Administrative

- MATLAB2 grades should be out before long
- MATLAB4
  - Update to part 2 posted
  - reminder: two parts, but PLEASE TURN IN SINGLE PDF
  - hints on fft usage for part1, fft output plotting for part 2
- Other?

EE-125:
Digital Signal Processing

Spectrum analysis using the DFT

Professor Tracey

Tufts

Left over from last lecture: Other algorithms to know about

- Goertzel (P&M 8.3)
  - Rewrite the DFT so we have a parallel bank of filters, each one of which gives the output for a single frequency in the DFT
  - Advantage is that we don't need to implement every frequency; so can be faster than FFT if we just need answers at a few frequencies
  - Classic use: processing of dial tones
- Chirp z-transform (P&M 8.3)
  - Lets us evaluate the transform at points other than the unit circle
  - Used in speech analysis (on-line, see "The Chirp z-Transform Algorithm—A Lesson in Serendipity")

Spectral analysis overview and motivation

- The DFT/FFT have two main uses
  - Fast FFT-based FIR filtering (overlap/add, etc)
  - Spectrum estimation / spectral analysis
- We may want to do spectral analysis in order to:
  - Learn something about a signal, either by human or automated analysis of the frequency content
  - Do processing in frequency domain (mp3, etc), then go back to time domain
- We'll consider three main topics
  - Deterministic, non-time-varying signals, possibly in random noise (this lecture and next)
  - Random processes / noise (periodograms)
  - Time-varying but non-random signals (spectrogram & wavelet)

Where we are in the class

		DFT/FFT		7.2-7.3	MATLAB4	MATLAB3
23-Oct	12	DFT/FFT	FFT algorithm	8.1-8.3 + video		
25-Oct	13	spectrum analysis	Spectral analysis using DFT (non-random signals). Window effects, leakage, resolution	7.4		
30-Oct	14	spectrum analysis	Window metrics	notes	MATLAB5	
1-Nov	15	spectrum analysis	Periodogram (random signals)	14.1, 14.2		MATLAB4 - **Nov2**
6-Nov	16	spectrum analysis	**Quiz 2.** Short-time Fourier transform, applications	notes		
8-Nov	17	spectrum analysis	Wavelet analysis	notes		
13-Nov	18	filter design	Filter specification, FIR design 1	9.1-9.3		MATLAB5
15-Nov			**TEST, through spectrum estimation**	10.1, 10.2.1-10.2.3		
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Outline

- Overview
- Basic idea: window the signal, take the DFT.
- Understanding the effects of the window:
  - Resolution: length and type of window
  - Basic window types
  - More window types
- Effects of window choice
  - Problems caused by mainlobe too wide
  - Problems caused by sidelobes too high
- Some common misconceptions
  - Spectrum is really sparse (picket fence effect)
  - Zero-padding improves spectral resolution (i..e, my resolution is what the FFT gives me)

Window design considerations

- The main issues are main lobe width and sidelobe height
- Main lobe determine how ‘smoothed’ the desired response is
  - Thus ideally, main lobe would be very narrow
  - This generally implies higher M
- The sidelobe determines ‘ringing’ in frequency
  - High sidelobes mean less attenuation of undesired frequencies
  - High frequency sidelobes result from sharp discontinuities in time

- Rectangular window: narrowest mainlobe, but high sidelobes
- Other windows; wider mainlobe but lower sidelobes
  - All are linear phase
  - All have smaller mainlobe as M increases (just like rect)
  - Design is generally trial-and-error

