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A more human-like portfolio optimization approach

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ABSTRACT

Black and Litterman proposed an improvement to the Markowitz portfolio optimization model. They suggested the construction of views to represent investor's opinion about the future of stocks' returns. However, conceiving these views can be quite confusing. It requires the investor to quantify several subjective parameters. In this article, we propose a new way of creating these views using Verbal Decision Analysis. Questionnaires were designed with the intent of making it easier for investors to express their vision about stocks. Following the ZAPROS methodology, the investor answers sets of questions allowing to determine a Formal Index of Quality (FIQ). The views are then derived from the resulting FIQ. Our approach was implemented and tested on data from the Brazilian Stocks. It allows investors to create a personal risk-return balanced portfolio without the help of an expert. The experiments show that the proposed method mitigates the impact of poor view estimation. Also, one must notice that the method is qualitative and its aim is to create a more efficient portfolio considering the investor's vision.

1. Introduction

The works of Markowitz (1952); 1959) have completely transformed the Portfolio Optimization field, and derivations from it are still used to construct almost every portfolio. Markowitz used the stocks' profitability variance as a measure of risk along with the expected returns of stocks for portfolio selection, defining an efficient frontier that determined which portfolio composition would have the highest expected value for a given level of risk.

Albeit revolutionary, Markowitz's work shows some major drawbacks in practical applications. The resulting portfolios can be counter-intuitive (Black & Litterman, 1992; Michaud, 1989), they tend to concentrate on a small subset of the available securities and do not seem to be quite diversified (Bera & Park, 2008; Tütüncü & Koenig, 2004). The optimal portfolio is also extremely sensitive to small variations in the input data (Erdogan, Goldfarb, & Iyengar, 2008; Michaud, 1989; Tütüncü & Koenig, 2004).

These practical disadvantages of the Markowitz model motivated Fisher Black and Robert Litterman to develop a new approach. Thus the Black–Litterman approach (Black & Litterman, 1991), which combines the expected equilibrium between returns estimated through the Capital Asset Pricing Model (CAPM) and views to optimize the portfolio. The views represent the investor's opinion about the stocks' future returns. This model yields more stable and diversified portfolios than the mean-variance standard model (Walters, 2011).

Black and Litterman's original paper (Black & Litterman, 1992) only explained the core aspects of their idea, leaving it to others to better explain the implication of their model. He and Litterman (2002); Satchell and Scowcroft (2000); Walters (2011) explain the Black–Litterman approach in further detail. Walters (2011) also constructed a framework[1] to use the model and other portfolio optimization techniques. Mankert (2010) sheds more light on the practical implications of the Black–Litterman approach. Other studies focus on extensions of the original model, like Fernandes, Fernandes, and Street (2013); Herold (2005); Idzorek (2007); Meucci (2008).

Also, Bertsimas, Gupta, and Paschalidis (2012) proposed a more general extension of the original Black–Litterman model that can incorporate investor opinion about volatility and construct estimators for more general notions of risk. Reinterpreting the problem through inverse optimization (Bertsimas et al., 2012) extends the traditional model creating a approach that can combine a greater variety of views.

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[1] That is available in www.blacklitterman.org

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The expression of the investor's preferences can be seen as a decision making process. Traditionally, decision-making scenarios involve the analysis of objects from several points of view and can be assisted by multi-criteria methodologies. These help generating knowledge about the decision context and, as a consequence, increase the confidence of those making decisions (Evangelou, Karacapilidis, & Khaled, 2005). There are multi-criteria methods based either on quantitative or qualitative analysis of the problem, and choosing the best approach is a great challenge. Examples of problem-solving using quantitative methods can be found in Castro, Pinheiro, and Pinheiro (2009); Pinheiro, Souza, and Castro (2008); Toncovich, Turón, Escobar, and Moreno-Jiménez (2011). Among those who apply qualitative methods, we have (Castro, Pinheiro, Dantas Pinheiro, & Tamanini, 2011; Mendes, Carvalho, Furtado, & Pinheiro, 2008; Tamanini, Carvalho, Castro, & Pinheiro, 2009; Tamanini, Castro, Pinheiro, & Pinheiro, 2011; Tamanini, Pinheiro, & Pinheiro, 2010).

The Verbal Decision Analysis is based on multi-criteria problem-solving through qualitative analysis method. One of the advantages of qualitative methods is that all the questioning in the process of eliciting preferences is made in the decision maker's native language. Moreover, verbal descriptions are used to measure preference levels. This procedure is psychologically valid, respecting the limitations of the human information processing system. This characteristic makes the incomparability cases (Tamanini, 2010) become almost unavoidable since the scale of preferences is purely verbal and consequently not an accurate way of estimating values. Therefore, the method may not be capable of achieving satisfactory results in some situations, presenting an incomplete solution to the problem.

Establishing views in the traditional quantitative way is not an easy task and an investor would need help from an expert in the process. That is why we chose a method to setting views using Verbal Decision Analysis (VDA). For this propose, we developed questionnaires that are intuitive and can be answered by anyone with basic knowledge of investment options without needing any further special training.

The propose of this paper is to develop a methodology to construct personalized portfolio base on the investor's opinions. Our problem is not a typical multi-criteria problem, being actually very different from normal VDA applications. This is one of the major difficulties that have to be overcome in order to create the Black–Litterman views.

