Administrative

- Reminder: tomorrow's office hours are 2-4, not 3-5
- TA can't do office hours this week, but can do makeup
- Switched the date of exam 3 as we discussed
- Review schedule – I'd like to move Quiz 2 from next Monday to next Wednesday
  - then it can cover this week's topics – better prep for exam 2, which is 2 weeks from today
  - Conflicts with other exams?
- Will post HW due next Monday on Trunk, and send email
- Will start uploading HW solutions, etc in preparation for the quiz

EE-125: Digital Signal Processing

Periodograms

Professor Tracey

Tufts

Youtube version of today’s lecture (Links are fixed now!)

Van Veen lectures relevant to today’s lecture are: Random processes:
https://youtu.be/Y2bab0U3Ji8

Periodogram:
https://youtu.be/Qs-Zai0F2Pw

INFO WE DIDN’T GET TO TODAY:
Averaged Periodogram: (not covered today!)
https://youtu.be/41OEvKRDJiY

Examples: https://youtu.be/u6TzodpIeDQ

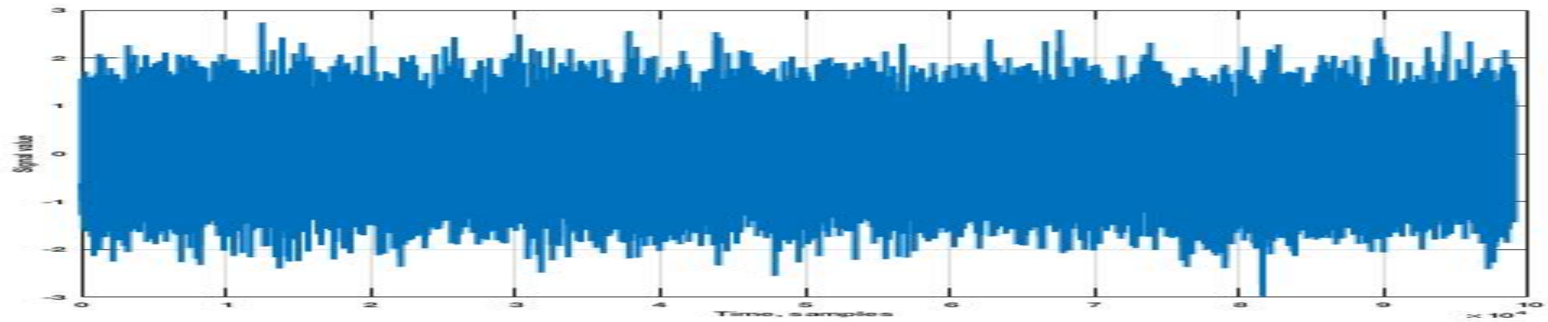
Reminder: last time

- The DFT/FFT have two main uses
  - Fast FFT-based FIR filtering (overlap/add, etc)
  - Spectrum estimation / spectral analysis
- We may want to do spectral analysis in order to:
  - Learn something about a signal, either by human or automated analysis of the frequency content
  - Do processing in frequency domain (mp3, etc), then go back to time domain
- We'll consider three main topics
  - Deterministic, non-time-varying signals, possibly in random noise
  - Random processes / noise (periodograms)
  - Time-varying but non-random signals (spectrograms)

Today's outline

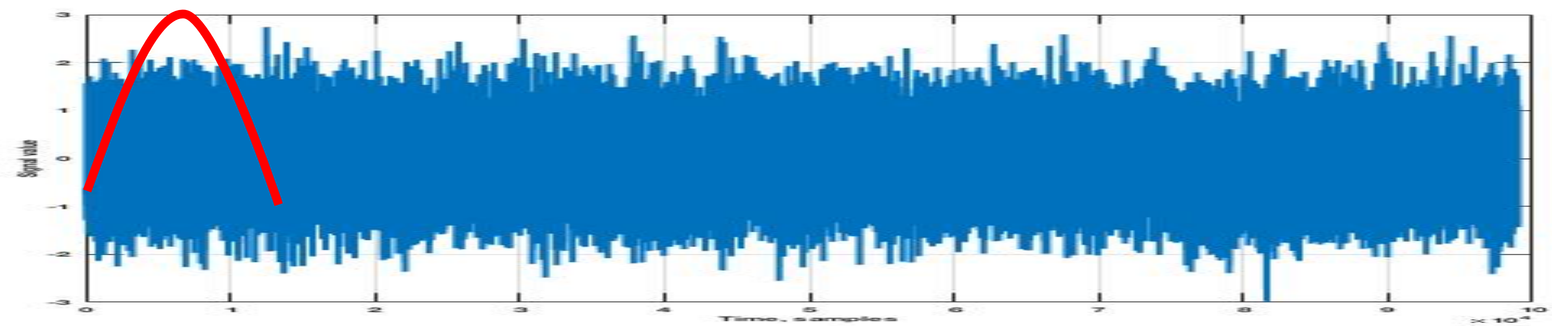
- Overview – where we are going
- Math preliminaries
- Simple approach – unaveraged periodogram
  - Adjustments to maintain amplitude
  - Problems with variance
- Periodogram averaging: Bartlett, Welch methods
- Advanced methods exist: high-resolution spectrum estimates. See P&M if interested

