Recitation 1

Histograms

Agenda

1. Images
2. Intensity Transformations
3. Histogram Equalization
4. Histogram Matching
5. Ex 1

Images

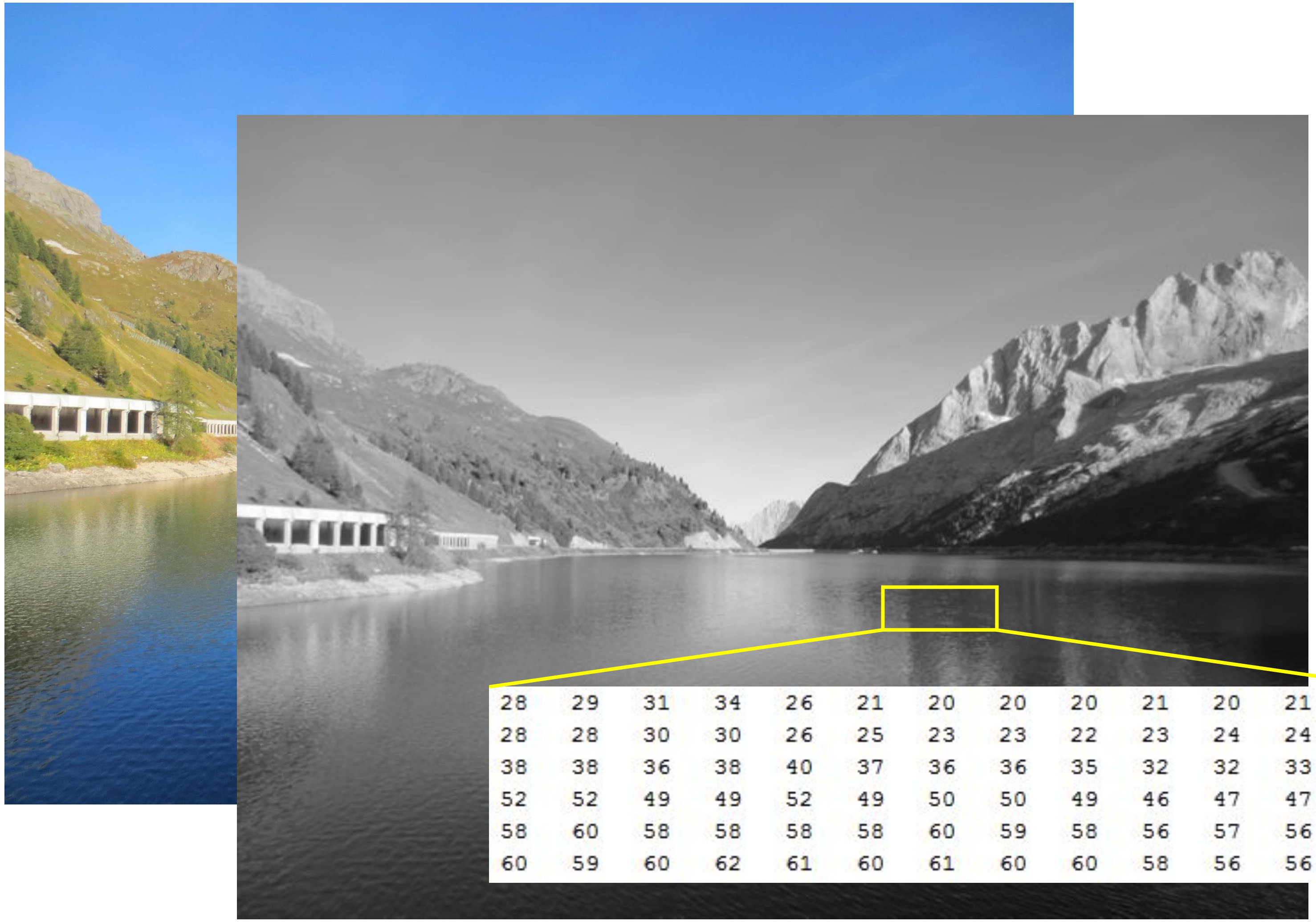
Intensity transformations

Histogram equalization

Histogram Matching

Ex 1

Images



28	29	31	34	26	21	20	20	20	21	20	21	21
28	28	30	30	26	25	23	23	22	23	24	24	25
38	38	36	38	40	37	36	36	35	32	32	33	34
52	52	49	49	52	49	50	50	49	46	47	47	45
58	60	58	58	58	58	60	59	58	56	57	56	55
60	59	60	62	61	60	61	60	60	58	56	56	56

Images

- Pixel (picture element) is the smallest single component of a digital image
- The number of pixels in an image is called the resolution

- In most cases:
  - Grayscale images have 8 bpp (256 intensities)
  - Color images have 24 bpp (each channel has 8 bpp)

(159)

(r,g,b) = (105,173,255)

(bpp = bits per pixel)

Images

- The value of a pixel describes how bright that pixel is.
- In cases of 8 bpp the range is between 0 and 255.

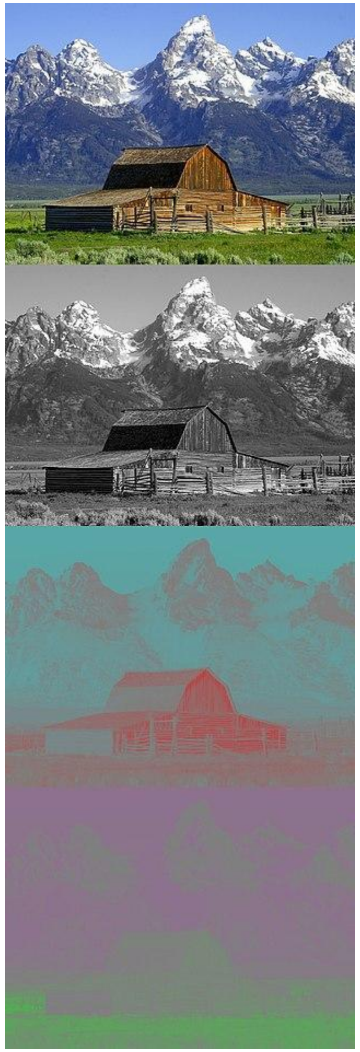
Representing Common Image Types

- **Intensity (or grayscale) images** -
  Either uint8 in the range {0,...,255} or floats in the range [0,1]