Moreover, the desired goal is to build a technique to support the creation of customized portfolios based on an individual's preferences. That is, we are trying to identify the profile by knowing his opinion about the stocks. Therefore, a comparison among performances of portfolios, in the present case, should consider only portfolios that are aligned with the individual preference. For this purpose, in the final part of Section 4 we compare the return of investing on the investor most preferred asset with our proposed approach.

In Section 2 we present a brief explanation of the Verbal Decision Analysis (VDA) framework used in this work. Section 3 brings a review of the Black–Litterman methodology. Finally, in Section 4 we report about the experiments made with Brazilian stocks while Section 5 brings a brief discussion regarding future works.

2. Verbal Decision Analysis

A decision may be defined as the result of a process of choice when someone is confronted with a problem or with an opportunity for creation, optimization or improvement of a given situation. On the other hand, decision making is a special activity of human behavior, aimed at the achievement of a given goal. It takes place in every activity of the human world, from simple daily problems to complex situations inside an organization. The conclusion of a decision making process can be an ordination of alternatives or the selection of a single alternative from a list of possible solutions for the problem.

Establishing its preferences and interests is usually enough to allow an individual to make decisions that solve simple problems. However, individuals often find it hard to separate emotions from reason. As a result, emotions often influence the decision making process (Larichev, 2001; Machado, Menezes, Tamanini, & Pinheiro, 2011). The decisions also involves several factors, some of which may not be measurable. Thus, when a decision maker needs to solve complex problems, covering many alternatives and a large volume of information that may not be measurable nor easily comparable, some methodologies exist to support the decision making process.

In order to solve a given problem, alternative solutions are taken into consideration. Such alternatives are defined and characterized according to a set of criteria, structured around its verbal and qualitative nature. There are a huge number of practical problems which is necessary to generate an ordinal scale of alternatives (Larichev & Moshkovich, 1997). The construction of such an ordinal scale is helpful in many situations, for example, to reject less preferable alternatives from a given set.

The Verbal Decision Analysis (VDA) framework is a set of methods defined to support the decision making process through the verbal representation of problems. Some methods that constitute the Verbal Decision Analysis framework are: ZAPROS-III, ZAPROS-LM, PACOM, and ORCLASS (Larichev & Moshkovich, 1997). According to Gomes, Moshkovich, and Torres (2010), in the majority of multi-criteria problems there is a set of alternatives that can be evaluated against the same set of characteristics (called criteria or attributes). The VDA framework is structured on the supposition that most decision making processes can be qualitatively described (Chrissis, Konrad, & Shrum, 2007). Although the decision maker's ability to choose is very dependent on the occasion and the stakeholders' interest, the methods to support decision making are universal.

Moreover, in Ustinovich and Kochin (2004) the analysis of a large amount of data-processing performed by human beings has shown that the psychologically correct operations are:

- Comparison of two assessments in verbal scale by two criteria;
- Assignment of multi-criteria alternatives to decision classes;
- Comparative verbal assessment of alternatives according to separate criteria.

This last operation is the only classification methodology within the VDA framework. The goal of the Verbal Decision Analysis framework is to establish a ranking of alternatives in order of preference.

The methods belonging to the Verbal Decision Analysis framework may be evaluated in light of their objectives:

- As a tool for ordinary classification, ORCLASS was one of the first methods designed to tackle classification problems. There are several other widely known methods for solving classification problems that can be applied and analyzed for future applications (Brasil, 2010; Brasil, Pinheiro, & Coelho, 2010; 2012) but that does not belong to Ustinovich and Kochin (2004) VDA framework;
- The other objective is to organize the solutions alternatives for the problem in a rank, from the most preferable to the least preferable one. Three methods are proposed within the VDA framework: ZAPROS-LM, ZAPROS-III, and PACOM. Although they have the same final goal, they have different purposes:

– PACOM is exclusively created to be applied according to pair compensation and consists in comparing the advantages and disadvantages of multi-attribute alternatives.
– The ZAPROS method was created to be applied by pair comparison and consists in comparing a pair of alternatives with the advantage of reaching a decision by using simple and understandable dialogue. It is also divided in two alternative methods:
  - ZAPROS-III differs from ZAPROS-LM in its level of treatment of inconsistence. ZAPROS-III can be considered an evolution of ZAPROS-LM in this concept.

2.1. Formal statement of the problem

The methodology follows the same problem formulation proposed by Larichev and Moshkovich (1997), where:

1. $k = \{c_1, c_2, \ldots, c_N\}$, representing a set of N criteria;
2. n_q represents the number of possible values on the scale of qth criterion, ($q \in k$); For the ill-structured problems, as in this case, usually $n_q \leq 4$;
3. $X_q = \{x_1, x_2, \ldots, x_{n_q}\}$ represents a set of values to the qth criterion, which is this criterion scale; $|X_q| = n_q$; The values of the scale are ranked from best to worst, and this order does not depend on the values of other scales;
4. $Y = X_1 \times X_2 \times \cdots \times X_N$ represents a set of vectors y_i, in such a way that: $y_i = \{y_{i1}, y_{i2}, \ldots, y_{iN}\}^\top$ and $y_i \in Y$, $y_{iq} \in X_q$, where $|Y| = \prod_{q=1}^{N} n_q$;
5. $A = \{a_i\}_1^t$ and $a_i \in Y$, where the set of t vectors represents the description of the real alternatives.

The order of the multi-criteria alternatives on set A is defined based on the decision maker's preferences.

2.2. The ZAPROS-III Method

According to Ustinovich and Kochin (2004), one of the most important features of ZAPROS methods is the use of psychologically grounded procedures for identifying the preferences. This method evaluates personal abilities and limitations of human information processing system. The disadvantages of the method also include the limited amount of attributes and difficulties in using quantitative criteria.