Some possible filters

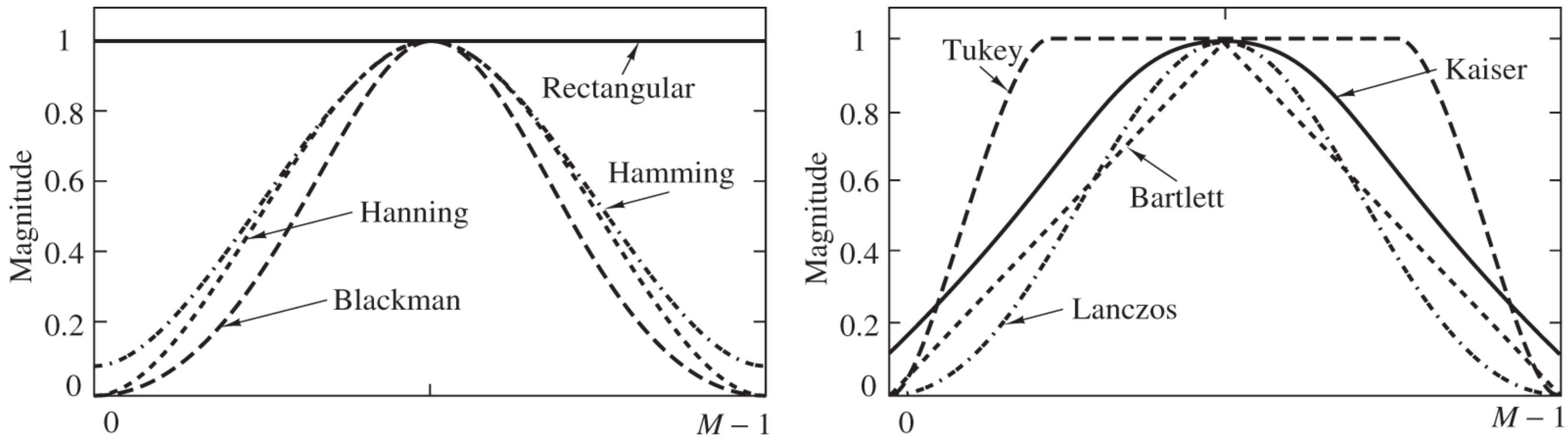

Figure 10.2.3 Shapes of several window functions.

[For lots of details on window types, see https://ccrma.stanford.edu/~jos/sasp/sasp.html](https://ccrma.stanford.edu/~jos/sasp/sasp.html)

Some common windows: Table 10.2 in book

Window type	~ main lobe	Peak sidelobe, dB
Boxcar	4 pi/M	-13
Bartlett (tri.)	8 pi / M	-26
Hanning	8 pi / M	-31
Hamming	8 pi /M	-41
Blackman	12 pi / M	-57

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Problems when mainlobes are too wide

- Our estimated spectrum is the true spectrum convolved with the window's spectrum

- When the window's mainlobe is narrower than the features in the true spectrum, our estimate can be reasonable

- When window mainlobe is wider, we mostly see the shape of the window

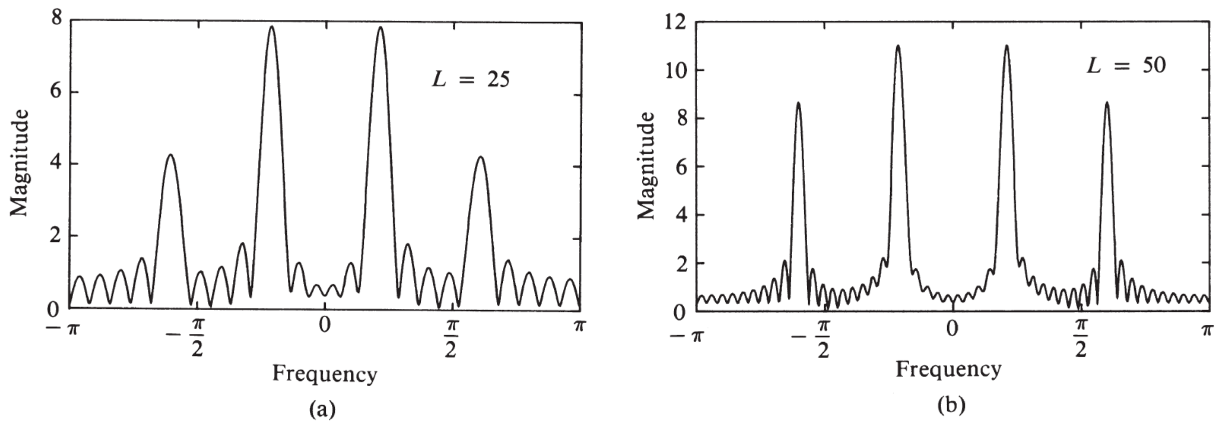
- If we have closely spaced tones, they may get smeared together if the mainlobe is too wide