- **Binary images** -
  boolean matrices of 0s and 1s

- **RGB images** -
  three channels

Color Spaces



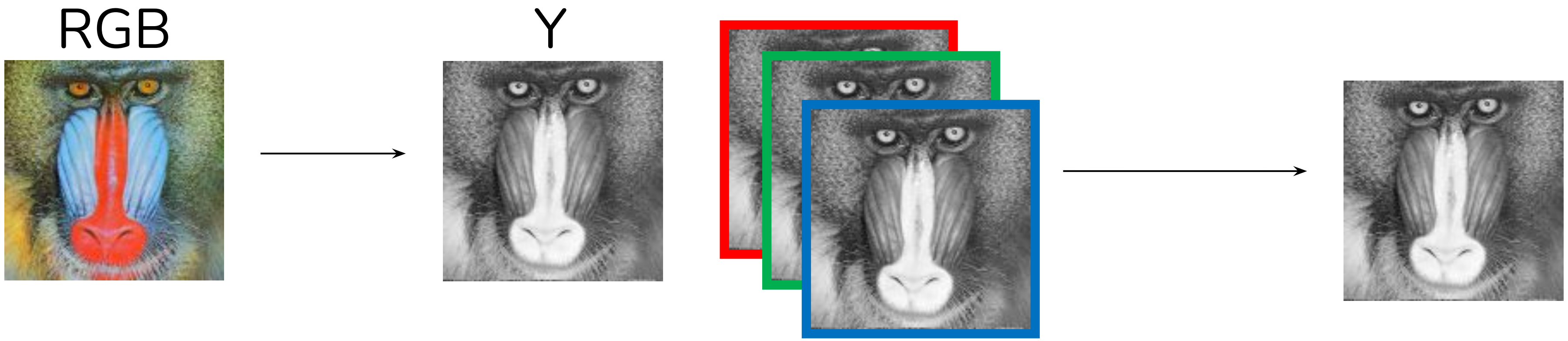
RGB

Y

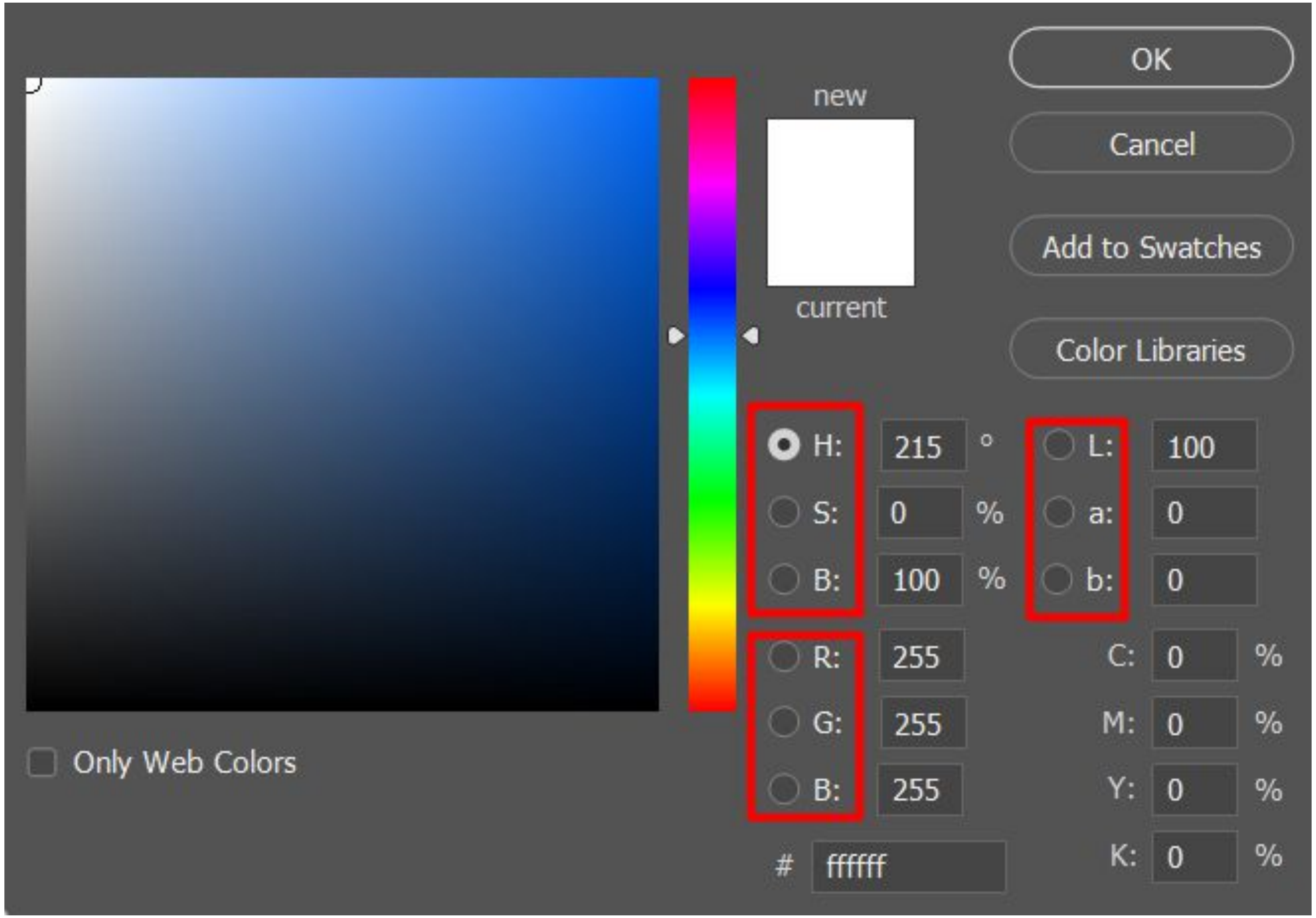
I

Q

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.321 \\ 0.212 & -0.523 & 0.311 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$



Color Spaces



Adobe Phtotoshop 2019

Images

Intensity Transformation

Histogram equalization

Histogram Matching

Ex 1

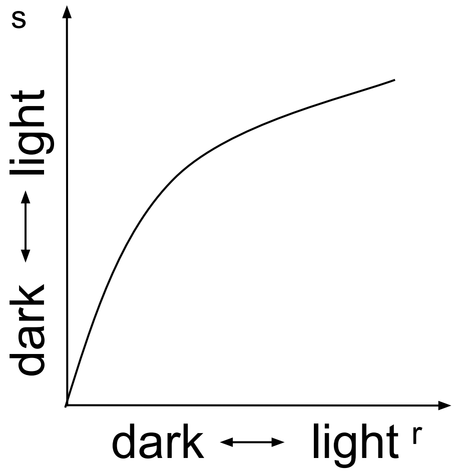
Intensity Transformation function

- In the case of a single-pixel transformation – we can use a lookup table to represent it