Furthermore, ZAPROS-III (Larichev, 2001) considers values known as Quality Variations (QV) or Quality Changing (QC) (Chrissis et al., 2007) and Formal Index of Quality (FIQ). The QV represents the distance between the evaluations of two criteria. The FIQ mainly aims at minimizing the number of comparable pairs of alternatives. The FIQ is used in the ranking of the alternatives.

According to Tamanini (2010), Fig. 1 presents a flowchart with steps for the application of the VDA method ZAPROS-III. As described in the figure, the method's application can be divided into four stages: Problem Formulation, Elicitation of Preferences/Comparison of Alternatives, Validation of the Decision maker's preferences, and Comparison of Alternatives.

A disadvantage of the method is that the number and values of criteria that can be handled are limited, in order to keep complexity under control.

Tamanini (2010) defends that although ZAPROS-III-i follows a procedure similar to its predecessor's to extract preferences, it also implements modifications that make it more efficient and more accurate regarding inconsistencies . The number of incomparable alternatives is essentially smaller than in previous ZAPROS (Chrissis et al., 2007).

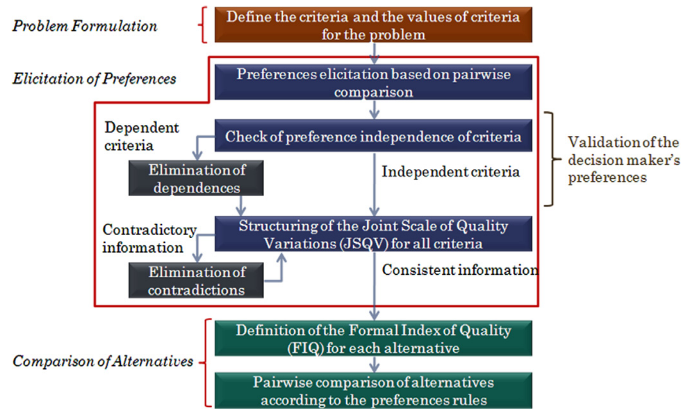

Fig. 1. Procedure to apply ZAPROS-III methodology.

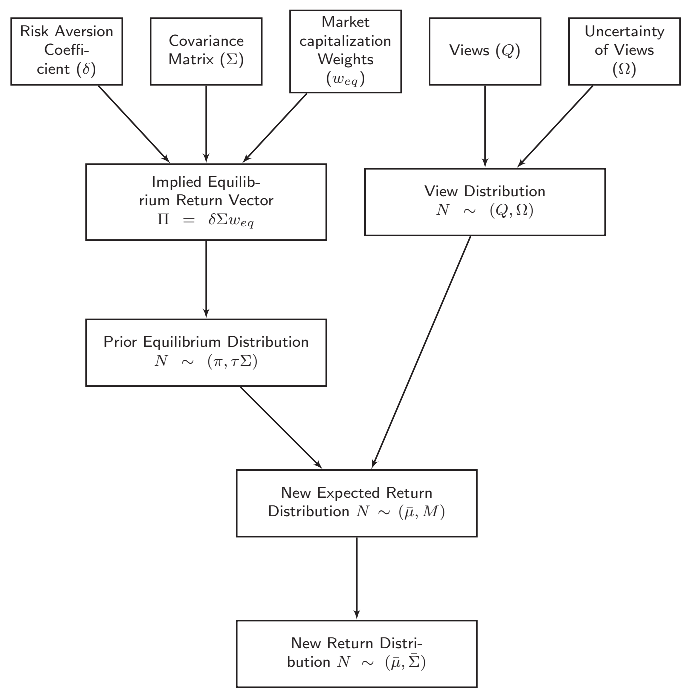

Fig. 2. Flowchart of Black–Litterman method (Idzorek, 2007).

3. Black–Litterman

The traditional portfolio approach proposed by Markowitz has some issues and does not consider the investor's vision of the market. Hence, the Black–Litterman method (Black & Litterman, 1991) was conceived to be a more practical and more flexible portfolio management method (Mankert, 2006). Its methodology begins by determining the equilibrium portfolio and the views of the investor, after these are combined to construct a new distribution of the stocks' returns. Using this new distribution, a portfolio optimization problem is formulated and a new optimal portfolio is obtained. A summary of the Black–Litterman model is present in Fig. 2.

The model proposed by Black and Litterman can be seen, in a rather simplistic way, as an adjustment in the prior distribution of the stocks' returns to adapt it to the investor's vision. Essentially, however, it combines the investor's views with the CAPM notion of market equilibrium (Black & Litterman, 1991; 1992).

3.1. Market equilibrium

The Black–Litterman assumption is that the a priori distributions of returns are consistent with market equilibrium. Considering that all investors' utility functions are the same, the CAPM theory shows that everyone should hold the same portfolio, the market portfolio w_{eq}. The market portfolio is the portfolio where the amount of stocks is proportional to its market value.

