Administrative

- Delay in Matlab2 grades; Matlab3 comments this week
- Matlab4 due on Thursday
- Trunk schedule change (fix!) for end-of term
  - no exam during finals week (last year's plan)
  - also no quiz 3; usually each quiz is 5% of grade. I'd like to keep each quiz at 5% of grade
- Thursday office hours: 3-4 ok, can't do 4-5. When to switch?

Clarifications on Matlab4

- Part 1
  - For non-fully-padded case; pad x and h to be same length as the longer of the two
  - For the h(n) and x(n) given, you won't be able to hear the time aliasing, but plot of differences should show where it is
- Part 2
  - Better to compare DCT and FFT-based compression for 'on-screen' signal
  - Note that Matlab's DCT is not just a different normalization, but actually a different variant of the DCT than covered in book. However, Matlab's documentation will give you the formulas you need, as the PDF states

EE-125:
Digital Signal Processing

Metrics for DFT-based spectrum analysis

Professor Tracey

Tufts

Reminder: from lecture on windowing

- The DFT/FFT have two main uses
  - Fast FFT-based FIR filtering (overlap/add, etc)
  - Spectrum estimation / spectral analysis
- We may want to do spectral analysis in order to:
  - Learn something about a signal, either by human or automated analysis of the frequency content
  - Do processing in frequency domain (mp3, etc), then go back to time domain
- We'll consider three main topics
  - Deterministic, non-time-varying signals, possibly in random noise
  - Time-varying but non-random signals (spectrograms)
  - Random processes / noise (periodograms)

Tufts

Reminder: Example: 3 sinusoids, 2 closely spaced, Rectangular window

1. When doing DFT, we are applying a window (even if just boxcar)
2. Thus, spectrum estimate is given by convolution: $V(\omega)= X(\omega) * W(\omega)$
3. By increasing the window length, we get smaller mainlobe, and can resolve signals
4. High sidelobes can distort signal & mask weak signals

