

# Administrative stuff

- Matlab4 & Matlab5 grades will be posted tonight
- Brief discussion of Matlab6
- Reminder of assignments for rest of term:
  - HW due next Monday
  - Matlab6 due next Monday
  - Exam 3 next Wednesday
  - Matlab7 due Dec 20

**EE-125:  
Digital Signal Processing**

**Lecture:  
IIR wrapup, and  
Sampling rate conversion**

**Professor Tracey**

**Tufts**

# Outline

- Brief wrap-up of IIRs from last time
- Changing the sampling rate (P&M 6.5, more detail in Chapter 11)
  - Motivation
  - Reducing sampling rate by an integer (downsampling)
  - Increasing sampling rate by an integer (upsampling)
  - Changing rate by a ratio of integers

# Wrapping up IIR

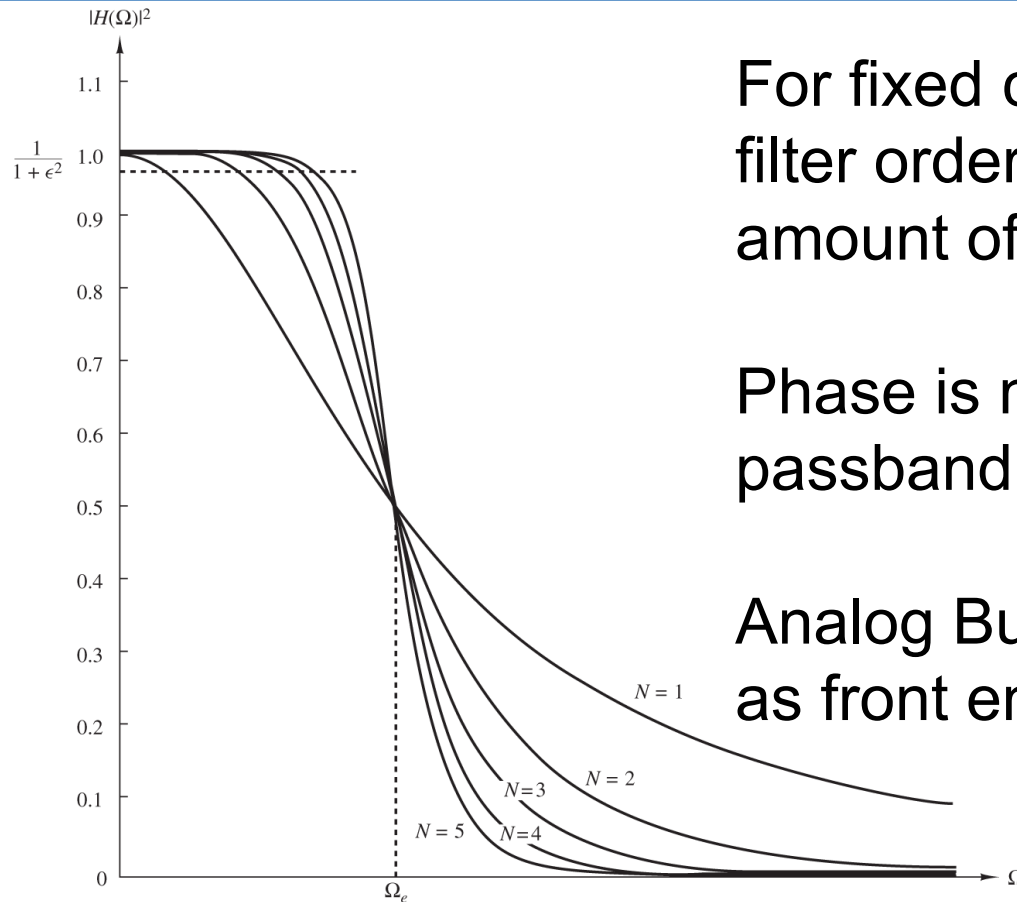
- Let's briefly review different commonly used IIR filter types
- I'll have a very easy multiple choice question about this on exam 3

# Common IIR Filter Types (P&M 10.3.4)

(KNOW THIS FOR EXAM!!!)