First we have to assume that the returns of the stocks r are normally distributed with mean $\mathbb{E}(r)$ and covariance matrix Σ i.e. $r \sim N(\mathbb{E}(r), \Sigma)$. When the market is efficient, the expected return for any asset has the following property

$$\mathbb{E}(r_i) - r_f = \beta_i(\mathbb{E}(r_m) - r_f) \tag{1}$$

The $\mathbb{E}(r_i)$ and $\mathbb{E}(r_m)$ are the asset i and market portfolio's expected returns, while r_f is the risk free asset return. Coefficient β_i is the covariance between asset i and the market portfolio returns, divided by the market portfolio variance

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} \tag{2}$$

Also, the market portfolio return is

$$r_m = \sum_{j=1}^{n} r_j x_{mj} \tag{3}$$

The risk equilibrium premium Π is the expected excess of return yielded by the risky stocks, which should perform better than the risk free stock. It is properly defined as the difference between the asset returns and risk free returns $\Pi_i = \mathbb{E}(r_i) - r_f$. Using the fact that

$$\sigma_{im} = \sum_{j=1}^{n} x_{mj}\sigma_{ji} \tag{4}$$

and (1) we have

$$\begin{aligned} \Pi_i &= \beta_i(\mathbb{E}[r_m] - r_f) \\ &= \frac{\sigma_{im}}{\sigma_m^2}(\mathbb{E}[r_m] - r_f) \\ &= \frac{\mathbb{E}[r_m] - r_f}{\sigma_m^2}\Big(\sum_{j=1}^{n} x_{mj}\sigma_{ji}\Big) \end{aligned} \tag{5}$$

With the risk aversion parameter δ

$$\delta = \frac{\mathbb{E}[r_m] - r_f}{\sigma_m^2} \tag{6}$$

the final result can be expressed in matrix form as

$$\Pi = \delta \Sigma x_m \tag{7}$$

A more detailed demonstration of these equations and more about the CAPM theory can be found in Elton, Gruber, Brown, and Goetzmann (2009) and Sharpe (1964). The result above can also be obtained by deriving the traditional quadratic utility function of the mean-variance model, assuming that all investors solve this problem.

Finally, we can define the prior distribution μ as the real return distribution with mean Π and variance $\tau\Sigma$

$$\begin{aligned} \mu &= \Pi + \epsilon_\pi \\ \epsilon_\pi &\sim N(0, \tau\Sigma) \end{aligned} \tag{8}$$

The τ is a small number that reflects the investor's uncertainty about prior return estimations (Walters, 2010). It is the most confusing parameter of the model and has several different calibration approaches. Further ahead we shall present Idzorek's technique to eliminate τ.

3.2. Specifying views

The views are the investor's vision regarding future market behavior. These views can be relative or absolute and need to be "fully invested". Hence, the sum of weights is zero for the relative view, and one for the absolute. An example of absolute view is "Stock A will return X percent" and of a relative view is "International stock will outperform domestic stock by Y percent". Furthermore, the confidence has to be defined by the investor, and this will change how much the view will affect the portfolio weights. The investor's view can be expressed as

$$P\mu = Q + \epsilon_q \tag{9}$$

Where P is the perspective of the investor and Q specifies the expected return of each view. The ϵ_q is an non-observable random and normally distributed vector with mean zero and a diagonal covariance matrix Ω that expresses the uncertainty of the views ($\epsilon_q \sim N(0, \Omega)$).

Considering v as the number of views and n the number of stocks, P will be a matrix $v \times n$ with p_i ($i \in \{1, \ldots, v\}$) representing a vector with n elements, Q a vector with v elements and Ω a $v \times v$ diagonal matrix

$$P^{\mathrm{T}} = [p_1, p_2, p_3, \ldots, p_v]$$
$$Q^{\mathrm{T}} = [q_1, q_2, q_3, \ldots, q_v]$$
$$\Omega = \begin{bmatrix} \omega_1 & 0 & \ldots & 0 \\ 0 & \omega_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \omega_v \end{bmatrix}$$

To better understand how to describe these views in matrix form, two examples of views were created: one relative and other absolute. In the first view, stock one will outperform stock two by 1 percent and in the second view, stock three will have return 2 percent.

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\bar{\mu} = \begin{bmatrix} 0.01 \\ 0.02 \end{bmatrix} + \epsilon_q$$

3.3. The estimation model

With the expected excess return and the views of the investor, it is possible to proceed to the next step of the Black–Litterman approach, which combines these two items. There are two ways to estimate the final model. The original (Black & Litterman, 1991) paper references the Theil's Mixed Estimation model (Theil, 1971), but there is also a Bayesian approach. The first method was chosen because it is easy to understand. By applying the identity matrix I, the problem can be seen in the matrix form

$$\begin{bmatrix} I \\ P \end{bmatrix}\bar{\mu} = \begin{bmatrix} \Pi \\ Q \end{bmatrix} + \begin{bmatrix} \epsilon_\pi \\ \epsilon_q \end{bmatrix} \tag{10}$$

Constructing the auxiliary matrices $A = [{I \atop P}]$, $b = [{\Pi \atop Q}]$ and $\epsilon = [{\epsilon_\pi \atop \epsilon_q}]$ we can reformulate the problem as

$$A\bar{\mu} = b + \epsilon \tag{11}$$

$$\epsilon \sim N(0, W),\ W = \begin{bmatrix} \tau\Sigma & 0 \\ 0 & \Omega \end{bmatrix} \tag{12}$$

Solving this system of equations using least squares, we have

$$\begin{aligned} \bar{\mu} &= (A^{\mathrm{T}}W^{-1}A)^{-1}A^{\mathrm{T}}W^{-1}b \\ &= [(\tau\Sigma)^{-1} + P^{\mathrm{T}}\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1} + P^{\mathrm{T}}\Omega^{-1}Q] \end{aligned} \tag{13}$$

