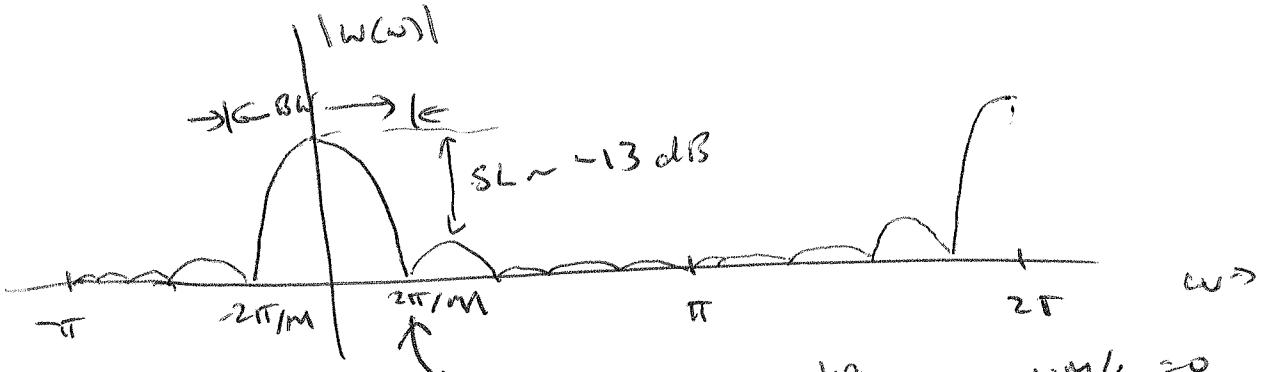
Some basic windows

① Boxcar / rectangular

$$w(n) = \begin{cases} 1, & n = 0, 1, \ldots M-1 \quad (M \text{ points}) \\ 0 & \text{else} \end{cases}$$

$$w(n) \longleftrightarrow W(\omega) = e^{-j\omega(M-1)/2} \; \frac{\sin(\omega M/2)}{\sin \omega/2}$$

so $$|W(\omega)| = \left| \frac{\sin(\omega M/2)}{\sin(\omega/2)} \right|$$



1st zero occurs when $\sin \omega M/2 = 0$

or $\omega M/2 = \pi \quad (\sin \pi = 0)$

or $\omega = 2\pi/M$

Bandwidth (BW)

can be measured as distance between zeros: so

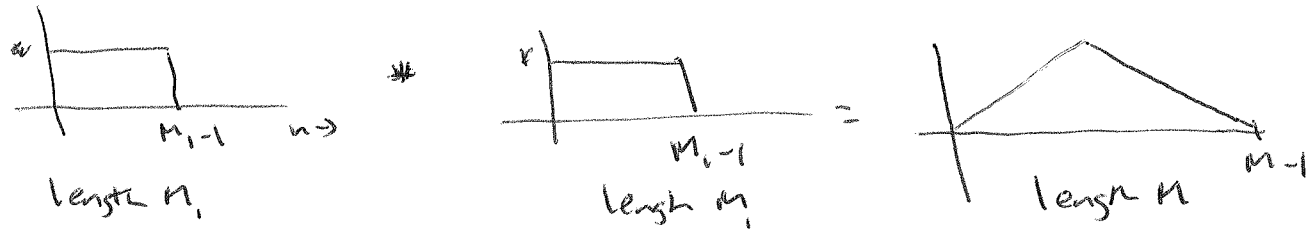
$$BW = 2\left(\frac{2\pi}{M}\right) = 4\pi/M \quad \text{for rectangular}$$

Sidelobe level

~ -13 dB for ~~the~~ boxcar independant of M (from calculation)

(2) Triangle / Bartlett window

note a triangle window is the convolution of two rectangles with half the length:



$M = M_1 + M_1 - 1$
$= 2M_1 - 1$

but convolution in ~~freq~~ time gives multiplication in ω:

$$|W_{tri}(\omega)| = \left| \frac{\sin(\omega M_1/2)}{\sin(\omega/2)} \right|^2$$

for length $M = 2M_1 - 1$ window

this means: (1) sidelobe is squared: (better)

$SL \sim -2 \times 13$ dB $= -26$ dB //

(2) resolution is worse:

~~HE~~ $BW \sim \frac{4\pi}{M_1} = \frac{4\pi}{M/2} = \frac{8\pi}{M}$

So, comparing M-point boxcar vs. tri:

Boxcar has twice as good resolution, but half as good (in dB) sidelobes

typical tradeoff: resolution ↑ (good) means sidelobes ↑ (bad)

(3) Many others. Raised cosines are good