$$\boxed{s = T(r)}$$

r	s
0	s_0
1	s_1
2	s_2
3	s_3
.	
.	
.	
.	
.	
.	
.	
.	
255	s_{255}

Example



r	s
0	0
1	48
2	60
3	67
.	
.	
.	
.	
.	
.	
.	
.	
255	255

$$s, r \in \{0, \dots, 255\}$$

$$s = 255\left(\frac{r}{255}\right)^{0.3}$$

$$\downarrow$$

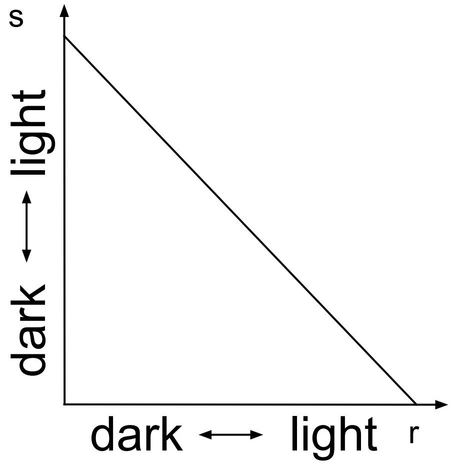
$$s, r \in [0,1]$$

$$s = r^{0.3}$$

Simple Examples of Intensity Transformations

- **Image negative**

$$s = 1 - r$$



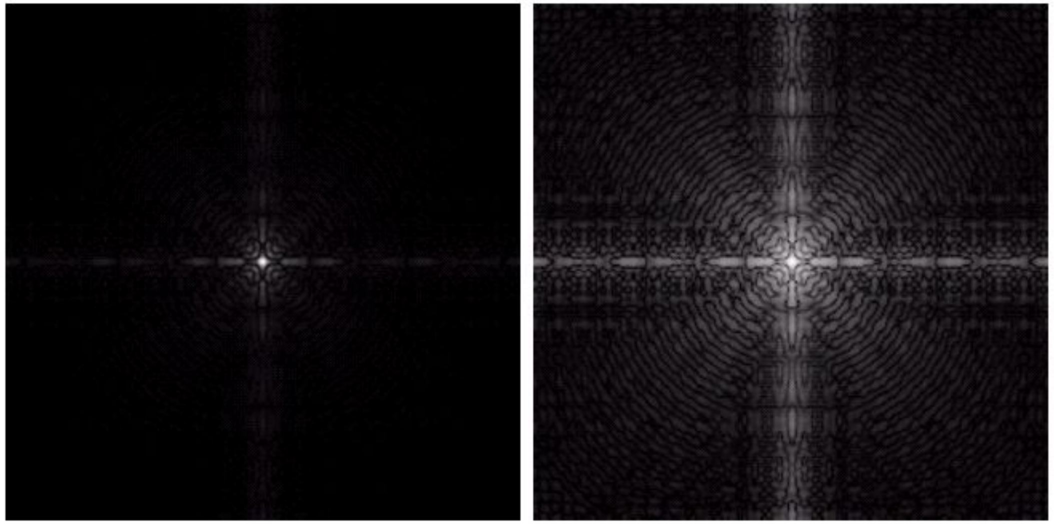
$s, r \in [0,1]$

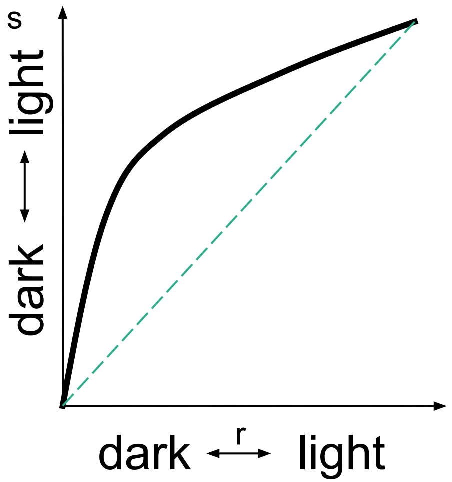
Simple Examples of Intensity Transformations

