600         207             6   105.000 0.62860
## 6        38.580      881.400         692             8   177.700 0.81120
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## 7       25.260      135.700         165           7    20.750 0.20720
## 8       28.800      461.600         726           6    47.090 0.35090
## 9       15.280       32.790          54           4     7.205 0.16980
## 10      43.330      225.700         135           7    99.080 0.89470
## 11       9.671      211.000         250           5    11.160 0.10600
## 12       2.467       39.390          55           5     3.304 0.10400
## 13       2.563       43.710          63           5     2.117 0.05001
## 14      10.950      364.800         453           5    24.090 0.14370
## 15       9.129      150.100         218           5    10.790 0.10250
## 16      20.220       31.640          45           4    12.170 0.31440
## 17      14.230      368.400         449           7    41.790 0.32290
## 18      10.900      726.700         940           6    37.430 0.14350
## 19      11.440      368.500         458           6    22.500 0.14640
## 20      10.150      279.800         418           6    20.620 0.13620
## 21       9.946      262.600         318           7    16.710 0.12740
## 22      10.050      152.000         215           4    10.430 0.10060
## 23       8.578      242.400         452           6    10.390 0.10560
## 24       8.342      107.800         173           4     6.092 0.07333
## 25      10.400      155.200         163           4    10.130 0.10690
## 26       7.301      102.800         166           4     5.552 0.07182
## 27       7.648      150.800         215           5     8.153 0.09182
## 28       9.597      499.500         809           7    22.100 0.12030
## 29      11.270      357.500         516           5    18.740 0.12570
## 30      10.170      138.400         209           4     8.332 0.10730
## 31      10.930      234.100         324           4    14.470 0.13820
## 32       6.655      106.400         180           5     4.518 0.07114
## 33       7.219      152.800         276           5     6.276 0.07565
## 34       7.969      324.800         480           4    16.420 0.11770
## 35      38.350       58.660          50           5    42.140 0.73540
## 36      29.930      127.700         140           5    28.530 0.35140
## 37      20.760       55.050          69           7    12.720 0.23980
## 38      14.620        8.109          19           2     5.050 0.12470
## 39      46.210      951.500         849           8   369.500 1.38300
## 40      41.460      302.100         216           7   193.800 1.51300
## 41      18.920      140.600         128           6    12.780 0.13140
## 42      24.630       61.060          63           6    20.760 0.26770
## 43      34.880      128.000          76           5    51.600 0.43570
## 44      33.400      363.200         345           6    90.480 0.54320
## 45      26.370       70.860          85           4    16.340 0.21960
## 46      13.000       85.100          54           6    16.180 0.23980
## 47      20.050      152.900         153           5    19.300 0.20900
## 48      11.590       27.580          32           4     4.839 0.10870
## 49      34.940      380.200         224           6    88.220 0.50480
## 50      43.010      254.600         162           6    97.010 0.67650
## 51      31.860       72.320          97           6    29.390 0.43460
## 52      30.130       94.740          86           4    45.470 0.83150
## 53      58.150      727.800         438           6   329.000 0.96550
## 54      31.080      731.300         939           8    95.680 0.56180
## 55      35.720      128.300         176           7    24.410 0.27710
## 56      10.350      114.800         178           5     6.659 0.13060
## 57      24.740      216.000         249           5    37.950 0.35540
## 58      25.410      185.200         131           5    53.710 0.61980
## 59      34.670      178.500         181           5    39.990 0.77920
## 60      44.320       67.920          72           3    41.540 0.92360
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## 61      10.080      69.580        101           5     4.881 0.11360
## 62      50.740     794.400        910           8   141.900 0.88370
## 63      16.410      67.440         83           5    13.590 0.21980
## 64      18.730      63.780         75           4    13.690 0.19330
## 65      14.880      69.390         86           5    13.550 0.19080
## 66      16.790      68.260         67           4    15.510 0.20990
## 67      18.520      84.880         88           5    19.610 0.24350
## 68      14.310      34.560         42           4     7.543 0.17640
## 69      15.950      58.300         72           5    11.410 0.18360
## 70      13.760      98.330        104           7    19.460 0.23330
## 71      18.970      76.220         89           4    17.990 0.23550
## 72      19.710      92.610         83           5    21.310 0.24770
## 73      18.440      72.380         87           4    16.110 0.21480
## 74      18.270      45.710         45           4    13.220 0.21370
## 75      43.620     942.000        820          11   276.800 1.09200
## 76      11.000    4192.000       6959           9   223.400 0.60530
## 77      19.800    5045.000       9818          11   410.600 0.74740
## 78      14.950    3724.000       6838          11   321.300 0.60820
## 79      16.810    5197.000       9933          11   397.100 0.81010
## 80       8.771    3512.000       5608          11   304.400 0.63950
## 81      16.870    6933.000      12198          10   464.000 0.95430
## 82      14.300    6215.000      12006          11   439.700 0.65480
## 83      16.250     602.100       1238           7    54.850 0.25680
## 84      13.740      58.370        103           4    11.660 0.18030
## 85      15.030     439.200        982           8    37.210 0.21210
## 86      17.000     187.500        380           5    23.200 0.19010
## 87      15.310     268.800        573           7    27.130 0.22550
## 88      16.430     230.300        456           6    28.080 0.25820
## 89      12.880     164.600        308           5    18.280 0.22670
## 90      10.750      57.430        107           4     8.564 0.13790
## 91      14.420     261.500        492           6    27.940 0.20170
## 92      16.170     156.000        325           6    18.830 0.20860
## 93      24.110     230.900        330           6    37.450 0.24460
## 94      27.190     198.900        349           5    37.120 0.23290
## 95      27.940     336.600        482           5    57.920 0.31610
## 96      27.190     144.100        193           5    30.550 0.26700
## 97      21.840     305.800        450           8    42.730 0.28490
## 98      25.740     233.000        392           5    38.210 0.28070
## 99      28.000     442.200        804           5    61.520 0.23820
## 100     26.590     310.900        457           6    57.900 0.29870
## 101     29.880     364.300        562           6    57.670 0.29180
## 102     22.110     410.000        643           8    56.760 0.25840
## 103     24.000     338.600        570           6    51.330 0.24900
## 104     24.830     268.000        511           7    44.570 0.30060
## 105      6.806     720.600       1506           7    53.690 0.15130
## 106      9.510    1422.000       2484           8    92.790 0.18620
## 107      5.648     780.100       1669           7    49.810 0.13320
## 108      9.684    1146.000       2026           7    77.460 0.18280
## 109      7.533    1257.000       2253           9    89.910 0.18200
## 110      5.849     396.700        889           6    24.940 0.11230
## 111      8.221    1362.000       2685           8   101.800 0.18210
## 112     12.510     285.000        453           6    28.080 0.16810
## 113     15.830     125.300        205           5    15.060 0.12860
## 114     13.220     168.700        295           5    22.150 0.16040
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## 115      22.140     1427.000         2516            10   101.500 0.40030
## 116      23.840      589.600          973             7    90.240 0.32190
## 117      20.790     3560.000         5124            11   467.600 0.90670
## 118      22.520     2359.000         3816             9   281.500 0.54270
## 119      22.460     3646.000         5514            12   547.700 1.03800
## 120      17.500      279.000          562             6    36.850 0.23260
## 121      17.080      230.500          267             5    33.130 0.21030
##     DBH_CYL      BETA BETA_CI_MIN BETA_CI_MAX     GAMMA GAMMA_CI_MIN
## 1   0.33810 0.6490960   0.5845871   0.7136049 0.5611569    0.4613768
## 2   1.02400 0.6829363   0.6644919   0.7013806 0.5280024    0.5013820
## 3   0.46040 0.6781915   0.6598275   0.6965556 0.5552287    0.5246141
## 4   0.17520 0.7267139   0.6543794   0.7990484 0.5239695    0.3687385
## 5   0.62860 0.6570512   0.6247534   0.6893489 0.4713205    0.4157894
## 6   0.81120 0.6600989   0.6445393   0.6756585 0.5102009    0.4821670
## 7   0.20700 0.7029662   0.6578722   0.7480603 0.5431863    0.4765990
## 8   0.35090 0.7146038   0.6943164   0.7348912 0.5783221    0.5475986
## 9   0.17040 0.6705480   0.6065800   0.7345160 0.4931497    0.3841572
## 10  0.89470 0.6188556   0.5813300   0.6563812 0.4599996    0.3974689
## 11  0.10720 0.7222208   0.6905464   0.7538953 0.6092744    0.5568585
## 12  0.10400 0.6445447   0.4808186   0.8082709 0.6560244    0.5039821
## 13  0.04873 0.7256554   0.6669573   0.7843535 0.5454019    0.4272953
## 14  0.14280 0.7303909   0.7076556   0.7531262 0.6216246    0.5834156
## 15  0.10280 0.7673100   0.7312611   0.8033589 0.6438713    0.5821492
## 16  0.30880 0.7219046   0.6389699   0.8048392 0.5067329    0.3783904
## 17  0.32850 0.6715765   0.6471342   0.6960188 0.5373326    0.5041769
## 18  0.15950 0.7592840   0.7444048   0.7741632 0.6123210    0.5863214
## 19  0.14690 0.7611710   0.7413187   0.7810233 0.6180822    0.5871875
## 20  0.13720 0.7376147   0.7116024   0.7636270 0.6255557    0.5871242
## 21  0.12720 0.7534069   0.7284452   0.7783686 0.6260007    0.5822447
## 22  0.09999 0.7472876   0.7111262   0.7834489 0.5342896    0.4796797
## 23  0.10270 0.7401108   0.7176338   0.7625878 0.5964353    0.5584638
## 24  0.07309 0.7311618   0.6966617   0.7656619 0.5582774    0.4863521
## 25  0.10850 0.7176169   0.6771900   0.7580438 0.5371953    0.4732867
## 26  0.07304 0.7303490   0.6834726   0.7772253 0.5951216    0.5224404
## 27  0.09184 0.6997881   0.6596545   0.7399218 0.6167503    0.5586555
## 28  0.12020 0.7388999   0.7218725   0.7559273 0.5938979    0.5657542
## 29  0.13020 0.7340843   0.7124660   0.7557025 0.5860334    0.5491777
## 30  0.10870 0.7081941   0.6729215   0.7434666 0.5353752    0.4822632
## 31  0.13720 0.7035987   0.6744694   0.7327280 0.6156953    0.5710943
## 32  0.06945 0.7213909   0.6758529   0.7669290 0.5548866    0.4958270
## 33  0.08847 0.7434797   0.7175324   0.7694271 0.6289580    0.5821704
## 34  0.11530 0.7140682   0.6923194   0.7358170 0.6185381    0.5808625
## 35  0.73540 0.7040124   0.6385130   0.7695119 0.4946511    0.3773533
## 36  0.35390 0.7058435   0.6697882   0.7418988 0.5368877    0.4698613
## 37  0.24250 0.7042605   0.6350028   0.7735182 0.4768613    0.3622901
## 38  0.12530 0.6738941   0.4401202   0.9076680 0.5514126    0.1501904
## 39  1.38300 0.6457680   0.6275156   0.6640203 0.5427405    0.5178528
## 40  1.51300 0.6301127   0.5994846   0.6607407 0.4990593    0.4476436
## 41  0.13180 0.7100492   0.6707628   0.7493356 0.5618156    0.4873966
## 42  0.26620 0.6810167   0.6349290   0.7271045 0.5375739    0.4359871
## 43  0.43730 0.6159506   0.5689065   0.6629947 0.4415232    0.3563256
## 44  0.58700 0.6688521   0.6454033   0.6923009 0.5634132    0.5244488
## 45  0.21780 0.6565289   0.6171838   0.6958739 0.5446181    0.4592488
## 46  0.24180 0.6436851   0.5708143   0.7165559 0.5547473    0.4584891
```

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## 47  0.21040 0.6657812   0.6215387   0.7100236 0.4872346    0.4238845
## 48  0.10880 0.6585725   0.5252952   0.7918498 0.5749416    0.3163984
## 49  0.50880 0.6460029   0.6209591   0.6710467 0.5294121    0.4851689
## 50  0.67080 0.6199519   0.5834273   0.6564765 0.5126401    0.4547075
## 51  0.42850 0.7471152   0.6844941   0.8097363 0.5076134    0.4087193
## 52  0.83150 0.6418346   0.5896907   0.6939785 0.5244859    0.4431741
## 53  0.96550 0.6301010   0.6087778   0.6514243 0.4822024    0.4467674
## 54  0.55600 0.7084476   0.6914726   0.7254227 0.5557824    0.5289760
## 55  0.27300 0.7611448   0.7174609   0.8048288 0.5892288    0.5109961
## 56  0.13300 0.7362041   0.6976248   0.7747835 0.6155313    0.5630367
## 57  0.34490 0.6525284   0.6210444   0.6840124 0.5322207    0.4874106
## 58  0.63580 0.6113037   0.5665172   0.6560901 0.4869757    0.4209128
## 59  0.77920 0.7291185   0.6966808   0.7615562 0.5113246    0.4576156
## 60  0.92360 0.6983667   0.6400457   0.7566877 0.4280813    0.3314873
## 61  0.11560 0.7573081   0.7096571   0.8049591 0.6097185    0.5317087
## 62  0.88370 0.7494984   0.7298389   0.7691579 0.5377073    0.5085931
## 63  0.21760 0.6369092   0.5858744   0.6879439 0.4897220    0.4045340
## 64  0.19430 0.6708943   0.6107871   0.7310015 0.4577842    0.3491516
## 65  0.19070 0.7016356   0.6472038   0.7560674 0.4860047    0.4105371
## 66  0.21110 0.6680015   0.6135700   0.7224329 0.5891002    0.5045293
## 67  0.25030 0.6364029   0.5865141   0.6862917 0.5853527    0.5132582
## 68  0.17530 0.6970691   0.6204967   0.7736414 0.5658850    0.4626472
## 69  0.18200 0.6863253   0.6441670   0.7284836 0.5493958    0.4750497
## 70  0.23360 0.6452292   0.6028924   0.6875659 0.5776612    0.5076921
## 71  0.23530 0.6547501   0.6047972   0.7047029 0.5024295    0.4351219
## 72  0.25070 0.6465228   0.6027381   0.6903075 0.5324089    0.4533876
## 73  0.21490 0.6579460   0.6144011   0.7014908 0.5304920    0.4578945
## 74  0.21150 0.6289622   0.5665758   0.6913487 0.4518016    0.3389051
## 75  0.89660 0.6457840   0.6317144   0.6598536 0.5663459    0.5428865
## 76  0.59560 0.7450442   0.7384493   0.7516391 0.6239168    0.6141901
## 77  0.72290 0.6832471   0.6770302   0.6894640 0.6460800    0.6380427
## 78  0.60940 0.6826143   0.6754853   0.6897432 0.6338689    0.6239483
## 79  0.82510 0.7027657   0.6969759   0.7085556 0.6253017    0.6176171
## 80  0.63850 0.6572001   0.6481528   0.6662475 0.6174567    0.6064280
## 81  0.85150 0.7060608   0.7006281   0.7114936 0.6311896    0.6236528
## 82  0.69460 0.6782085   0.6723961   0.6840209 0.6422233    0.6345449
## 83  0.26240 0.7299899   0.7156703   0.7443094 0.5970434    0.5751319
## 84  0.18380 0.6899783   0.6402771   0.7396795 0.5354659    0.4543294
## 85  0.20790 0.7360883   0.7192007   0.7529759 0.5982571    0.5731229
## 86  0.19380 0.7045495   0.6797866   0.7293124 0.5629518    0.5212687
## 87  0.22470 0.7275246   0.7045390   0.7505101 0.6207515    0.5875958
## 88  0.25830 0.7173536   0.6943588   0.7403484 0.5578278    0.5192471
## 89  0.23190 0.7379058   0.7064489   0.7693628 0.5863980    0.5430132
## 90  0.13720 0.7345077   0.6933677   0.7756478 0.6063617    0.5207253
## 91  0.20490 0.7257091   0.7054355   0.7459827 0.5709335    0.5338019
## 92  0.21390 0.7540472   0.7260213   0.7820730 0.5951981    0.5472266
## 93  0.24390 0.6984556   0.6738110   0.7231002 0.5544056    0.5171399
## 94  0.23660 0.6598613   0.6324707   0.6872519 0.5747381    0.5361387
## 95  0.33180 0.6460065   0.6217011   0.6703120 0.5824187    0.5484328
## 96  0.26580 0.6560867   0.6218145   0.6903588 0.5132842    0.4607153
## 97  0.28510 0.6949159   0.6732325   0.7165992 0.5531824    0.5202875
## 98  0.27420 0.7161359   0.6894637   0.7428080 0.5872773    0.5498641
## 99  0.23930 0.6848015   0.6630275   0.7065754 0.6115577    0.5832260
## 100 0.29800 0.6852602   0.6581909   0.7123295 0.5693618    0.5357902
```