Problems when sidelobes are too high

- We may have a very high-amplitude signal whose energy will "leak" across the whole spectrum
- This can interfere with (or completely mask) weak signals
- Examples:
  - Intentional jammers
  - Loud unintentional interferers (in the ocean, surface ships)
  - Tonal machinery noise in industrial applications
- You'll do one of these in MATLAB5

Example: 3 sinusoids, 2 closely spaced, Rectangular window

Top plots: mainlobe is wide, and we can't tell there are 2 tones



Bottom plot: high sidelobes mean we couldn't see any weak signals

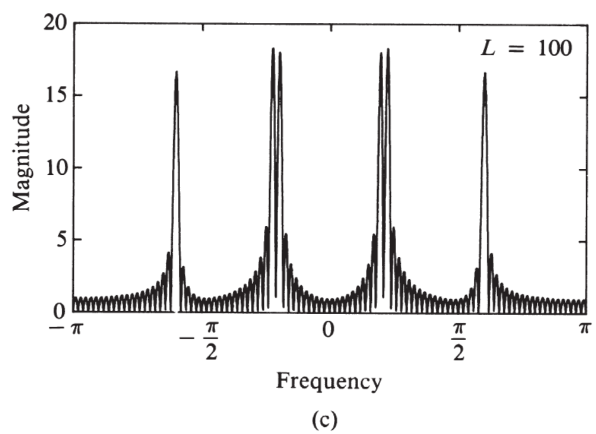
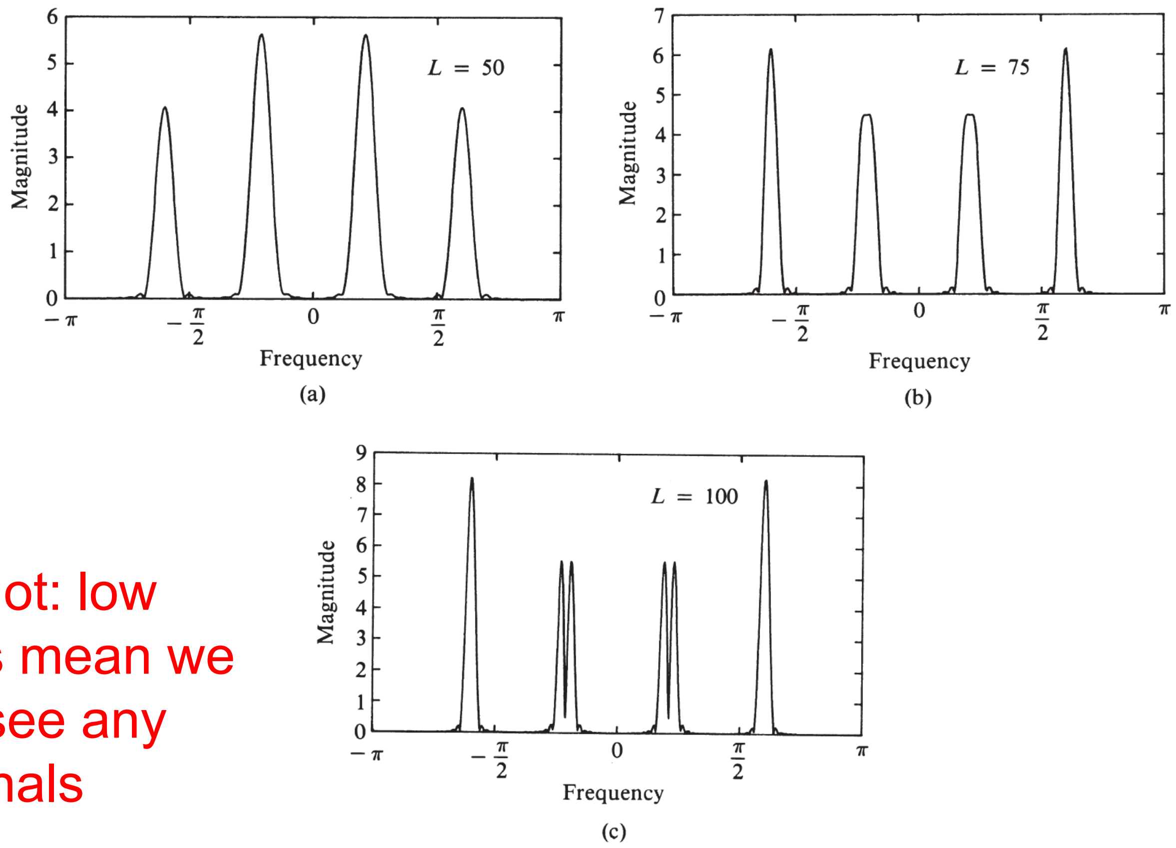


Figure 7.4.2 Magnitude spectrum for the signal given by (7.4.8), as observed through a rectangular window.