Log Transform

$$s = c * \log(1 + r)$$

$$c = \frac{255}{\log(1 + maxInpVal)}$$





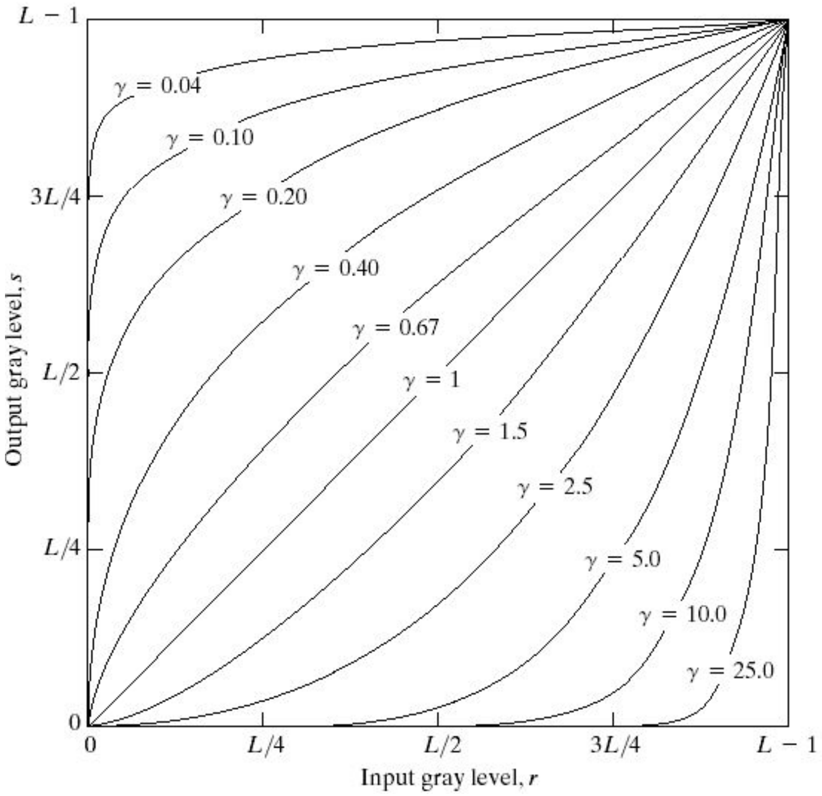
$s, r \in \{0, \dots, 255\}$

Simple Examples of Intensity Transformations

- **Power Law Transform**

$$s = c \cdot r^{\gamma}$$

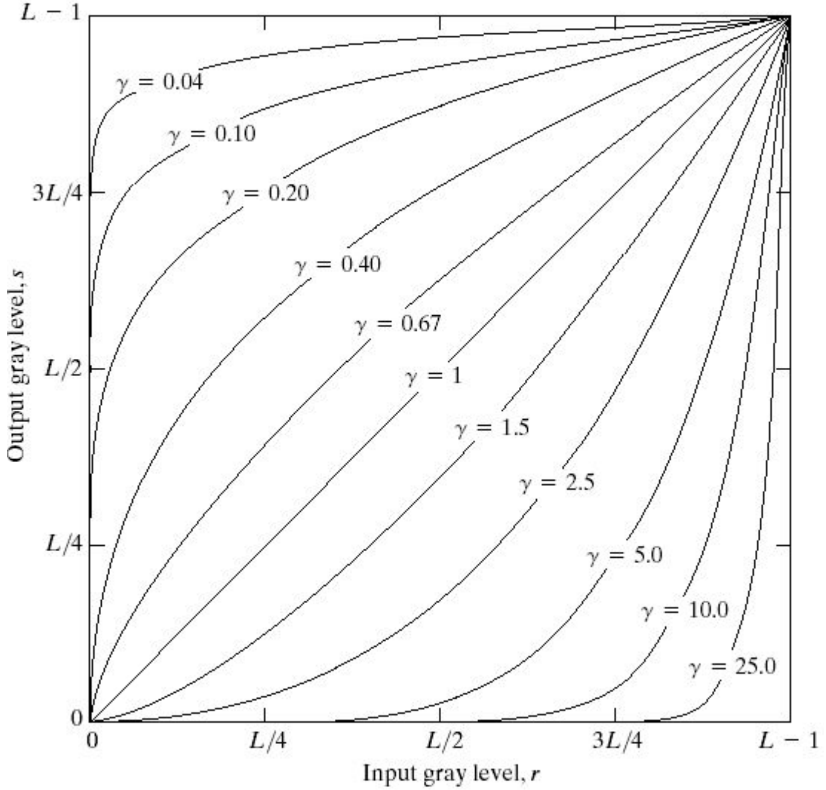
$\gamma = 2$



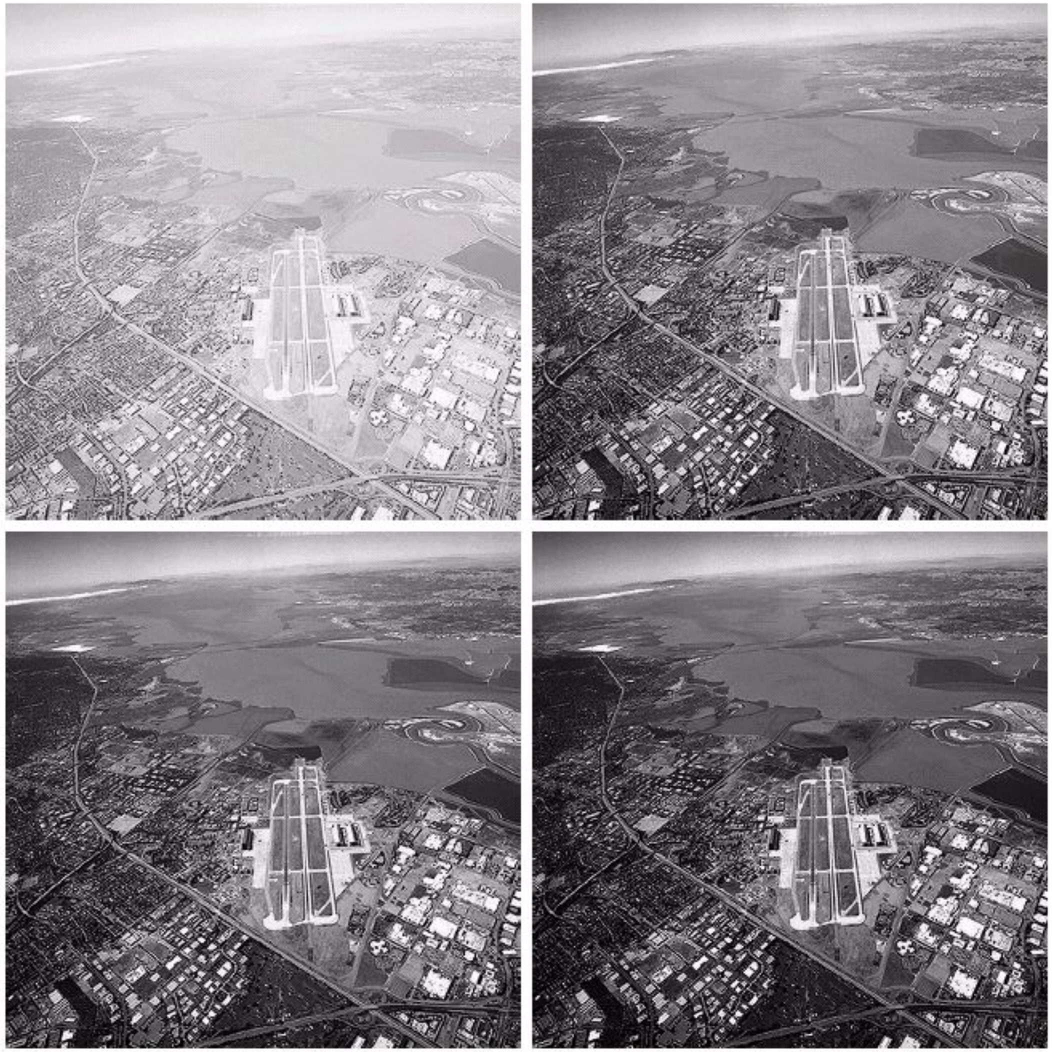
$s, r \in \{0, \ldots, 255\}$

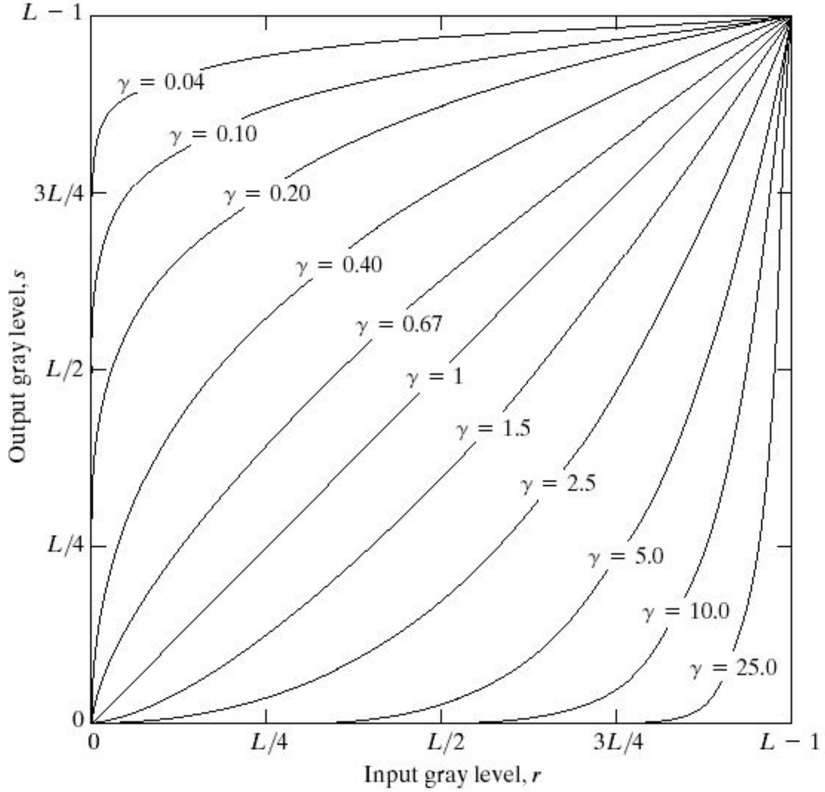
Gamma Correction - Example

What kind of correction should we try here?



Gamma Correction - Example





Gamma Correction

Images

Intensity Transformation

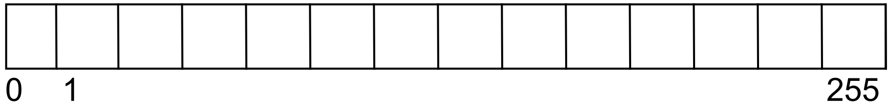
Histogram equalization

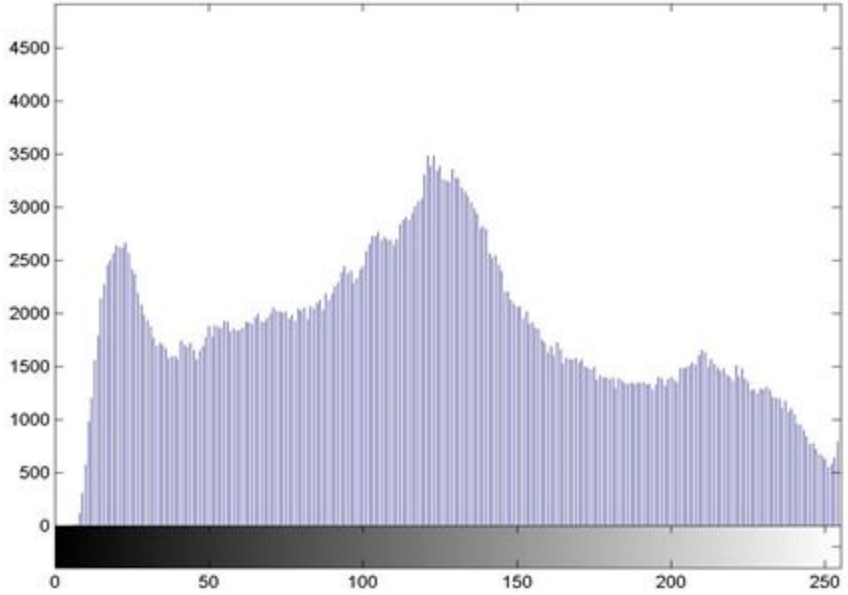
Histogram Matching

Ex 1

Histogram Processing

How many pixels intensity X has?





