Summary: FIR vs IIR systems

Consider 2 very similar systems w/ impulse response $h(n)$

$y(n) = a\,y(n-1) + b_0 x(n) + b_1 x(n-1)$

IIR → feedback means input can stay in system forever (resonance)

take z transform

$Y(z)(1-az^{-1}) = X(z)(b_0 + b_1 z^{-1})$

$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1}}{1 - a z^{-1}}$ ← zeros ← poles

pole at $z=a$

ROC $|z| > a$

stable if ROC includes u.c.

IIR → feedback (physically, resonance)

→ possible instability

Linear phase is ~~impossible~~ very hard

(poles change phase a lot)

$y(n) = b_0 x(n) + b_1 x(n-1)$

FIR → $h(n) \to 0$ after 2 samples

(moving average)

take z transform

$Y(z) = X(z)(b_0 + b_1 z^{-1})$

$H(z) = \frac{b_0 + b_1 z^{-1}}{1}$

$= \frac{b_0 z + b_1}{z}$

trivial pole $z=0$

ROC: $z \neq 0$

includes u.c. so stable

if we had $y(n) = b_0 x(n) + b_1 x(n+1)$

we'd get $H(z) = b_0 + b_1 z$

$|H(z)| \to \infty$ at $|z| \to \infty$

so ROC: $|z| \neq \infty$

includes u.c. so stable

☆ FIR → no feedback →

only trivial poles →

always stable.

Linear phase is easy

step 1 → symmetric real $h(n)$ → real $H(z)$

step 2 → delay to make causal → linear phase shift