Example: 3 sinusoids, 2 closely spaced, Hanning window

Top plots: mainlobe is wide, and we can't tell there are 2 tones



Bottom plot: low sidelobes mean we COULD see any weak signals

Figure 7.4.4 Magnitude spectrum of the signal in (7.4.8) as observed through a Hanning window.

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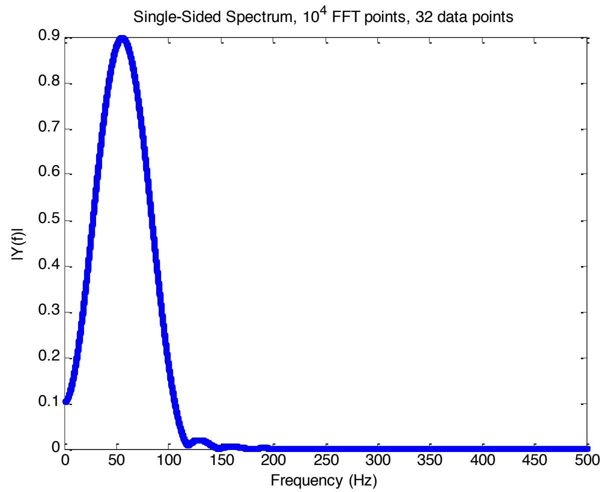
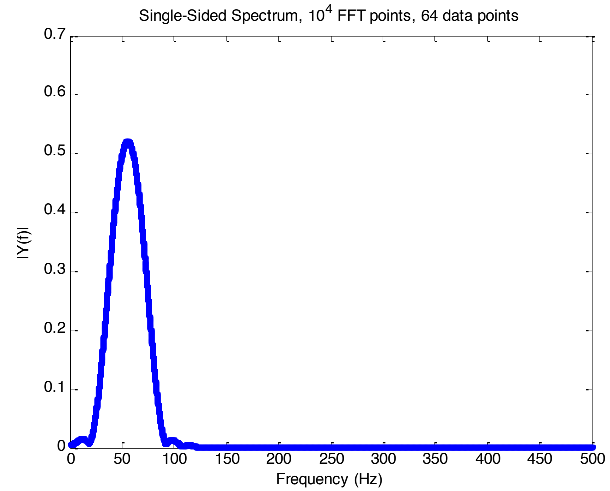
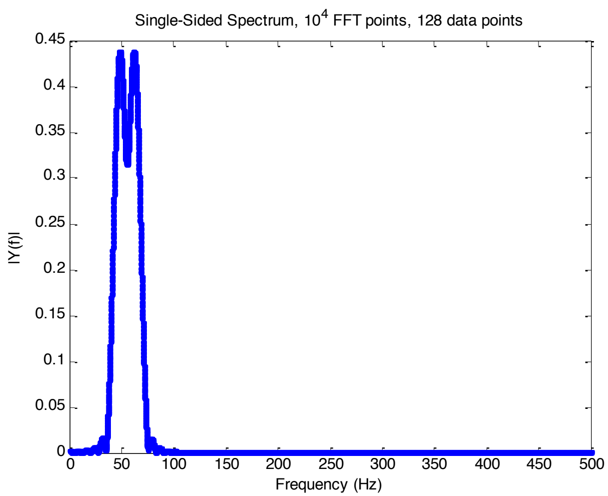
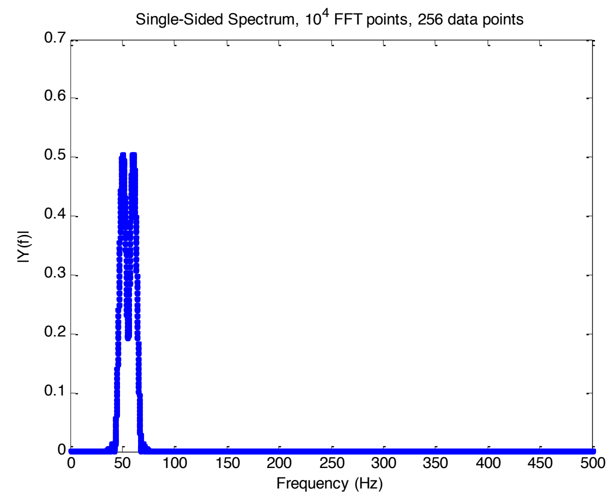
More FFT points due to zero padding ~= Better Resolution! Resolution depends on actual data length