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## 101 0.29260 0.7058407   0.6844623   0.7272190 0.5772626    0.5466580
## 102 0.26520 0.6815492   0.6602833   0.7028150 0.5633175    0.5346662
## 103 0.25190 0.6722738   0.6511642   0.6933834 0.5653314    0.5351638
## 104 0.29780 0.6776442   0.6501331   0.7051553 0.5466570    0.5126818
## 105 0.15270 0.6986859   0.6827867   0.7145851 0.6229967    0.6026444
## 106 0.19070 0.7093337   0.6984451   0.7202223 0.6065444    0.5902446
## 107 0.13350 0.7142885   0.7002669   0.7283100 0.6112541    0.5911622
## 108 0.18240 0.7151137   0.7035627   0.7266646 0.5904819    0.5718196
## 109 0.18340 0.7153130   0.7045616   0.7260645 0.5973898    0.5806178
## 110 0.10990 0.6996352   0.6793038   0.7199666 0.6143444    0.5872828
## 111 0.18140 0.7064258   0.6947806   0.7180711 0.6010483    0.5833541
## 112 0.17760 0.7232774   0.6949632   0.7515916 0.5739427    0.5357797
## 113 0.13070 0.7431416   0.7057503   0.7805328 0.6224085    0.5665973
## 114 0.15900 0.6917214   0.6536211   0.7298217 0.5990798    0.5460170
## 115 0.40800 0.7378356   0.7255535   0.7501176 0.5925469    0.5769539
## 116 0.32230 0.7379157   0.7203702   0.7554611 0.5865165    0.5591976
## 117 0.80730 0.7307589   0.7234649   0.7380529 0.5901176    0.5792781
## 118 0.54340 0.7366241   0.7279058   0.7453424 0.5972262    0.5853364
## 119 0.96260 0.7174126   0.7098858   0.7249393 0.6040907    0.5940252
## 120 0.23610 0.7446034   0.7203699   0.7688368 0.5586983    0.5215650
## 121 0.21330 0.7197280   0.6865815   0.7528744 0.5450028    0.4979787
##     GAMMA_CI_MAX     D_BETA   D_GAMMA     FIB_R     FIB_L SIMPLE_THETA
## 1      0.6609370 0.09484445 0.1635181 0.7724112 0.5545410    0.4806499
## 2      0.5546228 0.12537382 0.1768239 0.7054908 0.5028016    0.4946240
## 3      0.5858433 0.12022869 0.1950223 0.7088602 0.5118038    0.5077904
## 4      0.6792005 0.10511272 0.1343484 0.7662347 0.4730527    0.5395121
## 5      0.5268517 0.13715322 0.1484402 0.6763985 0.5165357    0.4353383
## 6      0.5382348 0.13038152 0.1815989 0.6894531 0.4805186    0.4609666
## 7      0.6097737 0.08057091 0.1490398 0.7868559 0.5579388    0.5270297
## 8      0.6090457 0.09312095 0.1717558 0.7605916 0.5528191    0.5683033
## 9      0.6021423 0.10195488 0.1400188 0.7271781 0.5498000    0.4601769
## 10     0.5225304 0.09959273 0.1403833 0.7357134 0.5101857    0.3992102
## 11     0.6616903 0.11041436 0.2290007 0.7494128 0.5133189    0.6046636
## 12     0.8080667 0.12371713 0.1871329 0.7271175 0.6082256    0.5331986
## 13     0.6635085 0.12076990 0.1718344 0.7394219 0.5107303    0.5555878
## 14     0.6598336 0.10427773 0.2274911 0.7632207 0.5271563    0.6279817
## 15     0.7055934 0.11568578 0.2513874 0.7557408 0.5387933    0.7145956
## 16     0.6350754 0.10945219 0.2052004 0.7750218 0.4741626    0.5205778
## 17     0.5704883 0.11509584 0.1765051 0.6960378 0.5362438    0.4890297
## 18     0.6383205 0.10951312 0.2424262 0.7594585 0.5119680    0.6656829
## 19     0.6489769 0.10795009 0.2332027 0.7550512 0.5300354    0.6749714
## 20     0.6639872 0.09637165 0.2283960 0.7780639 0.5348039    0.6431236
## 21     0.6697567 0.09729877 0.2033859 0.7881210 0.5570856    0.6698994
## 22     0.5888994 0.13469924 0.1722283 0.7432090 0.5505838    0.5731200
## 23     0.6344067 0.13179121 0.1984820 0.7343308 0.5211766    0.6196031
## 24     0.6302027 0.13751506 0.1900995 0.6956608 0.5577468    0.5732562
## 25     0.6011038 0.12793025 0.1935964 0.6979638 0.5294354    0.5394005
## 26     0.6678029 0.11602672 0.2449735 0.7404371 0.5361956    0.6040735
## 27     0.6748450 0.13150667 0.2703931 0.7278885 0.4764010    0.5789512
## 28     0.6220416 0.12933476 0.2245371 0.7285339 0.5152553    0.6154561
## 29     0.6228892 0.12129736 0.2082055 0.7281067 0.5390231    0.6013555
## 30     0.5884873 0.13136177 0.2246207 0.7066323 0.5035813    0.5271636
## 31     0.6602963 0.12781837 0.2334429 0.7098451 0.5081224    0.5834094
## 32     0.6139463 0.11966784 0.2089720 0.7341593 0.5381724    0.5580268
```

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## 33     0.6757457 0.15548053 0.2552273 0.6801185 0.4914234    0.6560674
## 34     0.6562137 0.16169033 0.2100797 0.6851266 0.5512691    0.6006734
## 35     0.6119490 0.17367767 0.2086774 0.6818696 0.4855286    0.4930551
## 36     0.6039141 0.10475089 0.2079708 0.7503539 0.4790050    0.5256332
## 37     0.5914326 0.09323776 0.1292149 0.7945204 0.5041230    0.4807701
## 38     0.9526347 0.11187476 0.1085341 0.6858076 0.6484365    0.5005986
## 39     0.5676281 0.14332961 0.1770971 0.6620296 0.5278870    0.4665289
## 40     0.5504750 0.14841825 0.2034846 0.6215585 0.4447949    0.4282007
## 41     0.6362345 0.11095237 0.2083656 0.7345257 0.4895027    0.5494959
## 42     0.6391608 0.10486022 0.1318990 0.7548835 0.5661632    0.4990168
## 43     0.5267208 0.12584303 0.1638204 0.6912049 0.4772630    0.3879479
## 44     0.6023776 0.10861883 0.1750572 0.7320450 0.5545503    0.5029634
## 45     0.6299875 0.09956728 0.1771957 0.7234361 0.5650736    0.4782807
## 46     0.6510056 0.10018202 0.2101708 0.7230494 0.5208866    0.4714216
## 47     0.5505847 0.10138413 0.1651672 0.7531574 0.5198829    0.4522694
## 48     0.8334849 0.15533881 0.1290068 0.6575409 0.4570770    0.4990806
## 49     0.5736553 0.10806372 0.1939966 0.7299978 0.5190333    0.4590711
## 50     0.5705727 0.12601220 0.2050962 0.7090183 0.4578541    0.4267076
## 51     0.6065075 0.08446557 0.1698543 0.7888778 0.5339946    0.5496340
## 52     0.6057977 0.12353805 0.2055051 0.7007197 0.4911056    0.4523910
## 53     0.5176375 0.11910004 0.1750730 0.6936360 0.4656132    0.4192909
## 54     0.5825889 0.11502893 0.2055008 0.7326721 0.5075616    0.5429054
## 55     0.6674615 0.09825441 0.2318750 0.7756171 0.5227783    0.6449057
## 56     0.6680259 0.10393420 0.1980751 0.7649752 0.5551670    0.6314166
## 57     0.5770308 0.11421744 0.2013728 0.7066609 0.5082825    0.4669235
## 58     0.5530385 0.12908148 0.1972162 0.6770162 0.4627348    0.4068090
## 59     0.5650337 0.09570103 0.1773878 0.7609542 0.5337052    0.5321305
## 60     0.5246752 0.13243110 0.1261244 0.6921071 0.4858776    0.4424916
## 61     0.6877282 0.08820073 0.1369007 0.7622237 0.5426498    0.6596826
## 62     0.5668215 0.09972856 0.1860900 0.7647893 0.5190189    0.5790011
## 63     0.5749100 0.14120702 0.1914720 0.6556785 0.5425011    0.4288816
## 64     0.5664168 0.13724183 0.1581861 0.7154239 0.5101740    0.4387994
## 65     0.5614722 0.12350070 0.1460996 0.7110534 0.5418137    0.4846440
## 66     0.6736711 0.11156957 0.2073166 0.7447451 0.5385155    0.5187883
## 67     0.6574472 0.11179275 0.2166593 0.7335156 0.4912393    0.4815570
## 68     0.6691227 0.08425010 0.1902013 0.7985085 0.5255339    0.5368631
## 69     0.6237419 0.09079010 0.1683297 0.7524175 0.5392546    0.5127802
## 70     0.6476302 0.10251366 0.1844078 0.7164990 0.5351855    0.4863961
## 71     0.5697371 0.10462114 0.1526797 0.7436494 0.5876029    0.4514725
## 72     0.6114303 0.12876498 0.2065496 0.6509396 0.5438038    0.4612872
## 73     0.6030895 0.09611513 0.1622714 0.7703640 0.5750832    0.4711392
## 74     0.5646982 0.12376664 0.1457542 0.6940484 0.4793365    0.4025525
## 75     0.5898053 0.10684978 0.1741008 0.7364501 0.5579708    0.4803080
## 76     0.6336435 0.10324263 0.2047141 0.7642067 0.5396315    0.6536894
## 77     0.6541174 0.12661540 0.2052788 0.6999272 0.5503202    0.5782832
## 78     0.6437895 0.12742592 0.2098564 0.6832321 0.5338351    0.5683566
## 79     0.6329863 0.11366590 0.1970368 0.7256837 0.5496806    0.5899203
## 80     0.6284854 0.12732548 0.1936002 0.6800844 0.5484502    0.5244441
## 81     0.6387264 0.12290743 0.2051311 0.7172000 0.5406212    0.5994752
## 82     0.6499017 0.12421242 0.2103621 0.6943717 0.5330190    0.5684235
## 83     0.6189550 0.10343919 0.1928565 0.7660754 0.5373042    0.6052552
## 84     0.6166024 0.13532686 0.1667823 0.7163486 0.5461289    0.5071283
## 85     0.6233913 0.09504182 0.1938220 0.7750185 0.5481962    0.6152858
## 86     0.6046348 0.10723067 0.1766426 0.7439049 0.5466632    0.5436642
```

```
## 87     0.6539071 0.10483116 0.1887998 0.7597876 0.5513491    0.6227516
## 88     0.5964084 0.09364570 0.1795747 0.7662508 0.5464530    0.5553718
## 89     0.6297827 0.09548955 0.1832691 0.7727124 0.5351997    0.6071532
## 90     0.6919981 0.11595945 0.1745617 0.7286420 0.5899744    0.6203280
## 91     0.6080652 0.10274349 0.2125052 0.7554576 0.5118550    0.5768081
## 92     0.6431697 0.08520857 0.2021333 0.7873963 0.5368208    0.6397524
## 93     0.5916713 0.11527837 0.1667861 0.7280727 0.5620004    0.5300806
## 94     0.6133375 0.10889518 0.1816342 0.6988395 0.5700163    0.5003618
## 95     0.6164045 0.10725228 0.1720942 0.7044217 0.5708391    0.4900447
## 96     0.5658530 0.12818518 0.1661122 0.6929333 0.5436890    0.4590833
## 97     0.5860772 0.10653477 0.1872455 0.7351353 0.5440401    0.5251129
## 98     0.6246905 0.10476929 0.1774377 0.7434551 0.5372860    0.5776087
## 99     0.6398894 0.11937524 0.1778999 0.7134816 0.5723888    0.5549623
## 100    0.6029335 0.11583515 0.1665151 0.7010466 0.5631462    0.5254489
## 101    0.6078671 0.10238213 0.1953728 0.7488784 0.5470398    0.5562133
## 102    0.5919688 0.11056657 0.1757415 0.7179493 0.5481051    0.5170095
## 103    0.5954989 0.11275035 0.1793985 0.7106405 0.5636451    0.5079779
## 104    0.5806321 0.11226492 0.1786418 0.7029410 0.5582121    0.5014813
## 105    0.6433490 0.12704958 0.1961184 0.7064188 0.5480644    0.5823191
## 106    0.6228442 0.13473608 0.2020751 0.7055534 0.5407174    0.5840296
## 107    0.6313460 0.11034791 0.1903920 0.7426872 0.5507859    0.5948846
## 108    0.6091441 0.13104407 0.1987701 0.7188503 0.5429631    0.5788556
## 109    0.6141619 0.12194657 0.1985301 0.7237181 0.5457164    0.5848082
## 110    0.6414059 0.12385306 0.1931149 0.7168550 0.5448771    0.5768573
## 111    0.6187424 0.12718725 0.1900659 0.7146852 0.5455905    0.5756299
## 112    0.6121058 0.08492373 0.1562740 0.7825510 0.5901912    0.5761101
## 113    0.6782197 0.11115655 0.1970296 0.7608364 0.5455117    0.6490774
## 114    0.6521426 0.12714492 0.1775307 0.7069026 0.5698039    0.5547376
## 115    0.6081400 0.08242626 0.1780331 0.7917673 0.5731032    0.6126489
## 116    0.6138354 0.09954695 0.1713130 0.7684451 0.5482278    0.6072750
## 117    0.6009570 0.08958928 0.1806251 0.7722524 0.5647869    0.6002434
## 118    0.6091160 0.09131078 0.1793151 0.7763506 0.5662596    0.6151387
## 119    0.6141561 0.10480314 0.1811064 0.7449327 0.5652808    0.5933264
## 120    0.5958315 0.09677485 0.1630239 0.7815983 0.5637728    0.5914463
## 121    0.5920269 0.09108083 0.1756760 0.7892534 0.5154482    0.5480601
##             WBE  NODE_WBE       THETA NODE_THETA  SYM_VOL   ASYM_VOL NETWORK_N
## 1     1.6917002 0.4111492   1.8091361  0.4078041 1.868476 0.05036251  5.459432
## 2     2.2628348 0.4152442   2.5067750  0.4266200 2.550107 0.07715922  2.000000
## 3     0.8152653 0.4205054   0.8661260  0.4302698 1.556030 0.07965858  1.584963
## 4    -6.8457443 0.4469940  -5.8719165  0.4372872 3.107848 0.05262817  5.523562
## 5     1.5086065 0.3848346   1.6582126  0.3932700 1.683184 0.07123979  7.426265
## 6     1.8669734 0.3934087   2.1067681  0.4017229 1.850238 0.07986317  7.149747
## 7     3.0812105 0.4158140   3.3578712  0.4121488 2.181892 0.04081800  6.426265
## 8     3.0918286 0.4527714   3.4892368  0.4491187 2.155524 0.05574753  6.918863
## 9     1.3842748 0.3814238   1.4623836  0.3762049 1.790843 0.04854229  4.247928
## 10    1.2014761 0.3609728   1.2624973  0.3617499 1.546659 0.04373447  6.149747
## 11    2.2077152 0.4569334   2.4941318  0.4420169 2.394310 0.08790106  2.584963
## 12    1.0527531 0.4055967   1.1168742  0.3843185 2.070687 0.07977095  1.000000
## 13    1.3678235 0.4399950   1.4572868  0.4469068 2.409797 0.07614622  1.000000
## 14   -9.5558453 0.4725436  -7.0411922  0.4608485 3.155990 0.08282497  4.392317
## 15    3.2478854 0.4926392   4.0412122  0.4737861 2.489023 0.10649357  3.459432
## 16   64.4684981 0.4409203 -42.1077811  0.4105875 2.791060 0.07699595  5.392317
## 17    0.6957348 0.4058813   0.7290555  0.4048777 1.477001 0.06880846  1.000000
## 18  155.7301630 0.4920468 -23.4541988  0.4710376 2.755788 0.09531975  5.977280
```

```