Histogram Processing

$$0 \le p(r_k) \le 1$$

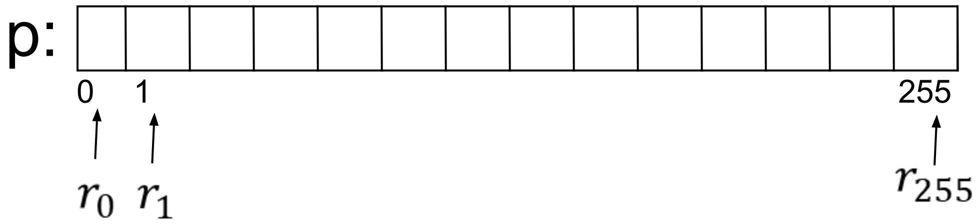


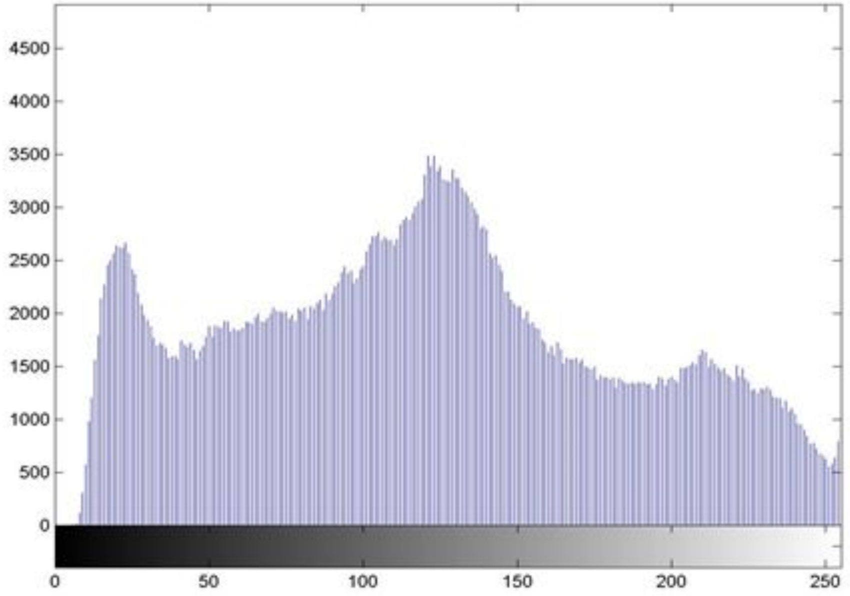
Image made of N pixels.

n_k *− How many pixels with intensity k*

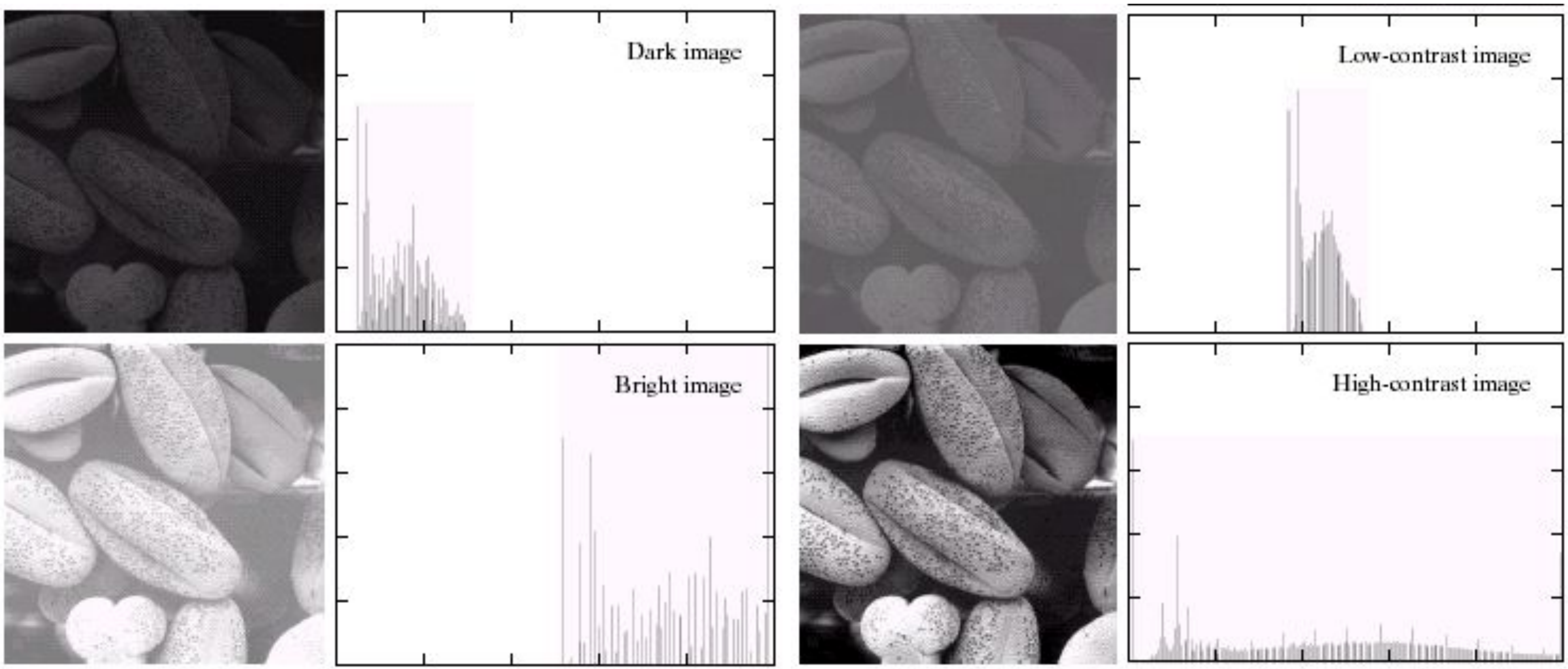
$$h(r_k) = n_k$$

$$\sum h(r_k) = N$$

To normalize: $p(r_k) = \dfrac{n_k}{N}$



Sample of Image Histograms

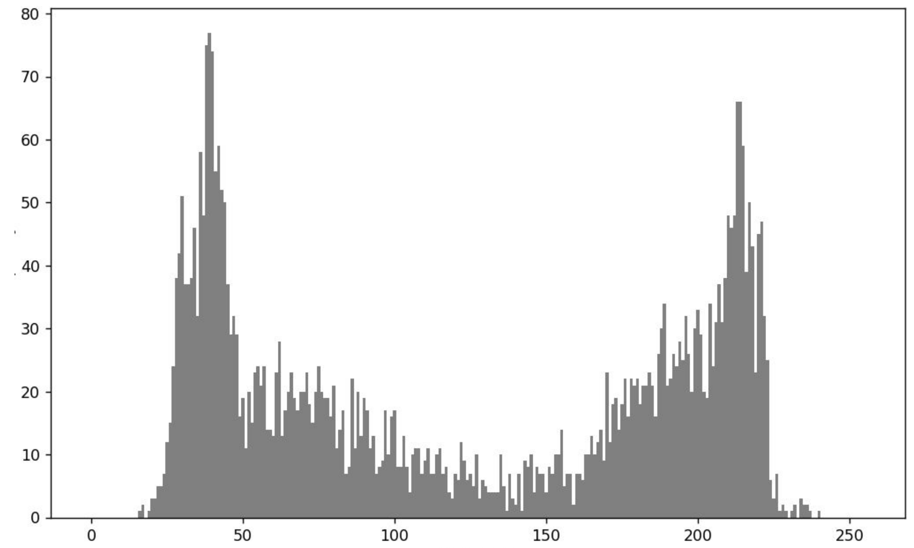


Histogram Equalization

Image made of N pixels.