All have same # FFT points (10^4) but different actual data lengths





Tufts

Following slides are ones we almost certainly won't get to

Deterministic tonal signals in random noise

- An example problem: sinusoid in noise e(n), with window w(n) applied:

$$x(t) = w(n)\,[A \sin(\omega_0 n) + e(n)]$$

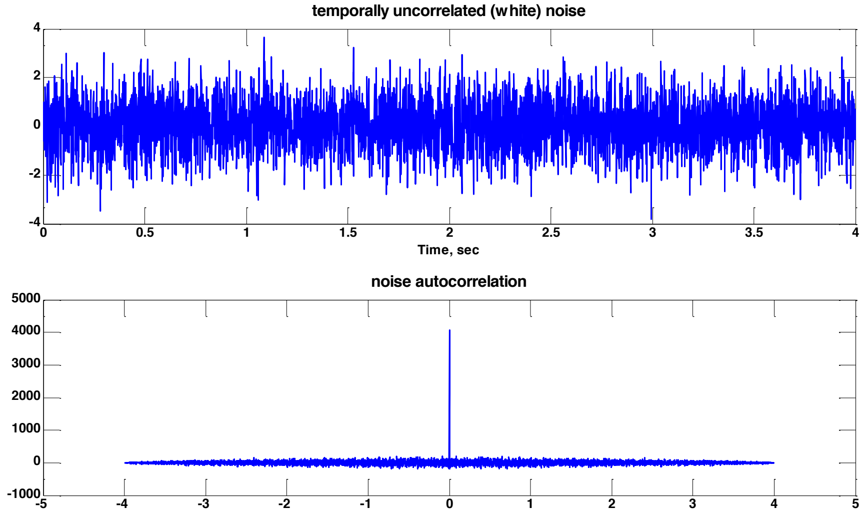
- If SNR is high, no problem; if SNR is poor, we need to consider how the window mainlobe size affects our results

AWGN – Additive Gaussian White Noise

- Additive – added to signal, passes through system

  y = h*(x+w) = h*x + h*w

- Gaussian – each individual sample is drawn from a Gaussian distribution: N(0, σ^2) (sigma*randn in Matlab)
- White – temporally uncorrelated; each time sample is unrelated to previous or next, so get "white" spectrum



$$\gamma_{ww}(l) = \sigma_w^2 \delta(l)$$

Noise equivalent-bandwidths:

On the use of windows for harmonic analysis with the Discrete Fourier Transform," F. Harris, Proc IEEE, 1978.

