There are a number of texts on discrete-time signals and systems. We mention as examples the books by McGillem and Cooper (1984), Oppenheim and Willsky (1983), and Siebert (1986). Linear constant-coefficient difference equations are treated in depth in the books by Hildebrand (1952) and Levy and Lessman (1961).

The last topic in this chapter, on correlation of discrete-time signals, plays an important role in digital signal processing, especially in applications dealing with digital communications, radar detection and estimation, sonar, and geophysics. In our treatment of correlation sequences, we avoided the use of statistical concepts. Correlation is simply defined as a mathematical operation between two sequences, which produces another sequence, called either the *crosscorrelation sequence* when the two sequences are different, or the *autocorrelation sequence* when the two sequences are identical.

In practical applications in which correlation is used, one (or both) of the sequences is (are) contaminated by noise and, perhaps, by other forms of interference. In such a case, the noisy sequence is called a *random sequence* and is characterized in statistical terms. The corresponding correlation sequence becomes a function of the statistical characteristics of the noise and any other interference.

The statistical characterization of sequences and their correlation is treated in Chapter 12. Supplementary reading on probabilistic and statistical concepts dealing with correlation can be found in the books by Davenport (1970), Helstrom (1990), Peebles (1987), and Stark and Woods (1994).

## Problems

**2.1** A discrete-time signal $x(n)$ is defined as

$$x(n) = \begin{cases} 1 + \frac{n}{3}, & -3 \le n \le -1 \\ 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

**(a)** Determine its values and sketch the signal $x(n)$.

**(b)** Sketch the signals that result if we:

1. First fold $x(n)$ and then delay the resulting signal by four samples.
2. First delay $x(n)$ by four samples and then fold the resulting signal.

**(c)** Sketch the signal $x(-n+4)$.

**(d)** Compare the results in parts (b) and (c) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$.

**(e)** Can you express the signal $x(n)$ in terms of signals $\delta(n)$ and $u(n)$?

**2.2** A discrete-time signal $x(n)$ is shown in Fig. P2.2. Sketch and label carefully each of the following signals.

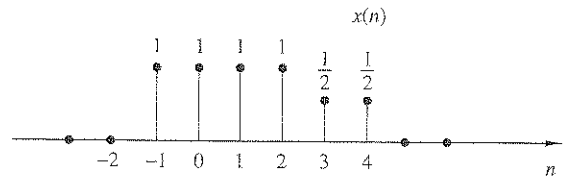


**Figure P2.2**

**(a)** $x(n-2)$ **(b)** $x(4-n)$ **(c)** $x(n+2)$ **(d)** $x(n)u(2-n)$ **(e)** $x(n-1)\delta(n-3)$
**(f)** $x(n^2)$ **(g)** even part of $x(n)$ **(h)** odd part of $x(n)$

**2.3** Show that

**(a)** $\delta(n) = u(n) - u(n-1)$

**(b)** $u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$

**2.4** Show that any signal can be decomposed into an even and an odd component. Is the decomposition unique? Illustrate your arguments using the signal

$$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$$

**2.5** Show that the energy (power) of a real-valued energy (power) signal is equal to the sum of the energies (powers) of its even and odd components.

**2.6** Consider the system

$$y(n) = \mathcal{T}[x(n)] = x(n^2)$$

**(a)** Determine if the system is time invariant.

**(b)** To clarify the result in part (a) assume that the signal

$$x(n) = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

is applied into the system.

(1) Sketch the signal $x(n)$.

(2) Determine and sketch the signal $y(n) = \mathcal{T}[x(n)]$.

(3) Sketch the signal $y_2'(n) = y(n-2)$.

(4) Determine and sketch the signal $x_2(n) = x(n-2)$.

(5) Determine and sketch the signal $y_2(n) = \mathcal{T}[x_2(n)]$.

(6) Compare the signals $y_2(n)$ and $y(n-2)$. What is your conclusion?

**(c)** Repeat part (b) for the system

$$y(n) = x(n) - x(n-1)$$

Can you use this result to make any statement about the time invariance of this system? Why?

**(d)** Repeat parts (b) and (c) for the system

$$y(n) = \mathcal{T}[x(n)] = nx(n)$$

**2.7** A discrete-time system can be

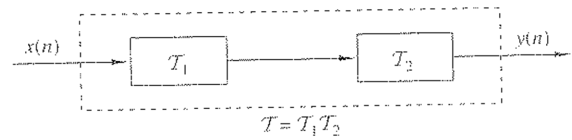


Figure P2.8

**2.9** Let $\mathcal{T}$ be an LTI, relaxed, and BIBO stable system with input $x(n)$ and output $y(n)$. Show that:

**(a)** If $x(n)$ is periodic with period $N$ [i.e., $x(n) = x(n+N)$ for all $n \geq 0$], the output $y(n)$ tends to a periodic signal with the same period.

