Administrative

- **MATLAB1**
  - See 'SubmittingProjects' folder for details on submitting
- **HW based on today's lecture is in today's folder – due next Monday**
  - Photos of two of the problems are uploaded separately
- **Change in office hours**
  - See doodle poll →

Tuesday 12-2 (current)	Thursday 3-5	Friday 1-3	Friday 3-5
2	8	8	9
☐	☐	☐	☐
		✓	✓
(✓)		✓	✓
	✓		✓
	✓	✓	✓
✓		(✓)	✓
	✓		
			✓
	✓		✓
	✓	✓	
	✓	✓	
	✓	(✓)	(✓)
	✓	✓	✓

EE-125: Digital Signal Processing

Lecture 4: Fourier Transform, Sampling review

Professor Tracey

Tufts

Lecture 4 : Outline

- Continue with Discrete-time Fourier transform (DTFT)
  - Transform pairs
  - Symmetry properties
  - Other properties

- Sampling theory
  - Matlab2 explores this topic

Reminder: different transforms

	Periodic	**Aperiodic**
CT	CT Fourier series x(t), c_k	CTFT x(t), X(f)
DT	DT Fourier series x(n), c_k	DTFT x(n), X(w)

Chap 7

Our focus now

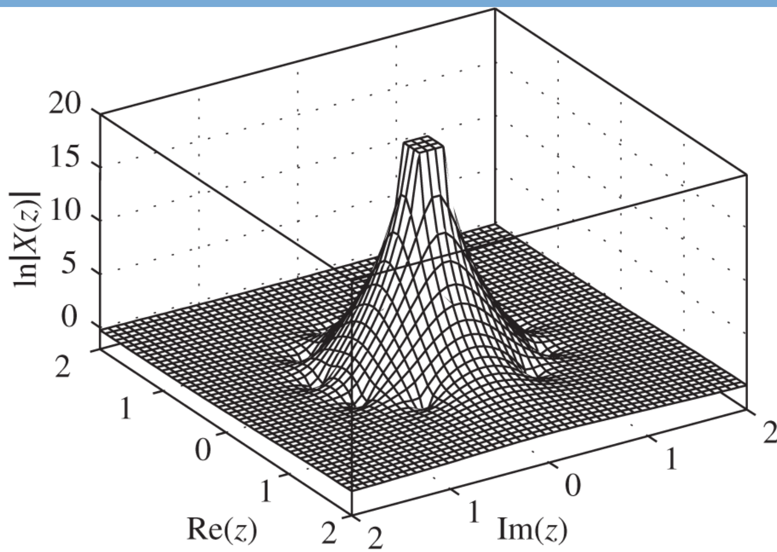
Definition of the DTFT:

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

Tufts

Relationship of Z to DTFT



Consequence of this:

DTFT is 2 pi periodic in radian frequency w

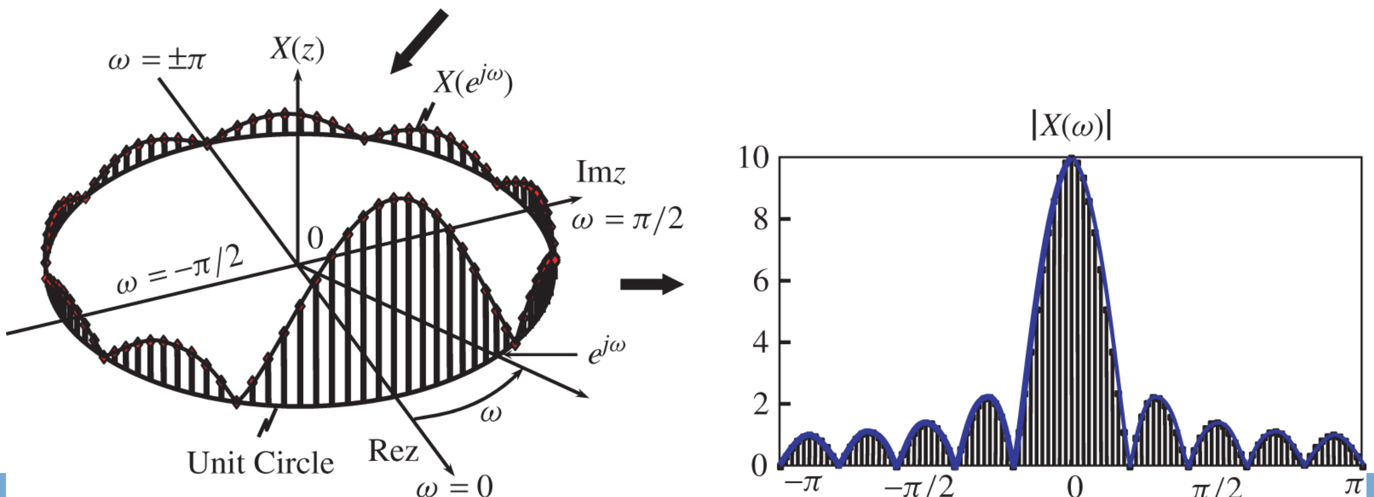


Figure 4.2.9 relationship between $X(z)$ and $X(\omega)$ for the sequence in Example 4.2.4, with $A = 1$ and $L = 10$

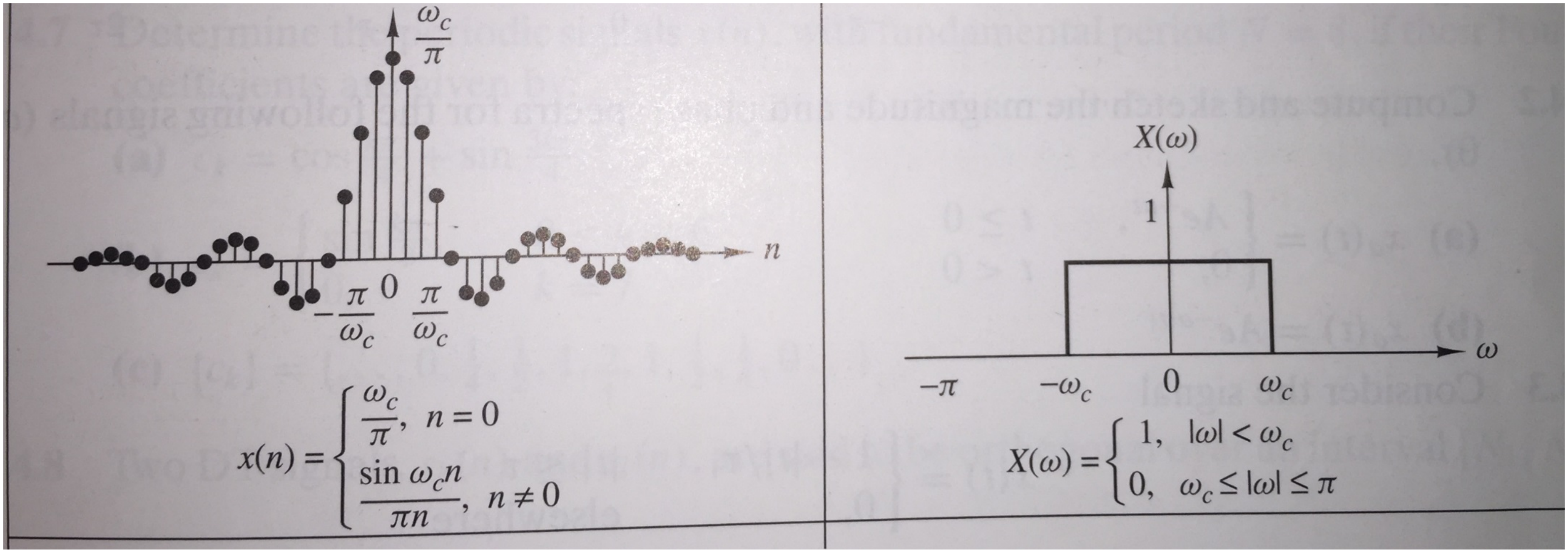
Transform pairs – more info

1) Transform pair video at
https://youtu.be/F1qfDnVNWPY

2) DemoDTFT codes on Trunk with today's notes
- Each has a line marked 'CHANGE THIS NUMBER' with the parameter you want to vary