## 19    9.7179833 0.4862786  18.0665229  0.4682903 2.717264 0.09105293  4.523562
## 20    4.6465554 0.4756076   5.7876714  0.4605610 2.565017 0.07656193  4.087463
## 21    3.3721447 0.4719846   3.9106346  0.4623942 2.490206 0.07149005  3.584963
## 22    3.1355016 0.4515005   3.7253376  0.4366185 2.491042 0.08873352  3.321928
## 23    2.3373206 0.4616324   2.6964659  0.4540178 2.436358 0.09815871  2.584963
## 24    1.8474006 0.4373261   2.0571334  0.4415031 2.502141 0.09756929  1.584963
## 25    2.9758887 0.4360413   3.5142894  0.4187708 2.440942 0.08867603  3.459432
## 26    2.1301199 0.4663014   2.4293247  0.4423844 2.422478 0.09905950  2.321928
## 27    3.4662740 0.4447027   4.3259877  0.4346736 2.981359 0.12086581  1.584963
## 28    2.2325929 0.4541444   2.5200205  0.4326222 2.932417 0.10570079  1.000000
## 29    5.1051923 0.4516292   6.7878830  0.4327655 2.670174 0.09140121  3.584963
## 30    2.5505157 0.4106679   2.9414767  0.3917612 2.774805 0.10206230  1.584963
## 31    2.6671710 0.4345120   3.1445981  0.4145184 2.586519 0.10409460  2.321928
## 32    2.2675448 0.4288252   2.5506149  0.4078808 2.551730 0.08805235  2.000000
## 33    4.5652953 0.4741680   7.1255874  0.4551838 2.647129 0.14842279  3.459432
## 34    1.9488865 0.4477491   2.2820393  0.4300850 2.427326 0.12936327  2.000000
## 35    3.7394633 0.4181669   5.3189778  0.4761277 2.330950 0.13190220  5.554589
## 36    3.7019449 0.4130036   4.3905294  0.4030087 2.459324 0.07328985  4.169925
## 37    1.5520293 0.4141318   1.6228418  0.4144554 2.145960 0.04222987  2.321928
## 38    0.6987622 0.3946907   0.7278075  0.3909009 1.152347 0.04653321  3.321928
## 39    0.8979224 0.4000525   0.9686900  0.4181352 1.514545 0.08786623  2.584963
## 40    1.0965948 0.3706064   1.2239501  0.3868833 1.443125 0.09810624  6.658211
## 41    4.1891289 0.4383350   5.2111758  0.4344704 2.410078 0.07949391  5.169925
## 42    1.9285945 0.4309069   2.0740230  0.4343695 1.939488 0.04949842  5.857981
## 43    1.5401834 0.3502309   1.6747550  0.3504932 1.759102 0.06477724  6.087463
## 44    1.8177787 0.4206331   2.0009171  0.4225280 1.806630 0.06416592  7.761551
## 45    0.9859005 0.4008417   1.0469576  0.4049587 1.365011 0.05713059  6.108524
## 46    1.4498369 0.4191025   1.5706131  0.4194879 1.745028 0.06534739  5.459432
## 47    2.1229639 0.3837536   2.2948007  0.3843938 2.206520 0.05461122  3.584963
## 48    1.8162968 0.4406616   2.0128124  0.4307968 2.121532 0.08053741  3.321928
## 49    1.3407436 0.3930546   1.4566387  0.3897591 1.584530 0.06653590  7.554589
## 50    1.1742977 0.3806346   1.2726476  0.4048951 1.722023 0.08037020  3.321928
## 51    1.5061402 0.4263620   1.5841553  0.4228418 2.186362 0.05011802  2.000000
## 52    0.9408080 0.3938049   1.0191624  0.3961385 1.393121 0.08118786  4.584963
## 53    0.8455075 0.3699540   0.8894037  0.3812874 1.604076 0.06623336  1.584963
## 54    9.0711813 0.4519873  15.7391584  0.4476480 2.482962 0.08169449  7.179909
## 55   48.5979967 0.4924479 -44.7590615  0.4930766 2.673885 0.08074060  6.700440
## 56    1.5939811 0.4861818   1.7395563  0.4829291 1.994618 0.07392261  3.459432
## 57    1.7307064 0.4001111   1.9145043  0.3971716 1.885810 0.07391966  5.459432
## 58    1.0542996 0.3661825   1.1482519  0.6151011 1.419940 0.07847574  6.321928
## 59    2.8796639 0.4508670   3.2522503  0.4445637 2.105792 0.05887676  7.199672
## 60    1.3817192 0.3701195   1.4766555  0.3739805 1.881576 0.06167409  3.321928
## 61    1.6423681 0.4931358   1.7401809  0.4938596 1.918848 0.04606364  4.459432
## 62  171.3130345 0.4670863 -32.8588452  0.4665067 2.605477 0.06633406  8.308339
## 63    1.4841843 0.3704525   1.6605049  0.3624602 1.738962 0.08841049  5.906891
## 64    1.2527308 0.3837725   1.3648053  0.3791116 1.624303 0.07550481  5.357552
## 65    1.4456435 0.4094151   1.5647929  0.4064182 1.760591 0.06546508  5.169925
## 66    1.1450181 0.4193740   1.2431543  0.4246088 1.562270 0.07647007  5.000000
## 67    1.0211334 0.4113350   1.1074554  0.4270017 1.404038 0.07628812  5.977280
## 68    1.2456922 0.4490150   1.3212601  0.4502432 1.630175 0.05271407  5.169925
## 69    1.1817761 0.4219842   1.2499477  0.4185879 1.569321 0.05101269  5.392317
## 70    0.9440365 0.4059847   1.0058955  0.4152938 1.319214 0.06093182  6.209453
## 71    0.7574568 0.4000561   0.7966203  0.4092627 1.148168 0.05283343  4.954196
## 72    0.9282953 0.3813509   1.0153953  0.3963176 1.306738 0.08643571  6.108524
```

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## 73    0.8925458 0.4043377   0.9383389  0.4033762 1.316404 0.05084872  5.000000
## 74    1.0310758 0.3608233   1.0966504  0.3698950 1.443956 0.05922624  5.321928
## 75    1.4006553 0.4087167   1.5165497  0.4126244 1.582285 0.06098494  8.731319
## 76    1.8384928 0.4903303   1.9979332  0.4815433 2.497596 0.07628742  1.584963
## 77    1.7696856 0.4570188   2.0020893  0.4507392 1.972445 0.09174957  4.523562
## 78    2.0717891 0.4484480   2.3991591  0.4390332 2.003855 0.09360026  5.614710
## 79    1.9326784 0.4619438   2.1507051  0.4561425 2.136717 0.07911532  3.584963
## 80    1.2091955 0.4277667   1.3094410  0.4243808 1.890732 0.08482069  2.321928
## 81    3.5719744 0.4676530   4.4616134  0.4621041 2.288228 0.09027501  5.857981
## 82    1.5769467 0.4507441   1.7496431  0.4437222 2.045608 0.09070261  3.000000
## 83    1.1901992 0.4668530   1.2647419  0.4685190 2.033928 0.07102635  1.584963
## 84    1.1095894 0.4278736   1.2078160  0.4372519 1.571574 0.08190396  4.392317
## 85    1.3703483 0.4743042   1.4587842  0.4746594 2.088895 0.06504655  2.000000
## 86    0.8817895 0.4459473   0.9237990  0.4422689 1.822530 0.06632700  1.000000
## 87    1.3252139 0.4880646   1.4250627  0.4840569 1.908519 0.07124054  2.807355
## 88    1.0624408 0.4351740   1.1139104  0.4334539 1.896363 0.05803697  1.584963
## 89    1.1109209 0.4816611   1.1751514  0.4819555 1.797244 0.06234809  2.321928
## 90    1.0475338 0.4652064   1.1141093  0.4631544 1.878839 0.07577886  1.584963
## 91    1.5689263 0.4463245   1.7013394  0.4410955 2.156306 0.07543296  2.321928
## 92    2.1586704 0.4900107   2.3486366  0.4840210 2.302847 0.06059217  3.000000
## 93    1.2698467 0.4321781   1.3726347  0.4390644 1.640646 0.06845144  5.285402
## 94    0.9513252 0.4108692   1.0139097  0.4121196 1.431199 0.06583648  4.169925
## 95    0.8144592 0.4081930   0.8583087  0.4150380 1.374817 0.06109385  2.807355
## 96    0.9889711 0.3871336   1.0667268  0.3929394 1.387914 0.07274852  5.643856
## 97    1.5465818 0.4295019   1.6896625  0.4265211 1.758113 0.06800603  6.169925
## 98    1.1393166 0.4574192   1.2152181  0.4553500 1.707807 0.06614455  3.169925
## 99    0.8105871 0.4510971   0.8540192  0.4499032 1.700933 0.07560202  1.000000
## 100   0.7692484 0.4265707   0.8097335  0.4340365 1.478869 0.06814918  1.584963
## 101   1.1148300 0.4509357   1.1955785  0.4506730 1.599016 0.06857668  4.087463
## 102   0.9692607 0.4274607   1.0278940  0.4298049 1.603087 0.06674617  2.584963
## 103   0.6554542 0.4160872   0.6868319  0.4192401 1.389283 0.06876670  1.000000
## 104   1.1579228 0.4120719   1.2370956  0.4165070 1.744787 0.06814066  3.000000
## 105   2.1003328 0.4579208   2.4056970  0.4541569 2.097894 0.08974829  4.523562
## 106 -29.3974553 0.4582588 -11.7266543  0.4479004 2.744722 0.09927373  7.169925
## 107   2.0784636 0.4594672   2.3131428  0.4533447 2.176880 0.07491303  3.700440
## 108   4.7292779 0.4502500   6.4647417  0.4387842 2.362042 0.09478824  6.392317
## 109   4.2165800 0.4526575   5.4665983  0.4474747 2.272268 0.08703861  7.209453
## 110   2.0637564 0.4520052   2.3345798  0.4474665 2.159073 0.08578272  3.807355
## 111   3.5527109 0.4479774   4.3728084  0.4409468 2.354658 0.08775426  5.000000
## 112   1.0208840 0.4637116   1.0632749  0.4635839 1.701017 0.04667413  2.321928
## 113   1.3020136 0.4839134   1.4126224  0.4764759 1.890814 0.08048323  2.807355
## 114   1.1136811 0.4402871   1.2055801  0.4444715 1.684057 0.08182373  3.169925
## 115   1.7142815 0.4721925   1.8186733  0.4705057 2.187422 0.05136141  2.584963
## 116   1.8520398 0.4673007   2.0242216  0.4646521 1.915503 0.06196103  5.781360
## 117   2.1335364 0.4674626   2.3504298  0.4655472 1.865503 0.05677363  9.033423
## 118   1.8565291 0.4780396   2.0327882  0.4778011 1.768946 0.05820311  9.310613
## 119   1.3720024 0.4686120   1.4756694  0.4670655 1.874903 0.06773775  3.321928
## 120   1.7453090 0.4600037   1.8894588  0.4581200 1.867649 0.05745410  5.857981
## 121   1.8598739 0.4324073   2.0117824  0.4305941 1.930806 0.05510703  5.584963
##     N_ASYM   N_MEAN    N_MIN    N_MAX    TIPS_MEAN TIPS_VARIANCE   TIPS_CV
## 1       NA 5.480794 4.000000 6.961589 2.038694e-04  1.534759e-07 0.7047448
## 2       NA 7.763854 7.291561 8.000000 2.352772e-04  1.410435e-04 1.0166416
## 3       NA 5.762951 5.000000 7.288852 1.121634e-04  5.202104e-07 0.9191356
## 4       NA 5.237593 5.000000 5.475187 4.814784e-05  1.135907e-07 0.8036760
```

```
## 5       NA 7.187927 6.000000 8.375854 1.316679e-04  1.006402e-05 0.9857686
## 6       NA 7.144899 6.000000 8.289798 1.378319e-04  6.090287e-07 0.9684384
## 7       NA 7.678067 7.140951 8.215183 7.451995e-05  2.941687e-07 0.8710581
## 8       NA 8.234827 7.884558 8.585095 5.823043e-05  8.176861e-08 0.7665602
## 9       NA 4.518099 3.972222 5.063976 3.279896e-05  3.387857e-09 0.7631335
## 10      NA 6.248808 5.500000 6.997616 2.501472e-04  2.869885e-06 0.8957381
## 11      NA 4.433033 4.000000 4.674099 3.591203e-05  3.057303e-09 0.6528664
## 12      NA 2.992214 2.000000 3.976642 9.674308e-05  2.136232e-08 0.5404566
## 13      NA 3.598888 3.000000 4.395554 2.363751e-05  5.671354e-09 0.7594727
## 14      NA 3.148708 2.000000 4.297415 5.646944e-05  8.754985e-09 0.6679411
## 15      NA 3.406930 2.500000 3.955847 6.935343e-05  6.357457e-09 0.6362265
## 16      NA 4.675171 3.953125 5.397217 3.545280e-05  5.066000e-09 0.9508013
## 17      NA 4.327719 4.000000 4.983156 4.746844e-05  3.533915e-08 0.8638375
## 18      NA 6.271747 6.160156 6.383338 4.057913e-05  3.948824e-09 0.6870583
## 19      NA 4.094753 3.500000 4.689506 4.885137e-05  3.936250e-09 0.6090489
## 20      NA 5.492024 5.000000 5.984048 6.849424e-05  1.671521e-08 0.6795485
## 21      NA 3.379319 1.000000 5.758639 6.048567e-05  1.086245e-08 0.6384426
## 22      NA 3.404824 2.000000 4.809649 5.271417e-05  6.779294e-09 0.6652070
## 23      NA 4.394425 4.000000 4.788849 1.620389e-05  1.970584e-09 0.7689426
## 24      NA 1.838225 1.000000 2.676450 3.864811e-05  3.082686e-09 0.6430116
## 25      NA 3.089260 2.500000 3.678520 6.558097e-05  1.082035e-08 0.6772867
## 26      NA 3.057162 2.000000 4.478649 3.366715e-05  1.427100e-09 0.6546963
## 27      NA 4.431849 3.500000 5.664898 3.766721e-05  2.856098e-09 0.6820076
## 28      NA 3.498277 2.000000 5.494830 2.562028e-05  1.127747e-09 0.6667504
## 29      NA 5.692059 4.750000 6.634118 3.391610e-05  1.041380e-08 0.6770358
## 30      NA 3.658774 3.317547 4.000000 3.734052e-05  5.387934e-09 0.5952288
## 31      NA 2.926129 1.000000 4.852259 4.606050e-05  8.004446e-09 0.6945701
## 32      NA 3.376924 2.666667 4.528530 1.694646e-05  4.727516e-10 0.5946609
## 33      NA 4.572862 3.000000 6.145724 1.545260e-05  5.805187e-10 0.6620732
## 34      NA 3.246645 2.000000 4.239936 3.513093e-05  2.054774e-09 0.6302412
## 35      NA 5.643634 5.000000 6.287268 1.621565e-04  3.745621e-05 1.1501611
## 36      NA 4.992440 4.486654 5.498226 7.974786e-05  2.030840e-08 0.7091960
## 37      NA 3.106835 2.000000 4.213670 4.088295e-05  2.681724e-09 0.6213280
## 38      NA 3.478472 3.000000 3.956944 4.340084e-05  2.432348e-09 0.7508769
## 39      NA 7.627556 7.000000 8.255113 2.685404e-04  9.112529e-05 0.9882058
## 40      NA 6.518546 6.412092 6.625000 2.512217e-04  5.327399e-05 1.3641152
## 41      NA 6.087522 6.012283 6.162760 4.354094e-05  6.204870e-08 0.8406227
## 42      NA 5.656616 5.560303 5.752930 5.576984e-05  3.641439e-06 1.0481944
## 43      NA 6.229879 5.700684 6.759074 1.480538e-04  5.343756e-06 0.9304212
## 44      NA 6.063240 5.213542 6.912938 1.020770e-04  1.560385e-06 1.0072927
## 45      NA 6.148288 5.590278 6.706299 4.336570e-05  3.165046e-08 0.9084205
## 46      NA 6.139052 5.878255 6.399848 7.889217e-05  1.885178e-07 0.9915071
## 47      NA 4.586851 3.500000 5.448054 6.483870e-05  2.378241e-08 0.8148723
## 48      NA 3.481035 3.000000 3.962071 1.274113e-04  1.238233e-07 0.8575710
## 49      NA 8.415894 7.839193 8.992594 1.485365e-04  1.107208e-06 0.8651118
## 50      NA 4.543399 3.000000 6.086798 3.003011e-04  2.444665e-06 0.9667192
## 51      NA 5.157097 4.500000 5.971290 2.518148e-05  8.115924e-09 0.8959229
## 52      NA 5.719079 5.261936 6.176222 1.393598e-04  3.199197e-07 0.9411473
## 53      NA 6.909299 6.000000 7.818599 4.172653e-04  1.324595e-05 0.9508240
## 54      NA 7.033437 5.000000 9.066874 8.110513e-05  1.074743e-07 0.8201297
## 55      NA 5.869154 5.000000 6.738307 4.302675e-05  2.178874e-08 0.7687088
## 56      NA 4.765653 3.833333 6.270946 1.284374e-05  8.855572e-10 0.7427948
## 57      NA 6.432887 5.500000 7.365774 6.199225e-05  2.792400e-07 0.9134644
## 58      NA 6.946083 6.411024 7.481141 8.581954e-05  1.182495e-06 1.0663441
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## 59      NA 7.751542 6.500000 9.003085 3.499371e-05  1.717191e-07 0.8611271
## 60      NA 5.996168 4.000000 7.992337 1.220825e-04  6.174144e-05 1.4988249
## 61      NA 4.101540 3.000000 5.203080 1.254150e-05  1.870529e-09 0.7904015
## 62      NA 7.022869 5.000000 9.045738 5.206489e-05  1.619269e-07 0.8148420
## 63      NA 5.358683 4.000000 6.717367 7.901171e-05  2.411347e-08 0.8552935
## 64      NA 6.566657 5.734375 7.398939 8.044947e-05  2.774554e-07 0.9694268
## 65      NA 5.745825 5.128133 6.363516 9.491774e-05  3.574973e-08 0.8507397
## 66      NA 6.138455 5.500000 6.776910 1.164500e-04  1.809529e-07 0.8330926
## 67      NA 7.480428 7.305908 7.654948 7.852308e-05  3.248220e-07 0.9164004
## 68      NA 6.602702 6.234375 6.971029 3.720103e-05  1.130951e-08 0.9141320
## 69      NA 5.351418 4.476562 6.226273 5.760226e-05  2.360229e-08 0.8331269
## 70      NA 6.339407 5.968171 6.710643 1.109365e-04  1.074825e-07 0.8842563
## 71      NA 6.472555 6.159532 6.785577 5.895156e-05  2.791620e-08 0.7776059
## 72      NA 5.399974 4.000000 6.799947 7.567941e-05  4.063249e-07 1.1352903
## 73      NA 5.692471 5.330729 6.054213 5.512812e-05  3.151838e-08 0.8642673
## 74      NA 6.920247 6.835938 7.004557 4.465302e-05  2.155019e-07 1.3037733
## 75      NA 6.043824 5.000000 7.087649 1.472283e-04  8.017017e-06 0.8692462
## 76      NA 4.385203 3.000000 5.770406 2.303103e-05  4.742065e-08 0.7164402
## 77      NA 6.798901 6.740778 6.857025 4.109204e-05  2.835030e-08 0.7341151
## 78      NA 7.774234 7.043931 8.504538 4.412354e-05  1.019862e-07 0.8021328
## 79      NA 5.943676 4.969102 6.918250 3.572693e-05  1.125313e-07 0.7463017
## 80      NA 5.085719 4.000000 6.142379 6.737372e-05  2.399472e-07 0.8413034
## 81      NA 7.285323 6.470472 8.100173 2.452417e-05  2.577771e-08 0.7449034
## 82      NA 7.694244 7.388488 8.000000 3.289980e-05  1.418096e-06 0.8175636
## 83      NA 5.133866 4.000000 6.267732 4.910603e-05  1.837146e-08 0.7462530
## 84      NA 3.891333 3.000000 4.782665 9.836065e-05  7.567812e-08 0.8646006
## 85      NA 4.754962 3.000000 6.509923 4.282685e-05  1.043026e-08 0.7263370
## 86      NA 5.404265 4.000000 6.212795 6.321741e-05  2.902310e-08 0.7829380
## 87      NA 6.011534 5.000000 7.023067 4.626515e-05  2.035621e-08 0.8129445
## 88      NA 4.812891 4.000000 5.625782 5.492863e-05  2.194325e-08 0.8332814
## 89      NA 5.789766 5.000000 6.579533 4.896076e-05  5.582269e-08 0.8123981
## 90      NA 5.494855 5.000000 5.989710 8.983216e-05  1.725672e-07 0.8403528
## 91      NA 6.559844 6.119688 7.000000 6.429881e-05  1.617052e-07 0.8162172
## 92      NA 4.718695 4.000000 5.437390 4.851667e-05  1.645832e-08 0.7729201
## 93      NA 5.245968 4.000000 6.491937 9.658693e-05  1.758909e-07 0.8574912
## 94      NA 5.080647 3.500000 6.661294 1.370785e-04  1.880939e-07 0.7820159
## 95      NA 5.557415 4.000000 7.114831 1.296925e-04  5.718490e-07 0.9140781
## 96      NA 4.765395 3.000000 6.530789 9.091331e-05  5.787834e-08 0.8855373
## 97      NA 6.549489 6.000000 7.098977 7.724497e-05  1.210938e-07 0.8221848
## 98      NA 6.282210 5.000000 7.564421 8.870456e-05  3.868673e-07 0.9500086
## 99      NA 5.324726 3.000000 6.798904 1.105172e-04  9.310134e-08 0.7851418
## 100     NA 6.389312 5.500000 7.667936 1.674386e-04  3.606992e-07 0.8885431
## 101     NA 4.259630 2.000000 6.519259 1.193415e-04  1.014616e-07 0.7580068
## 102     NA 5.779155 5.000000 6.558310 1.130839e-04  3.243873e-07 0.8168050
## 103     NA 4.110288 3.000000 5.330865 9.619864e-05  6.407093e-08 0.8313766
## 104     NA 6.464803 6.000000 6.929606 9.886493e-05  4.039322e-07 0.9450953
## 105     NA 6.736224 6.369887 7.102562 5.086571e-05  1.246920e-08 0.7110730
## 106     NA 8.203586 7.445733 8.961438 4.512947e-05  1.017613e-08 0.6953554
## 107     NA 7.048524 6.247268 8.387841 3.601068e-05  1.083607e-08 0.7290050
## 108     NA 7.774538 7.286922 8.262153 4.779872e-05  8.532788e-09 0.6802296
## 109     NA 7.729339 7.003648 8.455031 5.318735e-05  8.590206e-09 0.6883361
## 110     NA 5.313387 3.000000 6.880222 3.173431e-05  1.021150e-08 0.7394041
## 111     NA 7.900896 7.710068 8.091724 6.172884e-05  1.423878e-08 0.6562591
## 112     NA 6.567935 5.500000 8.703804 6.922161e-05  3.345033e-08 0.6957352
```

