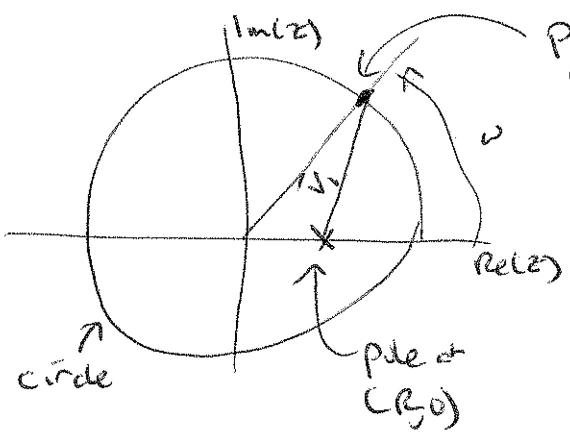
Detailed look: why does $|H(\omega)|$ change faster (near a pole) as pole approaches unit circle

example

pole on real axis $R<1$



$\vec{V}_1$ = vector from pole to point on circle (angle ω)

$$|v_1| = \sqrt{(\cos\omega - R)^2 + (\sin\omega - 0)^2}$$

$$= \sqrt{\cos^2\omega + \sin^2\omega + R^2 - 2R\cos\omega}$$

$$= \sqrt{1 + R^2 - 2R\cos\omega}$$

minimum at $\omega=0$: $|v_1| = \sqrt{1+R^2-2R} = \sqrt{(1-R)^2}$
$= (1-R)$ // makes sense...

for $\omega \neq 0$; as $\omega\uparrow$, $\cos\downarrow$, so $|v_1|\uparrow$

Rate of change with ω

$$\frac{d|v_1|}{d\omega} = \frac{1}{2}(1+R^2-2R\cos\omega)^{-1/2} \frac{d}{d\omega}(1+R^2-2R\cos\omega)$$

$$= \frac{+2R\sin\omega}{2\sqrt{1+R^2-2R\cos\omega}}$$

as R increases, (but is <1), the numerator $\uparrow$ and the denominator $\downarrow$

So, $\frac{d|v_1|}{d\omega}$ increases