EE-125: Digital Signal Processing

Lecture 9:

Filter design by pole/zero placement

Tufts

Administrative

- Will post HW due Monday – on this week's lectures
- Space available on tour of Haystack radio observatory – highly recommended. Let ECE office know if you want to go
  - Friday October, 27th 2017, starts @ 4 pm
  - MIT Haystack Observatory,99 Millstone Rd, Westford, MA 01886

Basic filter types - idealized

lowpass

highpass

bandpass

bandstop

all-pass

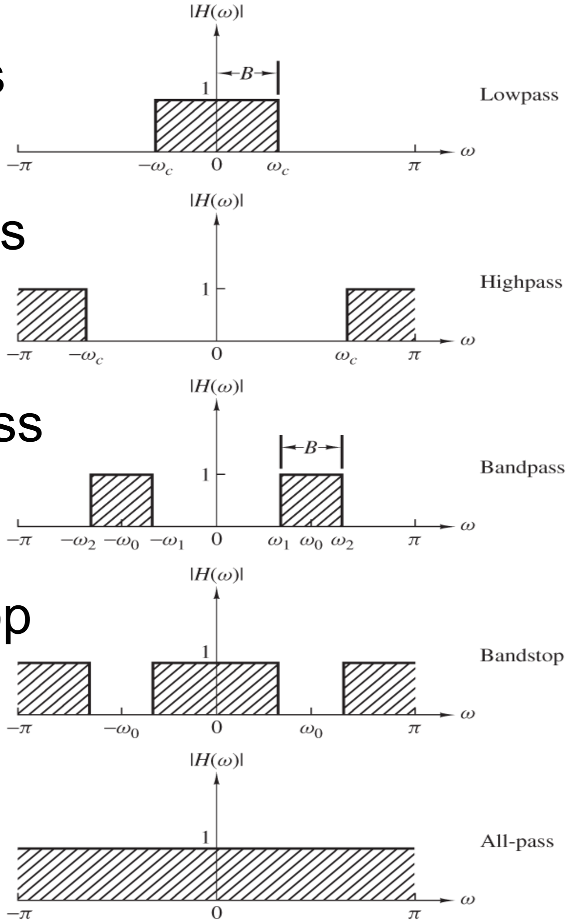

Figure 5.4.1 Magnitude responses for some ideal frequency-selective discrete-time filters.

- These filters have infinitely sharp transitions from ‘pass’ to ‘stop’ bands – unrealistic
- But, terminology is useful

Filter design by pole-zero placement

- Basic idea is as follows:
  - Put poles near frequencies you want to emphasize
  - Put zeros near frequencies you want to suppress
  - Add a constraint on the gain
  - Make sure filter is stable and has real-valued h(n)

- We'll review design of H(w) for:
  - Bandpass filters / digital resonators (P&M 5.4.3)
  - Notch filters (P&M 5.4.4)
  - Comb filters (P&M 5.4.5)
  - Allpass filters (P&M 5.4.6)

Comb filters – 1

- Definition: a comb filter is a filter where nulls (zeros) occur periodically across the frequency band – like teeth in a comb
- Design is a little different; we don't place poles/zeros directly
- Instead, consider a moving average FIR filter:

$$y(n) = \frac{1}{M+1} \sum_{k=0}^{M} x(n-k)$$

- System function of this is:

$$H(z) = \frac{1}{M+1} \frac{1 - z^{-(M+1)}}{1 - z^{-1}}$$

- Then $H(\omega)$ is:

$$H(\omega) = \frac{e^{-j\omega M/2}}{M+1} \frac{sin\omega(\frac{M+1}{2})}{sin\omega/2}$$

- We find zeros (nulls) from points where the sin goes to 0. Note the one pole is cancelled by a zero.

Comb filters - 2

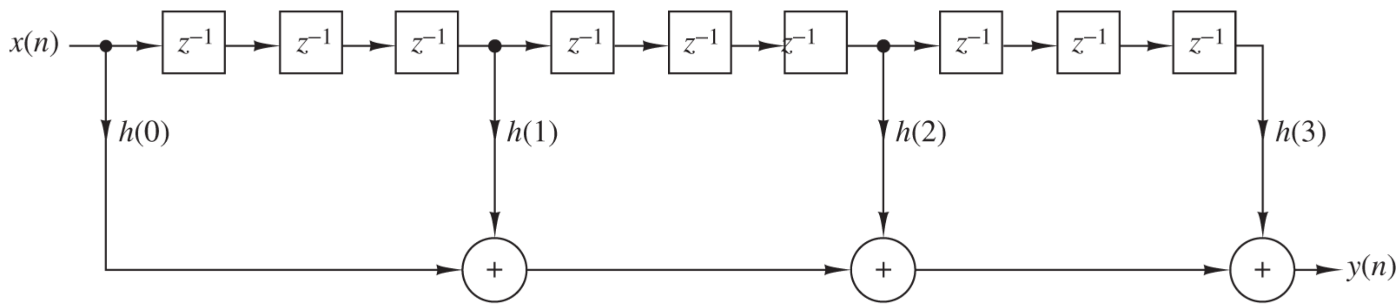

Figure 5.4.13 Realization of an FIR comb filter having $M = 3$ and $L = 3$.