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## 113     NA 4.199735 3.000000 5.399470 1.327453e-04  4.166119e-08 0.6391927
## 114     NA 5.384468 4.000000 6.768936 8.450538e-05  2.015932e-07 0.8448326
## 115     NA 7.109702 6.825632 7.393772 2.857852e-05  2.594670e-08 0.7089695
## 116     NA 6.588521 4.000000 9.177043 1.991885e-04  7.808415e-08 0.6732995
## 117     NA 8.205700 7.360590 9.050811 1.129235e-04  3.624850e-07 0.7404614
## 118     NA 8.426624 7.502496 9.350752 1.039373e-04  6.534421e-08 0.7583633
## 119     NA 7.135423 6.000000 8.270846 1.724924e-04  7.015621e-07 0.7713319
## 120     NA 7.393707 5.000000 9.787414 1.026410e-04  2.901235e-08 0.7134106
## 121     NA 6.684512 6.149529 7.219494 1.582167e-04  1.294355e-07 0.7842232
##      TIPS_GEO_CV TIPS_VOLUME TIPS_VOLUME_MEAN TIPS_BRANCH_RATIO EMPIRICAL
## 1   1.068184e+00    1.040586        0.9571338          1.124436 0.4822409
## 2   7.492404e+00    1.072532        1.1171078          1.444283 0.3475297
## 3   2.042764e+00    1.052496        1.1319599          1.300730 0.4357425
## 4   1.131901e+00    1.031411        1.1094090          1.170505 0.5204351
## 5   2.282063e+00    1.028369        1.0582747          1.328919 0.3865499
## 6   1.824931e+00    1.045341        1.0908660          1.262713 0.4433087
## 7   1.264156e+00    1.065284        1.1075608          1.393588 0.4466151
## 8   1.696617e+00    1.055619        1.1148436          1.268946 0.4810364
## 9   1.074358e+00    1.070411        1.1484157          1.368250 0.3060268
## 10  4.963160e+00    1.037152        1.0318296          1.299614 0.4045602
## 11  1.976094e+00    1.035830        1.0652128          1.185959 0.3898291
## 12  6.741610e-01    1.011849        1.1131882          1.339994 0.3169373
## 13  2.408278e+00    1.095084        1.0311651          1.277971 0.5594629
## 14  3.727049e+02    1.123454        0.8095932          1.349021 0.4421783
## 15  4.919677e+00    1.071607        0.9470900          1.137517 0.4195179
## 16  1.471348e+00    1.058890        1.2023608          1.188909 0.4770167
## 17  1.100090e+01    1.092774        1.1511605          1.321686 0.3548216
## 18  7.688622e+00    1.041560        0.9363401          1.153468 0.5275477
## 19  8.940306e-01    1.028992        1.0572660          1.083743 0.5260693
## 20  8.918434e-01    1.088531        1.1775959          1.368247 0.4133832
## 21  8.604651e-01    1.052253        1.1527067          1.234277 0.4270215
## 22  1.026326e+00    1.041146        1.0102023          1.166363 0.5121862
## 23  3.796229e+02    1.085186        0.9939962          1.337576 0.4064533
## 24  8.094965e-01    1.044610        1.0851697          1.110542 0.4941028
## 25  1.143913e+00    1.043539        1.0813549          1.113778 0.4495749
## 26  8.853715e-01    1.062839        1.1098329          1.248226 0.3861115
## 27  1.476955e+00    1.054499        0.9111109          1.194483 0.4833431
## 28  4.448193e+03    1.069742        0.9454885          1.208353 0.5055961
## 29  6.385785e+00    1.061983        1.0724910          1.238176 0.4580737
## 30  7.669304e-01    1.074944        1.1380693          1.102398 0.3887311
## 31  2.425828e+01    1.088088        1.0212622          1.243281 0.4136933
## 32  4.380065e+01    1.082324        1.0722851          1.324468 0.3746949
## 33  1.736679e+01    1.052556        1.0869840          1.163758 0.4617731
## 34  1.176559e+00    1.069431        1.1761480          1.217623 0.4367371
## 35  1.356061e+00    1.057184        1.2753011          1.452618 0.3571610
## 36  1.081863e+00    1.054735        1.0100211          1.193775 0.4267785
## 37  9.885008e-01    1.064264        1.1957095          1.215442 0.3856831
## 38  9.662664e-01    1.067445        1.1893967          1.179353 0.2630873
## 39  3.912196e+00    1.085757        1.1604767          1.415533 0.3445830
## 40  3.649075e+00    1.129995        1.3613350          1.558038 0.3440404
## 41  1.401252e+00    1.053513        1.1902096          1.196273 0.4874049
## 42  1.769547e+00    1.108363        1.4656768          1.569769 0.3959354
## 43  9.062713e+04    1.090833        1.0257339          1.439900 0.3406200
## 44  2.938247e+00    1.057759        1.1266585          1.330734 0.4607277
```

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## 45  8.906526e+00    1.062775        0.9867148          1.317238 0.4083859
## 46  2.248339e+00    1.102140        1.2490610          1.532616 0.3231394
## 47  1.820110e+00    1.046100        1.0629787          1.203492 0.4774071
## 48  1.286161e+00    1.100802        1.3230910          1.304529 0.2740333
## 49  1.850352e+00    1.044756        1.0511925          1.326526 0.4120560
## 50  1.945850e+00    1.040488        1.1301875          1.230033 0.4329979
## 51  1.537242e+00    1.127257        1.4020371          1.383140 0.4050833
## 52  3.012957e+00    1.072309        1.0227442          1.434585 0.3133050
## 53  1.995299e+00    1.060155        1.1519487          1.271457 0.4377630
## 54  4.502117e+01    1.056623        1.0481596          1.273104 0.4892243
## 55  1.026574e+00    1.023214        1.0446221          1.180609 0.5203264
## 56  1.659820e+01    1.039134        0.9786784          1.203932 0.4436512
## 57  2.581651e+00    1.081657        1.1413442          1.401813 0.3749769
## 58  5.447181e+00    1.075482        1.2501914          1.369894 0.3788761
## 59  3.501522e+02    1.042111        0.9844367          1.295789 0.4682096
## 60  2.848791e+00    1.082511        1.3680116          2.080706 0.2868305
## 61  1.700170e+00    1.019706        1.1722842          1.210734 0.4808165
## 62  9.389514e+04    1.020002        1.0524538          1.234563 0.3596485
## 63  3.726636e+00    1.095456        1.0682090          1.301082 0.4340990
## 64  1.038958e+03    1.117556        1.0330986          1.628300 0.3595741
## 65  2.240346e+00    1.047835        1.1505048          1.244095 0.4265392
## 66  1.431099e+00    1.075276        1.1283678          1.399919 0.3267807
## 67  6.780782e+00    1.059959        1.0160927          1.330496 0.4302877
## 68  2.104032e+00    1.051294        0.9879215          1.314622 0.4620222
## 69  1.863133e+00    1.097279        1.0414364          1.371126 0.3691852
## 70  2.651441e+00    1.046375        1.0960248          1.228374 0.4678139
## 71  1.835203e+00    1.088114        1.1283101          1.352293 0.3238206
## 72  1.924058e+00    1.063205        1.1701785          1.387402 0.4084954
## 73  7.709418e+00    1.044632        1.0835230          1.336917 0.3984438
## 74  2.941068e+00    1.066312        1.0427256          1.580186 0.3034862
## 75  1.589202e+00    1.047774        1.1272857          1.281702 0.4365733
## 76  5.666277e+06    1.055046        1.0889484          1.268063 0.4228000
## 77  1.726191e+05    1.083964        1.0780901          1.258770 0.4604897
## 78  5.038280e+03    1.108536        1.2159585          1.299009 0.4526894
## 79  1.859361e+03    1.063419        1.0859864          1.213880 0.4846304
## 80  5.955938e+01    1.082428        1.1164138          1.348947 0.4300120
## 81  6.015291e+05    1.037771        1.0484426          1.236754 0.3771286
## 82  8.378816e+04    1.092348        1.1255318          1.304128 0.4487346
## 83  1.293791e+00    1.045789        1.0540040          1.281572 0.4831421
## 84  1.558406e+00    1.072782        1.0170300          1.284496 0.4284364
## 85  1.456270e+00    1.066463        1.0918031          1.320281 0.4659360
## 86  1.338932e+00    1.097433        1.0968923          1.429873 0.3967394
## 87  1.571548e+00    1.067977        1.1331224          1.322889 0.4686044
## 88  2.231822e+00    1.079534        1.1445584          1.347992 0.4353764
## 89  1.306795e+00    1.066084        1.0869526          1.398977 0.4147497
## 90  1.704094e+00    1.041900        1.0733044          2.106275 0.2645434
## 91  1.333217e+00    1.068451        1.1034860          1.394631 0.4352428
## 92  1.757416e+00    1.066479        1.0958890          1.353012 0.5077169
## 93  1.737818e+00    1.065139        1.0894903          1.297917 0.4599472
## 94  2.212587e+00    1.058882        1.0443831          1.288980 0.4740282
## 95  5.393872e+03    1.082736        1.0940448          1.521432 0.3904500
## 96  1.832359e+00    1.061545        1.1903079          1.316230 0.4113576
## 97  4.652195e+02    1.052524        0.9446462          1.238405 0.5170632
## 98  2.315738e+00    1.060705        1.1105165          1.706052 0.3277607
```

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## 99  1.253342e+03    1.059894        1.0159308          1.335048 0.4365186
## 100 1.197076e+01    1.061428        1.0918962          1.428420 0.3897747
## 101 2.292186e+00    1.050953        1.0562425          1.273807 0.5036798
## 102 1.241577e+04    1.092029        1.0530715          1.364001 0.4485512
## 103 2.204399e+00    1.072687        1.0701131          1.340216 0.4326280
## 104 2.209351e+01    1.131460        1.3712033          1.404381 0.3818878
## 105 3.219140e+01    1.087407        1.1573031          1.263419 0.4696411
## 106 2.917272e+01    1.073538        1.0628586          1.199495 0.5123678
## 107 1.921725e+00    1.063905        1.0546524          1.320925 0.4492452
## 108 2.700675e+01    1.090398        1.0466421          1.266600 0.4863463
## 109 1.974107e+01    1.074295        1.0280438          1.237327 0.4900560
## 110 3.841923e+01    1.074726        1.1027403          1.293426 0.4796561
## 111 1.410086e+04    1.092661        1.0627941          1.234299 0.4700990
## 112 1.755310e+00    1.031846        1.0084122          1.260271 0.4828997
## 113 8.112138e-01    1.063373        1.1472713          1.253516 0.4439314
## 114 5.068153e+01    1.038824        1.0361660          1.923774 0.2852538
## 115 1.568430e+00    1.036656        1.0501363          1.315167 0.3802286
## 116 3.036174e+00    1.048714        0.9378997          1.159973 0.5092971
## 117 2.930088e+00    1.037944        1.0154272          1.227531 0.4587479
## 118 1.444941e+01    1.039841        1.0027451          1.203507 0.5515913
## 119 2.357187e+00    1.060692        1.0474085          1.282383 0.4911352
## 120 2.406807e+00    1.055781        0.9466316          1.276788 0.4993798
## 121 2.386924e+01    1.070055        1.0207224          1.310766 0.4872269
##     EMPIRICAL_CORRECTED EMPIRICAL_VOL EMPIRICAL_INTERCEPT NORMALISATION
## 1             0.7305490     0.7020556           -2.352775  0.0181432626
## 2             0.8514493     0.7938685           -2.103880  0.1572211060
## 3             0.7838618     0.7447647           -2.499848  0.0020985183
## 4             0.7445283     0.7218542           -2.546907  0.0198581968
## 5             0.7082066     0.6886698           -2.961034  0.0270712837
## 6             0.7530373     0.7203750           -2.645249  0.0200207346
## 7             0.9024422     0.8471375           -1.565179  0.0478033045
## 8             0.9023373     0.8547946           -1.499949  0.0214629455
## 9             0.6679831     0.6240433           -3.759962  0.0028536836
## 10            0.6710633     0.6470249           -3.269198  0.0188881315
## 11            0.7488936     0.7229887           -2.645976  0.0051427404
## 12            0.6076128     0.6004974           -3.463904  0.0142304206
## 13            0.8637193     0.7887240           -2.371605  0.1175571965
## 14            0.7899752     0.7031667           -2.570161  0.0096907255
## 15            0.6820519     0.6364760           -3.052820  0.0075479968
## 16            0.8561646     0.8085492           -2.273380  0.0067926700
## 17            0.6815420     0.6236804           -3.756101  0.0044836382
## 18            0.8002188     0.7682887           -2.060595  0.0278836954
## 19            0.7152376     0.6950854           -2.663458  0.0052516023
## 20            0.8071885     0.7415395           -2.227368  0.0158143317
## 21            0.7661373     0.7280925           -2.347469  0.0183805048
## 22            0.7845804     0.7535742           -2.190632  0.0091865407
## 23            0.7967201     0.7341781           -2.661774  0.0084533334
## 24            0.6960789     0.6663526           -3.080927  0.0022435732
## 25            0.7361767     0.7054614           -2.399028  0.0045307888
## 26            0.6721514     0.6324111           -3.399423  0.0049178044
## 27            0.8130914     0.7710688           -1.976806  0.0069942640
## 28            0.7853940     0.7341900           -2.448179  0.0017154322
## 29            0.7751658     0.7299233           -2.535846  0.0279569602
## 30            0.5688355     0.5291767           -4.300653  0.0081907429
```

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## 31          0.7571109     0.6958180          -2.793137  0.0166013496
