Administrative

- HW problems based on today's lecture (will also post on Trunk). Due next Monday
  - 3.32. Hint: look at section 3.5.3
  - 3.38, parts a,b,c, EXCEPT ONLY DO 2nd part: plot the pole-zero pattern and determine whether systems are stable
  - 3.30. Hint: if you aren't sure how to get started, pick a (simple) example of a particular real-valued, even sequence (say { -1, 0, 1^ , 0, -1} where '^' denotes n=0) and work that out

EE-125:
Digital Signal Processing

Lecture 6:
Z-transform of LTI systems

Professor Tracey

Tufts

Outline

- In lecture 2, we reviewed z-transforms for *signals*; today, we'll use them for *systems*
- CCDE's and rational functions (P&M 3.3)
  - Poles and zeros
  - Time-domain response vs. pole location
- Causality and stability (3.5.3)
- Pole-zero cancellation (3.5.4)

Geometric interpretation

- We can think of evaluating X(z) or H(z) in the complex plane.
- To calculate figures like this one, could write code like:

```
Npts=100;
Zreal=linspace(-2,2,Npts);
Zimag=linspace(-2,2,Npts);

for izR=1:Npts
  for izI = 1:Npts
      z =Zreal(izR)+j*Zimag(izI);
      H = (system function using z)
      Hmag(izR,izI) = abs(H);
  end
end
```

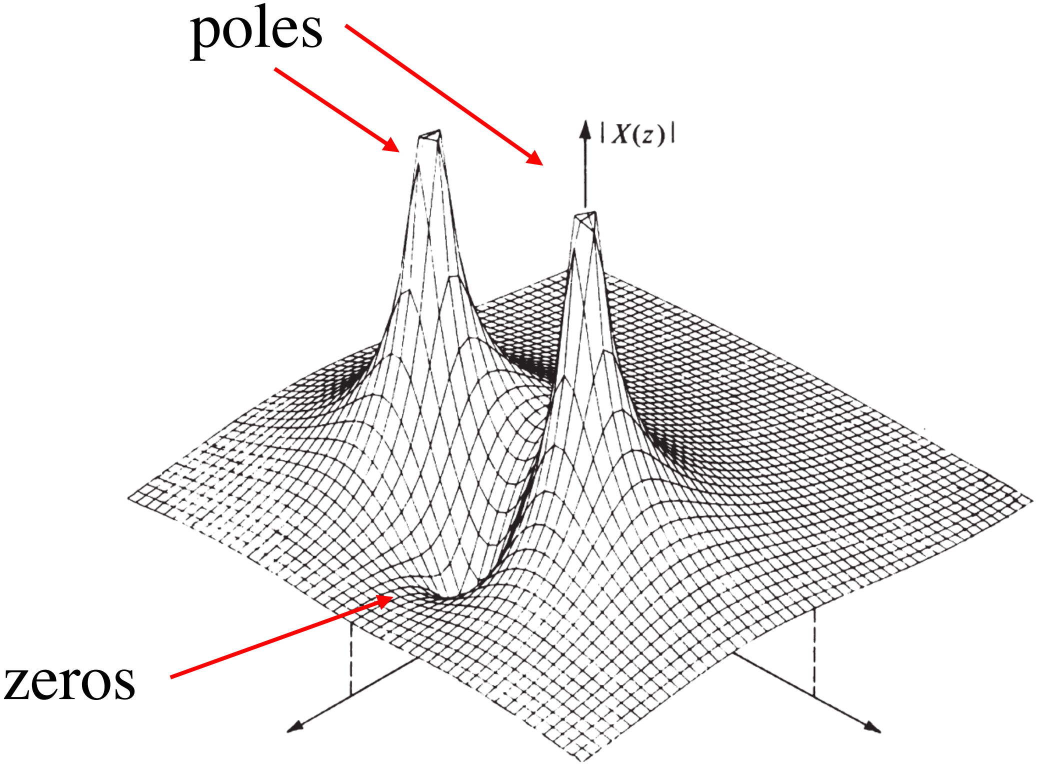

Figure 3.3.4 Graph of $|X(z)|$ for the z-transform in (3.3.3).