- We can expand on this by inserting L delays in between each h(n). This is equivalent to making a new hL(n) = [1 0 0 0 1 0 0 0 1 0 0 0 1]
- Mathematically, this is using the old h(n) but adjusting the exponent:

$$H_L(z) = \sum_{k=0}^{M} h(k) z^{-kL}$$

- Which leads to

$$H_L(\omega) = \frac{e^{-j\omega LM/2}}{M+1} \frac{sin\omega L(\frac{M+1}{2})}{sin\omega L/2}$$

- Used to a) cancel out signals with harmonics or b) do audio effects
- Flanging: a comb filter where L varies over time, changing the ‘comb’

All-pass filters (P&M 5.4.6)

- Definition: $|H_{AP}(\omega)|$ = constant
- Simplest (but most useful?) case: time delay
- More interesting example: 1 zero, 1 pole

$$H(z) = \frac{z^{-1} - a}{1 - az^{-1}}$$

- Magnitude of this is:

$$\begin{aligned} \mid H_{ap}(\omega) \mid^2 &= H(z)H(z^{-1}) \mid_{z=e^{j\omega}} \\ &= \left(\frac{z^{-1} - a}{1 - az^{-1}}\right)\left(\frac{z - a}{1 - az}\right) \mid_{z=e^{j\omega}} \\ &= 1 \end{aligned}$$

All-pass filters, con't

- More general case: poles/zeros can be real or complex, and there can be many of them:

$$H_{ap}(z) = \prod_{k=1}^{N_R} \frac{z^{-1} - \alpha_k}{1 - \alpha_k z^{-1}} \prod_{k=1}^{N_C} \frac{(z^{-1} - \beta_k)(z^{-1} - \beta_k^*)}{(1 - \beta_k z^{-1})(1 - \beta_k^* z^{-1})}$$

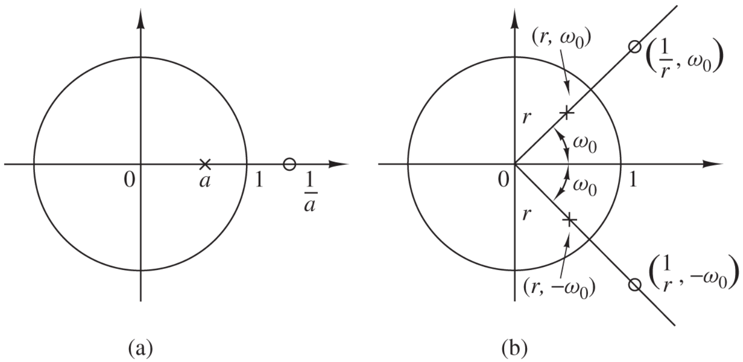

Figure 5.4.16 Pole–zero patterns of (a) a first-order and (b) a second-order all-pass filter.

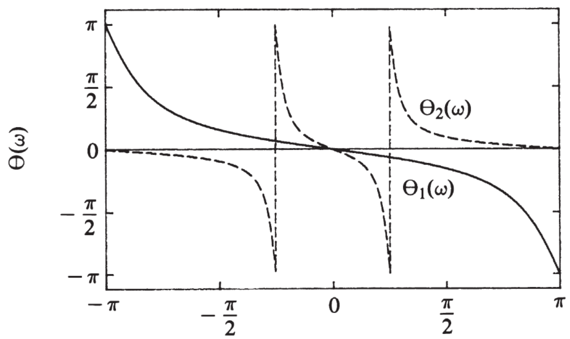

Figure 5.4.17 Frequency response characteristics of an all-pass filter with system functions (1) $H(z) = (0.6 + z^{-1})/(1 + 0.6z^{-1})$, (2) $H(z) = (r^2 - 2r\cos\omega_0 z^{-1} + z^{-2})/\ (1 - 2r\cos\omega_0 z^{-1} + r^2 z^{-2})$, $r = 0.9$, $\omega_0 = \pi/4$.

FIR vs IIR

IIR

- Feedback means input can influence the system forever
- Poles (resonances) are not just at origin
  - Potential instability
  - But, can have fast changes near points of interest
- Linear phase is very hard to do-thus, group delay not constant

FIR

- No feedback means system has limited memory
- Poles (resonances) are only at origin or infinity
  - ALWAYS stable
- Linear phase is very easy – can get systems with constant group delay (no dispersion)