## 32          0.6664922     0.6157971          -3.770125  0.0031114137
## 33          0.6904780     0.6560012          -3.386311  0.0069775784
## 34          0.7069506     0.6610530          -3.084677  0.0026639022
## 35          0.6609213     0.6251715          -2.910062  0.0311446722
## 36          0.6481096     0.6144765          -3.369441  0.0019910901
## 37          0.6358612     0.5974654          -3.716524  0.0005250380
## 38          0.4716272     0.4418283          -5.403592  0.0016165813
## 39          0.7299916     0.6723344          -2.857535  0.0118572112
## 40          0.7847500     0.6944722          -2.903043  0.0321164722
## 41          0.8201780     0.7785173          -2.247266  0.0317896726
## 42          0.8423814     0.7600231          -2.473485  0.0296198075
## 43          0.7130607     0.6536848          -3.097506  0.0227909990
## 44          0.7930979     0.7497906          -2.430461  0.0248327149
## 45          0.7870582     0.7405692          -2.796662  0.0137206428
## 46          0.7092121     0.6434865          -3.324801  0.0215261421
## 47          0.8323390     0.7956589          -2.222879  0.0031564842
## 48          0.7326642     0.6655732          -2.553533  0.0036495208
## 49          0.7846654     0.7510517          -2.560567  0.0302301085
## 50          0.7055210     0.6780674          -2.789634  0.0016293222
## 51          0.8105111     0.7190120          -2.743311  0.0006672975
## 52          0.6898898     0.6433687          -3.351854  0.0060313064
## 53          0.7257387     0.6845588          -2.721137  0.0212992984
## 54          0.8397019     0.7947033          -1.912007  0.0107591722
## 55          0.7679518     0.7505287          -2.094689  0.0119947949
## 56          0.8223920     0.7914203          -2.321161  0.0012053869
## 57          0.8136325     0.7522090          -2.528466  0.0098802352
## 58          0.7064646     0.6568817          -3.375906  0.0092848812
## 59          0.8353333     0.8015783          -2.123444  0.0119562418
## 60          0.8461374     0.7816432          -2.150085  0.0251683335
## 61          0.7711556     0.7562528          -2.611086  0.0075540224
## 62          0.8324371     0.8161134          -1.766977  0.0165092711
## 63          0.7467936     0.6817192          -2.971472  0.0170205450
## 64          0.8579237     0.7676782          -2.368006  0.0390721306
## 65          0.7450652     0.7110522          -2.687027  0.0176822417
## 66          0.6729754     0.6258629          -3.184241  0.0226472699
## 67          0.8629813     0.8141645          -2.351198  0.0188738231
## 68          0.9389472     0.8931344          -1.671939  0.0146575812
## 69          0.8729132     0.7955251          -2.201591  0.0205806179
## 70          0.7921594     0.7570512          -2.560306  0.0287399232
## 71          0.6932472     0.6371092          -3.731726  0.0085158494
## 72          0.8280133     0.7787900          -2.577594  0.0282579586
## 73          0.7772864     0.7440767          -2.833371  0.0091842611
## 74          0.7683951     0.7206099          -3.204859  0.0177992011
## 75          0.7601209     0.7254626          -2.658137  0.0322950020
## 76          0.8313364     0.7879625          -2.072859  0.0005047401
## 77          0.7618214     0.7028104          -2.785087  0.0020681450
## 78          0.7982607     0.7201035          -2.664008  0.0142550826
## 79          0.7817085     0.7350899          -2.537770  0.0003563001
## 80          0.8490443     0.7843886          -2.129172  0.0002690096
## 81          0.7952490     0.7663049          -2.283682  0.0048253617
## 82          0.8138935     0.7450863          -2.494682  0.0933101549
## 83          0.8787422     0.8402672          -1.691557  0.0019678352
## 84          0.7968634     0.7428012          -2.519778  0.0117831423
```

```
## 85            0.8616689     0.8079690          -1.990059  0.0078493318
## 86            0.8313274     0.7575199          -2.415440  0.0036900899
## 87            0.8720428     0.8165367          -1.931459  0.0090719420
## 88            0.8159298     0.7558165          -2.433826  0.0013831199
## 89            0.8854900     0.8306003          -1.807681  0.0176095505
## 90            0.9026069     0.8663086          -1.555556  0.0185878713
## 91            0.8816782     0.8251928          -1.851463  0.0368197411
## 92            0.9087302     0.8520849          -1.538526  0.0034516610
## 93            0.8128528     0.7631428          -2.485565  0.0124134566
## 94            0.7821326     0.7386398          -2.599911  0.0177722982
## 95            0.8453855     0.7807862          -2.330688  0.0292616579
## 96            0.7344491     0.6918683          -3.131490  0.0116144657
## 97            0.7908906     0.7514230          -2.398302  0.0255815420
## 98            0.8502738     0.8016117          -2.038485  0.0141265615
## 99            0.7997200     0.7545282          -2.266445  0.0061890671
## 100           0.7681200     0.7236665          -2.592500  0.0158367129
## 101           0.8274943     0.7873752          -2.151644  0.0105066336
## 102           0.8307088     0.7607024          -2.387415  0.0192005186
## 103           0.7956575     0.7417427          -2.695042  0.0045758132
## 104           0.7835201     0.6924858          -3.007428  0.0181036639
## 105           0.8162862     0.7506725          -2.326390  0.0373388488
## 106           0.8049334     0.7497949          -2.295214  0.0593027259
## 107           0.8905568     0.8370641          -1.658296  0.0188565495
## 108           0.7924024     0.7267090          -2.466535  0.0473524958
## 109           0.8381932     0.7802265          -2.075356  0.0516474846
## 110           0.8473458     0.7884296          -2.106393  0.0235487226
## 111           0.8075765     0.7390914          -2.226606  0.0251061978
## 112           0.8582831     0.8317939          -1.692607  0.0200399349
## 113           0.7641348     0.7185950          -2.301669  0.0139348552
## 114           0.8431814     0.8116694          -1.895413  0.0273064402
## 115           0.8692837     0.8385458          -1.752446  0.0043158274
## 116           0.8102004     0.7725655          -1.893924  0.0166516246
## 117           0.8497717     0.8187063          -1.736927  0.0449528012
## 118           0.8878474     0.8538302          -1.531049  0.0621860975
## 119           0.8590577     0.8099034          -1.778470  0.0124900218
## 120           0.9084147     0.8604197          -1.372399  0.0305649941
## 121           0.8677729     0.8109613          -1.850578  0.0536677547
##     EMPIRICAL_VOL_MEAN EMPIRICAL_VOL_TIP EMPIRICAL_TIP_MEAN    CI_MIN    CI_MAX
## 1            0.4867699        0.6926161          0.5422489 0.4199934 0.5537143
## 2            0.4358676        0.7018620          0.5019313 0.3278736 0.3683643
## 3            0.4856940        0.6418161          0.5667835 0.4150012 0.4575205
## 4            0.5278566        0.6453728          0.6091720 0.4601418 0.5886289
## 5            0.4430386        0.6004960          0.5136936 0.3541853 0.4218719
## 6            0.4766324        0.6362576          0.5597714 0.4252567 0.4621269
## 7            0.5160673        0.7270455          0.6223973 0.4026761 0.4953485
## 8            0.5304189        0.7611920          0.6104094 0.4569922 0.5063456
## 9            0.3290164        0.5388030          0.4187211 0.2557369 0.3662059
## 10           0.4290787        0.5695310          0.5257720 0.3670810 0.4458662
## 11           0.4345694        0.6761723          0.4623215 0.3504359 0.4336507
## 12           0.3980353        0.5210947          0.4246942 0.2223447 0.4517726
## 13           0.5137381        0.7330780          0.7149774 0.4712787 0.6641479
## 14           0.4208140        0.7708738          0.5965079 0.4158142 0.4702141
## 15           0.4474852        0.6611758          0.4772086 0.3764814 0.4674740
## 16           0.5434715        0.7167553          0.5671296 0.4101677 0.5547608
```

```
## 17          0.3872609          0.5530256          0.4689627 0.3301113 0.3813816
## 18          0.5338677          0.7602786          0.6085092 0.5065243 0.5494436
## 19          0.5455188          0.6654419          0.5701240 0.5000955 0.5533921
## 20          0.4329942          0.6371699          0.5656102 0.3844914 0.4444460
## 21          0.4393708          0.6423007          0.5270627 0.3943606 0.4623874
## 22          0.5162631          0.7153537          0.5973948 0.4709132 0.5570766
## 23          0.4134720          0.7057590          0.5436620 0.3780186 0.4370269
## 24          0.4799461          0.6326458          0.5487220 0.4354341 0.5606763
## 25          0.4420785          0.6705234          0.5007267 0.4100756 0.4928790
## 26          0.3691896          0.5702185          0.4819543 0.3431427 0.4344609
## 27          0.4539327          0.7870185          0.5773450 0.4398211 0.5311719
## 28          0.4888950          0.7292152          0.6109387 0.4793669 0.5332604
## 29          0.4699519          0.6722754          0.5671760 0.4291626 0.4889323
## 30          0.3942071          0.5101388          0.4285362 0.3510424 0.4304661
## 31          0.4328484          0.6783182          0.5143369 0.3759456 0.4552313
## 32          0.3630368          0.5593736          0.4962716 0.3365380 0.4171781
## 33          0.4756778          0.6082866          0.5373922 0.4274465 0.4988562
## 34          0.4540535          0.5887060          0.5317811 0.4098698 0.4653655
## 35          0.3876736          0.4783027          0.5188185 0.3106010 0.4107005
## 36          0.4135359          0.5788098          0.5094776 0.3883534 0.4690056
## 37          0.3887172          0.5267220          0.4687756 0.3153375 0.4717213
## 38          0.2651422          0.3982189          0.3102727 0.1905400 0.3632567
## 39          0.3992369          0.5774839          0.4877688 0.3264530 0.3637199
## 40          0.3988453          0.5390700          0.5360282 0.3152339 0.3754793
## 41          0.5171716          0.6871625          0.5830693 0.4428916 0.5363921
## 42          0.4335085          0.5515707          0.6215269 0.3508430 0.4468232
## 43          0.3809409          0.5906357          0.4904589 0.3002016 0.3864804
## 44          0.4847651          0.6352715          0.6131060 0.4382662 0.4843402
## 45          0.4487467          0.6890406          0.5379412 0.3686243 0.4524363
## 46          0.3454598          0.5112204          0.4952486 0.2761569 0.3781150
## 47          0.4740034          0.7349679          0.5745555 0.4359185 0.5228442
## 48          0.3173951          0.5559508          0.3574844 0.2077761 0.3614192
## 49          0.4498784          0.6683741          0.5466029 0.3828034 0.4435440
## 50          0.4519053          0.5936128          0.5326016 0.3922559 0.4779717
## 51          0.4191051          0.5822547          0.5602871 0.3378341 0.4857192
## 52          0.3549116          0.5920127          0.4494628 0.2688574 0.3651006
## 53          0.4595723          0.5931075          0.5565967 0.4141054 0.4627722
## 54          0.5074816          0.7202501          0.6228335 0.4701515 0.5090709
## 55          0.5366990          0.6809564          0.6143022 0.4807828 0.5631224
## 56          0.5002217          0.7688882          0.5341260 0.4004026 0.4915712
## 57          0.4376511          0.6509814          0.5256476 0.3435206 0.4093137
## 58          0.4094203          0.5365256          0.5190200 0.3398768 0.4223505
## 59          0.5233922          0.7444707          0.6067008 0.4265451 0.5139437
## 60          0.3133675          0.5293296          0.5968099 0.2389980 0.3442361
## 61          0.5553069          0.6465111          0.5821408 0.4157004 0.5561325
## 62          0.5235287          0.7577839          0.4440087 0.3389901 0.3815659
## 63          0.4424017          0.6240212          0.5647985 0.3906917 0.4823290
## 64          0.3836502          0.6822434          0.5854945 0.3111406 0.4155471
## 65          0.4302995          0.6220475          0.5306552 0.3726572 0.4882119
## 66          0.3496306          0.5369356          0.4574666 0.2842932 0.3756179
## 67          0.4379085          0.7624782          0.5724963 0.3780898 0.4896919
## 68          0.4639253          0.8394756          0.6073847 0.4025250 0.5303138
## 69          0.4407519          0.7491055          0.5061996 0.3222176 0.4229990
## 70          0.4972822          0.6782908          0.5746505 0.4178016 0.5238127
```

```
## 71          0.3462097         0.5671321          0.4379004 0.2806329 0.3736545
## 72          0.4678939         0.6503612          0.5667474 0.3663187 0.4555282
## 73          0.4360075         0.6566840          0.5326861 0.3435313 0.4621339
## 74          0.3725642         0.6328406          0.4795646 0.2500362 0.3683622
## 75          0.4755417         0.6273120          0.5595570 0.4198984 0.4539103
## 76          0.5076944         0.7197859          0.5361372 0.4141970 0.4315818
## 77          0.4737910         0.6461812          0.5796507 0.4538677 0.4672083
## 78          0.4702117         0.6327957          0.5880476 0.4443969 0.4611367
## 79          0.5070367         0.6757278          0.5882829 0.4776454 0.4917175
## 80          0.4559502         0.6997736          0.5800634 0.4206372 0.4395957
## 81          0.4897130         0.7192420          0.4664154 0.3713202 0.3830279
## 82          0.4719801         0.6715784          0.5852073 0.4421314 0.4554365
## 83          0.5345346         0.7621645          0.6191813 0.4637668 0.5033268
## 84          0.4490813         0.7029459          0.5503246 0.3730843 0.4920006
## 85          0.5062787         0.7234982          0.6151665 0.4432763 0.4897540
## 86          0.4394727         0.6795865          0.5672872 0.3634373 0.4330930
## 87          0.5145004         0.7144942          0.6199115 0.4387176 0.5005272
## 88          0.4716619         0.6562012          0.5868839 0.4045080 0.4686004
## 89          0.4559307         0.7372477          0.5802252 0.3772395 0.4559897
## 90          0.3038021         0.7613132          0.5572013 0.2153864 0.3249195
## 91          0.4601690         0.7235745          0.6070029 0.4052452 0.4674609
## 92          0.5273541         0.7489251          0.6869468 0.4662958 0.5528174
## 93          0.4828625         0.6787852          0.5969731 0.4309649 0.4908786
## 94          0.4807071         0.6646019          0.6110130 0.4417380 0.5086788
## 95          0.4238693         0.6784034          0.5940430 0.3626296 0.4204047
## 96          0.4397490         0.5830777          0.5414412 0.3796071 0.4457637
## 97          0.5199148         0.7059324          0.6403338 0.4932097 0.5420705
## 98          0.3991782         0.6762225          0.5591767 0.2992274 0.3590148
## 99          0.4500554         0.6944940          0.5827733 0.4145444 0.4596575
## 100         0.4166995         0.6349681          0.5567620 0.3607225 0.4211667
## 101         0.5099581         0.7026738          0.6415906 0.4791176 0.5295011
## 102         0.4551257         0.7000380          0.6118245 0.4220662 0.4766982
## 103         0.4542479         0.6618844          0.5798151 0.4075278 0.4592742
## 104         0.4116692         0.5650583          0.5363160 0.3508512 0.4156699
## 105         0.4988421         0.6731092          0.5933534 0.4518673 0.4881140
## 106         0.5207037         0.7048235          0.6145827 0.4990974 0.5259910
## 107         0.5051249         0.7687855          0.5934191 0.4324841 0.4666559
## 108         0.4912252         0.6755735          0.6160064 0.4726476 0.5004421
## 109         0.5113283         0.7417036          0.6063594 0.4761216 0.5043983
## 110         0.5132692         0.7088093          0.6203999 0.4559398 0.5046061
## 111         0.4716445         0.7044637          0.5802425 0.4562828 0.4843336
## 112         0.5517411         0.7666266          0.6085845 0.4519401 0.5159800
## 113         0.4698916         0.6355970          0.5564751 0.4015991 0.4907259
## 114         0.3496732         0.7338830          0.5487638 0.2524012 0.3223825
## 115         0.5069404         0.7718003          0.5000642 0.3665497 0.3944181
## 116         0.5113255         0.7696141          0.5907707 0.4905603 0.5287496
## 117         0.5433260         0.7715084          0.5631272 0.4493440 0.4683486
## 118         0.5917392         0.8001778          0.6638438 0.5410795 0.5623073
## 119         0.5254201         0.7379957          0.6298236 0.4820478 0.5003939
## 120         0.5169520         0.8325151          0.6376020 0.4733502 0.5268407
## 121         0.5045516         0.7420690          0.6386405 0.4499200 0.5276274
##     CI_VOL_MIN CI_VOL_MAX CI_MA_MIN CI_MA_MAX PATH_FRAC ASYM_FRAC BRANCH_ANGLE
## 1    0.5891759  0.8365617 0.4093662 0.5788091       NaN 0.4200000     61.63048
## 2    0.7674933  0.8211500 0.4154236 0.4573176       NaN 0.4796875     79.05586
```