Fourier transform pairs – boxcar in frequency (Table 4.6 in book)

REMEMBER:
REPEATS EVERY 2p

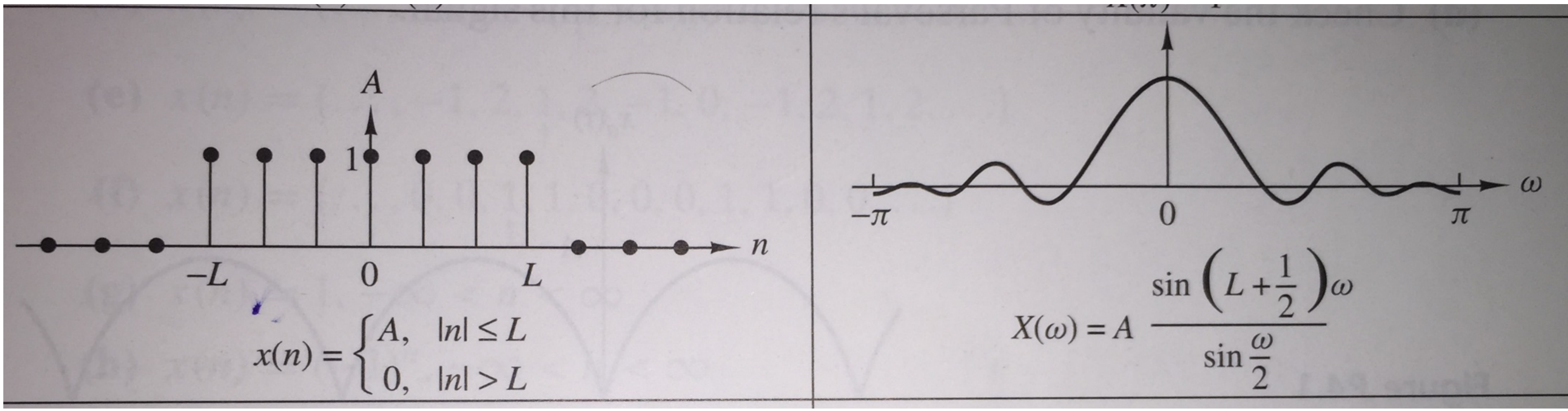


$$x(n) = \begin{cases} \frac{\omega_c}{\pi}, & n = 0 \\ \frac{\sin \omega_c n}{\pi n}, & n \neq 0 \end{cases}$$

$$X(\omega) = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & \omega_c \leq |\omega| \leq \pi \end{cases}$$

Form: sin() / time

Fourier transform pairs – boxcar in time (Table 4.6 in book)



Form: sin() / sin ()

Why?

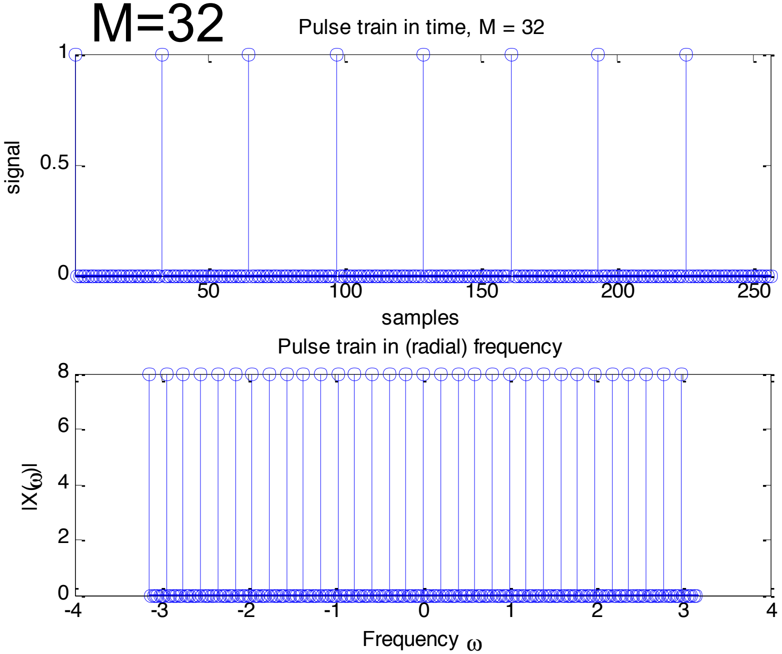
Fourier transform pairs – Pulse trains (see book)