The variance can also be adjusted to reflect the change in the return data. Hence, the variance of the returns relative to the new data is

$$M = (A^{\mathrm{T}}W^{-1}A)^{-1}$$
$$= [(\tau\Sigma)^{-1} + P^{\mathrm{T}}\Omega^{-1}P]^{-1} \tag{14}$$

With this value M, the actual new variance $(\bar{\Sigma})$ can be evaluated as (He & Litterman, 2002)

$$\bar{\Sigma} = \Sigma + M \tag{15}$$

The final step in this process is to solve the mean-variance model by using the posterior distribution of the Black–Litterman. Having the new vector of expected returns and the covariance matrix, the new optimal portfolio can be estimated using the standard mean-variance method

$$\max w^{\mathrm{T}}\bar{\mu} - \frac{\delta}{2}w^{\mathrm{T}}\bar{\Sigma}w \tag{16}$$

The solution obtained using the first-order conditional is

$$w^{*} = \frac{1}{\delta}\bar{\Sigma}^{-1}\bar{\mu} \tag{17}$$

3.4. Idzorek

Idzorek describes an easy way to determine the level of trust by specifying the confidence level of each view as a percentage. This method is deemed to be much more intuitive (Walters, 2011).

Another problem commonly found in the Black–Litterman model is the determination of τ (Walters, 2010). Idzorek calibrates the confidence of a view so that w/τ ratio is equal to the variance of the portfolio view $(p^{\mathrm{T}}\Sigma p)$ (He & Litterman, 1999), rendering the scalar value of τ irrelevant. Idzorek still presents his formulas with τ, but it can be removed in order to simplify the equations (Walters, 2011).

4. Experiments with Brazilian stocks

Our process of composing a portfolio is divided in two stages: VDA and Black–Litterman. In the first step, the investor must answer a series of questions which will be used to create the views which, in turn, will be used in the Black–Litterman to build the new portfolio. Therefore, we apply the proposed methodology to construct the view of the Black–Litterman model by using these questionnaires.

4.1. Construction of the views

Two different sets of questions were prepared. One of them is used to identify what are the investor's preferences regarding specific sectors of the financial market, while the other aims at mapping the investor's perspective regarding the companies he/she intends to invest in.

The questionnaire about the sectors contains 3 questions. The first one on how the domestic scenario is favorable to that sector, the second essentially the same as the first but regarding the external scenario, and the last one on the growth expectation for that sector. It was conceived simple, so it can be answered by most people.

The other questionnaire, about the stocks, has 7 questions regarding risk, reliability, expected growth, innovation, profitability, management, and company employees. It was also conceived to be as simple as possible, comprising only a few questions.

In order to construct the views based on the answers given in the questionnaires, we use the FIQ of the ZAPROS-III method. We consider the FIQ as a rating through which we can quantify not only the classification of stocks, but also how much one stock is better than another one. The FIQ has to be transformed into a standard for the views, and the values are normalized between 0 and 1 to create an absolute view that represents the investor's perspective.

Table 1
FIQ and sector of the stocks

Stock	Sector	FIQ stock
Petrobras	Oil, gas and biofuels	19
ItaúUnibanco	Financial	15
Bradesco	Financial	26
Banco do Brasil	Financial	19
Vale	Mining	11
Itaúsa	Financial	31
Eletrobras	Electricity	39
Sid. Nacional	Steel mill and metallurgy	31
Cemig	Electricity	31
Oi	Telecommunications	46

For questionnaires such as the one about sectors, in which an alternative represents multiple stocks, we chose to equally divide the value attributed to the sector among the stocks. For example, if the value of the sector is considered to be 0.5 and we have two stocks, each one will have a value of 0.25.

Because confidence is a parameter that is somewhat complicated to determine even as a percentage, we decided to insert one more question in the questionnaires, in order to gauge how confident the participant is with his/her answers, thus obtaining the confidence of the view. To discretize the values, this question has four possible answers (very little confidence, little confidence, reasonably confident and very confident), which are associated with 25, 50, 75, and 100 percent of confidence, respectively.

The last parameter of the view is the expected return. To have sufficient impact on the portfolio, we chose 0.5 percent as its value. This value was chosen based on the expected return of the stocks and would be better calculated automatically, but it was not possible to conceive a general formula which was appropriate for every case.

4.2. Results

To better understand how this methodology would behave in practice, a test program was conceived to work with the Aranaú (Tamanini, 2010) and Akutan (Walters, 2011) frameworks. After the questionnaires have been filled out, the program generates a graphical report showing the optimal portfolio and its details.

The Black–Litterman analytical resolution of the optimal portfolio has some limitations: even while using a Lagrangian decomposition, like in Silva, Lee, and Pornrojnangkool (2009), the resulted formulation still cannot assure that the stocks's percentages are positive. Because of this limitation, the Jay Walters framework has to be extended to solve the problem using the CPLEX[2] solver.

We chose the 10 major companies negotiated in the Brazilian market[3]: Petrobras, Itaú Unibanco, Bradesco, Banco do Brasil, Vale, Itaúsa, Eletrobras, Sid. Nacional, Cemig and Oi. For each of these companies, we chose the stock with the highest negotiated volume to construct our portfolio and define the corresponding sector. These companies operate in the following sectors: electricity, financial, mining, oil, gas and biofuels, steel mill and metallurgy, and telecommunications. The covariances between the assets and the questionnaires are presented in the appendix.

After answering the questions, we obtained the FIQ values for the stocks Table 1 and for the sectors Table 2. The lower the FIQ,

[2] Version 12.4.0.0.
[3] In 2012 according to Forbes.