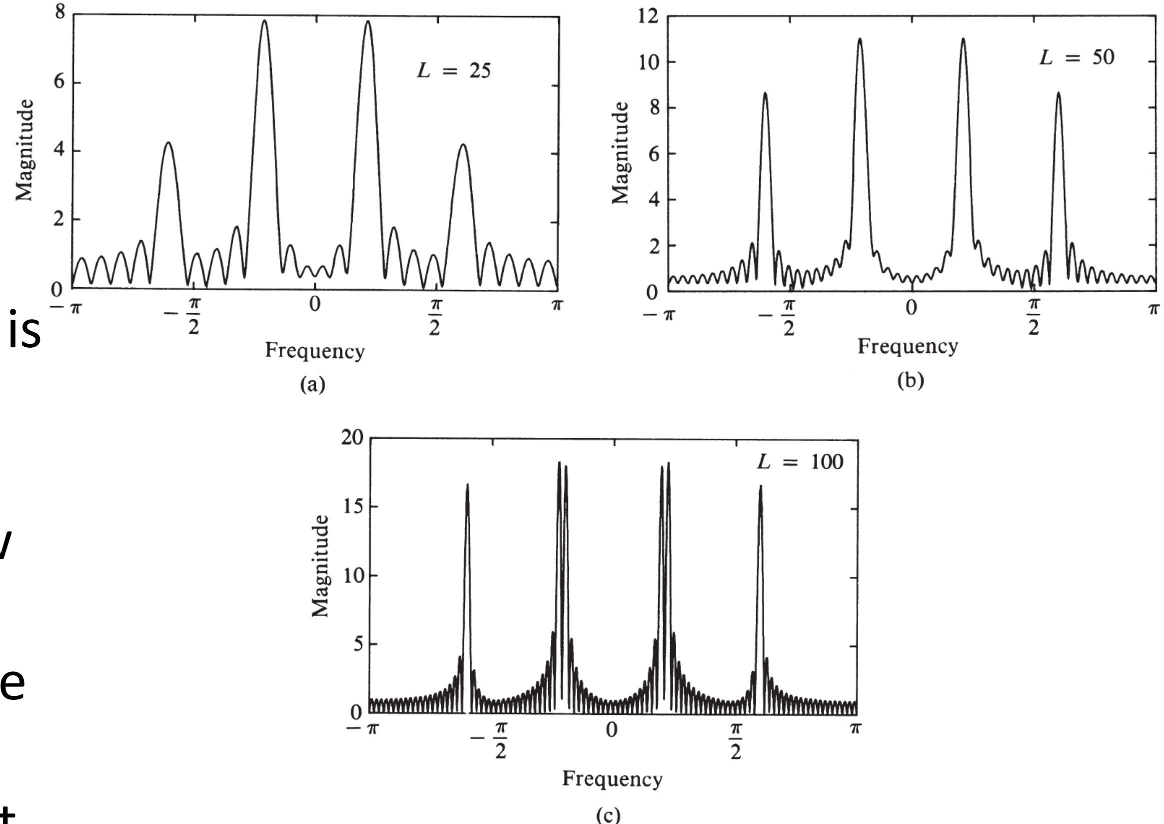


Figure 7.4.2 Magnitude spectrum for the signal given by (7.4.8), as observed through a rectangular window.

Reminder: Example: 3 sinusoids, 2 closely spaced, Hanning window

1. By using other windows, we can suppress sidelobes at cost of widening the main lobe
2. Intuition: window helps make signal look smoothly periodic
3. Mainlobe / sidelobe tradeoff

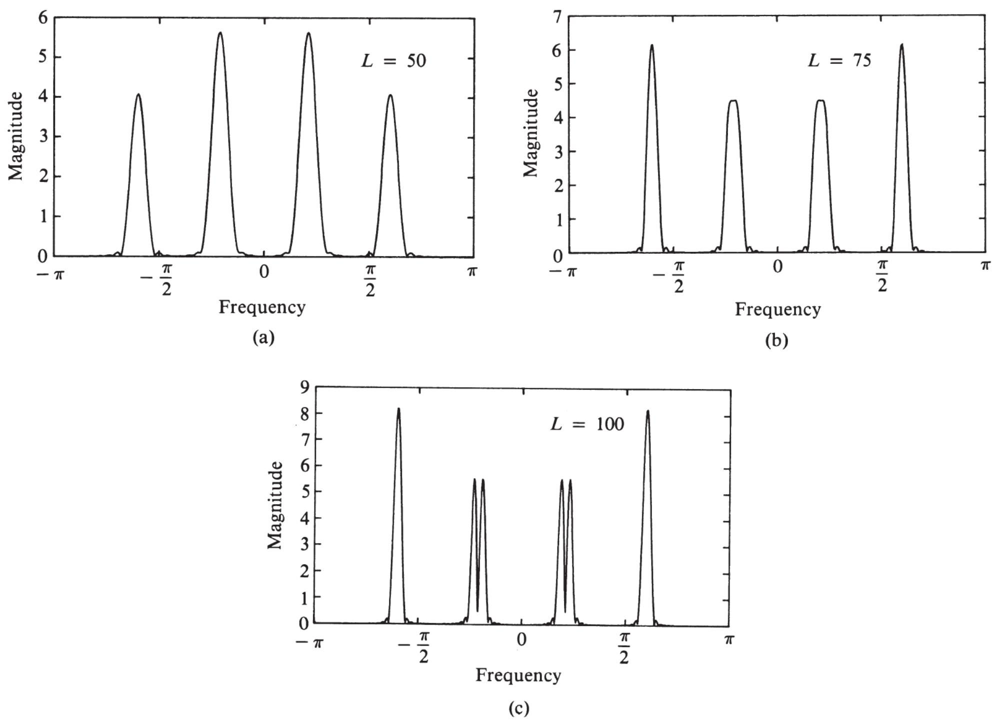


Figure 7.4.4 Magnitude spectrum of the signal in (7.4.8) as observed through a Hanning window.

Some common windows: Table 10.2 in book

Window type	Approx. main lobe width	Peak sidelobe, dB
Boxcar (rectangle)	4 pi/M	-13
Bartlett (triangle)	8 pi / M	-26
Hanning	8 pi / M	-31
Hamming	8 pi /M	-41
Blackman	12 pi / M	-57

Outline for today

- Some common misconceptions
  - –Spectrum is really sparse (picket fence effect)
  - –Zero-padding improves spectral resolution (i..e, my resolution is what the FFT gives me)
- Harris paper
  - –Famous paper on window design for spectrum estimation

Window Figures of Merit:

"On the use of windows for harmonic analysis with the Discrete Fourier Transform," F. Harris, Proc IEEE, 1978.