Type	Characteristic
Butterworth	Monotonic, smooth response; <i>maximally flat</i> at $F=0$ , Infinity (N-1 zero derivatives)
Chebyshev Type I	Minimized absolute difference in <b>passband</b> ; passband ripple, monotonic in stopband
Chebyshev Type II	Minimized absolute difference in <b>stopband</b> ; via stopband ripple, monotonic in passband
Elliptical	Lowest filter order for given transition band; equiripple in both stop and pass band; optimized for magnitude (phase may be distorted)
Bessel	Linear phase within passband; at expense of magnitude (rolloff is slower than others)

# Butterworth



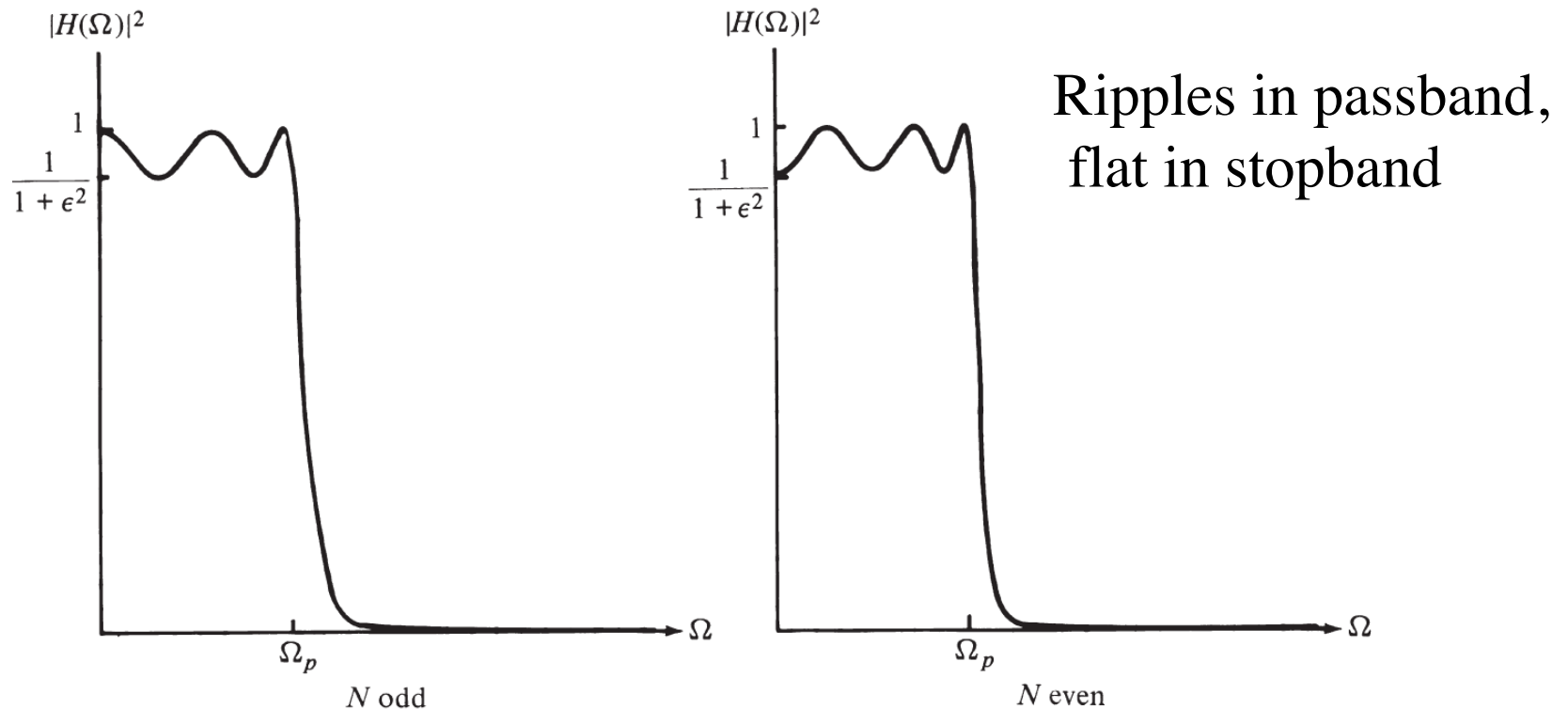
For fixed cutoff frequency,  
filter order  $N$  determines  
amount of attenuation

Phase is not linear in  
passband, but is not too bad..