Table 2
FIQ of the sectors.

Sector	FIQ sector
Oil, gas and biofuels	12
Financial	1
Mining	7
Electricity	3
Steel mill and metallurgy	8
Telecommunications	6

Table 3
The expected return of the stocks.

Stock	Exp. ret.
Petrobras	−0.6107
ItaúUnibanco	0.1759
Bradesco	−0.145
Banco do Brasil	0.4877
Vale	−0.3962
Itaúsa	0.1924
Eletrobras	−0.1139
Sid. Nacional	−0.6274
Cemig	0.3786
Oi	−0.5714

Table 4
A summary of the views data.

Stock	View sector	View stocks
Petrobras	0.14	0.00
ItaúUnibanco	0.16	0.08
Bradesco	0.10	0.08
Banco do Brasil	0.14	0.08
Vale	0.18	0.14
Itaúsa	0.08	0.08
Eletrobras	0.04	0.13
Sid. Nacional	0.08	0.11
Cemig	0.08	0.13
Oi	0.00	0.17
Confidence	75 percent	75 percent
Return	−0.0013	−0.00083

the better the alternative. Therefore, these values were normalized using the difference between the maximum values of the companies.

The same normalization is done with the sector FIQ, but with the values being distributed for all the stocks in the sector.

The expected return were estimated as the mean of the daily returns for February 2013, and these values are shown in Table 3. The returns vary greatly, but this was not specifically for this month, as the Brazilian market was experiencing some instability.

Considering a confidence level of 75 percent for both the stocks and sector questionnaires. Finally the views are composed by the confidence level, the return and the normalized FIQ values for both the stocks and the sectors, which can be seen summarized in Table 4.

Inputting the calculated views into the Black–Litterman model, we obtain the optimal portfolio of the Fig. 4. To analyze how the portfolio changes, the equilibrium portfolio is presented in Fig. 3.

To analyze the sensitivity of our method, experiments were conducted, as we shall see. However, we must emphasize that an improvement in the qualification of an asset does not necessarily mean an increase of its percentage in the optimal portfolio, as this variation also depends on the correlation and on the stocks' return rates.

Answering the questionnaires with better expectations regarding the growth, the risk, the innovation, the profitability and the employees of Sid. Nacional, we obtained the portfolio in Fig. 5.

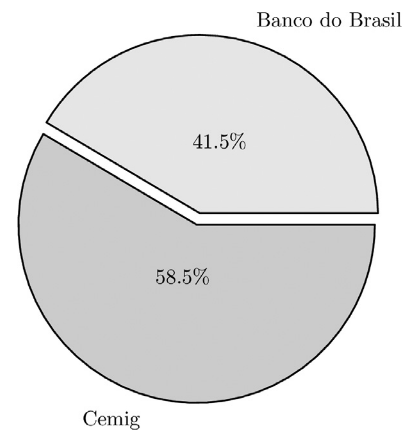

Fig. 3. Equilibrium portfolio.

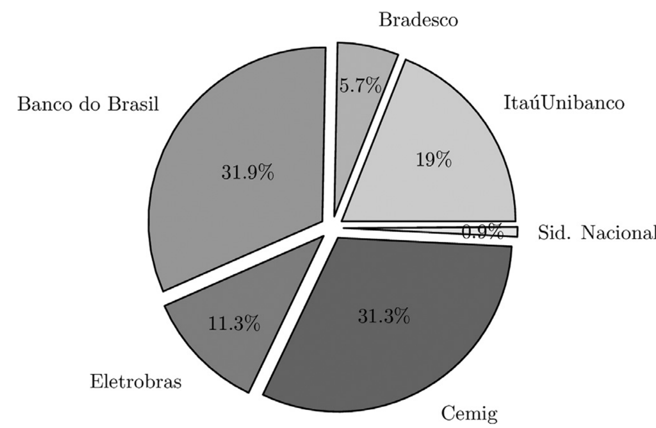

Fig. 4. The Black–Litterman portfolio with our views.

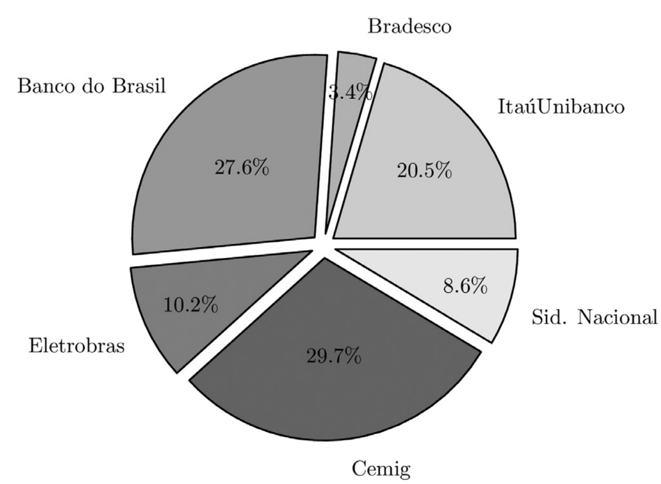

Fig. 5. Result of the increase in the qualification of Sid. Nacional.

The participation of Sid. Nacional's stocks in the portfolio increased from 0.9 percent to 8.6 percent. The increment was small because of the stock's equilibrium return and high correlation with Eletrobras.

