Frequency sampling filter design

Before, we've looked at $H_d(\omega)$ forms that give analytic results for $h_d(n)$.

→ Now, we will specify $H_d(\omega)$ at a collection of points in frequency.

→ we will then inverse transform (IDFT) on the computer to find $h(n)$.

Several variations:

1) specify $H_d(\omega)$ at evenly spaced samples.
   → Make sure $H_d(\omega)$ is conjugate symmetric
   → Make sure $H_d(\omega)$ has a linear phase shift consistent with desired shift for causality

   → take IDFT / IFFT

2) Do the above but use many points in frequency* → gives long $h_d(n)$

   Then, apply window

$$h(n) = \underbrace{h_d(n)}_{\text{long, finite}} \; \underbrace{w(n)}_{\text{shorter}}$$

This is matlab's 'fir2'

* we'll see why later.

<u>more variations</u>

3) specifying $H_d(\omega)$ at unequally spaced points (not in book)

\# points = filter ~~order~~ length

at each point, we know

$$H(\omega_k) = \sum_{n=0}^{L-1} h(n)\, e^{-j\omega_k n}$$

if ~~filter is linear phase & even length~~, eqn is ~~in ter~~

we can collect L points, L eqns and solve.

Note we'll generally impose linear phase, etc. by turning the $e^{-j\omega_k n}$ into a cos( )

4) Develop specialized formulas — P&M 10.2.3

The idea is: pick the form we want (real, even or odd length, symmetric or antisymmetric)

For each case, specialized equations are found that relate sampled points to $h(n)$

<u>question</u>: 1) which points do we sample
2) transition band behavior

(5) FIR design by frequency sampling

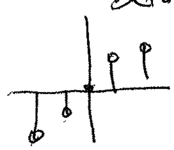
<u>Example:</u> (may want to copy, as related to HW)

Say we want <u>antisymmetric</u> filter $M=5$, with

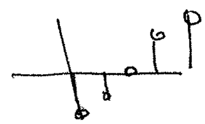
$$|H(\omega)| = 1 \text{ at } \omega = \pi/2 \text{ and } \omega = \pi/4$$

odd, antisymmetric filter always has middle point $=0$

before shift



causal, shifted version



also, because of symmetry, $M=5$ odd, antisymm. filter has only 2 degrees of freedom.

the (real) response is:

$$H_R(\omega) = \sum_{n=0}^{4} h(n)\, e^{-j\omega(n-2)}$$

$$= 2\sum_{n=0}^{1} h(n) \sin\omega(2-n) \quad \Leftarrow \text{book, eq. } 10.2.12$$

<u>note:</u> for $\omega=0$ or $\omega=\pi$, $H_R(\omega)=0$!
for any odd, antisymmetric filter.
Similar properties hold for ~~even~~, other combinations

plus in $\omega = \pi/2$:

$$2\{h(0)\underbrace{\sin\tfrac{\pi}{2}2}_{0} + h(1)\overbrace{\sin\tfrac{\pi}{2}(2-1)}^{1}\} = 1$$

so $2h(1) = 1$
$h(1) = 1/2$

plus in $\omega = \pi/4$

$$2[h(0)\sin\tfrac{\pi}{2} + h(1)\sin\tfrac{\pi}{4}] = 1$$

$$h(0) = \tfrac{1}{2} - \tfrac{1}{2}\sin\tfrac{\pi}{4} = 0.1464$$

So $h = [0.1464 \quad 0.5 \quad 0 \quad -0.5 \quad -0.1464]$

Frequency sampling examples – matlab

1) strength – arbitrary shapes allowed
2) danger – oscillations between sampled points