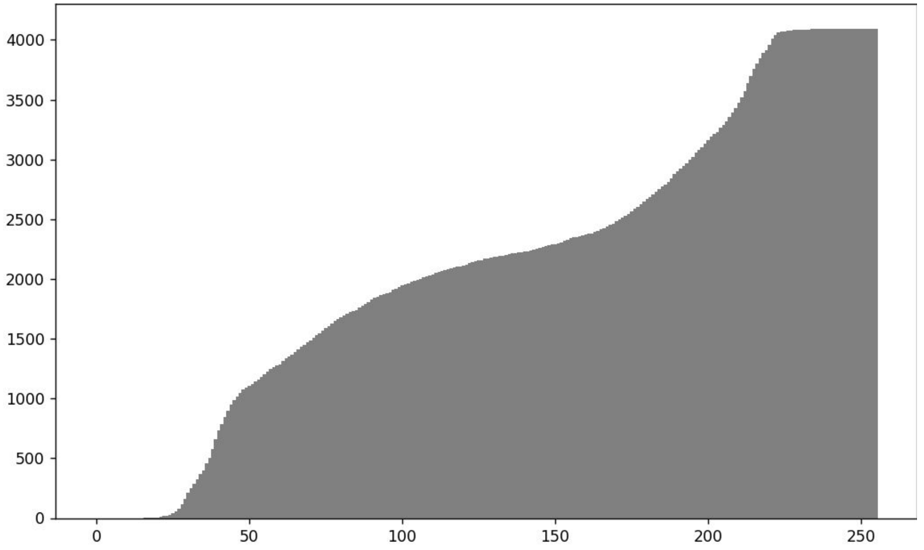
n_k – *How many pixels with intensity k*

$$h(r_k) = n_k \qquad \sum_{k=0}^{255} h(r_k) = N \qquad p(r_k) = \frac{n_k}{N}$$



cumulative histogram

$$C(k) = \sum_{i=0}^{k} n_i \qquad C(255) = \sum_{i=0}^{255} n_i = N$$



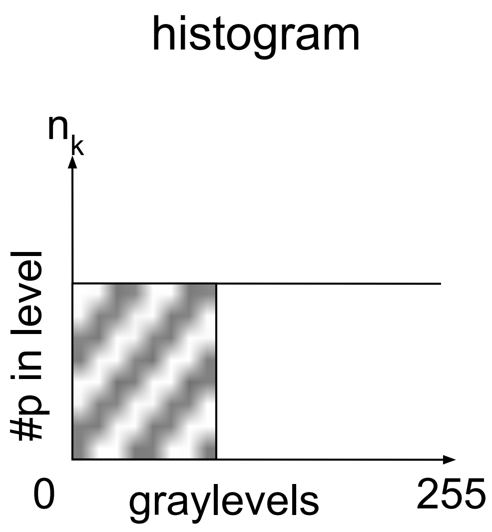
Histogram Equalization

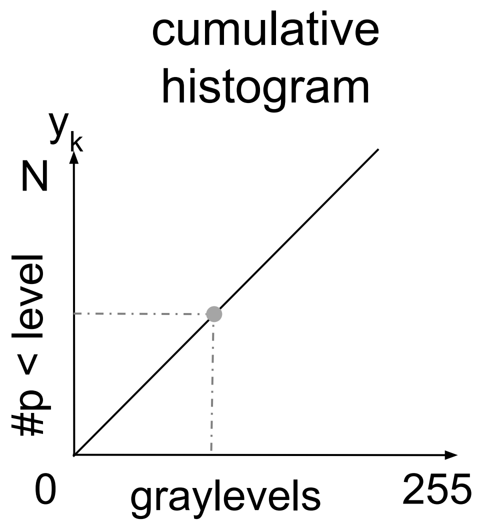
Motivation: improve the image contrast.

Goal: make equal use of all gray levels.

N pixels in range 0, … , K (assume K=255).

Uniform histogram ↔ linear cumulative histogram



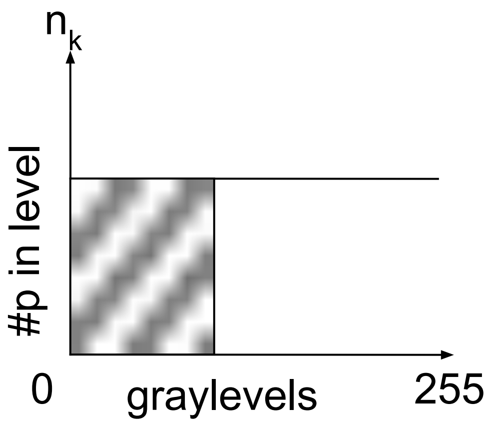


Histogram Equalization

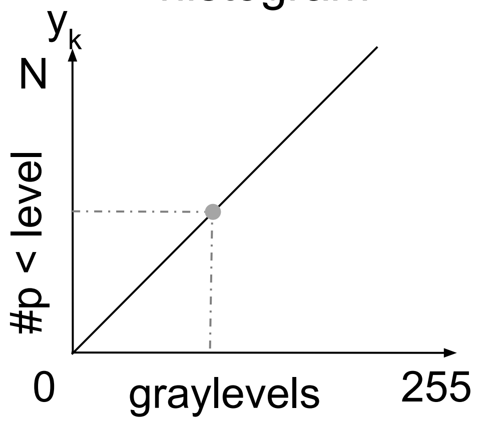
Ideally, we would like to equally use all gray levels

N pixels in range 0, ... K (assume K=255).

