EE-125: Digital Signal Processing

Review Topic: Z Transform

MATLAB assignments

- The ‘publish’ command can be a handy way to do your assignments
  - see script ‘publish_example_script.m’ in Lecture 3 folder on Trunk

- Getting matlab:

  https://it.tufts.edu/sw-matlabstudent

  Reactivating last year’s license may be tricky….

Z-transform

- Covering material from P&M 4.1
- Big picture: The Z-transform is like the Laplace transform, but for discrete-time (DT) signals and systems
- Why "z" transform?

Outline

- Definition of z transform
- Definition of Region of Convergence (ROC)
  - Points in z plane where transform X(z) doesn't blow up (sequence converges)
  - *Always* need to specify ROC
  - ROC for FIR vs IIR
- Z-transform properties

Z-transform is like the Laplace transform, but for discrete time

Continuous time (CT): Laplace (1782)

x(t) < ------ > X(s)

Math: use Laplace to solve constant-coefficient differential equations

Used to a) **analyze systems** (pole/zero plots) and
b) **understand their stability**

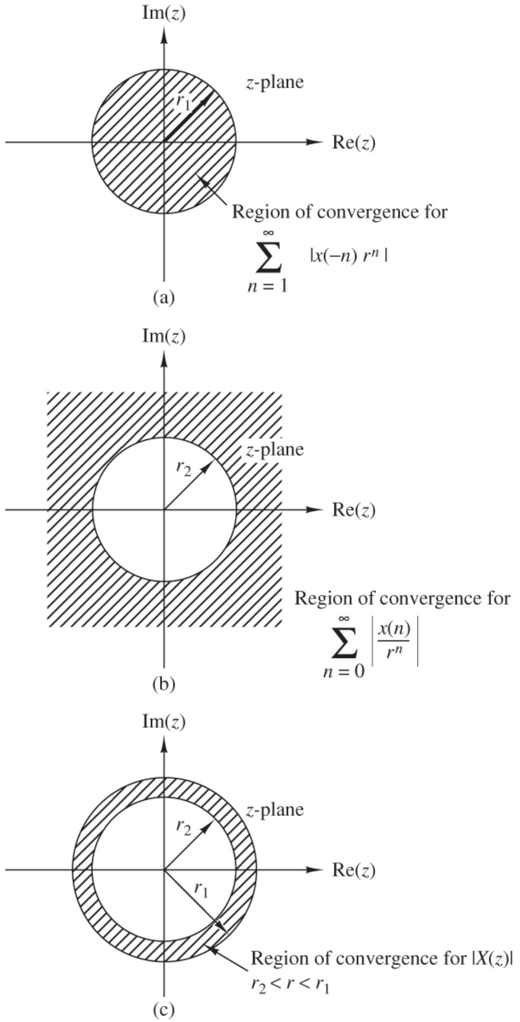
Discrete time (DT): Z (1950's)

x(n) < ------ > X(z)

Math: use Z to solve constant-coefficient difference equations

Used to a) **analyze systems** (pole/zero plots) and
b) **understand their stability**

Regions of Convergence



anticausal

causal

Two-sided

Figure 3.1.1 Region of convergence for $X(z)$ and its corresponding causal and anticausal components.

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Common z transforms

TABLE 3.3 Some Common z-Transform Pairs

	Signal, $x(n)$	z-Transform, $X(z)$	ROC
1	$\delta(n)$	1	All z
2	$u(n)$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3	$a^n u(n)$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
4	$na^n u(n)$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
5	$-a^n u(-n-1)$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
6	$-na^n u(-n-1)$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
7	$(\cos\omega_0 n)u(n)$	$\dfrac{1-z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\lvert z\rvert > 1$
8	$(\sin\omega_0 n)u(n)$	$\dfrac{z^{-1}\sin\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\lvert z\rvert > 1$
9	$(a^n\cos\omega_0 n)u(n)$	$\dfrac{1-az^{-1}\cos\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
10	$(a^n\sin\omega_0 n)u(n)$	$\dfrac{az^{-1}\sin\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$

Z-transform: Brute force

- $y(n) = 2\,(1/2)^{(n-2)}\ u(n-2)$. Find $Y(z)$.

brute force:

$$y = 2\left(\tfrac{1}{2}\right)^{-2}\left(\tfrac{1}{2}\right)^{n} u(n-2)$$

$$= 8\left(\tfrac{1}{2}\right)^{n} u(n-2)$$

$$Y(z) = 8 \sum_{n=2}^{\infty} \left(\tfrac{1}{2}\right)^{n} z^{-n}$$

$$= 8 \sum_{n=2}^{\infty} \left(\frac{1}{2z}\right)^{n}$$

Change variables: $m = n-2$, so $n = m+2$

$$Y(z) = 8 \sum_{m=0}^{\infty} \left(\tfrac{1}{2} z^{-1}\right)^{m+2}$$

$$= 8\left(\tfrac{1}{2} z^{-1}\right)^{2} \sum_{m=0}^{\infty} \left(\tfrac{1}{2} z^{-1}\right)^{m}$$

$$= 2z^{-2} \frac{1}{1-\tfrac{1}{2}z^{-1}} \quad |z| > \tfrac{1}{2}$$

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Example of using Z properties