Analog Butterworths often used  
as front end in biomed systems

Figure 10.3.10 Frequency response of Butterworth filters.

# Chebyshev, type I – poles only



**Figure 10.3.11** Type I Chebyshev filter characteristic.

# Chebyshev, type II – poles and zeros

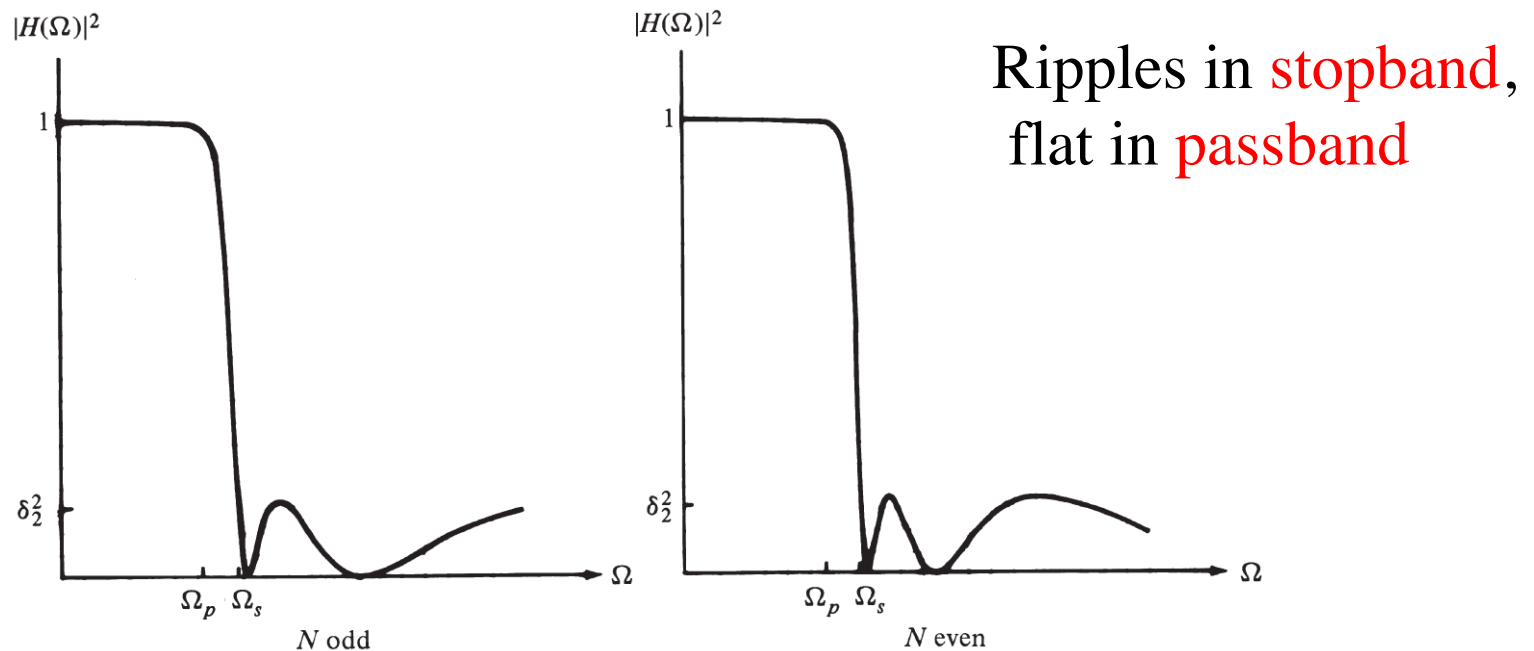
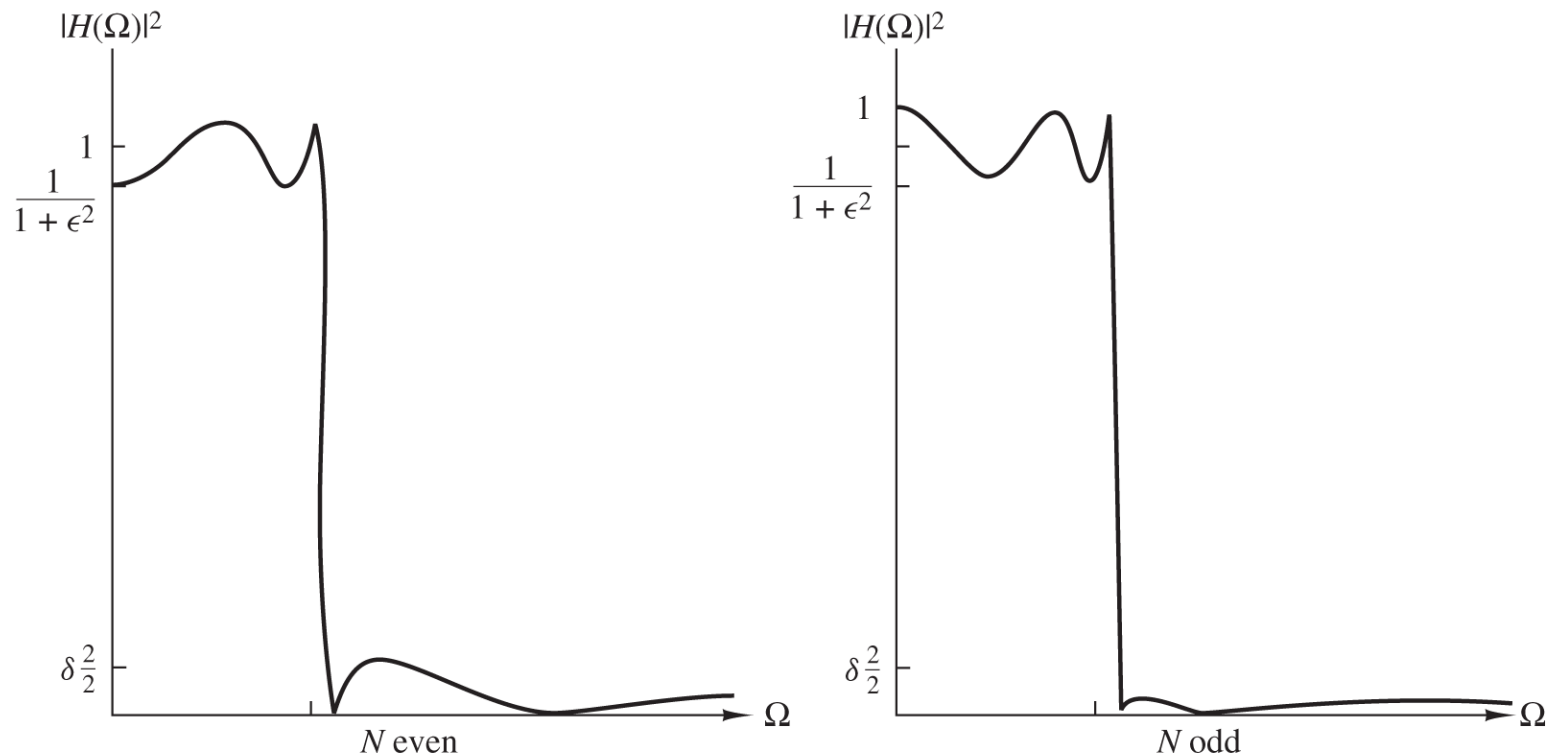


