Q) Do circularly even (or odd) sequences need to have an odd number of samples, like the examples in class yesterday?

A) No – see below for an example. The trick is that if the number of samples is even, one of the values will be unconstrained by symmetry conditions.

Let's consider the case of signals with circular even symmetry (basically the same argument can be made for odd).

Let's say we have a signal with N samples, starting at n=0 and going up to n = N-1 (so the values are x(0) through x(N-1). The definition of a circular even sequence is one where

$x(n) = x((N-n)_N)$

where the subscript N means 'modulo' – so we calculate N-n and than find (N-n) modulo N. The modulo operation is what makes things wrap around….

Now, let's look at two cases (the simplest possible)

Case 1: N =3, so there are three values: x(0), x(1), and x(2)

Plugging into the equation above, we see

$x(1) = x((3-1)_3) = x(2)$

$x(2) = x((3-2)_3) = x(1)$

so, x(0) can be any number, and x(1)=x(2).

If we want a circular odd function, then we just need x(1) =-x(2)

Case 2: N =4, so there are four values: x(0), x(1), x(2), and x(3)

Plugging into the equation above, we see

$x(1) = x((4-1)_4) = x(3)$

$x(2) = x((4-2)_4) = x(2)$ (so x(2) just has to equal itself, which is easy to accomplish!)

$x(3) = x((4-3)_4) = x(1)$

so, x(0) and x(2) can be any numbers, and x(1)=x(3).

If we want a circular odd function, then we just need x(1) =-x(3)