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## 3    0.7169258  0.7736847 0.4653179 0.5069623       NaN 0.4894515     60.62307
## 4    0.6157891  0.8461883 0.4548968 0.6125184       NaN 0.2553191     71.36005
## 5    0.6516505  0.7277921 0.4177713 0.4698341       NaN 0.4722222     77.22856
## 6    0.6930932  0.7487307 0.4587902 0.4951685       NaN 0.4627575     66.84857
## 7    0.7822883  0.9173625 0.4698482 0.5668331       NaN 0.4273504     81.86761
## 8    0.8169246  0.8944201 0.5052512 0.5568401       NaN 0.3761996     76.62915
## 9    0.5047707  0.7714987 0.2726944 0.3969712       NaN 0.3250000     70.08179
## 10   0.5907356  0.7086779 0.3939164 0.4673798       NaN 0.4545455     65.22704
## 11   0.6599085  0.7920988 0.3943480 0.4788930       NaN 0.3886010     62.68537
## 12   0.4935739  0.7305838 0.2957395 0.5357150       NaN 0.4333333     59.38111
## 13   0.6575049  0.9461307 0.4256305 0.6200843       NaN 0.4074074     62.93923
## 14   0.6538263  0.7562305 0.3930001 0.4505963       NaN 0.4474394     65.70282
## 15   0.5803603  0.6980175 0.4032535 0.4965684       NaN 0.3987730     58.59976
## 16   0.6751163  0.9683543 0.4546641 0.6496253       NaN 0.3414634     65.69337
## 17   0.5907885  0.6584036 0.3608519 0.4156026       NaN 0.4401042     65.16839
## 18   0.7358603  0.8021461 0.5113971 0.5573257       NaN 0.3721228     60.92219
## 19   0.6581341  0.7341114 0.5186064 0.5738278       NaN 0.3484848     69.15991
## 20   0.6889514  0.7981417 0.4029516 0.4652766       NaN 0.3481013     60.84133
## 21   0.6673723  0.7943373 0.4045860 0.4771462       NaN 0.4120000     55.98105
## 22   0.6862598  0.8274914 0.4726859 0.5638576       NaN 0.3620690     58.83318
## 23   0.6865018  0.7851654 0.3835169 0.4457668       NaN 0.3988604     66.71086
## 24   0.5834241  0.7610687 0.4226018 0.5450718       NaN 0.3834586     64.68080
## 25   0.6195943  0.8032286 0.3966359 0.4927273       NaN 0.4087591     59.98800
## 26   0.5496676  0.7276103 0.3246136 0.4198867       NaN 0.4166667     60.64999
## 27   0.6889076  0.8630287 0.4078669 0.5052012       NaN 0.3583815     57.03926
## 28   0.6962522  0.7741951 0.4630735 0.5161564       NaN 0.3344000     66.14954
## 29   0.6860792  0.7765692 0.4410044 0.5007995       NaN 0.3834951     66.35829
## 30   0.4751579  0.5893366 0.3540096 0.4389690       NaN 0.3764706     56.54935
## 31   0.6369367  0.7601425 0.3929161 0.4768392       NaN 0.3934426     63.28214
## 32   0.5474864  0.6926311 0.3246080 0.4060150       NaN 0.3676471     66.80965
## 33   0.6092762  0.7063095 0.4393884 0.5149643       NaN 0.3722944     62.13709
## 34   0.6221411  0.7023986 0.4259202 0.4840450       NaN 0.3090909     56.96891
## 35   0.5709065  0.6845943 0.3503538 0.4289687       NaN 0.5434783     73.65882
## 36   0.5420793  0.6965427 0.3708048 0.4611912       NaN 0.3865546     76.18535
## 37   0.4834693  0.7383405 0.3159318 0.4782712       NaN 0.3965517     71.01880
## 38   0.2615299  0.7464241 0.1845460 0.3809369       NaN 0.5000000     69.24745
## 39   0.6512213  0.6941320 0.3806037 0.4187822       NaN 0.4737609     71.44053
## 40   0.6590864  0.7317579 0.3704127 0.4294604       NaN 0.5459184     66.61561
## 41   0.7045350  0.8602684 0.4698578 0.5692498       NaN 0.4016393     72.54640
## 42   0.6829444  0.8458010 0.3892162 0.4828412       NaN 0.3606557     63.45219
## 43   0.5875693  0.7272397 0.3371871 0.4303721       NaN 0.4246575     55.23209
## 44   0.7147527  0.7865461 0.4606861 0.5101027       NaN 0.4634921     64.52188
## 45   0.6599924  0.8309833 0.3989059 0.5048149       NaN 0.4473684     64.91733
## 46   0.5440175  0.7611426 0.2949761 0.4045836       NaN 0.4897959     76.41985
## 47   0.7080135  0.8941539 0.4249534 0.5287151       NaN 0.4351145     69.03775
## 48   0.4853262  0.9127627 0.2488306 0.4048523       NaN 0.4615385     62.42343
## 49   0.7025024  0.8029563 0.4205797 0.4812180       NaN 0.4471154     52.86924
## 50   0.6130090  0.7500304 0.4091672 0.4991074       NaN 0.4866667     67.71481
## 51   0.6229873  0.8298376 0.3483513 0.5042297       NaN 0.4285714     85.34869
## 52   0.5771626  0.7171692 0.3093238 0.4072180       NaN 0.5974026     68.72997
## 53   0.6504353  0.7204725 0.4349431 0.4855962       NaN 0.4871134     70.18293
## 54   0.7636167  0.8270554 0.4869528 0.5288757       NaN 0.4070681     81.78184
## 55   0.6939682  0.8116991 0.4982722 0.5780893       NaN 0.3819444     79.96336
## 56   0.7221702  0.8673109 0.4533447 0.5519458       NaN 0.3961039     64.68683
```

```
## 57   0.7049259  0.8026636 0.4035734 0.4746062       NaN 0.4093023     67.50188
## 58   0.6003899  0.7186888 0.3693951 0.4537824       NaN 0.5634921     66.53194
## 59   0.7495318  0.8572388 0.4847686 0.5650932       NaN 0.3941176     62.08283
## 60   0.6882204  0.8877477 0.2686467 0.3655329       NaN 0.4482759     77.19414
## 61   0.6958341  0.8219177 0.4977370 0.6195356       NaN 0.4565217     69.78224
## 62   0.7950995  0.8376826 0.5036446 0.5441977       NaN 0.4149856     83.30010
## 63   0.5942526  0.7820598 0.3937493 0.4970657       NaN 0.5128205     62.25747
## 64   0.6750614  0.8730019 0.3289011 0.4475129       NaN 0.3731343     60.02040
## 65   0.6155468  0.8213757 0.3740724 0.4949781       NaN 0.4871795     69.03131
## 66   0.5435686  0.7206164 0.3032157 0.4031504       NaN 0.3870968     66.72513
## 67   0.7181768  0.9229814 0.3830083 0.5006780       NaN 0.5443038     65.51819
## 68   0.7404536  1.0772978 0.3806380 0.5654367       NaN 0.5897436     56.81554
## 69   0.7128439  0.8877962 0.3781807 0.5136757       NaN 0.4393939     64.26472
## 70   0.6737886  0.8506028 0.4432880 0.5578531       NaN 0.3913043     58.17409
## 71   0.5588127  0.7263759 0.2976764 0.4026560       NaN 0.4625000     68.86953
## 72   0.7137101  0.8498041 0.4311820 0.5077316       NaN 0.4533333     59.56305
## 73   0.6627291  0.8354094 0.3832717 0.4959995       NaN 0.4500000     70.01013
## 74   0.6246818  0.8312690 0.3206519 0.4328810       NaN 0.4285714     70.88444
## 75   0.7004300  0.7513897 0.4588519 0.4928386       NaN 0.4303279     67.44124
## 76   0.7778176  0.7982397 0.4986692 0.5168829       NaN 0.3545963     77.20586
## 77   0.6941218  0.7116077 0.4668641 0.4808207       NaN 0.3619632     75.60624
## 78   0.7096499  0.7307110 0.4616038 0.4789801       NaN 0.3536263     78.63810
## 79   0.7260624  0.7442297 0.4998099 0.5143681       NaN 0.3819502     76.96260
## 80   0.7695829  0.7994791 0.4462649 0.4658457       NaN 0.3994307     76.07288
## 81   0.7596639  0.7730039 0.4835820 0.4959218       NaN 0.3793751     75.00925
## 82   0.7362746  0.7540035 0.4651511 0.4789094       NaN 0.3730749     76.79873
## 83   0.8146843  0.8666535 0.5159556 0.5537826       NaN 0.4106776     85.64177
## 84   0.6571777  0.8395805 0.3898649 0.5172921       NaN 0.4390244     81.25427
## 85   0.7784973  0.8385564 0.4844252 0.5291181       NaN 0.4316644     85.73021
## 86   0.7150398  0.8025238 0.4050447 0.4768271       NaN 0.4237288     80.71717
## 87   0.7799460  0.8548441 0.4842930 0.5465921       NaN 0.4124169     83.34672
## 88   0.7133098  0.8008562 0.4396472 0.5060079       NaN 0.4213483     79.64517
## 89   0.7785855  0.8860899 0.4162474 0.4993974       NaN 0.4448980     81.63575
## 90   0.7737876  0.9698922 0.2546330 0.3624657       NaN 0.3000000     81.96024
## 91   0.7798529  0.8731688 0.4299350 0.4925292       NaN 0.3779528     80.57553
## 92   0.7953631  0.9128519 0.4885961 0.5691865       NaN 0.4128788     82.13620
## 93   0.7191368  0.8098416 0.4522857 0.5155064       NaN 0.4747475     77.60794
## 94   0.6886370  0.7922734 0.4482534 0.5155105       NaN 0.4270463     69.55276
## 95   0.7443920  0.8189598 0.3966477 0.4529591       NaN 0.5164557     74.78103
## 96   0.6418228  0.7458160 0.4080434 0.4739181       NaN 0.4615385     73.94842
## 97   0.7164700  0.7880813 0.4936384 0.5475899       NaN 0.4176904     78.02426
## 98   0.7685535  0.8360919 0.3714630 0.4289613       NaN 0.3569132     83.08984
## 99   0.7221750  0.7883310 0.4284262 0.4727765       NaN 0.4307458     72.84763
## 100  0.6867698  0.7625455 0.3902812 0.4449062       NaN 0.4414894     77.97147
## 101  0.7514237  0.8250469 0.4842886 0.5369883       NaN 0.4350515     73.33488
## 102  0.7275133  0.7954055 0.4292071 0.4826094       NaN 0.4897579     77.67329
## 103  0.7069627  0.7782338 0.4277792 0.4823544       NaN 0.4516807     74.79393
## 104  0.6556687  0.7313702 0.3791619 0.4469634       NaN 0.4348958     79.30309
## 105  0.7249352  0.7773235 0.4805882 0.5177892       NaN 0.3655822     68.69027
## 106  0.7308732  0.7692066 0.5065327 0.5352712       NaN 0.3657363     60.11580
## 107  0.8120538  0.8628448 0.4878532 0.5230080       NaN 0.3654588     74.00749
## 108  0.7063743  0.7476292 0.4766118 0.5062866       NaN 0.3668070     61.85413
## 109  0.7593182  0.8017106 0.4967231 0.5263629       NaN 0.3853311     67.08912
## 110  0.7564280  0.8217850 0.4896685 0.5380073       NaN 0.3932749     72.39469
```

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## 111  0.7176998  0.7611205 0.4573111 0.4864271       NaN 0.3603604     60.54593
## 112  0.7947265  0.8705902 0.5231078 0.5819418       NaN 0.4178674     77.84770
## 113  0.6591890  0.7833547 0.4272704 0.5167643       NaN 0.3354037     72.59517
## 114  0.7663580  0.8596599 0.3183650 0.3840602       NaN 0.3696682     83.81704
## 115  0.8221097  0.8553105 0.4924738 0.5218319       NaN 0.4165701     74.86940
## 116  0.7387810  0.8078950 0.4888742 0.5348078       NaN 0.3671988     68.68507
## 117  0.8076123  0.8299528 0.5340832 0.5527286       NaN 0.3870156     67.87044
## 118  0.8399613  0.8679280 0.5809660 0.6027121       NaN 0.4010152     65.63206
## 119  0.7980147  0.8219692 0.5160519 0.5349584       NaN 0.3997642     71.70441
## 120  0.8142690  0.9091862 0.4872578 0.5484558       NaN 0.3960396     71.63409
## 121  0.7499869  0.8768930 0.4628062 0.5500624       NaN 0.3878505     69.16258
##     SITE LIDAR_ID CENSUS_ID    SPP  SOURCE           GENUS        SPECIES
## 1    CAM        1        NA PENMAC    MOMO    Pentaclethra    macrophylla
## 2    CAM      100        NA ENTCYL    MOMO Entandrophragma    cylindricum
## 3    CAM      101        NA BAPLEP    MOMO          Baphia    leptobotrys
## 4    CAM      102        NA ERIOBL    MOMO        Eribroma       oblongum
## 5    CAM      103        NA TERSUP    MOMO      Terminalia        superba
## 6    CAM      104        NA DUBMAC    MOMO        Duboscia     macrocarpa
## 7    CAM      105        NA ERIOBL    MOMO        Eribroma       oblongum
## 8    CAM      106        NA MACBAR    MOMO       Macaranga        barteri
## 9    CAM       11        NA ENTCYL    MOMO Entandrophragma    cylindricum
## 10   CAM       18        NA TERSUP    MOMO      Terminalia        superba
## 11   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 12   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 13   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 14   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 15   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 16   CAM       28        NA PETMAC    MOMO   Petersianthus    macrocarpus
## 17   CAM       29        NA BAPLEP    MOMO          Baphia    leptobotrys
## 18   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 19   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 20   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 21   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 22   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 23   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 24   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 25   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 26   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 27   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 28   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 29   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 30   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 31   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 32   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 33   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 34   GER       NA        NA PRUAVI    HACK          Prunus          avium
## 35   CAM       32        NA TERSUP    MOMO      Terminalia        superba
## 36   CAM       52        NA PYCANG    MOMO      Pycnanthus     angolensis
## 37   CAM       54        NA MANALT    MOMO        Mansonia      altissima
## 38   CAM       59        NA TERSUP    MOMO      Terminalia        superba
## 39   CAM        6        NA TRISCL    MOMO    Triplochiton    scleroxylon
## 40   CAM       60        NA ERYSUA    MOMO   Erythrophleum     suaveolens
## 41   CAM       61        NA CYLGAB    MOMO    Cylicodiscus     gabunensis
## 42   CAM       62        NA CYLGAB    MOMO    Cylicodiscus     gabunensis
```

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## 43   CAM       65        NA CYLGAB    MOMO    Cylicodiscus     gabunensis