Figure 10.3.13 Type II Chebyshev filters.



# Elliptical filters – optimally fast transition, ripple in both stop and pass bands



**Figure 10.3.14** Magnitude-squared frequency characteristics of elliptic filters.

# Bessel – linear phase in passband

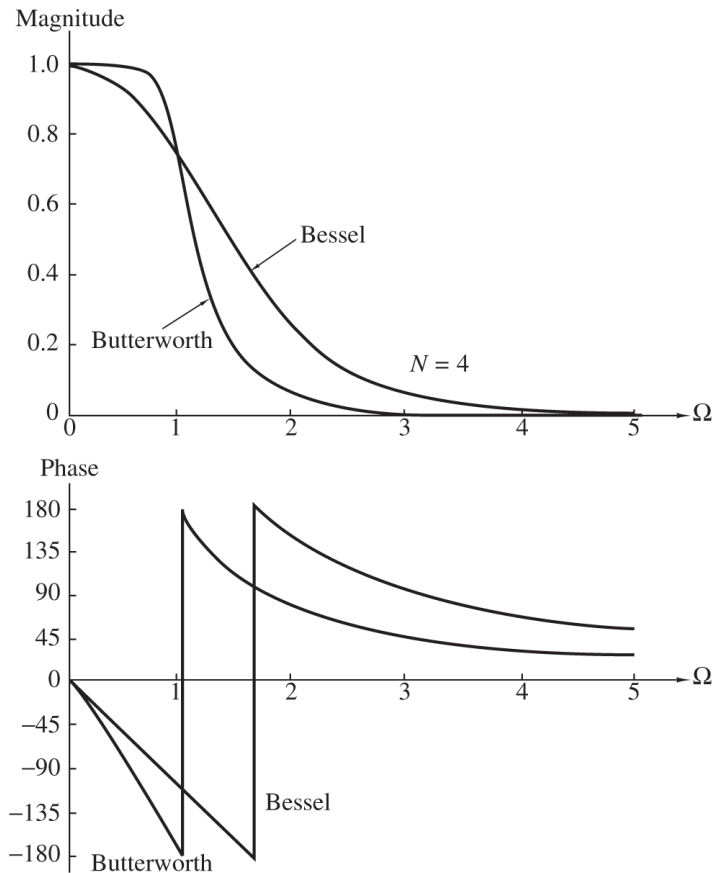


Figure 10.3.15 Magnitude and phase responses of Bessel and Butterworth filters of order  $N = 4$ .

Cost of linear phase:  
Slower rolloff in amplitude

Interesting if you are a lot  
about phase but can't do  
filtfilt 'trick'

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# Some final points on IIR...

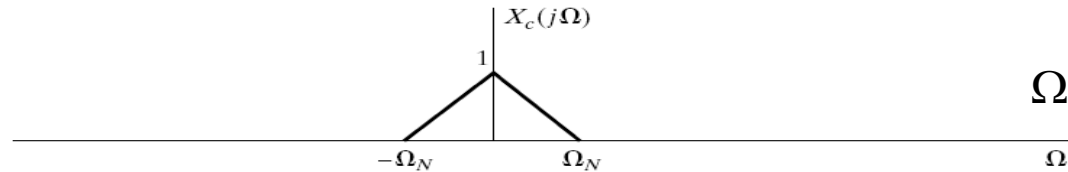
- Mapping above let us translate existing analog designs into digital filters. What if we want to start from scratch?
- If we know the desired impulse response  $h(n)$ , we can come up with a pole/zero (or all-pole model that fits it).
  - Example: Prony's method models data as sum of damped exponentials. Can be useful in modeling data (tires, ocean modes)
- If we have a desired frequency response, we can use the Yule-Walker method. Comes up with IIR that matches desired; may not be optimal
- An interesting article on the history and stability issues in IIR: "The rise and fall of recursive digital filters," Charles M. Rader, IEEE Signal Processing magazine, 2006

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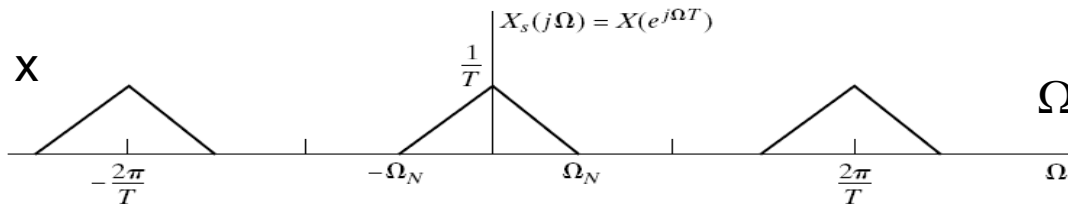
# Frequency domain look at downsampling (DS): no prefiltering / no aliasing

