EE 133 - Digital Image Processing
Department of Electrical and Computer Engineering
Tufts University Spring 2017
Problem Set #2

Distributed: Feb. 2, 2017 Due: Feb. 16, 2017

Problem 2.1

Implement from scratch the following morphological operations:

- Erosion
- Dilation
- Opening
- Closing
- Thinning
- Skeletons

Verify that the methods work on `image1` located in the file `ps02s117_image.mat` distributed with this problem set. Additionally, use morphological methods to "denoise" (in the binary sense) and then find the skeletons of the "true" objects located in `image1`.

Problem 2.2

Implement morphological reconstruction by dilation and answer the following questions using `image3` in `ps02s117_image.mat` as the mask image.

1. Take as a marker image a line down the middle of the image. What is the result of reconstruction by dilation when the structuring element is a 3×3 box? What about a 7×7 box? How does the shape of the structuring element impact the result of the reconstruction by dilation?

2. Design a processing method based on reconstruction by dilation that finds all connected components in a binary image that contain vertical structures that are 10 or more pixels high.

Problem 2.3

Erosion of a set A by a structuring element B is a subset of A as long as the origin of B is contained in B. Give an example in which the erosion $A \check{\ominus} B$ lies outside or partially outside of A.

Problem 2.4

Show that another definition of erosion is

$$A \check{\ominus} B = \bigcup_{b \in B} (A)_{-b} \tag{2.4.1}$$

Problem 2.5

In addition to morphological reconstruction by dilation and the related operation of opening by reconstruction, one can define morphological reconstruction by erosion and closing by reconstruction. To do so, we start with the notion of geodesic erosion of size b defined recursively as

$$E_G^{(n)}(F) = E_G^{(1)}\left[E_G^{(n-1)}(F)\right] \tag{2.5.1}$$

$$E_G^{(1)}(F) = (F \check{\ominus} B) \cup G \tag{2.5.2}$$

where F is the marker image, G the mask image, and B the structuring element. Running the recursion until the point where $E_G^{(n)}(F) = E_G^{(n-1)}(F)$ defines $R_B^E(F)$, morphological reconstruction by erosion of a mask image G from a marker image F. Finally, closing by reconstruction is defined as

$$C_R^{(n)}(F) = R_F^E\left[F \oplus nB\right] \tag{2.5.3}$$

where $F \oplus nB$ indicated dilation of F by B n times.

1. Explain what morphological reconstruction by erosion does both in words and with a simple example or two.
2. If opening by reconstruction preserves the shapes of image components that remain after erosion, what does closing by reconstruction do? Again, please provide a brief explanation and illustration.

Problem 2.6

Problem 4.17 on page 211 in the Birchfield text.

Problem 2.7

Problem 4.24 on page 212 in the Birchfield text

Problem 2.8

Read sections 4.1.1 and 4.1.2 of the Birchfield text. On page 136 and 137, the author states "Similarly, by combining the commutativity and duality properties, it is easy to show that Minkowski subtraction can be computed by leaving $\neg\mathcal{A}$ stationary and instead shifting $\neg\mathcal{B}$." The mathematical form of this is provided by equation (4.33) in the text which states $\mathcal{A} \ominus \mathcal{B} = \neg\mathcal{B} \ominus \neg\mathcal{A}$.

1. Prove the two duality equations in equations (4.28) and (4.29) on page 135 of the text.
2. Prove formally that $\mathcal{A} \ominus \mathcal{B} = \neg\mathcal{B} \ominus \neg\mathcal{A}$.
3. Do problem 4.22 parts (a), (b), and (f) on page 212 of the Birchfield text.

Problem 2.9

Compute by hand the 2D convolution of the following two images

$$f(m,n) = \begin{cases} \max(m,-n) & |m+n| < 2 \\ 0 & \text{else} \end{cases}$$

$$h(m,n) = \begin{cases} m + (-1)^n & m,n \in \{0,1\} \\ 0 & \text{else} \end{cases}$$

Verify your results using the Matlab function `conv2`.