- See book for derivation: result is

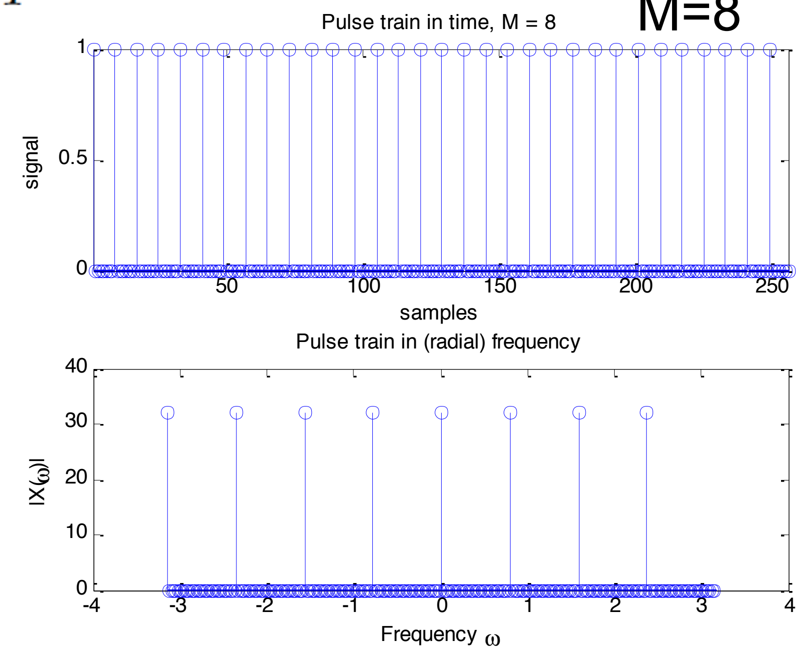
$$x(n) = \sum_{k=-\infty}^{\infty} \delta(n - kM) \Leftrightarrow X(\omega) = \frac{2\pi}{M} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_M)$$