Analog (CT) signal



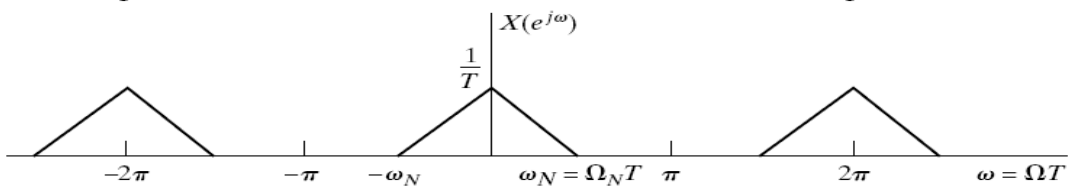
$\Omega = 2\pi F$ ,  $F$  in Hz

Analog (CT) signal x pulse train (CTFT)



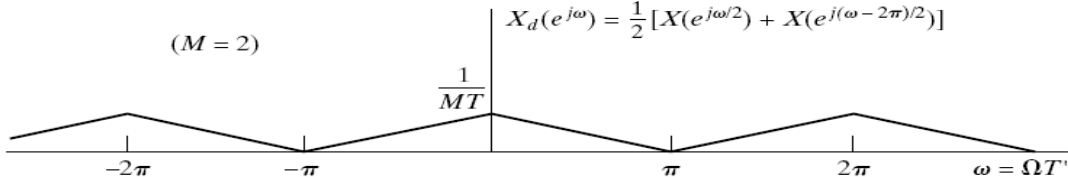
$\Omega = 2\pi F$ ,  $F$  in Hz

Sampled DT signal



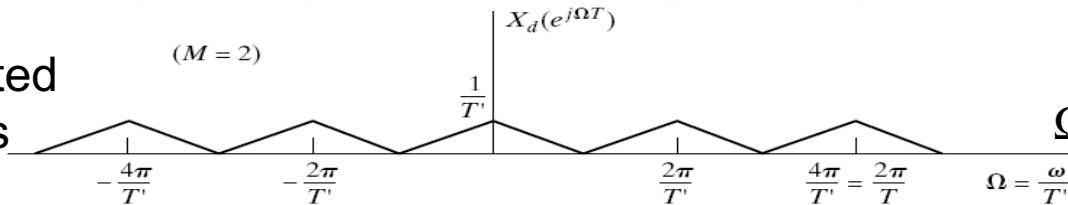
$(M = 2)$

DS, no prefilter



$(M = 2)$

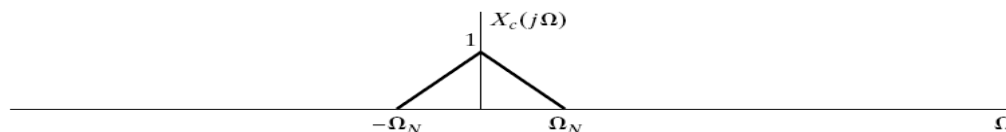
Equivalent converted back to continuous frequency