TABLE I
WINDOWS AND FIGURES OF MERIT

WINDOW		HIGHEST SIDE-LOBE LEVEL (dB)	SIDE-LOBE FALL-OFF (dB/OCT)	COHERENT GAIN	EQUIV NOISE BW (BINS)	3.0-dB BW (BINS)	SCALLOP LOSS (dB)	WORST CASE PROCESS LOSS (dB)	6.0-dB BW (BINS)	OVERLAP CORRELATION (PCNT) 75% OL	OVERLAP CORRELATION (PCNT) 50% OL
RECTANGLE		−13	−6	1.00	1.00	0.89	3.92	3.92	1.21	75.0	50.0
TRIANGLE		−27	−12	0.50	1.33	1.28	1.82	3.07	1.78	71.9	25.0
$\cos^{\alpha}(x)$	α = 1.0	−23	−12	0.64	1.23	1.20	2.10	3.01	1.65	75.5	31.8
HANNING	α = 2.0	−32	−18	0.50	1.50	1.44	1.42	3.18	2.00	65.9	16.7
	α = 3.0	−39	−24	0.42	1.73	1.66	1.08	3.47	2.32	56.7	8.5
	α = 4.0	−47	−30	0.38	1.94	1.86	0.86	3.75	2.59	48.6	4.3
HAMMING		−43	−6	0.54	1.36	1.30	1.78	3.10	1.81	70.7	23.5
RIESZ		−21	−12	0.67	1.20	1.16	2.22	3.01	1.59	76.5	34.4
RIEMANN		−26	−12	0.59	1.30	1.26	1.89	3.03	1.74	73.4	27.4
DE LA VALLE-POUSSIN		−53	−24	0.38	1.92	1.82	0.90	3.72	2.55	49.3	5.0
TUKEY	α = 0.25	−14	−18	0.88	1.10	1.01	2.96	3.39	1.38	74.1	44.4
	α = 0.50	−15	−18	0.75	1.22	1.15	2.24	3.11	1.57	72.7	36.4
	α = 0.75	−19	−18	0.63	1.36	1.31	1.73	3.07	1.80	70.5	25.1
BOHMAN		−46	−24	0.41	1.79	1.71	1.02	3.54	2.38	54.5	7.4
POISSON	α = 2.0	−19	−6	0.44	1.30	1.21	2.09	3.23	1.69	69.9	27.8
	α = 3.0	−24	−6	0.32	1.65	1.45	1.46	3.64	2.08	54.8	15.1
	α = 4.0	−31	−6	0.25	2.08	1.75	1.03	4.21	2.58	40.4	7.4
HANNING-POISSON	α = 0.5	−35	−18	0.43	1.61	1.54	1.26	3.33	2.14	61.3	12.6
	α = 1.0	−39	−18	0.38	1.73	1.64	1.11	3.50	2.30	56.0	9.2
	α = 2.0	NONE	−18	0.29	2.02	1.87	0.87	3.94	2.65	44.6	4.7
CAUCHY	α = 3.0	−31	−6	0.42	1.48	1.34	1.71	3.40	1.90	61.6	20.2
	α = 4.0	−35	−6	0.33	1.76	1.50	1.36	3.83	2.20	48.8	13.2
	α = 5.0	−30	−6	0.28	2.06	1.68	1.13	4.28	2.53	38.3	9.0
GAUSSIAN	α = 2.5	−42	−6	0.51	1.39	1.33	1.69	3.14	1.86	67.7	20.0
	α = 3.0	−55	−6	0.43	1.64	1.55	1.25	3.40	2.18	57.5	10.6
	α = 3.5	−69	−6	0.37	1.90	1.79	0.94	3.73	2.52	47.2	4.9
DOLPH-	α = 2.5	−50	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Window Figures of Merit:

"On the use of windows for harmonic analysis with the Discrete Fourier Transform," F. Harris, Proc IEEE, 1978.