Overview: Lecture in 3 slides



How to find frequency content of (random) signal above? 10,000 points of data

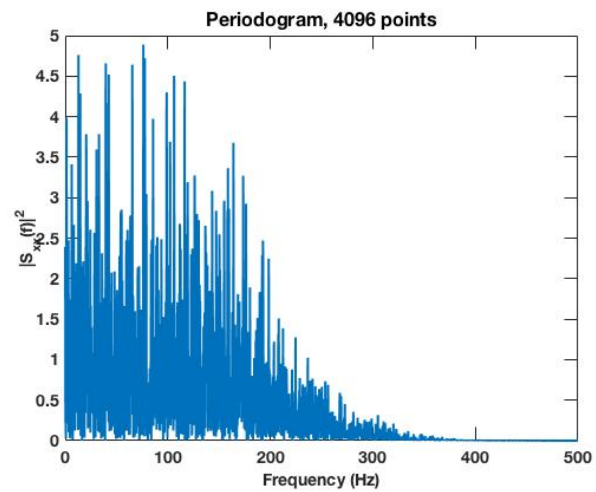
Overview: Lecture in 3 slides



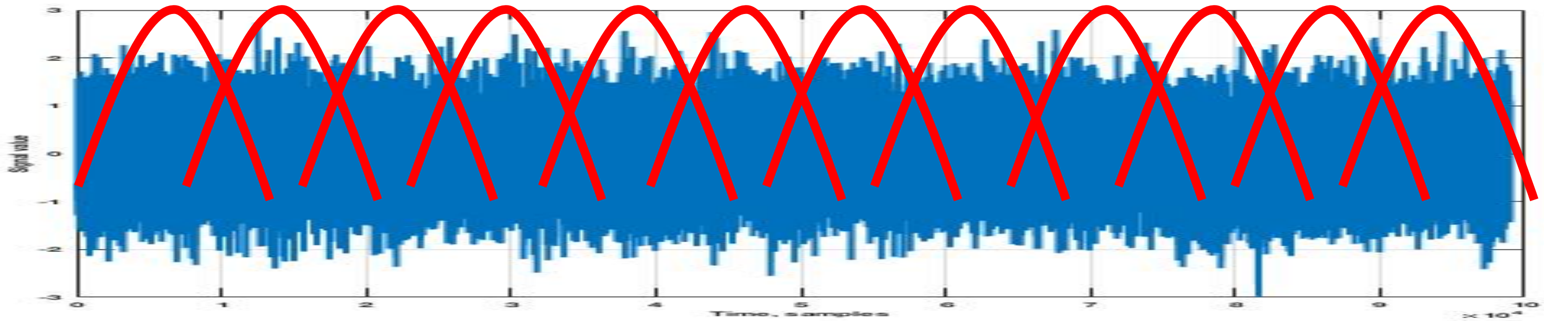
How to find frequency content of (random) signal above? 10,000 points of data

A1) We could apply a window to extract part of signal, take DFT, and plot magnitude-squared

Works, but variance is very high!

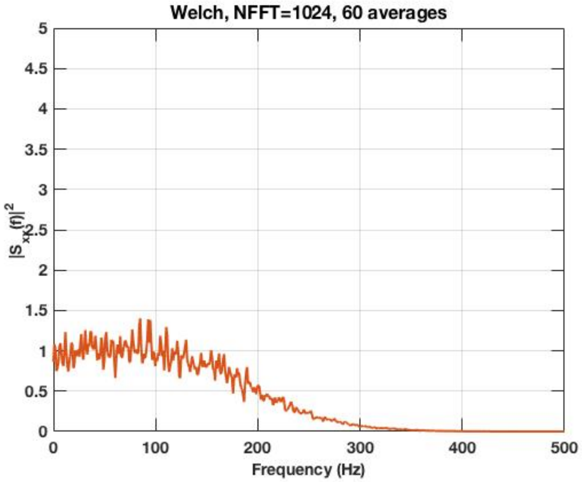


Overview: Lecture in 3 slides



How to find frequency content of (random) signal above? 10,000 points of data

A2) We could get take many windowed DFTs, and average their power to reduce the variance
!