Time-domain behavior: single real pole

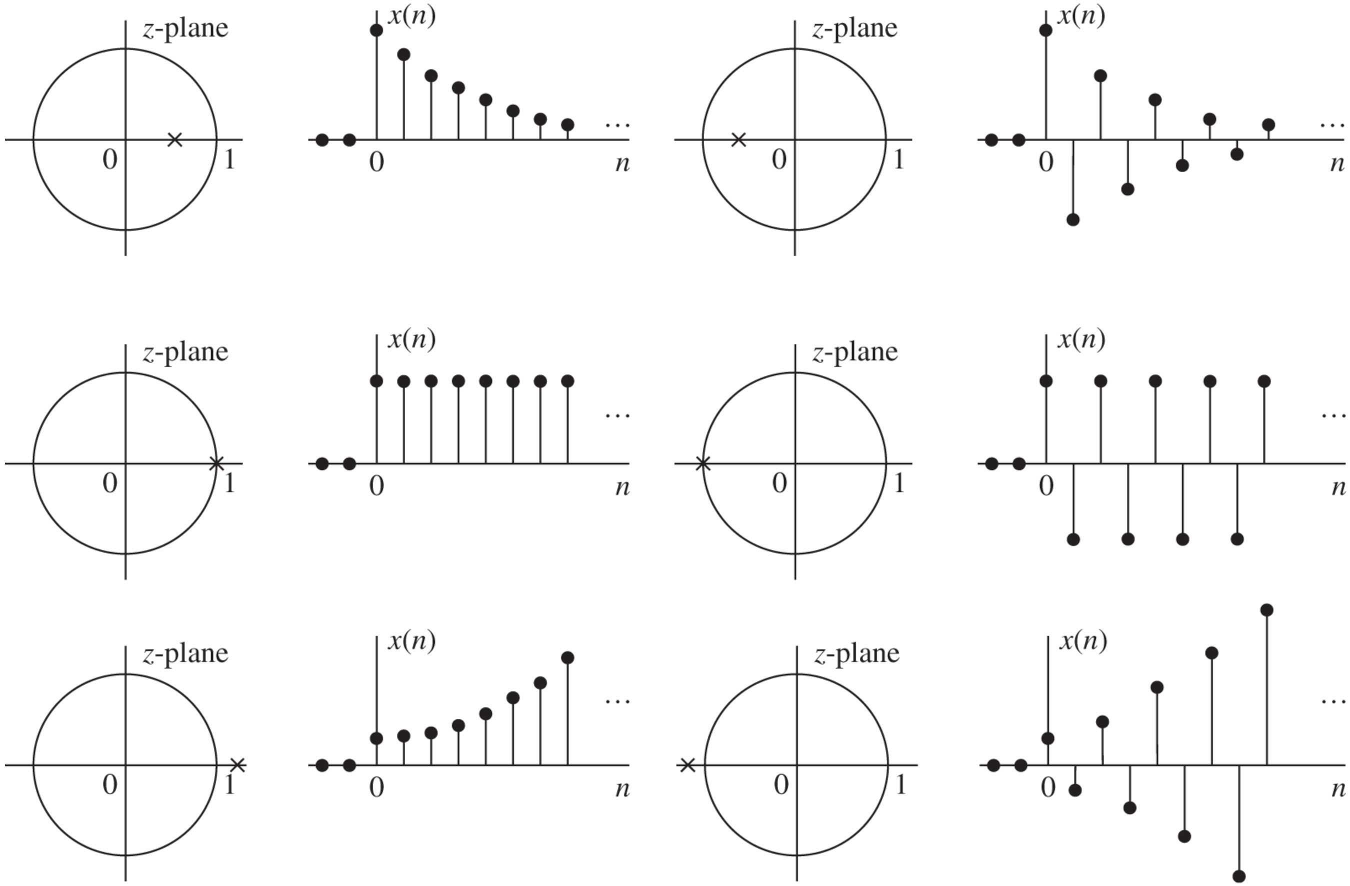

Figure 3.3.5 Time-domain behavior of a single-real-pole causal signal as a function of the location of the pole with respect to the unit circle.

Here, we could be describing a signal x(n) or an impulse response h(n); math is same

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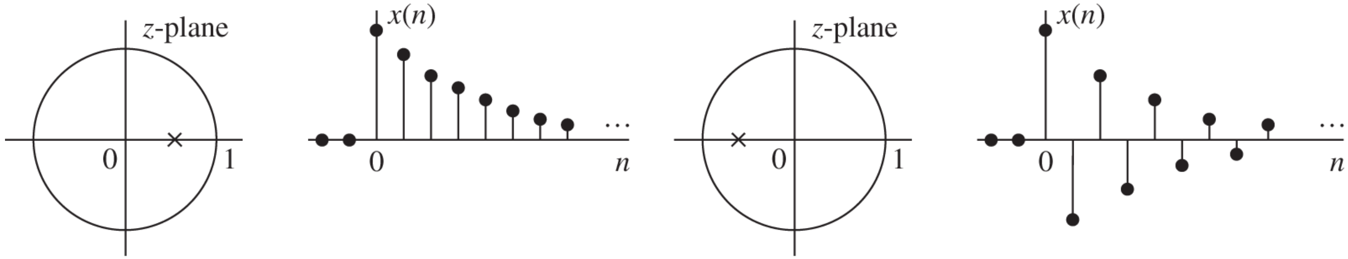
Time-domain behavior: single real pole