**(b)** If $x(n)$ is bounded and tends to a constant, the output will also tend to a constant.

**(c)** If $x(n)$ is an energy signal, the output $y(n)$ will also be an energy signal.

**2.10** The following input–output pairs have been observed during the operation of a time-invariant system:

$$x_1(n) = \{\underset{\uparrow}{1}, 0, 2\} \xleftrightarrow{\mathcal{T}} y_1(n) = \{\underset{\uparrow}{0}, 1, 2\}$$

$$x_2(n) = \{\underset{\uparrow}{0}, 0, 3\} \xleftrightarrow{\mathcal{T}} y_2(n) = \{\underset{\uparrow}{0}, 1, 0, 2\}$$

$$x_3(n) = \{\underset{\uparrow}{0}, 0, 0, 1\} \xleftrightarrow{\mathcal{T}} y_3(n) = \{1, \underset{\uparrow}{2}, 1\}$$

Can you draw any conclusions regarding the linearity of the system. What is the impulse response of the system?

**2.11** The following input–output pairs have been observed during the operation of a linear system:

$$x_1(n) = \{-1, \underset{\uparrow}{2}, 1\} \xleftrightarrow{\mathcal{T}} y_1(n) = \{1, \underset{\uparrow}{2}, -1, 0, 1\}$$

$$x_2(n) = \{1, \underset{\uparrow}{-1}, -1\} \xleftrightarrow{\mathcal{T}} y_2(n) = \{-1, \underset{\uparrow}{1}, 0, 2\}$$

$$x_3(n) = \{0, \underset{\uparrow}{1}, 1\} \xleftrightarrow{\mathcal{T}} y_3(n) = \{\underset{\uparrow}{1}, 2, 1\}$$

Can you draw any conclusions about the time invariance of this system?

**2.12** The only available information about a system consists of $N$ input–output pairs, of signals $y_i(n) = \mathcal{T}[x_i(n)]$, $i = 1, 2, \ldots, N$.

**(a)** What is the class of input signals for which we can determine the output, using the information above, if the system is known to be linear?

**(b)** The same as above, if the system is known to be time invariant.

**2.13** Show that the necessary and sufficient condition for a relaxed LTI system to be BIBO stable is

$$\sum_{n=-\infty}^{\infty} |h(n)| \leq M_h < \infty$$

for some constant $M_h$.

**2.14** Show that:

**(a)** A relaxed linear system is causal if and only if for any input $x(n)$ such that

$$x(n) = 0 \text{ for } n < n_0 \Rightarrow y(n) = 0 \qquad \text{for } n < n_0$$

**(b)** A relaxed LTI system is causal if and only if

$$h(n) = 0 \qquad \text{for } n < 0$$

**2.15**

**(a)** Show that for any real or complex constant $a$, and any finite integer numbers $M$ and $N$, we have

$$\sum^{N} n = M a^n = \begin{cases} \dfrac{a^M - a^{N+1}}{1-a}, & \text{if } a \neq 1 \\ N - M + 1, & \text{if } a = 1 \end{cases}$$

**(b)** Show that if $|a| < 1$, then

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

**2.16** **(a)** If $y(n) = x(n) * h(n)$, show that $\sum_y = \sum_x \sum_h$, where $\sum_x = \sum_{n=-\infty}^{\infty} x(n)$.

**(b)** Compute the convolution $y(n) = x(n) * h(n)$ of the following signals and check the correctness of the results by using the test in (a).

(1) $x(n) = \{1, 2, 4\}$, $h(n) = \{1, 1, 1, 1, 1\}$

(2) $x(n) = \{1, 2, -1\}$, $h(n) = x(n)$

(3) $x(n) = \{0, 1, -2, 3, -4\}$, $h(n) = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$

(4) $x(n) = \{1, 2, 3, 4, 5\}$, $h(n) = \{1\}$

(5) $x(n) = \{\underset{\uparrow}{1}, -2, 3\}$, $h(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$

(6) $x(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$, $h(n) = \{1, \underset{\uparrow}{-2}, 3\}$

(7) $x(n) = \{\underset{\uparrow}{0}, 1, 4, -3\}$, $h(n) = \{\underset{\uparrow}{1}, 0, -1, -1\}$

(8) $x(n) = \{\underset{\uparrow}{1}, 1, 2\}$, $h(n) = u(n)$

(9) $x(n) = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$, $h(n) = \{1, -2, -3, \underset{\uparrow}{4}\}$

(10) $x(n) = \{1, 2, \underset{\uparrow}{0}, 2, 1\} h(n) = x(n)$

(11) $x(n) = (\frac{1}{2})^n u(n)$, $h(n) = (\frac{1}{4})^n u(n)$