where $$\omega_M \equiv \frac{2\pi}{M}$$

M=32



M=8



DTFT Symmetry Properties

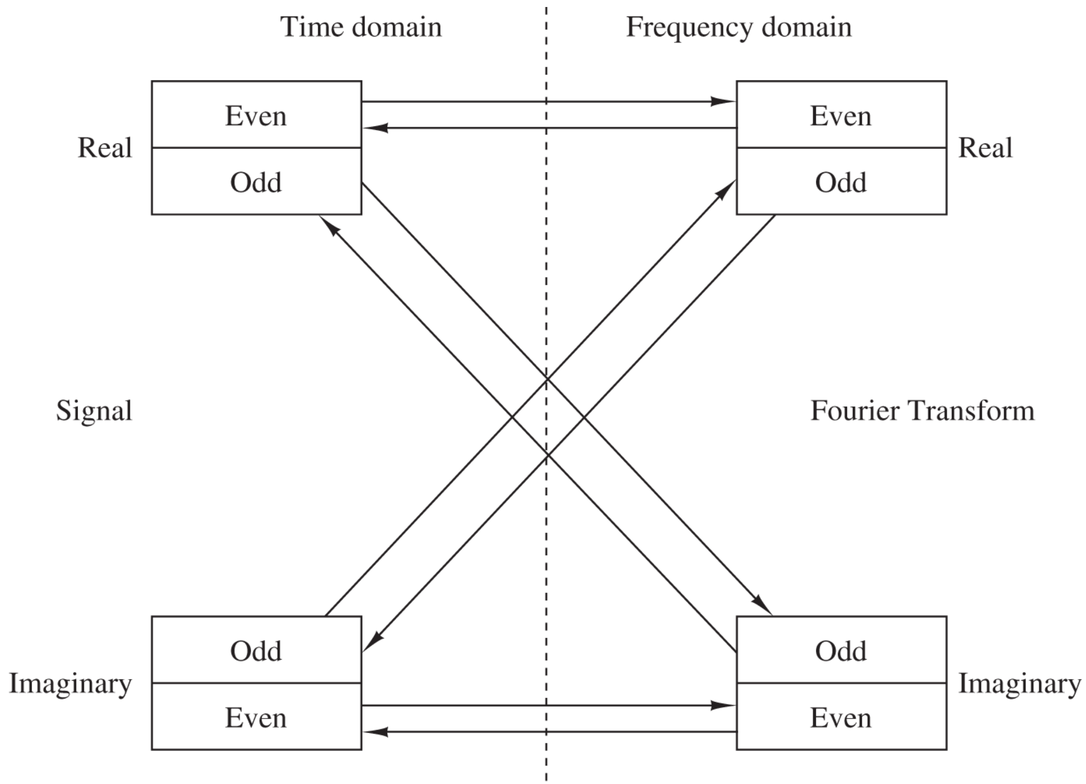


Figure 4.4.2 Summary of symmetry properties for the Fourier transform.

- **Most important**: the FT of a real signal has a complex conjugate F.T. i.e. H(w)* = H(-w). Will appear on exams! (and real life)

DTFT Symmetry Properties

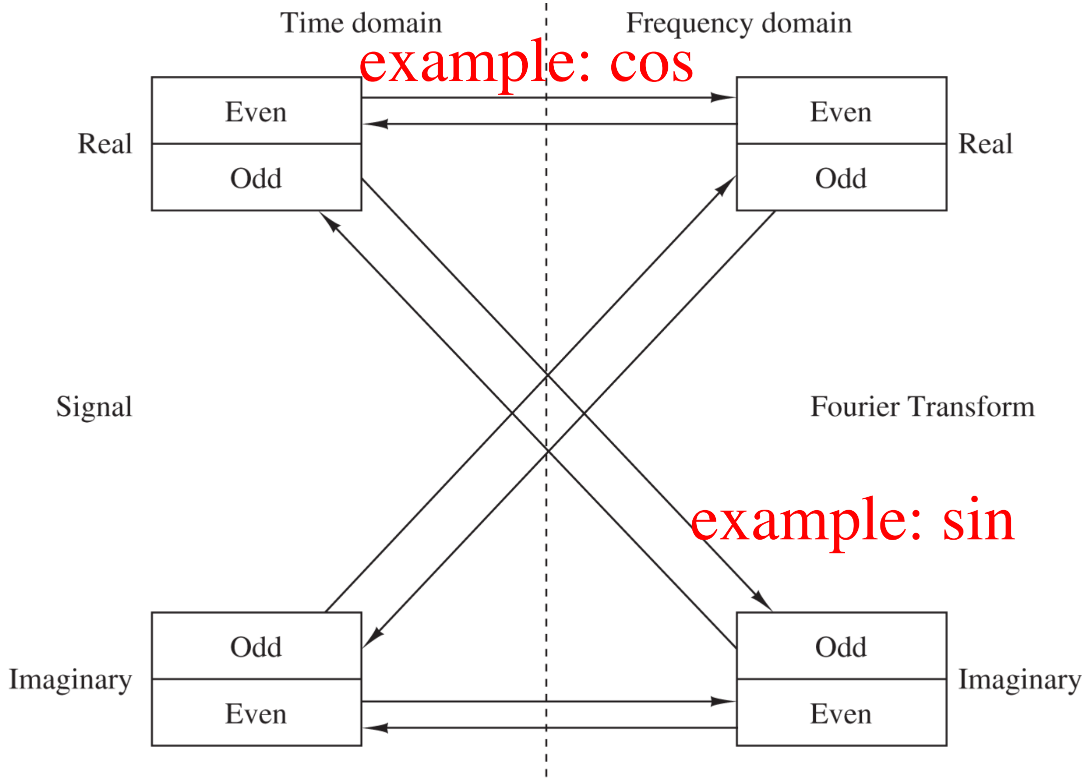


Figure 4.4.2 Summary of symmetry properties for the Fourier transform.