Take-aways on time-domain behavior:

1) Causal signals with poles inside the unit circle are always bounded in time
2) Rate of decay depends on distance of pole from z=1
3) Zeros have less dramatic effects (we'll see later)

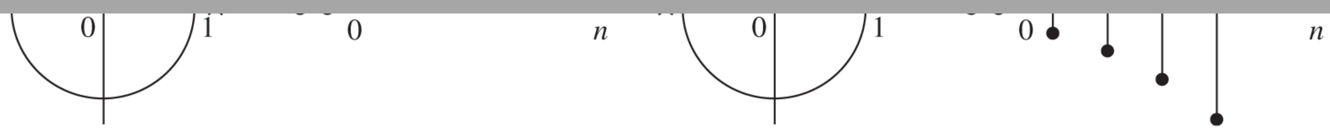


Figure 3.3.5 Time-domain behavior of a single-real-pole causal signal as a function of the location of the pole with respect to the unit circle.

Here, we could be describing a signal x(n) or an impulse response h(n); math is same

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Time-domain behavior: double complex pole

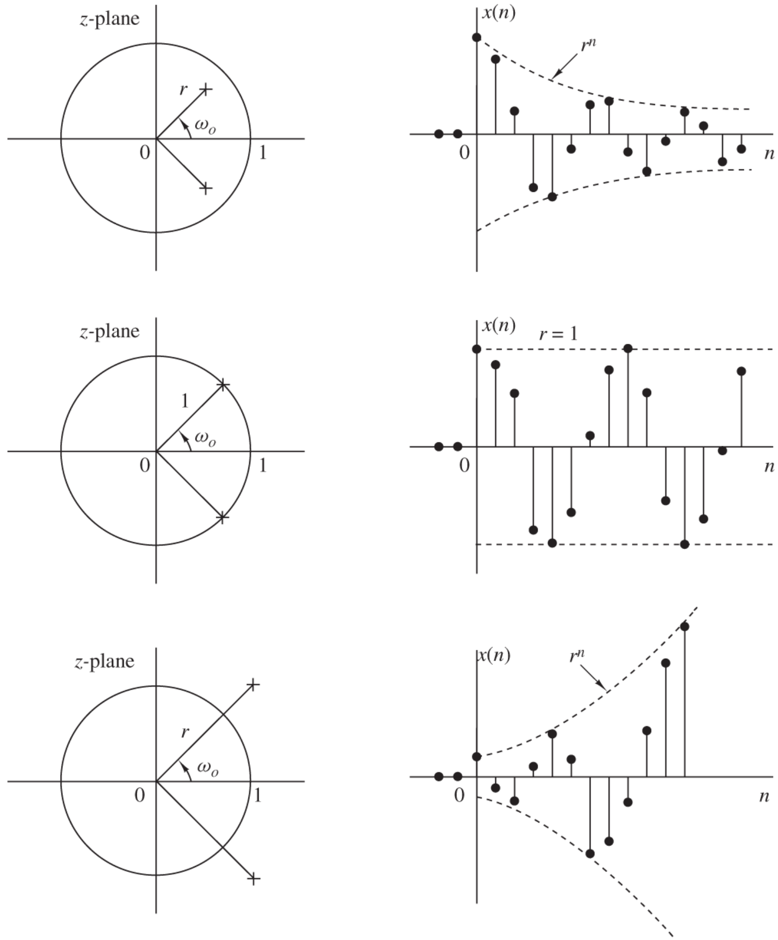

Figure 3.3.7 A pair of complex-conjugate poles corresponds to causal signals with oscillatory behavior.

Here, we could be describing a signal x(n) or an impulse response h(n); math is same

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Time-domain behavior: double real pole