TABLE I
WINDOWS AND FIGURES OF MERIT

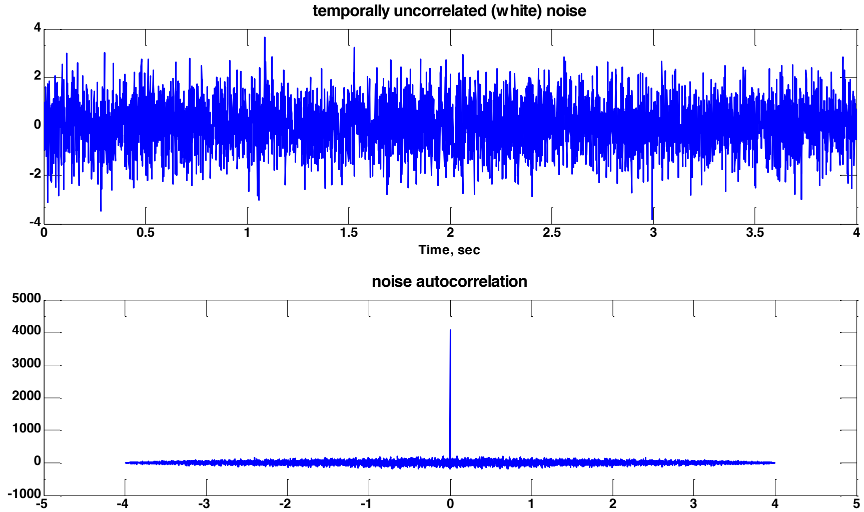
WINDOW		HIGHEST SIDE-LOBE LEVEL (dB)	SIDE-LOBE FALL-OFF (dB/OCT)	COHERENT GAIN	EQUIV NOISE BW (BINS)	3.0-dB BW (BINS)	SCALLOP LOSS (dB)	WORST CASE PROCESS LOSS (dB)	6.0-dB BW (BINS)	OVERLAP CORRELATION (PCNT) 75% OL	OVERLAP CORRELATION (PCNT) 50% OL
RECTANGLE		−13	−6	1.00	1.00	0.89	3.92	3.92	1.21	75.0	50.0
TRIANGLE		−27	−12	0.50	1.33	1.28	1.82	3.07	1.78	71.9	25.0
$\cos^{\alpha}(x)$ HANNING	$\alpha = 1.0$	−23	−12	0.64	1.23	1.20	2.10	3.01	1.65	75.5	31.8
	$\alpha = 2.0$	−32	−18	0.50	1.50	1.44	1.42	3.18	2.00	65.9	16.7
	$\alpha = 3.0$	−39	−24	0.42	1.73	1.66	1.08	3.47	2.32	56.7	8.5
	$\alpha = 4.0$	−47	−30	0.38	1.94	1.86	0.86	3.75	2.59	48.6	4.3
HAMMING		−43	−6	0.54	1.36	1.30	1.78	3.10	1.81	70.7	23.5
RIESZ		−21	−12	0.67	1.20	1.16	2.22	3.01	1.59	76.5	34.4
RIEMANN		−26	−12	0.59	1.30	1.26	1.89	3.03	1.74	73.4	27.4
DE LA VALLE-POUSSIN		−53	−24	0.38	1.92	1.82	0.90	3.72	[illegible]	[illegible]	5.0
TUKEY	$\alpha = 0.25$	−14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	44.4
	$\alpha = 0.50$	−15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	36.4
	$\alpha = 0.75$	−19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	25.1
BOHMAN		−46	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7.4
POISSON	$\alpha = 2.0$	−19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	27.8
	$\alpha = 3.0$	−24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15.1
	$\alpha = 4.0$	−31	−6	0.25	2.08	1.75	1.03	4.21	2.58	40.4	7.4
HANNING-POISSON	$\alpha = 0.5$	−35	−18	0.43	1.61	1.54	1.26	3.33	2.14	61.3	12.6
	$\alpha = 1.0$	−39	−18	0.38	1.73	1.64	1.11	3.50	2.30	56.0	9.2
	$\alpha = 2.0$	NONE	−18	0.29	2.02	1.87	0.87	3.94	2.65	44.6	4.7
CAUCHY	$\alpha = 3.0$	−31	−6	0.42	1.48	1.34	1.71	3.40	1.90	61.6	20.2
	$\alpha = 4.0$	−35	−6	0.33	1.76	1.50	1.36	3.83	2.20	48.8	13.2
	$\alpha = 5.0$	−30	−6	0.28	2.06	1.68	1.13	4.28	2.53	38.3	9.0
GAUSSIAN	$\alpha = 2.5$	−42	−6	0.51	1.39	1.33	1.69	3.14	1.86	67.7	20.0
	$\alpha = 3.0$	−55	−6	0.43	1.64	1.55	1.25	3.40	2.18	57.5	10.6
	$\alpha = 3.5$	−69	−6	0.37	1.90	1.79	0.94	3.73	2.52	47.2	4.9
DOLPH-	$\alpha = 2.5$	−50	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Important note; Harris paper assumes no zero-padding