- **Most important**: the FT of a real signal has a complex conjugate F.T. i.e. H(w)* = H(-w). Will appear on exams! (and real life)

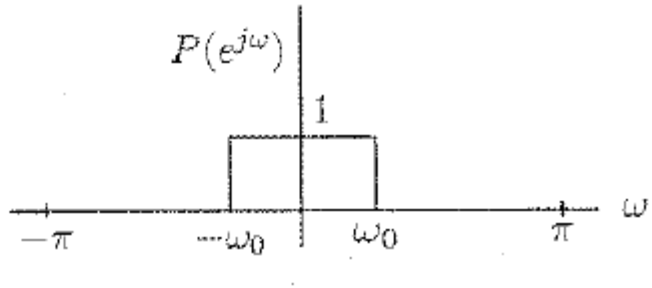
DTFT properties (see P&M Table 4.5)

- Linearity: a x1(n) + b x2(n) <-> a X1(w) + b X2(w)
- Time shifting: x(n-k) <-> exp(-j w k) X(w)
- Frequency shift: exp(j w0 n) x(n) < - > X(w−w0)
- Time reversal: x(-n) <-> X(-w)
- Parseval: S(energy in time) = S (energy in frequency)
- Convolution and multiplication relations - Very useful!

x1 * x2 <-> X1(w) X2(w)
x1(n) x2(n) <-> X1(w) * X2(w)

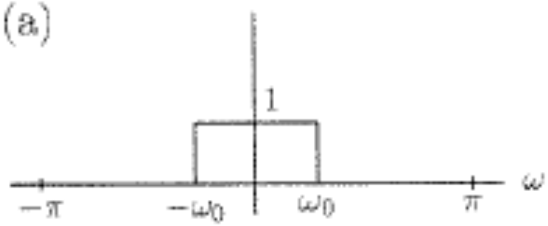
Convolution <-> Multiplication

- Say p(n) has the following transform, P(w):

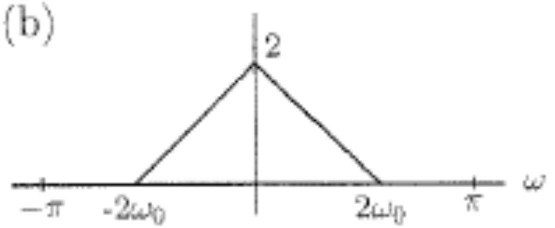


- Which of these is the Fourier transform of p(n) * p(n) ?

(a)



(b)

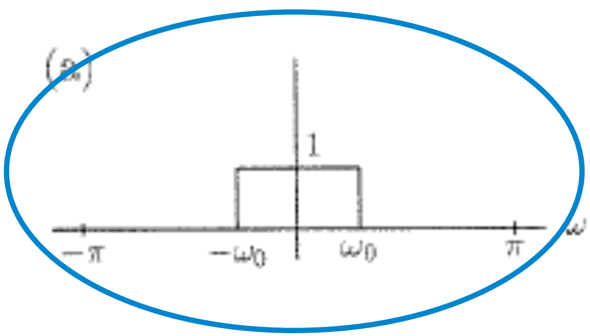


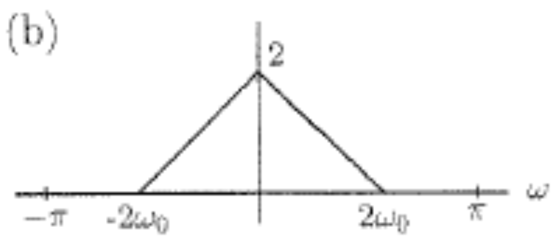
Convolution <-> Multiplication

p*p is convolution in time – so multiply in frequency

Note: Linear systems can attenuate or boost response at a frequency, but can't shift energy to new frequencies

- What is Fourier transform of p * p ?