The same thing happens if we increase the qualification of Oi, as it can be seen in Fig. 6. In this case, the questionnaire's, in Fig. A.8, answers changes from A3, B2, C2, D2, E2 and F3 to A1, B1, C1, D1, E1 and F1. The correlation between Oi and Bradesco is negative, which explains why Bradesco's percentage also increases.

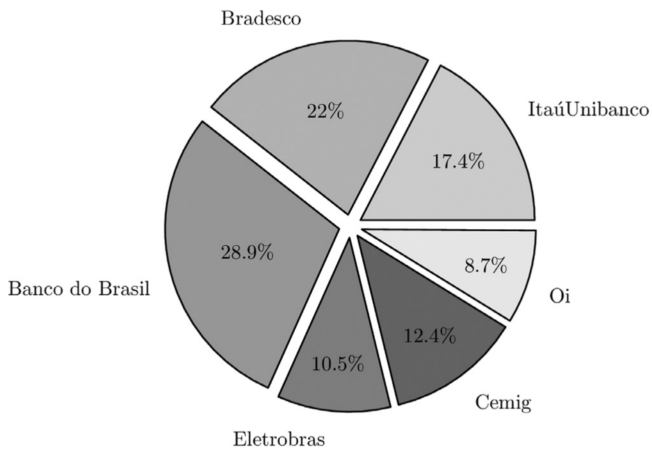

Fig. 6. Result of the increase in the qualification of Oi.

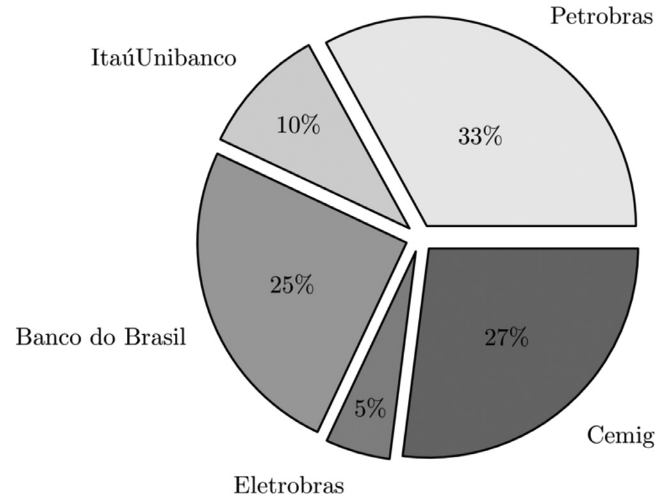

Fig. 7. Result of right scenario, where the investor guess is right.

Table 5
stocks returns for the period.

Asset	Ret. (percent)
Petrobras	4
ItaúUnibanco	−6
Bradesco	−13
Banco do Brasil	−7
Vale	−7
Itausa	−7
Eletrobras	−18
Sid. Nacional	−2
Cemig	−3
Oi	−44

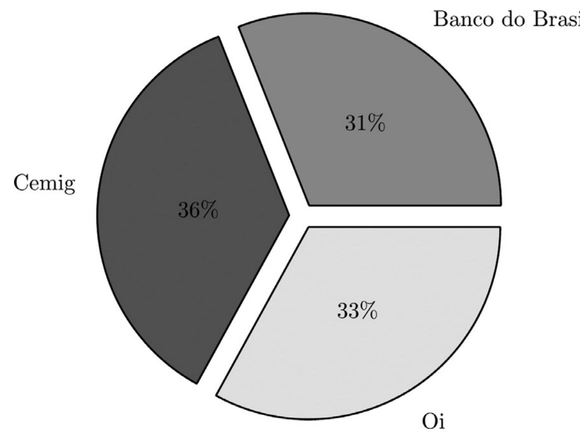

Fig. 8. Result of wrong scenario, where the investor guess is wrong.

Table 6
Sectors returns for the period.

Sector	Ret. (percent)
Oil, gas and biofuels	4
Financial	−8
Mining	−7
Electricity	−10
Steel mill and metallurgy	−2
Telecommunications	−44

Table 7
Return for the different scenarios and the market portfolio.

Portfolio	Ret. (percent)
Previous	93.04
Market portfolio	95.19
Right scenario	97.46
Wrong scenario	82.35

We have similar behavior when we increase the qualifications of the sectors, but in this case the change is less significant due to the return of the sectors' view and because the increase is distributed among all the sector's stocks.

It is possible to get similar behavior when the qualifications of the sectors are increased, but the changes on the optimal portfolio are less significant, due to the return of sectors view and because the increase is distributed among all the sector's stocks.

To analyze how the resultant portfolio would perform in different situations it was simulated two different views considering the future return of the stocks. It is assumed that the investor answers the questionnaires knowing the asset that will have the highest return and that is the only asset that he/she wants to invest. In the first scenario we have the best possible outcome, i.e. the investor guess was right and he/she invested on asset with highest return. In the second scenario the asset behavior is the contrary of what the investor was expecting, resulting on the worst portfolio return among all.

The performance was evaluated for a period of 6 months from February to September of 2013. In Table 5 the returns of the stocks are presented for this period and Table 6 shows the returns for the sectors.

Considering these values for the first scenario, the highest asset return is Petrobras and the sector with the highest return is Oil, gas and biofuels. For the second scenario, the worst asset return is Oi and the sector with the worst return is Telecommunications. Resulting portfolios obtained by answering the questionnaires, in accordance with those scenarios, are presented in Figs. 7 and 8.

Table 7 also compares these two portfolios with the Market Portfolio (optimized portfolio with the best Sharpe Ratio) and the portfolio generated before (Previous).