## 44   CAM       66        NA ERYSUA    MOMO   Erythrophleum     suaveolens
## 45   CAM       68        NA ERYSUA    MOMO   Erythrophleum     suaveolens
## 46   CAM       69        NA DUBMAC    MOMO        Duboscia     macrocarpa
## 47   CAM       71        NA ANNCHL    MOMO       Annickia     chlorantha
## 48   CAM       72        NA ANNCHL    MOMO       Annickia     chlorantha
## 49   CAM       74        NA ERYSUA    MOMO   Erythrophleum     suaveolens
## 50   CAM       75        NA PTESOY    MOMO     Pterocarpus       soyauxii
## 51   CAM       76        NA PYCANG    MOMO     Pycnanthus     angolensis
## 52   CAM       79        NA ERIOBL    MOMO       Eribroma       oblongum
## 53   CAM        8        NA TERSUP    MOMO     Terminalia        superba
## 54   CAM       80        NA PYCANG    MOMO     Pycnanthus     angolensis
## 55   CAM       83        NA PTESOY    MOMO     Pterocarpus       soyauxii
## 56   CAM       87        NA PETMAC    MOMO   Petersianthus    macrocarpus
## 57   CAM       88        NA MACBAR    MOMO      Macaranga        barteri
## 58   CAM       92        NA BAPLEP    MOMO         Baphia    leptobotrys
## 59   CAM       93        NA TERSUP    MOMO     Terminalia        superba
## 60   CAM       94        NA MANALT    MOMO       Mansonia      altissima
## 61   CAM       95        NA PETMAC    MOMO   Petersianthus    macrocarpus
## 62   CAM       98        NA PTESOY    MOMO     Pterocarpus       soyauxii
## 63   CHI       NA        NA ERYFOR    HACK   Erythrophleum         fordii
## 64   CHI       NA        NA ERYFOR    HACK   Erythrophleum         fordii
## 65   CHI       NA        NA ERYFOR    HACK   Erythrophleum         fordii
## 66   CHI       NA        NA ERYFOR    HACK   Erythrophleum         fordii
## 67   CHI       NA        NA ERYFOR    HACK   Erythrophleum         fordii
## 68   CHI       NA        NA ERYFOR    HACK   Erythrophleum         fordii
## 69   CHI       NA        NA ERYFOR    HACK   Erythrophleum         fordii
## 70   CHI       NA        NA ERYFOR    HACK   Erythrophleum         fordii
## 71   CHI       NA        NA ERYFOR    HACK   Erythrophleum         fordii
## 72   CHI       NA        NA ERYFOR    HACK   Erythrophleum         fordii
## 73   CHI       NA        NA ERYFOR    HACK   Erythrophleum         fordii
## 74   CHI       NA        NA ERYFOR    HACK   Erythrophleum         fordii
## 75   CAM       63        NA CYLGAB    MOMO    Cylicodiscus     gabunensis
## 76   ALB       NA        NA ULMAME ALBERTA           Ulmus      americana
## 77   ALB       NA        NA ULMAME ALBERTA           Ulmus      americana
## 78   ALB       NA        NA ULMAME ALBERTA           Ulmus      americana
## 79   ALB       NA        NA ULMAME ALBERTA           Ulmus      americana
## 80   ALB       NA        NA ULMAME ALBERTA           Ulmus      americana
## 81   ALB       NA        NA ULMAME ALBERTA           Ulmus      americana
## 82   ALB       NA        NA ULMAME ALBERTA           Ulmus      americana
## 83   CHI       NA        NA PINMAS    HACK           Pinus     massoniana
## 84   CHI       NA        NA PINMAS    HACK           Pinus     massoniana
## 85   CHI       NA        NA PINMAS    HACK           Pinus     massoniana
## 86   CHI       NA        NA PINMAS    HACK           Pinus     massoniana
## 87   CHI       NA        NA PINMAS    HACK           Pinus     massoniana
## 88   CHI       NA        NA PINMAS    HACK           Pinus     massoniana
## 89   CHI       NA        NA PINMAS    HACK           Pinus     massoniana
## 90   CHI       NA        NA PINMAS    HACK           Pinus     massoniana
## 91   CHI       NA        NA PINMAS    HACK           Pinus     massoniana
## 92   CHI       NA        NA PINMAS    HACK           Pinus     massoniana
## 93   GER       NA        NA QUEPET    HACK         Quercus        petraea
## 94   GER       NA        NA QUEPET    HACK         Quercus        petraea
## 95   GER       NA        NA QUEPET    HACK         Quercus        petraea
## 96   GER       NA        NA QUEPET    HACK         Quercus        petraea
```

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## 97   GER       NA        NA QUEPET    HACK         Quercus        petraea
## 98   GER       NA        NA QUEPET    HACK         Quercus        petraea
## 99   GER       NA        NA QUEPET    HACK         Quercus        petraea
## 100  GER       NA        NA QUEPET    HACK         Quercus        petraea
## 101  GER       NA        NA QUEPET    HACK         Quercus        petraea
## 102  GER       NA        NA QUEPET    HACK         Quercus        petraea
## 103  GER       NA        NA QUEPET    HACK         Quercus        petraea
## 104  GER       NA        NA QUEPET    HACK         Quercus        petraea
## 105  ALB       NA        NA ULMAME ALBERTA           Ulmus      americana
## 106  ALB       NA        NA ULMAME ALBERTA           Ulmus      americana
## 107  ALB       NA        NA ULMAME ALBERTA           Ulmus      americana
## 108  ALB       NA        NA ULMAME ALBERTA           Ulmus      americana
## 109  ALB       NA        NA ULMAME ALBERTA           Ulmus      americana
## 110  ALB       NA        NA ULMAME ALBERTA           Ulmus      americana
## 111  ALB       NA        NA ULMAME ALBERTA           Ulmus      americana
## 112  WYT     1551      3720 ACERPS  OXFORD            Acer pseudoplatanus
## 113  WYT     1777      4984 ACERPS  OXFORD            Acer pseudoplatanus
## 114  WYT     2225      3682 ACERPS  OXFORD            Acer pseudoplatanus
## 115  WYT      229      4707 ACERPS  OXFORD            Acer pseudoplatanus
## 116  WYT      424      3614 ACERPS  OXFORD            Acer pseudoplatanus
## 117  WYT       45      4278 ACERPS  OXFORD            Acer pseudoplatanus
## 118  WYT      600      4588 ACERPS  OXFORD            Acer pseudoplatanus
## 119  WYT       81      3704 QUERRO  OXFORD         Quercus          robur
## 120  WYT      869      4937 ACERPS  OXFORD            Acer pseudoplatanus
## 121  WYT      990      4561 ACERPS  OXFORD            Acer pseudoplatanus
##                          BINOMIAL     VOL_PRES  LENGTH_SLOPE  FULL_VOL TIP_SLOPE
## 1      Pentaclethra macrophylla -0.236247470 -2.815874e-01 0.5962637 1.1493473
## 2   Entandrophragma cylindricum  0.168764357  6.845620e-03 0.7957638 0.9132607
## 3             Baphia leptobotrys  0.054736659  2.899577e-02 0.7446150 1.0512748
## 4              Eribroma oblongum -0.050600517  2.868170e-02 0.7178322 1.1471598
## 5             Terminalia superba -0.009735488 -7.670244e-02 0.6746298 1.1307029
## 6           Duboscia macrocarpa  0.098078452  3.611658e-02 0.7018743 1.0315346
## 7              Eribroma oblongum  0.008970928 -8.718092e-02 0.8403407 1.0541616
## 8              Macaranga barteri  0.116365295 -1.297702e-02 0.8236977 1.0393853
## 9   Entandrophragma cylindricum -0.625272625 -3.517464e-01 0.5744959 0.9865643
## 10            Terminalia superba  0.222349842  1.314895e-01 0.6350293 1.0414615
## 11                 Prunus avium -0.225710998  1.743131e-01 0.7481140 1.0708090
## 12                 Prunus avium -0.773910716 -2.543432e-01 0.6777109 1.1551894
## 13                 Prunus avium  0.153569332  4.042559e-01 0.8815299 1.1198106
## 14                 Prunus avium -0.184343231  5.116903e-03 0.7273352 1.0513899
## 15                 Prunus avium -0.339704171  1.934514e-02 0.6880348 1.0692652
## 16    Petersianthus macrocarpus -0.120019803 -3.896410e-02 0.6742764 1.1510548
## 17            Baphia leptobotrys -0.085118691  6.816390e-02 0.6044463 0.8706150
## 18                 Prunus avium -0.017685472  1.796051e-01 0.7967238 1.0671274
## 19                 Prunus avium -0.107116294  1.640677e-02 0.7214400 1.0461215
## 20                 Prunus avium  0.003274072  1.704559e-01 0.7862600 0.9436198
## 21                 Prunus avium -0.268982127  1.746188e-02 0.7232032 1.0100568
## 22                 Prunus avium -0.104273915  4.867834e-02 0.7613074 1.0859198
## 23                 Prunus avium -0.055206216  2.901430e-01 0.7268153 0.9326610
## 24                 Prunus avium -0.071856511  2.808788e-01 0.7463290 0.9622041
## 25                 Prunus avium -0.261678545  9.317889e-02 0.6226624 0.9958511
## 26                 Prunus avium -0.151564579  2.015300e-01 0.6749968 1.0829367
## 27                 Prunus avium -0.195803375  1.232846e-01 0.7843082 1.1438143
## 28                 Prunus avium -0.085719657  2.608578e-01 0.7825678 1.1083609
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## 29                Prunus avium -0.033336863  1.824909e-01 0.7844831 1.0140325
## 30                Prunus avium -0.193408403  3.315828e-01 0.5193192 1.0276134
## 31                Prunus avium -0.054857241  3.103322e-01 0.6858595 1.0957563
## 32                Prunus avium -0.360512080  1.375135e-01 0.6489506 1.0184027
## 33                Prunus avium -0.173095275  1.420537e-01 0.6688195 0.9530084
## 34                Prunus avium -0.265056321  1.072398e-01 0.6862818 1.1132258
## 35           Terminalia superba -0.305637165 -2.919754e-01 0.6320904 1.1853165
## 36         Pycnanthus angolensis -0.070635132 -9.128962e-02 0.6436237 1.0073644
## 37            Mansonia altissima -0.341776357 -2.154805e-05 0.5549901 0.9659770
## 38           Terminalia superba -0.976855697 -4.265043e-01 0.2604923 1.1510219
## 39     Triplochiton scleroxylon  0.043412765  2.958213e-03 0.6551686 0.9418087
## 40     Erythrophleum suaveolens  0.061965839 -9.894636e-04 0.6902215 1.0571171
## 41      Cylicodiscus gabunensis  0.013737304  6.968958e-02 0.8417420 1.0168631
## 42      Cylicodiscus gabunensis -0.080475483 -9.123347e-02 0.7717729 1.0835827
## 43      Cylicodiscus gabunensis -0.009529223 -1.183910e-01 0.6894382 1.0177662
## 44     Erythrophleum suaveolens  0.059432415  4.151120e-02 0.7189757 1.0312270
## 45     Erythrophleum suaveolens -0.057107894 -1.530760e-01 0.7539852 1.0497697
## 46          Duboscia macrocarpa -0.130148680 -1.895716e-01 0.5509300 0.9811445
## 47          Annickia chlorantha  0.220626804  2.509753e-01 0.8651959 0.8655458
## 48          Annickia chlorantha -1.126383201 -3.478819e-01 0.7275912 1.0345591
## 49     Erythrophleum suaveolens  0.032296211 -2.369024e-02 0.7039387 1.0716295
## 50         Pterocarpus soyauxii  0.144307651 -1.987496e-02 0.6976087 0.9325883
## 51         Pycnanthus angolensis -0.075343161 -2.216719e-01 0.6281567 0.8509809
## 52             Eribroma oblongum -0.057271113 -1.038059e-01 0.6131622 0.9330924
## 53           Terminalia superba  0.104620418  4.397556e-02 0.6791947 1.1181926
## 54         Pycnanthus angolensis  0.079491656  1.746422e-02 0.8143538 1.0553459
## 55          Pterocarpus soyauxii -0.113852109 -1.643505e-01 0.7251775 1.0956349
## 56    Petersianthus macrocarpus -0.105429743  2.547377e-02 0.8112248 1.1192750
## 57             Macaranga barteri  0.085079398  1.106496e-01 0.7329318 1.0507530
## 58           Baphia leptobotrys -0.013892688 -1.030297e-01 0.6469239 1.2270696
## 59           Terminalia superba  0.115050318  1.767417e-02 0.8059056 1.0476990
## 60           Mansonia altissima -0.088440855 -4.520439e-02 0.8077522 1.3074907
## 61    Petersianthus macrocarpus  0.114046425  8.627816e-02 0.7825695 1.1325274
## 62         Pterocarpus soyauxii  0.048647443 -2.360924e-02 0.8144094 1.0580214
## 63          Erythrophleum fordii -0.175813508 -1.893080e-01 0.5479963 1.0220102
## 64          Erythrophleum fordii  0.048677425  2.209358e-03 0.7346867 1.1491000
## 65          Erythrophleum fordii -0.074591869 -1.183431e-01 0.6212685 1.0350963
## 66          Erythrophleum fordii -0.473525164 -5.142371e-01 0.6624912 0.9898115
## 67          Erythrophleum fordii -0.061781135 -1.484003e-01 0.7539178 1.0810113
## 68          Erythrophleum fordii -0.044860644 -7.147155e-02 0.8315195 1.0503654
## 69          Erythrophleum fordii -0.179570225 -1.663539e-01 0.7846194 1.0959499
## 70          Erythrophleum fordii  0.091978279 -4.715157e-02 0.7144183 1.0485661
## 71          Erythrophleum fordii  0.010814310  1.214467e-03 0.5777996 1.0324565
## 72          Erythrophleum fordii  0.088971060  6.130754e-02 0.7743645 1.0730656
## 73          Erythrophleum fordii  0.070914267  1.707756e-02 0.6992230 1.0552380
## 74          Erythrophleum fordii  0.040431496 -9.127431e-02 0.7161208 1.2699572
## 75       Cylicodiscus gabunensis  0.102347551  8.535895e-03 0.6892561 1.0668716
## 76              Ulmus americana  0.165565869  9.916831e-02 0.8016040 0.8979560
## 77              Ulmus americana -0.040480920  3.492906e-02 0.6827402 0.9136843
## 78              Ulmus americana -0.025115615  9.876851e-02 0.6901939 0.9031741
## 79              Ulmus americana  0.028648742  5.094275e-02 0.7293575 0.9315087
## 80              Ulmus americana  0.191803305  1.627080e-01 0.7794255 0.9466556
## 81              Ulmus americana  0.054500269  4.716195e-02 0.7607343 0.9705499
## 82              Ulmus americana  0.053769289  1.184120e-01 0.7318741 0.9034971
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## 83             Pinus massoniana  0.158989438  5.107761e-02 0.8711720 1.0151298
## 84             Pinus massoniana -0.002884005 -4.599389e-02 0.7449819 1.1254350
## 85             Pinus massoniana  0.259862192  8.391092e-02 0.8179026 0.9049255
## 86             Pinus massoniana  0.164491937  4.620511e-02 0.7792282 0.9875318
## 87             Pinus massoniana  0.144801786  1.656889e-02 0.8203577 1.0089542
## 88             Pinus massoniana  0.085024610  1.814244e-02 0.7523409 1.0106494
## 89             Pinus massoniana  0.110408450  2.740820e-02 0.8118874 1.0651784
## 90             Pinus massoniana  0.147072120  2.179682e-01 0.8580225 0.9131383
## 91             Pinus massoniana  0.216051152  1.560157e-01 0.8316665 0.9814799
## 92             Pinus massoniana  0.200172963  5.683366e-02 0.8531320 0.9283542
## 93              Quercus petraea  0.080026671  4.092249e-02 0.7342115 1.0389700
## 94              Quercus petraea  0.152740827  1.689310e-01 0.7340577 0.8247783
## 95              Quercus petraea  0.094757721  1.324452e-01 0.7709931 0.8132592
## 96              Quercus petraea  0.043067718  3.871432e-02 0.6933143 0.9018902
## 97              Quercus petraea  0.053559043  1.652966e-02 0.7302146 0.9506086
## 98              Quercus petraea  0.193779865  9.496733e-02 0.7970773 0.8516179
## 99              Quercus petraea  0.036325900  2.539913e-02 0.7295440 0.7785410
## 100             Quercus petraea -0.030681891  1.294745e-02 0.7180034 0.8685458
## 101             Quercus petraea  0.131203864  9.695483e-02 0.7702557 0.8698003
## 102             Quercus petraea  0.133941820  5.103401e-02 0.7700128 0.7994284
## 103             Quercus petraea  0.135417994  1.189896e-01 0.7479805 0.9143571
## 104             Quercus petraea  0.144552902  1.295591e-01 0.7121048 0.7796767
## 105             Ulmus americana  0.118047769  7.521935e-02 0.7527525 0.9855236
## 106             Ulmus americana  0.126710347  1.457288e-01 0.7639782 0.9724406
## 107             Ulmus americana  0.235169529  1.678077e-01 0.8642433 0.9629809
## 108             Ulmus americana  0.098126284  1.003797e-01 0.7552695 0.9781170
## 109             Ulmus americana  0.187221479  1.446541e-01 0.7779201 0.9787046
## 110             Ulmus americana  0.155543653  1.277281e-01 0.8117483 0.9456699
## 111             Ulmus americana  0.159795079  1.155124e-01 0.7275871 1.0163993
## 112         Acer pseudoplatanus  0.123490352 -4.474166e-02 0.8421849 1.0146402
## 113         Acer pseudoplatanus  0.076746767  6.075956e-02 0.7306230 0.8838387
## 114         Acer pseudoplatanus  0.135467243  3.807517e-02 0.8172227 0.7960240
## 115         Acer pseudoplatanus  0.251364146  2.780505e-02 0.8312040 1.0115010
## 116         Acer pseudoplatanus  0.184881877  7.019814e-02 0.7431044 1.0476667
## 117         Acer pseudoplatanus  0.162020159  2.111358e-02 0.8155293 1.0192154
## 118         Acer pseudoplatanus  0.174688681  1.693090e-02 0.8487411 1.0281376
## 119               Quercus robur  0.187570523  5.075930e-02 0.8016759 0.9640992
## 120         Acer pseudoplatanus  0.104531336 -6.777217e-02 0.8594262 1.1000312
## 121         Acer pseudoplatanus  0.219355840  9.740830e-02 0.7534861 1.0736010
```
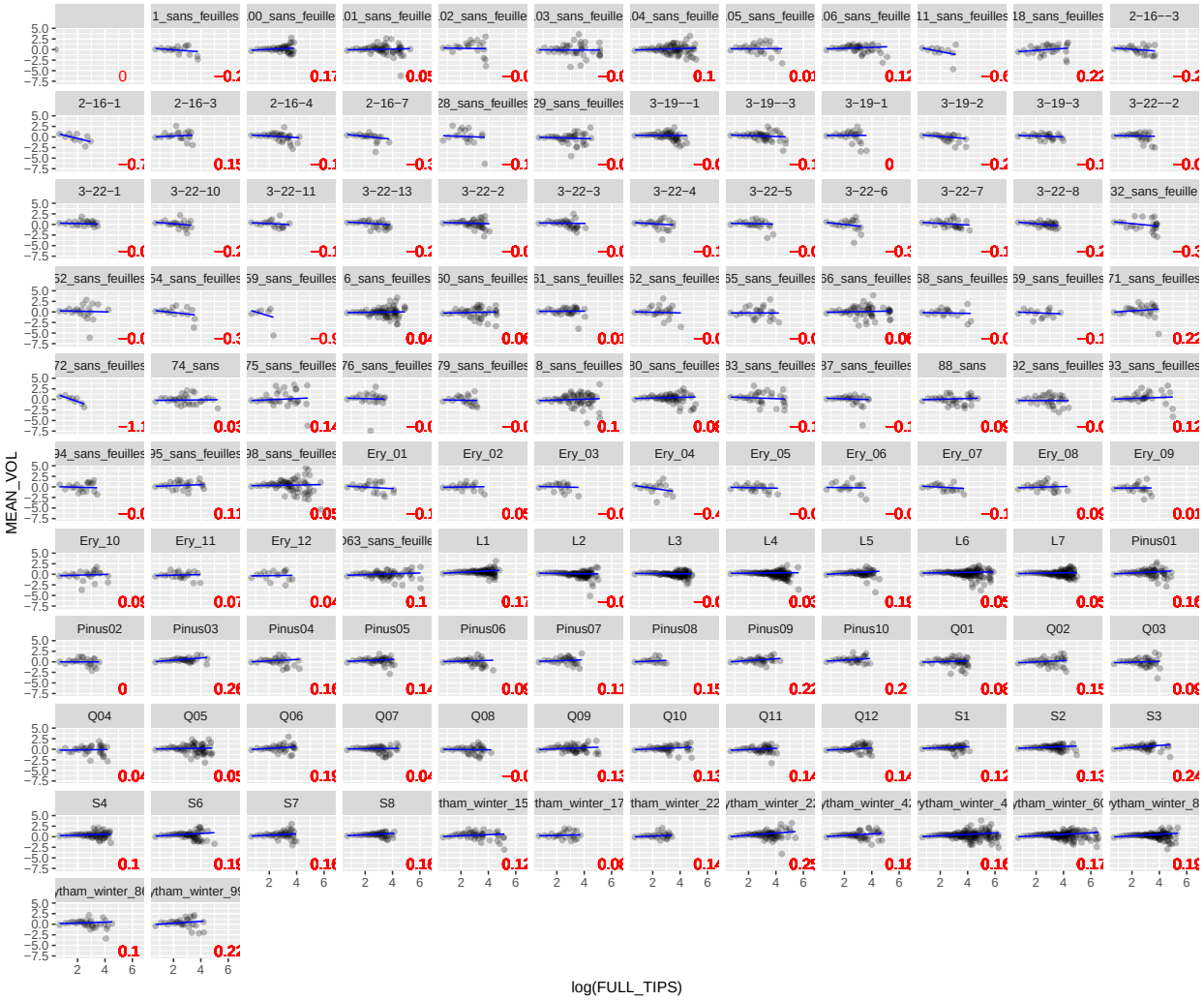
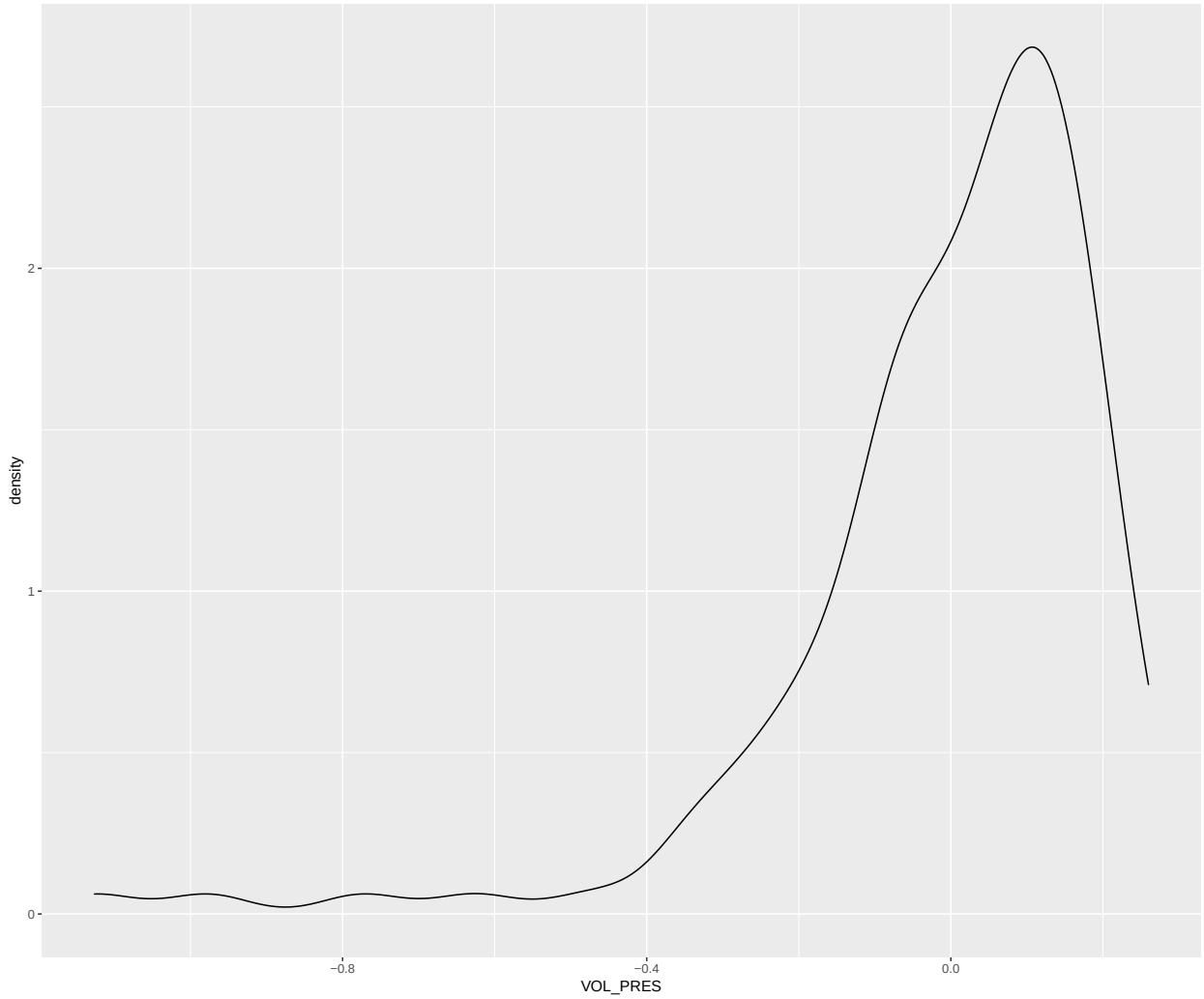

Thus, using distal tips as a category to define branching generations in a tree, we group branches and measure the ratio between the volume of a branch and the total volume of its children. For each branching generation, we average this ratio, then regress against the tip generations using a weighted OLS regression to adjust for the number of nodes in each branching generation. This yields broadly flat curves across the tree dataset, indicating a kind of space-filling emerging from heterogeneous branching nodes across the trees.