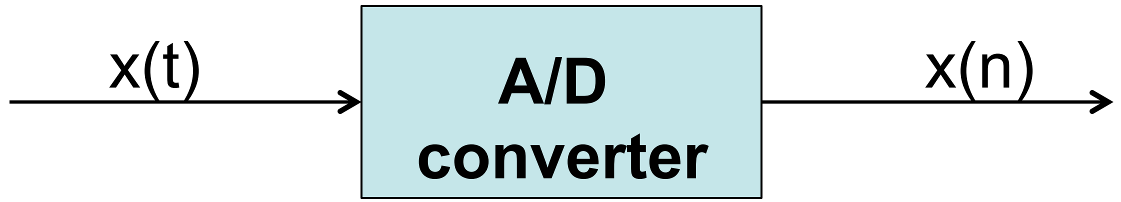
![Tufts]

Lecture 4 : Outline

- Finish Discrete-time Fourier transform (DTFT)
  - Definition
  - Useful transform pairs
  - Symmetry properties
  - Other properties

- Sampling theory

Questions (from last lecture)



Continuous time (CT)

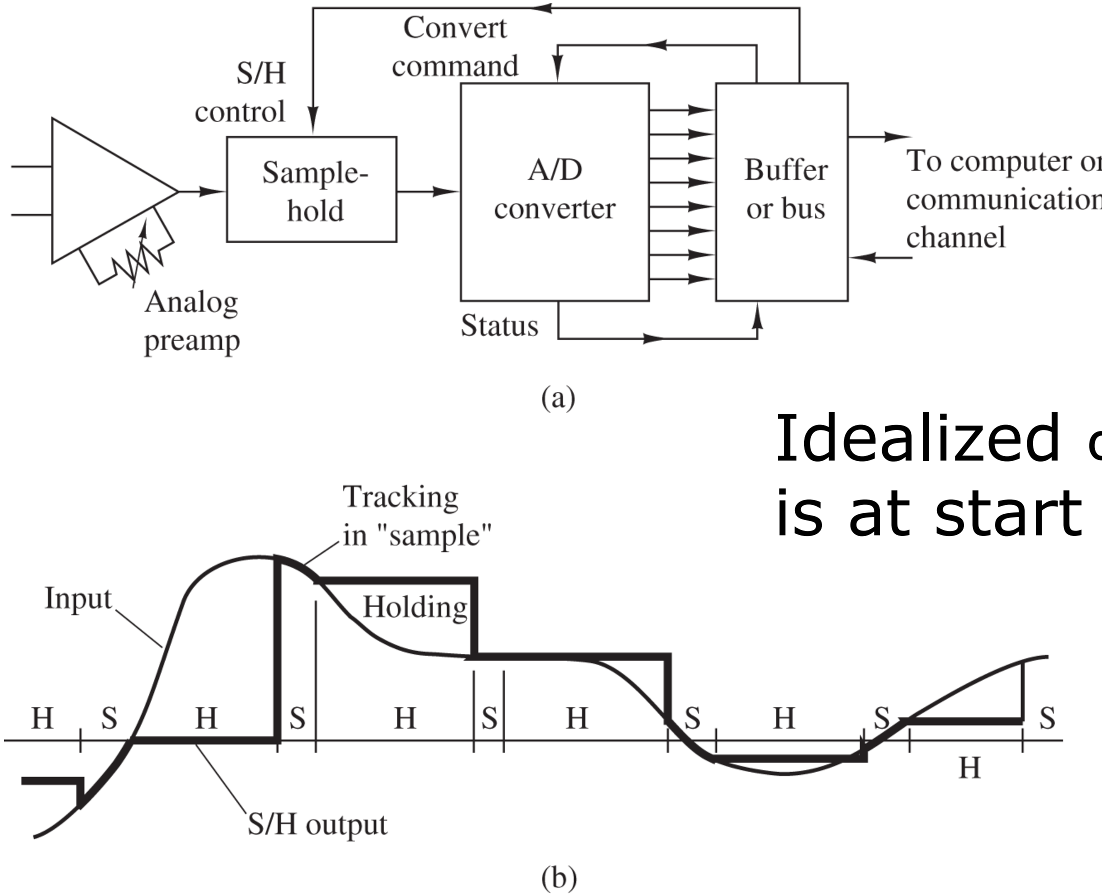
x(t), X(f)