Analyzing the worst scenario, the problem of allocating the portfolio entirely in an active disregarding the risk becomes clear. Should the investor put everything on Oi, he/she would have lost 44 percent of the initial investment. Using the proposed methodology, losses would decrease to 18 percent. However, the gains from choosing to invest in Petrobras would decrease from 4 percent to −3 percent. These results suggest the risk of the portfolio decreases when the methodology is applied.

5. Conclusions

Although it is undeniable that the Black–Litterman technique is able to mitigate most of the shortcomings of the traditional method proposed by Markowitz, the construction of views can be a confusing process and depends largely on the investor's ability and knowledge.

We propose a new way to solve this problem by using questionnaires and the ZAPROS-III method to construct the views. Techniques like the VDA enable us to transform the answers provided in the sets of questions into views of the Black–Litterman model. A case study based on Brazilian stocks was conducted as a demonstration of how to use the methodology, and encouraging results were obtained.

The most important advantage of the proposed method is that it allows to create a portfolio based on the investor's own opinion and without the help of an expert. It also makes it easier for the investor to manifest his/her own opinion and facilitates the process, allowing more frequent portfolio modifications. Using different scenarios in the experiment we observe how our method mitigates the impact of poor estimation, when the investor's vision about the future is quite wrong. Without the method, it can often lead to very bad outcomes. Also, when the investor is right about his/her guess the method does not reduce too much the final portfolio return.

While the proposed approach presents a lot of advantages, it has also its limitations. The ZAPROS method is not adequate for a large number of alternatives or criteria, rendering this method applicable only to cases involving a few stocks and questions. This shortcoming is a relative one, as the focus of this work is to create a simple method. Besides, asking an investor too many questions may not be reasonable.

It would be interesting to have further work done in the sense of proposing or developing new ways to improve the acquisition of the necessary parameters for the Black–Litterman views, like the expected return. Future work could also put other risk functions to the test, such as the conditional value at risk, and analyze the model's behavior. Also the construction of the views can be done in multilevel using ORCLASS, or some other similar method, to select the stocks of each view and then use ZAPROS to determine the weight of the stocks in the views.

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Appendix A. Questionnaires

The questions used in this work are presented in Tables A.8 and A.9.

Appendix B. Covariances

The matrix of covariances is spited in two parts, in Tables B.10 and B.11.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.ejor.2016.06.018

Table A.8
Questionnaire about the stocks.

Criteria	Possible values
A. Risk	A1. Low risk A2. Medium risk A3. High risk
B. Stability	B1. Company with years of market experience and tradition B2. Company with some market time B3. Company with little market time
C. Expected growth	C1. Company with promising future and accelerated growth C2. Company that is expected some growth C3. Company which is not expected growth
D. Innovation	D1. The company invests heavily in R&D and always comes up with new ideas D2. The company invests little in R&D and new ideas usually arise D3. The company does not invest in R&D and almost never comes up new ideas
E. Profitability	E1. Company always transfers profits to shareholders E2. Company usually transfers profits to shareholders E3. Company almost never transfers profits to shareholders
F. Employees	F1. Highly qualified employees that are always motivated F2. Good employees that are usually motivated F3. Employees without much qualification and lack of motivation

Table A.9
Questionnaire about the sectors.

Criteria	Possible values
A. Internal market	A1. Very favorable to industry A2. Favorable to industry A3. Not favorable to industry
B. External market	B1. Very favorable to industry B2. Favorable to industry B3. Not favorable to industry
C. Expected growth	C1. It is expected a high growth C2. It is expected some growth C3. It is expected little or none growth

Table B.10
The first part of the matrix of covariances.

	Petrobras	ItaúUni.	Bradesco	B. do Brasil	Vale
Petrobras	0.00017	0.00011	0.00010	0.00005	0.00001
ItaúUnibanco	0.00011	0.00038	0.00032	0.00013	−0.00001
Bradesco	0.00010	0.00032	0.00032	0.00010	0.00003
Banco do Brasil	0.00005	0.00013	0.00010	0.00027	0.00005
Vale	0.00001	−0.00001	0.00003	0.00005	0.00029
Itausa	0.00011	0.00038	0.00032	0.00013	0.00000
Eletrobras	0.00009	0.00003	0.00001	0.00009	0.00027
Sid. Nacional	0.00001	0.00004	0.00002	0.00003	0.00016
Cemig	−0.00001	0.00008	0.00007	−0.00003	−0.00001
Oi	−0.00003	−0.00005	−0.00004	0.00003	−0.00001

Table B.11
The second part of the matrix of covariances.

	Petrobras	ItaúUni.	Bradesco	B. do Brasil	Vale
Petrobras	0.00011	0.00009	0.00001	−0.00001	−0.00003
ItaúUnibanco	0.00038	0.00003	0.00004	0.00008	−0.00005
Bradesco	0.00032	0.00001	0.00002	0.00007	−0.00004
Banco do Brasil	0.00013	0.00009	0.00003	−0.00003	0.00003
Vale	0.00000	0.00027	0.00016	−0.00001	−0.00001
Itausa	0.00037	0.00002	0.00004	0.00007	−0.00005
Eletrobras	0.00002	0.00057	0.00023	0.00008	0.00000
Sid. Nacional	0.00004	0.00023	0.00022	0.00007	−0.00006
Cemig	0.00007	0.00008	0.00007	0.00015	−0.00004
Oi	−0.00005	0.00000	−0.00006	−0.00004	0.00026

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