MEAN_VOL
log(FULL_TIPS)
1_sans_feuilles
00_sans_feuille
01_sans_feuille
02_sans_feuille
03_sans_feuille
04_sans_feuille
05_sans_feuille
06_sans_feuille
11_sans_feuilles
18_sans_feuilles
2−16−−3
2−16−1
2−16−3
2−16−4
2−16−7
28_sans_feuilles
29_sans_feuilles
3−19−−1
3−19−−3
3−19−1
3−19−2
3−19−3
3−22−−2
3−22−1
3−22−10
3−22−11
3−22−13
3−22−2
3−22−3
3−22−4
3−22−5
3−22−6
3−22−7
3−22−8
32_sans_feuille
52_sans_feuilles
54_sans_feuilles
59_sans_feuilles
6_sans_feuilles
60_sans_feuilles
61_sans_feuilles
62_sans_feuilles
65_sans_feuilles
66_sans_feuilles
68_sans_feuilles
69_sans_feuilles
71_sans_feuilles
72_sans_feuilles
74_sans
75_sans_feuilles
76_sans_feuilles
79_sans_feuilles
8_sans_feuilles
80_sans_feuilles
83_sans_feuilles
87_sans_feuilles
88_sans
92_sans_feuilles
93_sans_feuilles
94_sans_feuilles
95_sans_feuilles
98_sans_feuilles
Ery_01
Ery_02
Ery_03
Ery_04
Ery_05
Ery_06
Ery_07
Ery_08
Ery_09
Ery_10
Ery_11
Ery_12
963_sans_feuille
L1
L2
L3
L4
L5
L6
L7
Pinus01
Pinus02
Pinus03
Pinus04
Pinus05
Pinus06
Pinus07
Pinus08
Pinus09
Pinus10
Q01
Q02
Q03
Q04
Q05
Q06
Q07
Q08
Q09
Q10
Q11
Q12
S1
S2
S3
S4
S6
S7
S8
tham_winter_15
tham_winter_17
tham_winter_22
ytham_winter_2
ytham_winter_4
ytham_winter_4
ytham_winter_6
ytham_winter_8
ytham_winter_8
ytham_winter_9



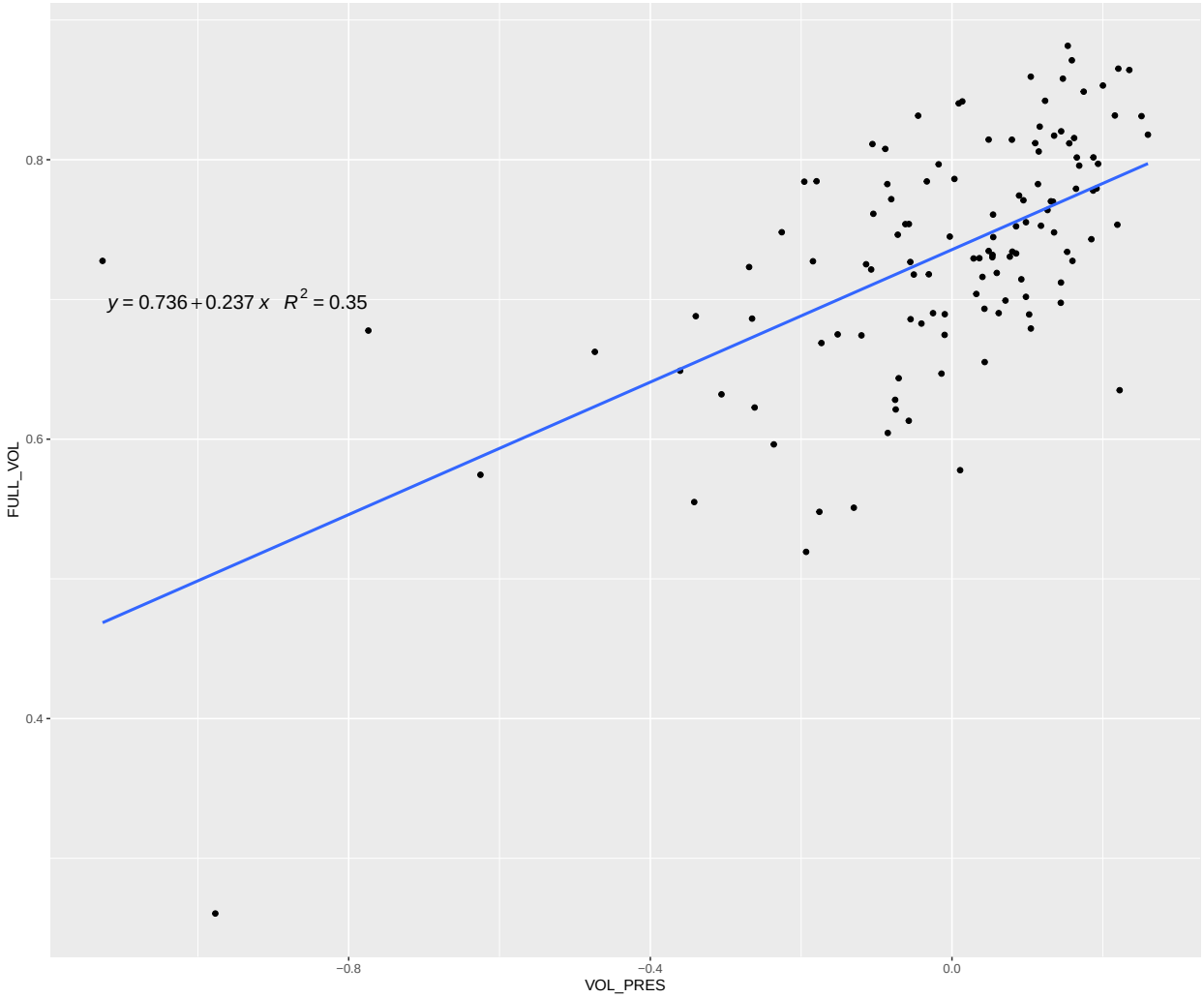

```
##
##  One Sample t-test
##
## data:  tt$VOL_PRES
## t = -0.58904, df = 120, p-value = 0.5569
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -0.05149497  0.02788051
## sample estimates:
##   mean of x
## -0.01180723
```

The reason this approach reveals approximate space-filling/volume-preservation is that averaging all nodes in a given branching level effectively accounts for variability in length asymmetry at a given level of the hierarchy, which otherwise pollutes measurements of the symmetrical γ scaling ratio. Another novel approach, subtly different from the definition of space-filling above, is using the volume of branches ($r^2 l\pi$) as an expression of space-filling rather than only the length of branches. While dimensionally the same (m^3), this measurement frees trees to fill spaces other than perfect spheres implied by the l_k^3 term in the original definition of space filling. It allows permits accounting for known covariation between radii and lengths in these branching networks. This may be a necessary feature of detecting space-filling in highly asymmetrical tree crowns.

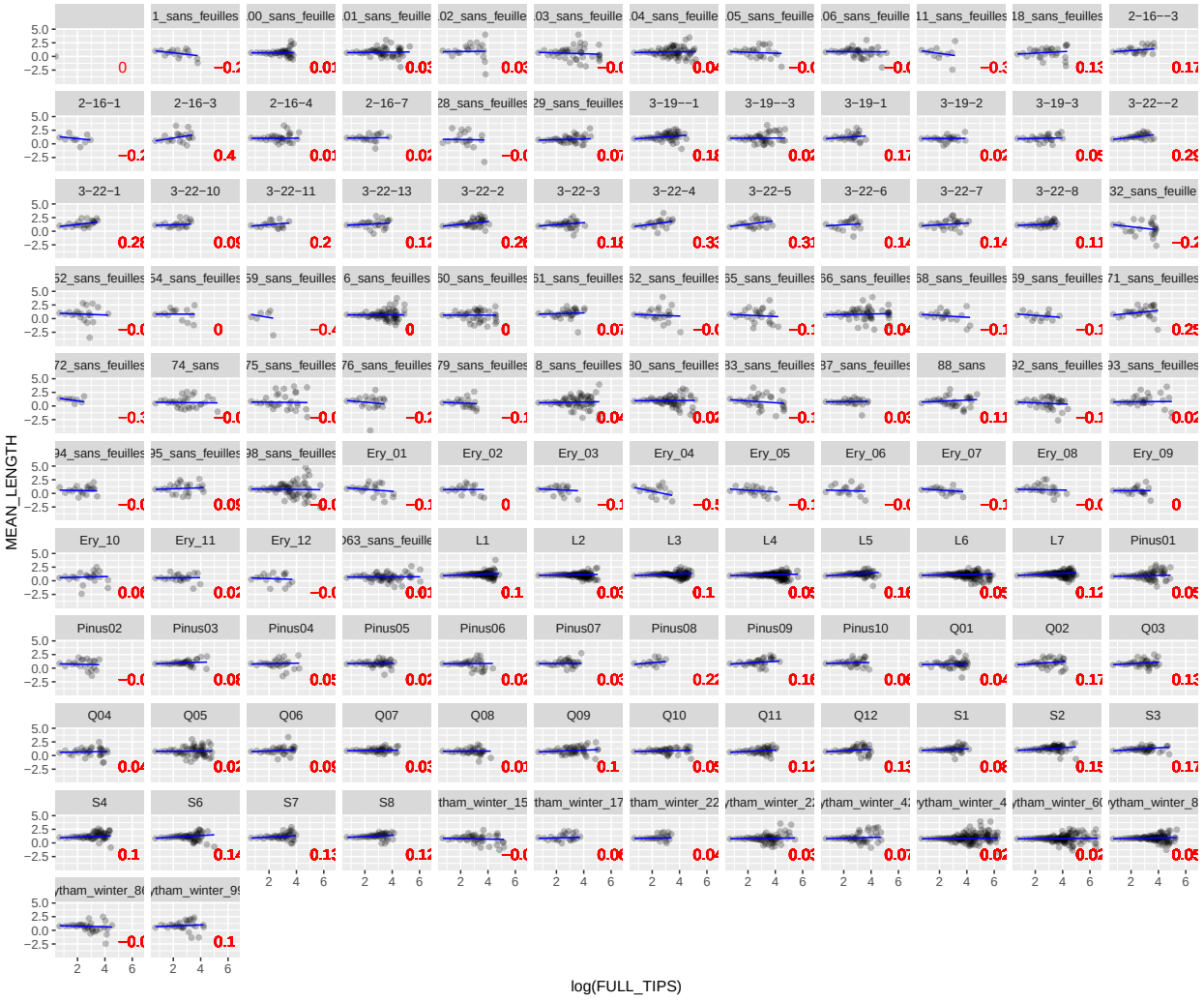
Alternatively, this plot corresponds to the expression

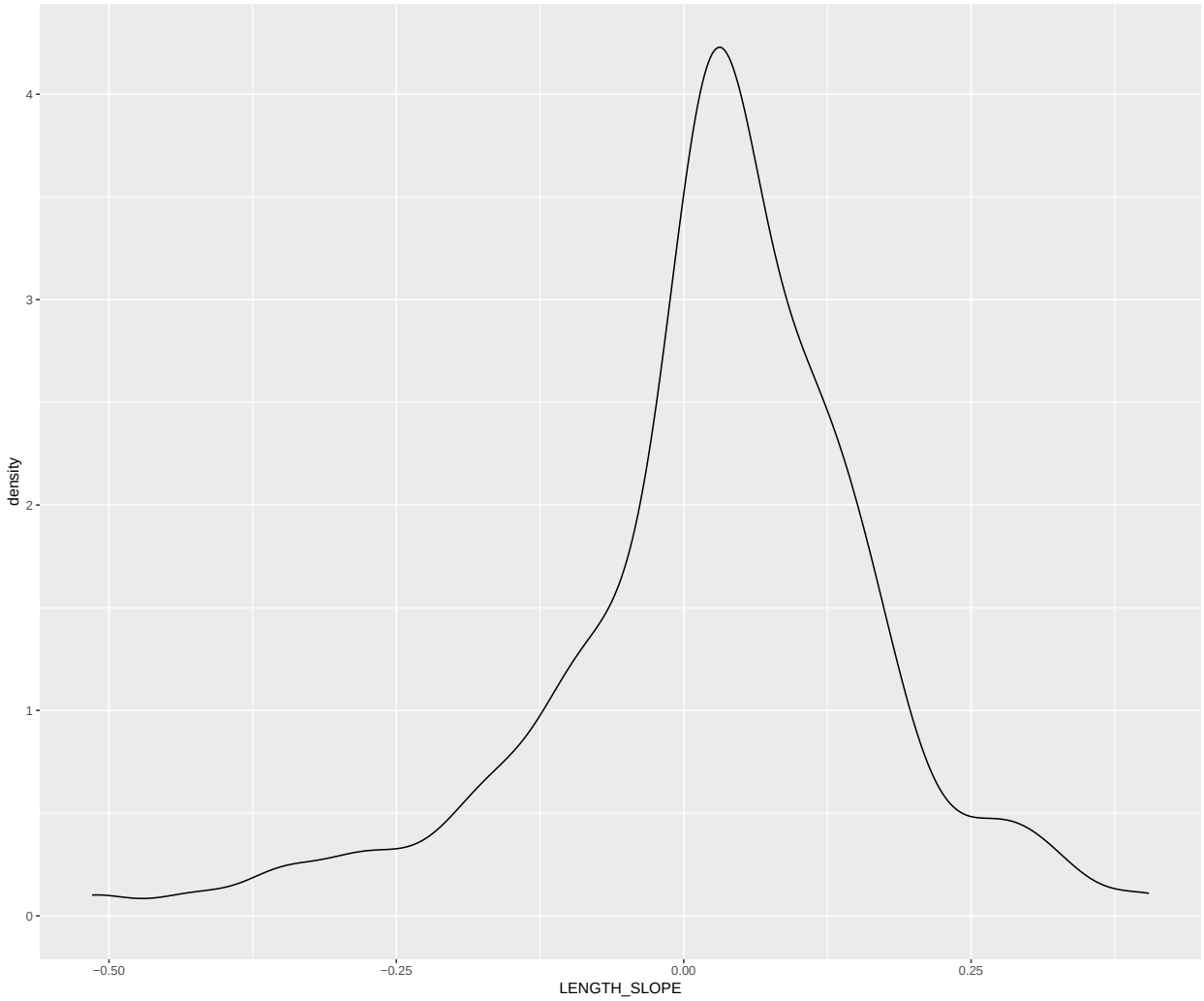
$$n_k r_k^2 l_k = n_{k+1} r_{k+1}^2 l_{k+1} 1 = n\beta^2\gamma \tag{4}$$

which appears in the derivation of the metabolic scaling exponent and could be interpreted as a 'volume-preservation' condition that jointly expresses how area-preservation and space-filling lead to 3/4 scaling. It is indicative that the volume preservation slopes above are moderately related to the volume scaling exponents we are claiming describe metabolic scaling



However, performing the same algorithm on branch lengths (i.e. comparing the ratio of parent length to the sum of child lengths 'length preservation') reveals similarly flat curves, albeit with more variability. Across trees, the slopes are not different from 0, the theoretical expectation for space-filling.


MEAN_LENGTH
log(FULL_TIPS)
1_sans_feuilles
00_sans_feuille
01_sans_feuille
02_sans_feuille
03_sans_feuille
04_sans_feuille
05_sans_feuille
06_sans_feuille
11_sans_feuilles
18_sans_feuilles
2−16−−3
2−16−1
2−16−3
2−16−4
2−16−7
28_sans_feuilles
29_sans_feuilles
3−19−−1
3−19−−3
3−19−1
3−19−2
3−19−3
3−22−−2
3−22−1
3−22−10
3−22−11
3−22−13
3−22−2
3−22−3
3−22−4
3−22−5
3−22−6
3−22−7
3−22−8
32_sans_feuille
52_sans_feuilles
54_sans_feuilles
59_sans_feuilles
6_sans_feuilles
60_sans_feuilles
61_sans_feuilles
62_sans_feuilles
65_sans_feuilles
66_sans_feuilles
68_sans_feuilles
69_sans_feuilles
71_sans_feuilles
72_sans_feuilles
74_sans
75_sans_feuilles
76_sans_feuilles
79_sans_feuilles
8_sans_feuilles
80_sans_feuilles
83_sans_feuilles
87_sans_feuilles
88_sans
92_sans_feuilles
93_sans_feuilles
94_sans_feuilles
95_sans_feuilles
98_sans_feuilles
Ery_01
Ery_02
Ery_03
Ery_04
Ery_05
Ery_06
Ery_07
Ery_08
Ery_09
Ery_10
Ery_11
Ery_12
963_sans_feuille
L1
L2
L3
L4
L5
L6
L7
Pinus01
Pinus02
Pinus03
Pinus04
Pinus05
Pinus06
Pinus07
Pinus08
Pinus09
Pinus10
Q01
Q02
Q03
Q04
Q05
Q06
Q07
Q08
Q09
Q10
Q11
Q12
S1
S2
S3
S4
S6
S7
S8
tham_winter_15
tham_winter_17
tham_winter_22
ytham_winter_2
ytham_winter_4
ytham_winter_4
ytham_winter_6
ytham_winter_8
ytham_winter_8
ytham_winter_9




```
##
##  One Sample t-test
##
## data:  tt$LENGTH_SLOPE
## t = 1.8773, df = 120, p-value = 0.06291
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -0.001357427  0.050993590
## sample estimates:
##  mean of x
## 0.02481808
```

Very simply then: since the distribution of these trees exhibit both area-preservation and space-filling, it is not surprising that the distribution of 'metabolic scaling exponents' is centered around the 3/4 expectation. Still, the goal of linking branching traits at fine (node-level) scales to whole-network properties such as space-filling and 3/4 scaling poses future opportunities for theory development.