Discrete time (DT)

x(n), X(w)

1) How are CT quantities (time *t*, frequency *f*) related to DT quantities (sample *n*, radial frequency w)?
2) How does the process of sampling affect the frequency response?

Practical A/D: Sample and Hold



Idealized d(t) sample is at start of 'hold'

Figure 6.3.1 (a) Block diagram of basic elements of an A/D converter; (b) time-domain response of an ideal S/H circuit.

Practical A/D: quantization

- A/D output is in integer "counts", or levels
- Spacing of levels is the quantization step:
  D = (Xmax-Xmin)/(L-1)

 for L levels (figure shows L=8)

Lots more detail in P&M 6.3

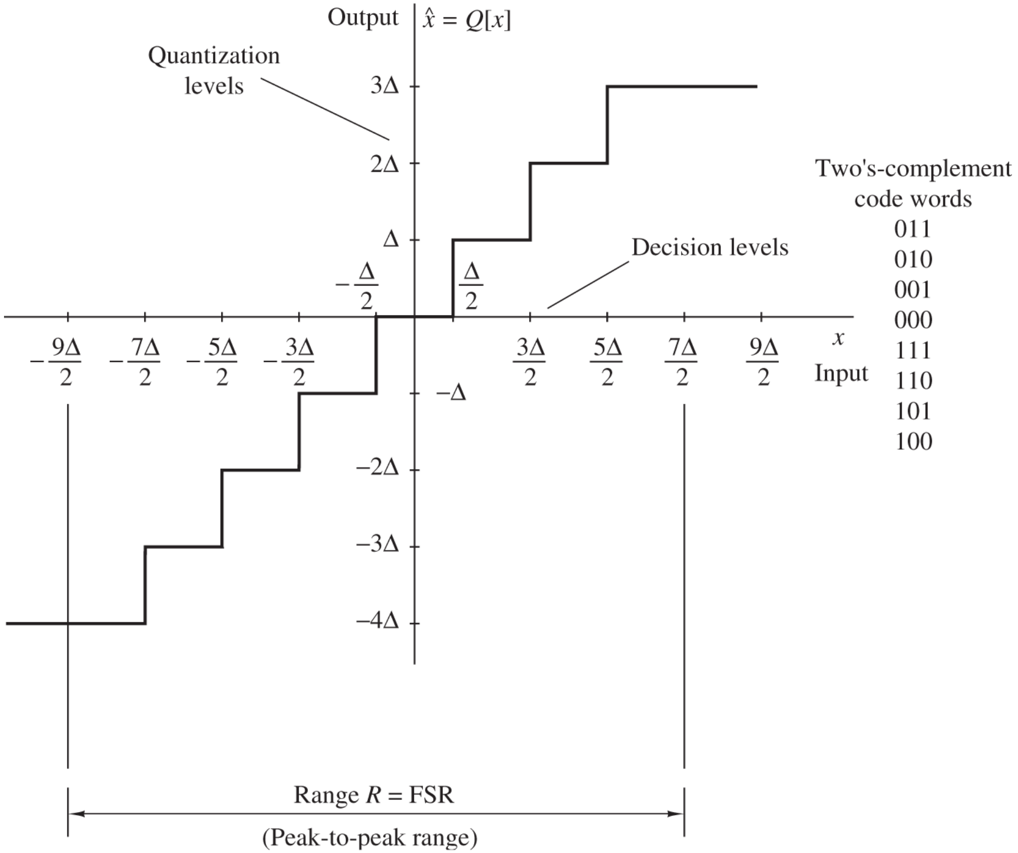


Figure 6.3.3 Example of a midtread quantizer.