<u>OR</u>: remember $a^n u(n) \Longleftrightarrow \frac{1}{1-az^{-1}}, \ |z|>a$

time-shift:

$$\left(\frac{1}{2}\right)^{n-2} u(n-2) \Longleftrightarrow z^{-2}\frac{1}{1-\frac{1}{2}z^{-1}}, \ |z|>\frac{1}{2}$$

<u>linearity</u>: multiply by 2

$$Y(z) = 2z^{-2}\frac{1}{1-\frac{1}{2}z^{-1}} \quad |z|>\frac{1}{2}$$

Inverse Z-transform

- Sadly, we won’t do much with inverse Z-transform in this class: mainly we will use the Z transform to analyze X(z)

- Inverse Z-transforms can be done by:
  - Inspection (answer is easy for finite-length sequences)
  - Known Z-transform pairs (Table 3.3 of book, for example)
  - Partial fractions
  - Contour integration

Review: regions of Convergence

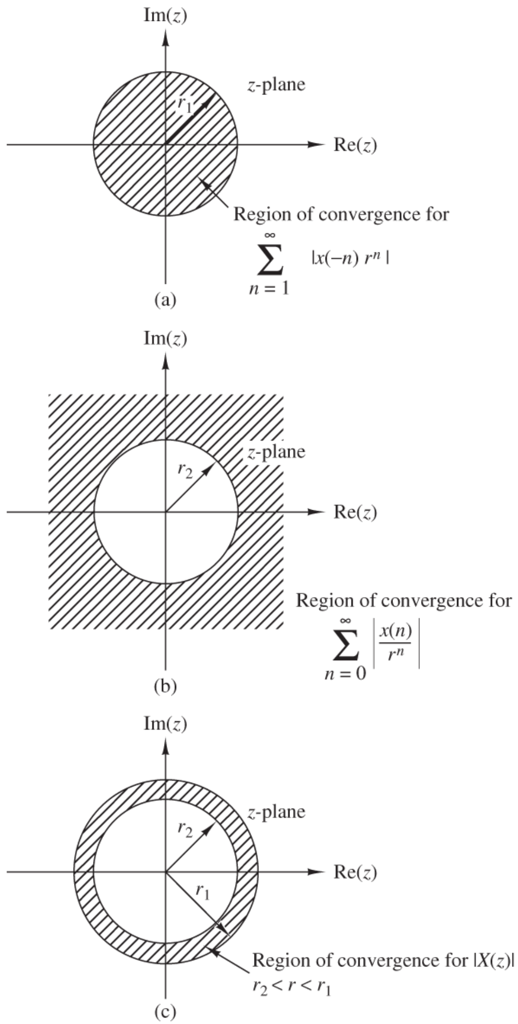

Figure 3.1.1 Region of convergence for $X(z)$ and its corresponding causal and anticausal components.

Anticausal signals
(turn off at n=0)
ROC: |z| < |a|

Causal signals
(turn off at n>=0)
ROC: |z| > |a|

Two-sided signals
(nonzero both sides of n=0)

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Z-transform: Brute force

- $y(n) = 2\,(1/2)^{(n-2)}\; u(n-2)$. Find $Y(z)$.

brute force: $y = 2\left(\tfrac{1}{2}\right)^{-2}\left(\tfrac{1}{2}\right)^{n} u(n-2)$

$= 8\left(\tfrac{1}{2}\right)^{n} u(n-2)$

$$Y(z) = 8 \sum_{n=2}^{\infty} \left(\frac{1}{2}\right)^{n} z^{-n}$$

$$= 8 \sum_{n=2}^{\infty} \left(\frac{1}{2z}\right)^{n}$$

Change variables: $m = n-2$, ~~so~~ so $n = m+2$

$$Y(z) = 8 \sum_{m=0}^{\infty} \left(\frac{1}{2} z^{-1}\right)^{m+2}$$

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Example of using Z properties

- $y(n) = 2\,(1/2)^{(n-2)}\ u(n-2)$. Find $Y(z)$.

<u>OR</u>: remember $a^n u(n) \Longleftrightarrow \frac{1}{1-az^{-1}}, \ |z| > a$

time-shift:

$$\left(\frac{1}{2}\right)^{n-2} u(n-2) \Longleftrightarrow z^{-2}\frac{1}{1-\frac{1}{2}z^{-1}}, \ |z| > \frac{1}{2}$$

<u>linearity</u>: multiply by 2

$$Y(z) = 2z^{-2}\frac{1}{1-\frac{1}{2}z^{-1}} \quad |z| > \frac{1}{2}$$