Binary Arithmetic (part 1) [@achongsBiz]

In this article we'll describe the simplest forms of binary addition and subtraction. In these operations we'll deal with positive and whole numbers only.

1 Binary Addition

(I) We first establish some basic binary results. The decimal equivalent is listed on the right:

Binary	**Decimal**
$0+0=0$	$0+0=0$
$0+1=1$	$0+1=1$
$1+1=10$	$1+1=2$
$10+1=11$	$2+1=3$

Just like with regular decimal addition, we stack both binary numbers and add the columns. If we need to perform a $1+1$ or $10+1$, a 1 is sent to the carry for the next column.

(II) Let's see a simple example, the decimal operation $2+3=5$. We know that 2 in binary is 10 and 3 in binary is 11, so:

$$\begin{array}{ccc} & 1 & 0 \\ + & 1 & 1 \\ \hline 1 & 0 & 1 \end{array}$$

On the first column we have $0+1=1$, next we have have $1+1=10$. We can confirm that this result is 5 since $(1)(2)^0+(1)(2)^2=5$.

(III) Let's see what happens with $14+5=9$. In binary 14 is 1110 and 5 is 101:

$$\begin{array}{ccccc} & 1 & & & \\ & 1 & 1 & 1 & 0 \\ + & & 1 & 0 & 1 \\ \hline 1 & 0 & 0 & 1 & 1 \end{array}$$

So we can follow our rules fairly well until we get to the third column where we need to do $1+1=10$. In this case we will record the 0 and the 1 moves to the carry position of the next column. The operation on the last column thus becomes $1+1=10$.

(IV) Now let's try $14+15$. In binary, 14 is 1110 and 15 is 1111:

$$\begin{array}{ccccc} & 1 & 1 & & \\ & 1 & 1 & 1 & 0 \\ + & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 0 & 1 \end{array}$$

On the second column we will record the 0 but move the 1 to the third column's carry position. On the third column we will have to calculate $1+1+1$. In binary, $1+1=10$ and $10+1=11$. We record the 1 and move a 1 to the fourth column's carry. On the last column we will again compute $1+1+1$.

2 Binary Subtraction

(I) Just like with addition, we'll list out some basic operations first.

Binary	**Decimal**
$0 - 0 = 0$	$0 - 0 = 0$
$1 - 1 = 0$	$1 - 1 = 0$
$1 - 0 = 1$	$1 - 0 = 1$

(II) Let's examine the case of $3 - 1 = 2$. In binary 3 is 11. This is a simple example without the need to borrow:

$$\begin{array}{cccc} & & 1 & 1 \\ - & & & 1 \\ \hline & & 1 & 0 \end{array}$$

This is consistent with our expectations as 2 in binary is 10.

(III) If we encounter a $0 - 1$, we will need to borrow two 1's from some column towards the left. Let's consider the case of $14 - 3 = 11$. In binary 14 is 1110. From the start, we need to borrow from the next column:

$$\begin{array}{ccccc} & & & & 1 \\ & & & & 1 \\ & 1 & 1 & \cancel{1} & 0 \\ - & & & 1 & 1 \\ \hline & & & & 1 \end{array}$$

Since the second column is now zero, we proceed to borrow from the third column:

$$\begin{array}{ccccc} & & & 1 & \\ & & & 1 & \\ & 1 & \cancel{1} & \cancel{1} & 0 \\ - & & & 1 & 1 \\ \hline & 1 & 0 & 1 & 1 \end{array}$$

(IV) Let's examine one final example, $17 - 15 = 2$. In binary, 17 is 10001 and 15 is 1111.

$$\begin{array}{cccccc} & & \cancel{1} & \cancel{1} & 1 & \\ & & 1 & 1 & 1 & \\ & \cancel{1} & 0 & 0 & 0 & 1 \\ - & & 1 & 1 & 1 & 1 \\ \hline & 0 & 0 & 0 & 1 & 0 \end{array}$$

We are able to calculate the first column, but since the second, third, and fourth columns are zero's we can only borrow from the last column. So:

- The fifth column loses a 1, and the fourth gains two 1's.
- The fourth column loses a 1, and the third column gains two 1's.
- The third column loses a 1, and the second column gains two 1's.

(V) To conclude, let's discuss why lending a 1 results in two 1's on the right. If the right most column is $n = 0$, then the decimal value of a 1 on column n is 2^n. On the example above, 17 has a 1 on column n=4 with a decimal value of $2^4 = 16$. If we transfer this amount to column n=3, the transferred amount can be represented with two 1's, each with a decimal value of $2^3 = 8$.