Some math preliminaries…

- Random processes used to model complex or noise-like signals
- Characterized by statistical properties
  - Mean, Variance
  - Autocorrelation

- We have to *estimate* these (example: sample mean, sample variance)
- We like estimators that are:
  - **Unbiased**: expected value of estimator = true value
  - **Asymptotically unbiased**: estimate -> true value as # data points L -> infinity
  - **Consistent**: variance of estimator -> 0 as # data points L -> infinity
- Example: https://onlinecourses.science.psu.edu/stat414/node/167

Ergodic signals

- For ergodic signals, averaging over time is same as averaging over ensemble (of different realizations)

- generally means the signal's statistical characteristics aren't changing over time (wide-sense stationary means mean, var constant)

- Example ergodic signal:
  - x = 2 + randn(1,1001)

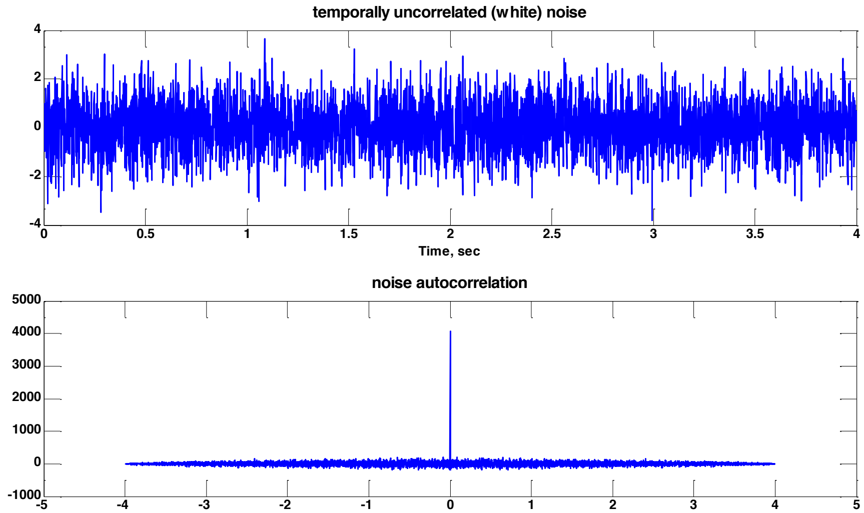
- Example non-ergodic signal:
  - x = (0:1000)./4 + randn(1,1001)

AWGN – Additive Gaussian White Noise

- Additive – added to signal, passes through system

$$y = h*(x+w_{in}) = h*x + w$$

- Gaussian – each individual sample is drawn from a Gaussian distribution: $N(0, \sigma^2)$ (sigma*randn in Matlab)
- White – temporally uncorrelated; each time sample is unrelated to previous or next, so get "white" spectrum



We saw that power spectrum is Fourier transform of auto-correlation

$$\gamma_{ww}(l) = \sigma_w^2 \delta(l)$$

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Yet more math preliminaries… notation

- An energy signal is a signal with finite energy (basically, finite length signals)
- A power signal is a signal with finite power
  - Power = energy / unit time
  - Power signals can be infinitely long, but *average* energy is finite (example: infinitely long cosine signal)
- Energy Spectral Density is for an "energy signal"
  - $S_{xx}(F) = |X(F)|^2$ (discussed in P&M 14.1)
- Power Spectral Density (PSD) is for a "power signal"
  - Called $\Gamma_{xx}(F)$ (discussed in P&M 14.2)
  - For random processes, we are generally talking about PSD

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A sample problem

- We sample a random signal at Fs = 1000 Hz
- We want to use a Hanning window to suppress sidelobes
- We want to be able to resolve tones spaced 5 Hz apart, defining mainlobe width as distance between nulls
- How many seconds of data should we measure to get a periodogram whose quality factor is at least Q = 50, if….
  - we don't use any overlap?
  - we use 50% overlap?
- Does this answer change if we zero-pad the FFT's used to estimate the periodogram?