AWGN – Additive Gaussian White Noise

- Additive – added to signal, passes through system

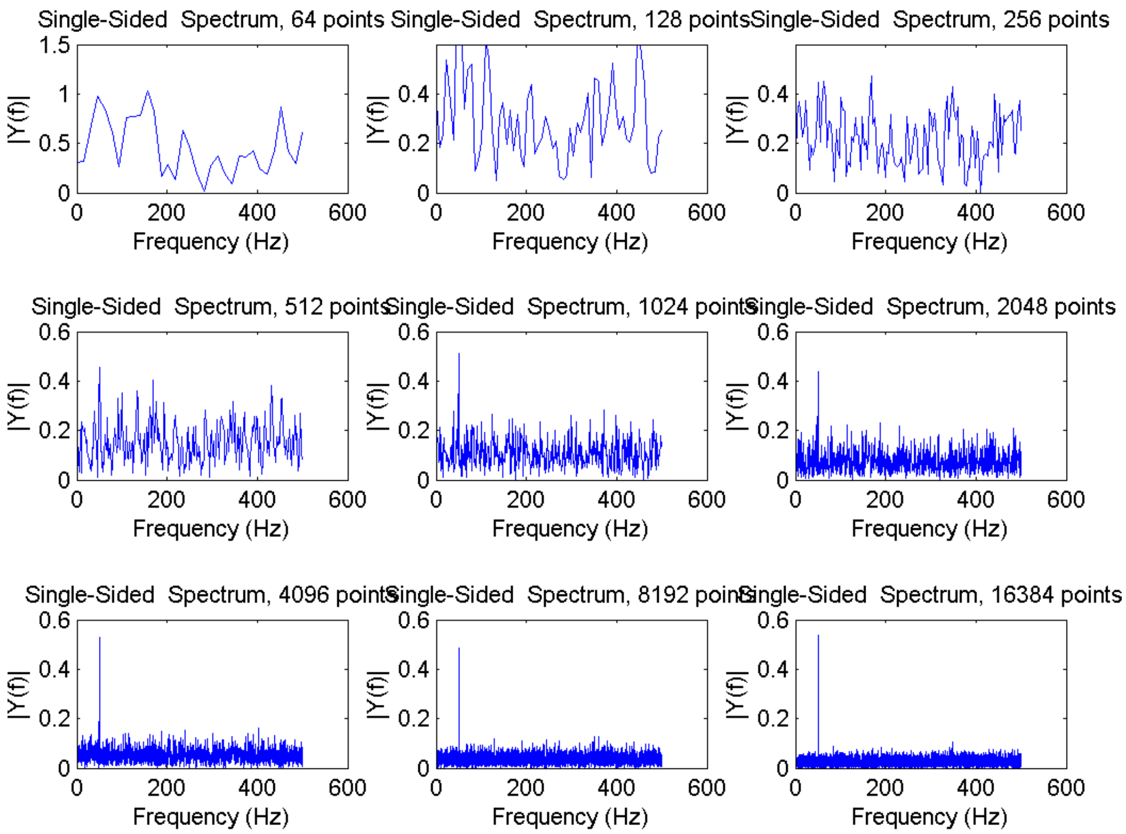
  $y = h*(x+w_{in}) = h*x + w$
- Gaussian – each individual sample is drawn from a Gaussian distribution: $N(0, \sigma^2)$ (sigma*randn in Matlab)
- White – temporally uncorrelated; each time sample is unrelated to previous or next, so get "white" spectrum



We saw that power spectrum is Fourier transform of auto-correlation

$$\gamma_{ww}(l) = \sigma_w^2 \delta(l)$$

Example of integration time benefit for tonal signal in noise



- Code integration_time_example.m uploaded to Trunk