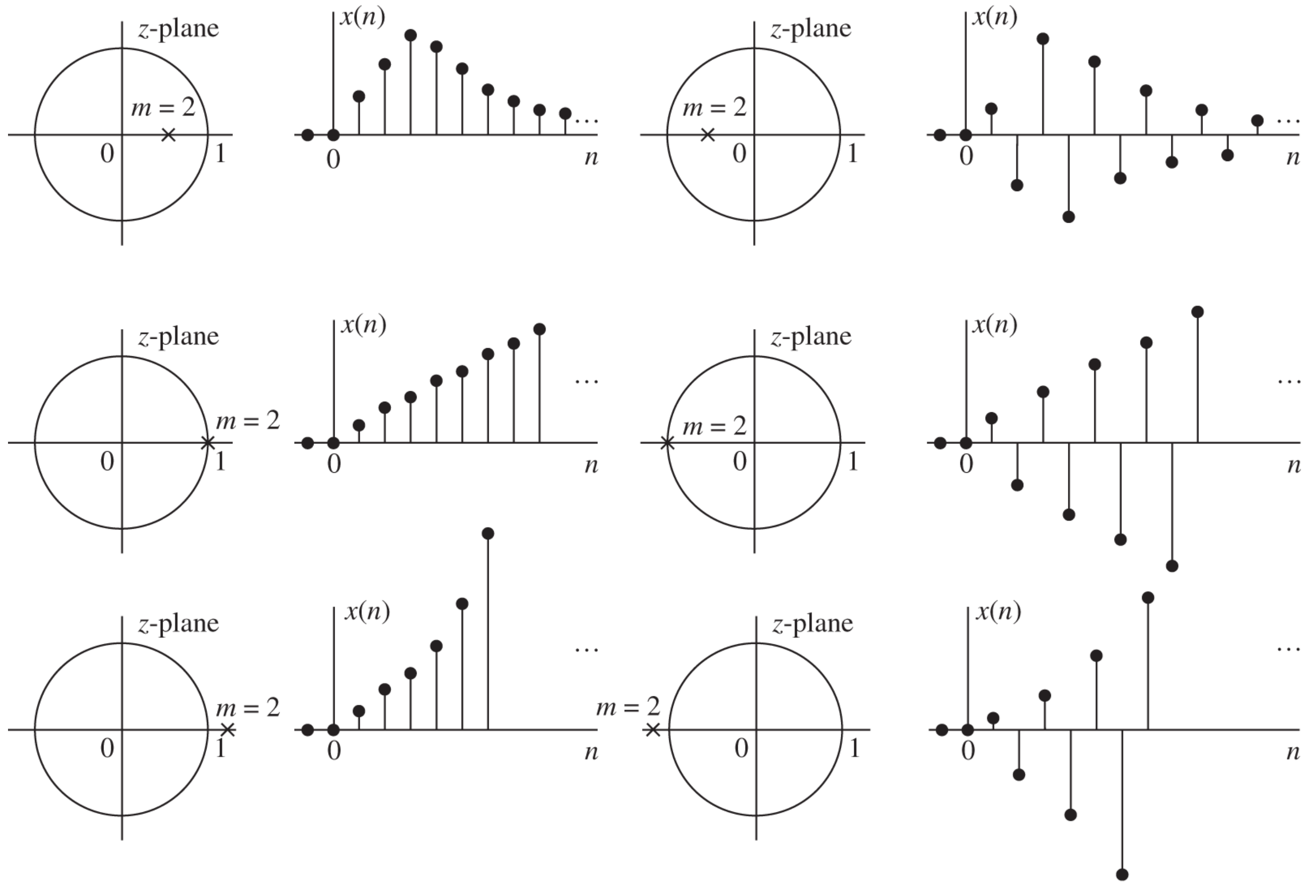

Figure 3.3.6 Time-domain behavior of causal signals corresponding to a double ($m = 2$) real pole, as a function of the pole location.

Here, we could be describing a signal x(n) or an impulse response h(n); math is same

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General case for rational functions - 1

- Start with a difference equation (almost always**, set a0=1**)

$$a_0 y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

- Take the Z-transform, and find H as a *ratio* (rational function):

$$H(z) \;=\; \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}}$$

- Pull out the highest power, and divide through by b0 and a0:

$$H(z) \;=\; \frac{b_0 z^{-M}}{a_0 z^{-N}} \frac{\sum_{k=0}^{M} b_k / b_0 z^{M-k}}{\sum_{k=0}^{N} a_k / a_0 z^{N-k}}$$

Rational functions - 2

- Factor the numerator and denominator into poles, zeros

$$H(z) = \frac{b_0}{a_0} z^{-M+N} \frac{(z-z_1)(z-z_2)\ldots(z-z_M)}{(z-p_1)(z-p_2)\ldots(z-p_N)}$$

- Finally, rewrite, giving:

M zeros

$$H(z) = G z^{(-M+N)} \frac{\Pi_{k=1}^{M}(z-z_k)}{\Pi_{k=1}^{N}(z-p_k)}$$

N poles

Gain G

If we specify poles/zeros, we know H to within a constant. We can find G if given a constraint; for example, H(z=1) = 1

|N-M|-th order zero at z=0, if N>M

|N-M|-th order pole at z=0, if M>N

Counting these, total # poles always = total # zeros