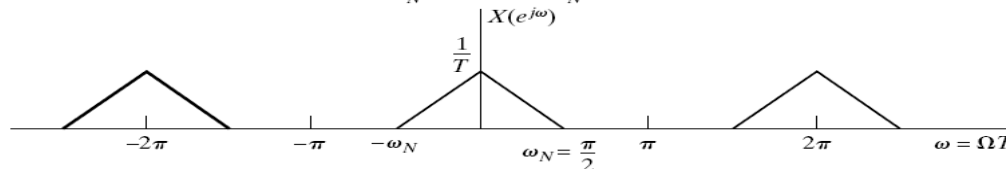
$\Omega = 2\pi F$ ,  $F$  in Hz

# Frequency domain look at downsampling (DS): with (possible) aliasing

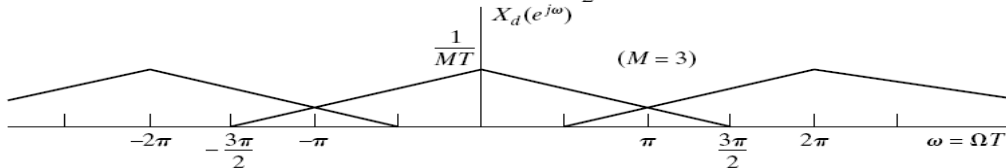
Analog (CT) signal



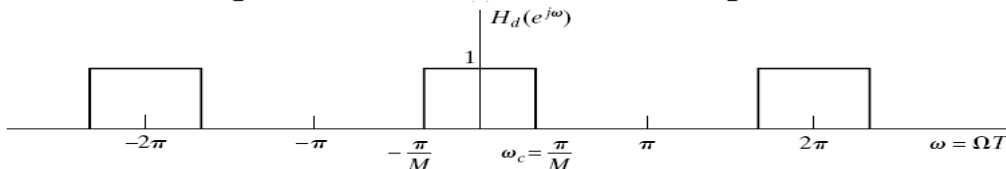
Sampled DT signal



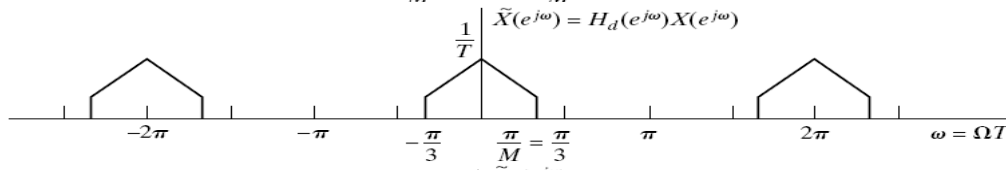
DS, no prefilter



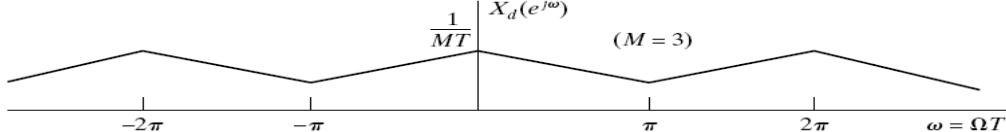
Ideal prefilter shape



Effect of prefilter  
(before DS)



DS, after prefilter



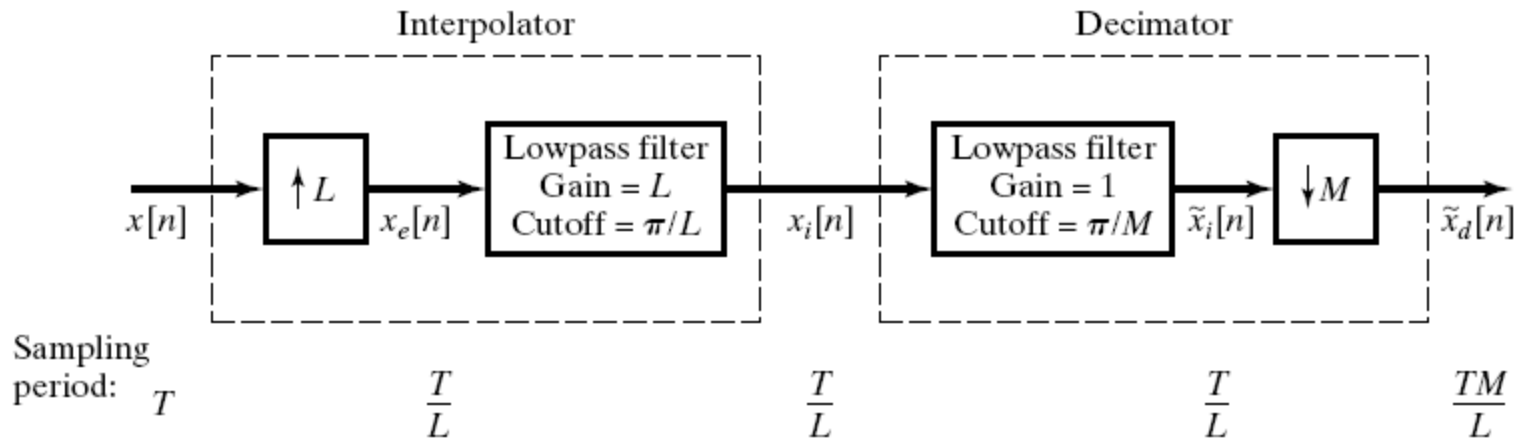
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# Changing Sampling rate by fraction (L/M) – matlab 'resample' function

We could go up by L, down by M.....



But easier is to combine the filters