TABLE I
WINDOWS AND FIGURES OF MERIT

WINDOW		HIGHEST SIDE-LOBE LEVEL (dB)	SIDE-LOBE FALL-OFF (dB/OCT)	COHERENT GAIN	EQUIV NOISE BW (BINS)	3.0-dB BW (BINS)	SCALLOP LOSS (dB)	WORST CASE PROCESS LOSS (dB)	6.0-dB BW (BINS)	OVERLAP CORRELATION (PCNT) 75% OL	50% OL
RECTANGLE		−13	−6	1.00	1.00	0.89	3.92	3.92	1.21	75.0	50.0
TRIANGLE		−27	−12	0.50	1.33	1.28	1.82	3.07	1.78	71.9	25.0
$\cos^{\alpha}(X)$ HANNING	$\alpha = 1.0$	−23	−12	0.64	1.23	1.20	2.10	3.01	1.65	75.5	31.8
	$\alpha = 2.0$	−32	−18	0.50	1.50	1.44	1.42	3.18	2.00	65.9	16.7
	$\alpha = 3.0$	−39	−24	0.42	1.73	1.66	1.08	3.47	2.32	56.7	8.5
	$\alpha = 4.0$	−47	−30	0.38	1.94	1.86	0.86	3.75	2.59	48.6	4.3
HAMMING		−43	−6	0.54	1.36	1.30	1.78	3.10	1.81	70.7	23.5
RIESZ		−21	−12	0.67	1.20	1.16	2.22	3.01	1.59	76.5	34.4
RIEMANN		−26	−12	0.59	1.30	1.26	1.89	3.03	1.74	73.4	27.4
DE LA VALLE-POUSSIN		−53	−24	0.38	1.92	1.82	0.90	3.72	2.55	49.3	5.0
TUKEY	$\alpha = 0.25$	−14	−18	0.88	1.10	1.01	2.96	3.39	1.38	74.1	44.4
	$\alpha = 0.50$	−15	−18	0.75	1.22	1.15	2.24	3.11	1.57	72.7	36.4
	$\alpha = 0.75$	−19	−18	0.63	1.36	1.31	1.73	3.07	1.80	70.5	25.1
BOHMAN		−46	−24	0.41	1.79	1.71	1.02	3.54	2.38	54.5	7.4
POISSON	$\alpha = 2.0$	−19	−6	0.44	1.30	1.21	2.09	3.23	1.69	69.9	27.8
	$\alpha = 3.0$	−24	−6	0.32	1.65	1.45	1.46	3.64	2.08	54.8	15.1
	$\alpha = 4.0$	−31	−6	0.25	2.08	1.75	1.03	4.21	2.58	40.4	7.4
HANNING-POISSON	$\alpha = 0.5$	−35	−18	0.43	1.61	1.54	1.26	3.33	2.14	61.3	12.6
	$\alpha = 1.0$	−39	−18	0.38	1.73	1.64	1.11	3.50	2.30	56.0	9.2
	$\alpha = 2.0$	NONE	−18	0.29	2.02	1.87	0.87	3.94	2.65	44.6	4.7
CAUCHY	$\alpha = 3.0$	−31	−6	0.42	1.48	1.34	1.71	3.40	1.90	61.6	20.2
	$\alpha = 4.0$	−35	−6	0.33	1.76	1.50	1.36	3.83	2.20	48.8	13.2
	$\alpha = 5.0$	−30	−6	0.28	2.06	1.68	1.13	4.28	2.53	38.3	9.0
GAUSSIAN	$\alpha = 2.5$	−42	−6	0.51	1.39	1.33	1.69	3.14	1.86	67.7	20.0
	$\alpha = 3.0$	−55	−6	0.43	1.64	1.55	1.25	3.40	2.18	57.5	10.6
	$\alpha = 3.5$	−69	−6	0.37	1.90	1.79	0.94	3.73	2.52	47.2	4.9
DOLPH-CHEBYSHEV	$\alpha = 2.5$	−50	0	0.53	1.39	1.33	1.70	3.12	1.85	69.6	22.3
	$\alpha = 3.0$	−60	0	0.48	1.51	1.44	1.44	3.23	2.01	64.7	16.3
	$\alpha = 3.5$	−70	0	0.45	1.62	1.55	1.25	3.35	2.17	60.2	11.9
	$\alpha = 4.0$	−80	0	0.42	1.73	1.65	1.10	3.48	2.31	55.9	8.7
KAISER-BESSEL	$\alpha = 2.0$	−46	−6	0.49	1.50	1.43	1.46	3.20	1.99	65.7	16.9
	$\alpha = 2.5$	−57	−6	0.44	1.65	1.57	1.20	3.38	2.20	59.5	11.2
	$\alpha = 3.0$	−69	−6	0.40	1.80	1.71	1.02	3.56	2.39	53.9	7.4
	$\alpha = 3.5$	−82	−6	0.37	1.93	1.83	0.89	3.74	2.57	48.8	4.8
BARCILON-TEMES	$\alpha = 3.0$	−53	−6	0.47	1.56	1.49	1.34	3.27	2.07	63.0	14.2
	$\alpha = 3.5$	−58	−6	0.43	1.67	1.59	1.18	3.40	2.23	58.6	10.4
	$\alpha = 4.0$	−68	−6	0.41	1.77	1.69	1.05	3.52	2.36	54.4	7.6