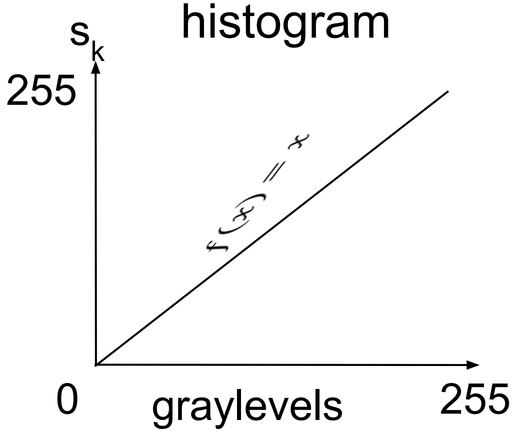
histogram



cumulative histogram



normalized cumulative histogram

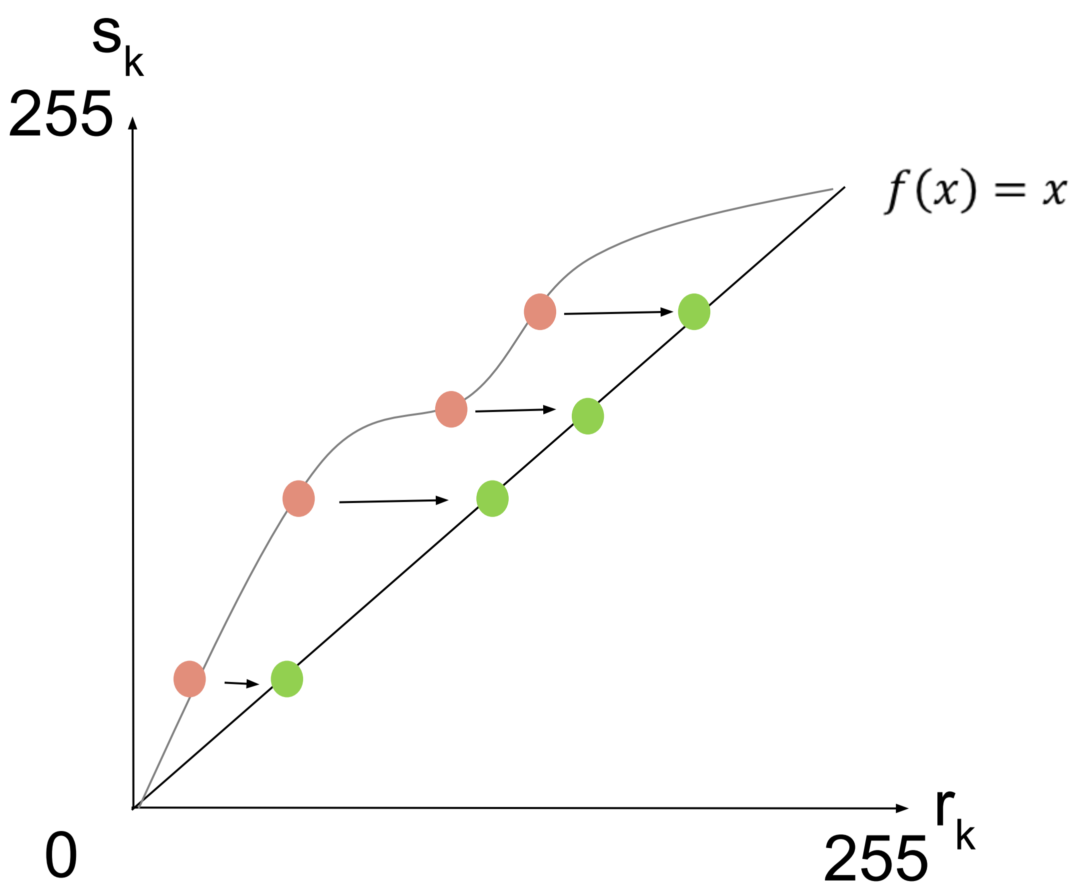


n_k is the number of

$$y_k = \sum_{i=0}^{k} n_i$$

$$s_k = \frac{255}{N} y_k = \frac{255}{N} \sum_{i=0}^{k} n_i$$

Histogram Equalization



Histogram Equalization: Algorithm

1. Compute the image histogram (np.histogram)
2. Compute the cumulative histogram (np.cumsum)
3. Normalize the cumulative histogram (divide by the total number of pixels)
4. Multiply the normalized histogram by the maximal gray level value (Z-1)

cumulative histogram C*(k)*
Let *m* be first grey level for which C*(m)≠0*

$$T(k) = round\left(255 * \frac{C(k)}{C(255)}\right) = round\left(255 * \frac{C(k)}{N}\right)$$

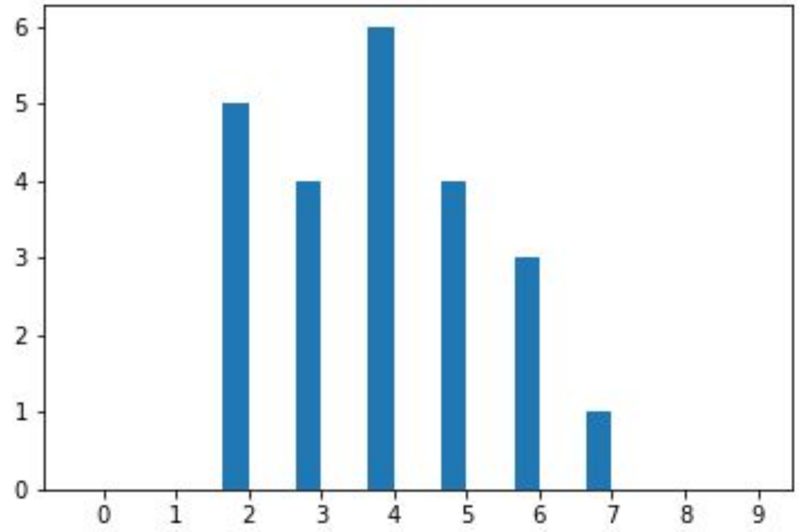
Histogram Equalization: Algorithm

cumulative histogram *C(k)*

$$T(k) = round\left(9 * \frac{C(k)}{C(9)}\right)$$

h:

0	1	2	3	4	5	6	7	8	9
0	0	5	4	6	4	3	1	0	0



Histogram Equalization: Algorithm

cumulative histogram *C(k)*

$$T(k) = round\left(9 * \frac{C(k)}{C(9)}\right)$$

h:

0	1	2	3	4	5	6	7	8	9
0	0	5	4	6	4	3	1	0	0

h cumulative:

0	1	2	3	4	5	6	7	8	9
0	0	5	9	15	19	22	23	23	23

Histogram Equalization: Algorithm

cumulative histogram *C(k)*

$$T(k) = round\left(9 * \frac{C(k)}{C(9)}\right)$$

h:

0	1	2	3	4	5	6	7	8	9
0	0	5	4	6	4	3	1	0	0

h cumulative:

0	1	2	3	4	5	6	7	8	9
0	0	5	9	15	19	22	23	23	23

T:

0	1	2	3	4	5	6	7	8	9
0	0	2	4	6	7	9	9	9	9

New h:

0	1	2	3	4	5	6	7	8	9
0	0	5	0	4	0	6	4	0	4

Example

$$T(2) = round\left(9 * \frac{C(2)}{C(9)}\right)$$

$$= round\left(9 * \frac{5}{23}\right) = round(1.9) = 2$$

Not חח"ע

Histogram Equalization: Algorithm

cumulative histogram *C(k)*

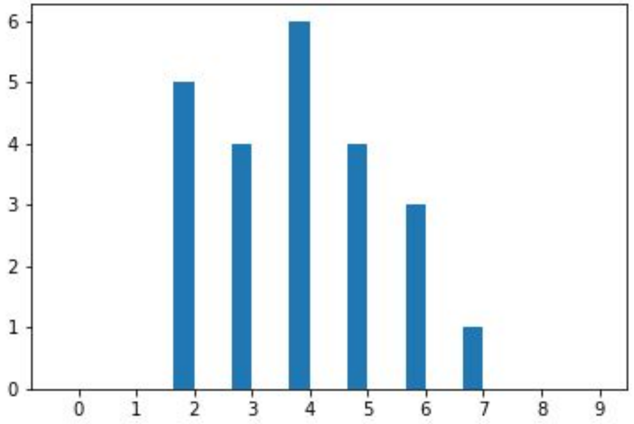
$$T(k) = round\left(9 * \frac{C(k)}{C(9)}\right)$$

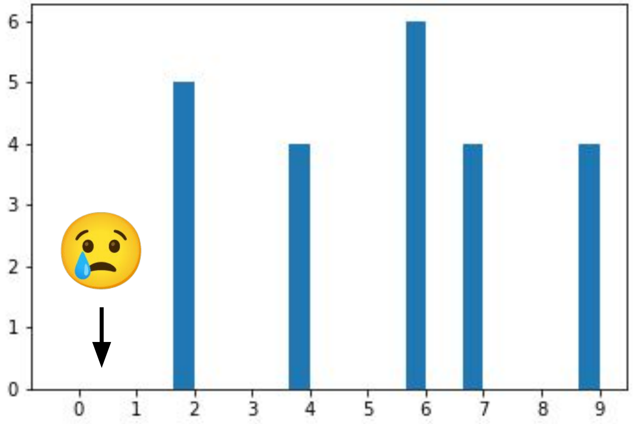
h:

0	1	2	3	4	5	6	7	8	9
0	0	5	4	6	4	3	1	0	0

New h:

0	1	2	3	4	5	6	7	8	9
0	0	5	0	4	0	6	4	0	4





Histogram Equalization: Algorithm

cumulative histogram *C(k)*
Let *m* be first grey level for which C*(m)≠0*

$$T(k) = round\left(9 * \frac{C(k) - \boldsymbol{C}(\boldsymbol{m})}{C(9)}\right)$$

h:

0	1	2	3	4	5	6	7	8	9
0	0	5	4	6	4	3	1	0	0

h cumulative:

0	1	2	3	4	5	6	7	8	9
0	0	5	9	15	19	22	23	23	23

T:

0	1	2	3	4	5	6	7	8	9
0	0	0	2	4	5	7	7	7	7

New h:

0	1	2	3	4	5	6	7	8	9
5	0	4	0	6	4	0	4	0	0

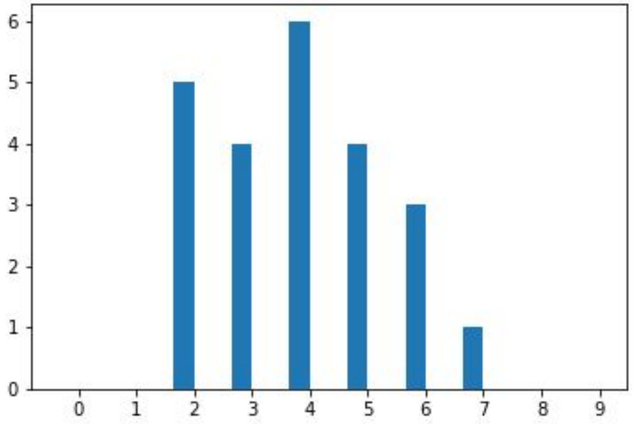
Histogram Equalization: Algorithm

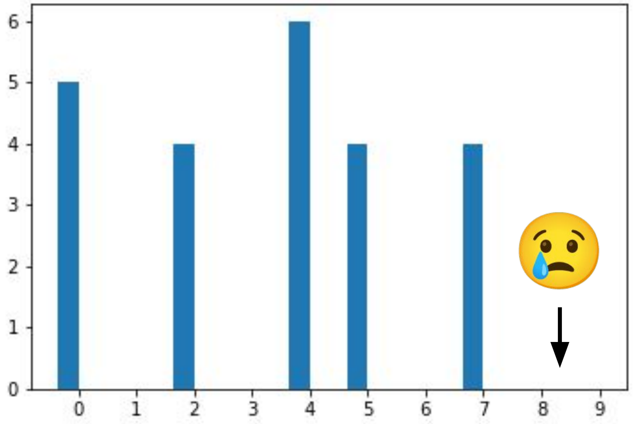
cumulative histogram *C(k)*
Let *m* be first grey level for which C*(m)≠0*

$$T(k) = round\left(9 * \frac{C(k) - \boldsymbol{C(m)}}{C(9)}\right)$$

	0	1	2	3	4	5	6	7	8	9
h:	0	0	5	4	6	4	3	1	0	0

	0	1	2	3	4	5	6	7	8	9
New h:	5	0	4	0	6	4	0	4	0	0





Histogram Equalization: Algorithm

cumulative histogram *C(k)*
Let *m* be first grey level for which C*(m)≠0*

$$T(k) = round\left(9 * \frac{C(k) - C(m)}{C(9) - \boldsymbol{C(m)}}\right)$$

h:

0	1	2	3	4	5	6	7	8	9
0	0	5	4	6	4	3	1	0	0

h cumulative:

0	1	2	3	4	5	6	7	8	9
0	0	5	9	15	19	22	23	23	23

T:

0	1	2	3	4	5	6	7	8	9
0	0	0	2	5	7	8	9	9	9

New h:

0	1	2	3	4	5	6	7	8	9
5	0	4	0	0	6	0	4	3	1

Histogram Equalization: Algorithm

cumulative histogram *C(k)*
Let *m* be first grey level for which C*(m)≠0*

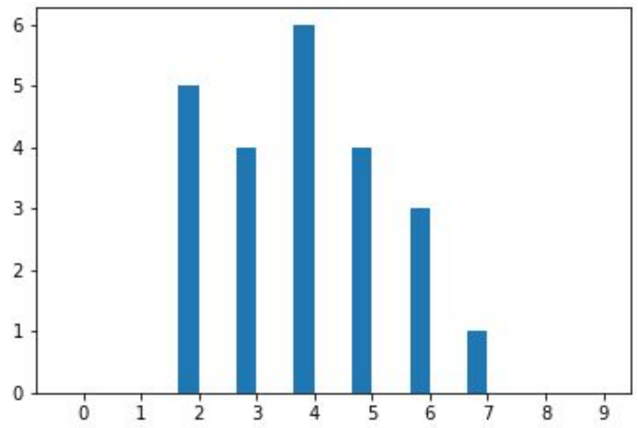
$$T(k) = round\left(9 * \frac{C(k) - C(m)}{C(9) - \boldsymbol{C(m)}}\right)$$

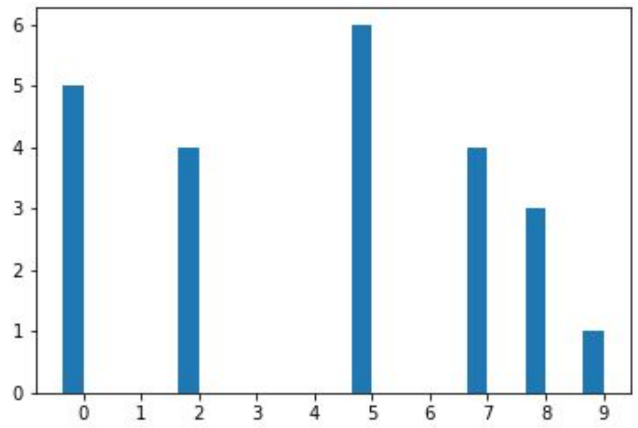
h:

0	1	2	3	4	5	6	7	8	9
0	0	5	4	6	4	3	1	0	0

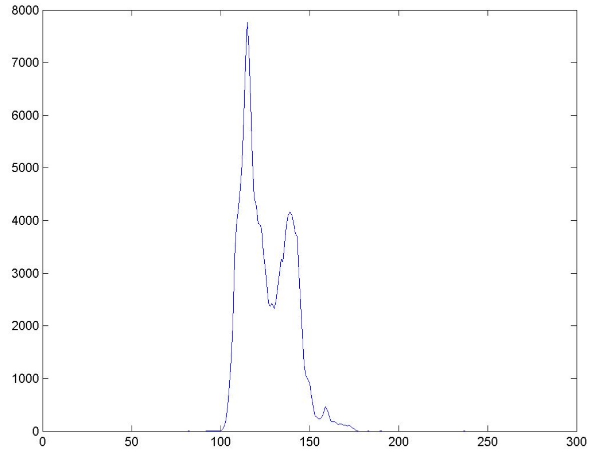
New h:

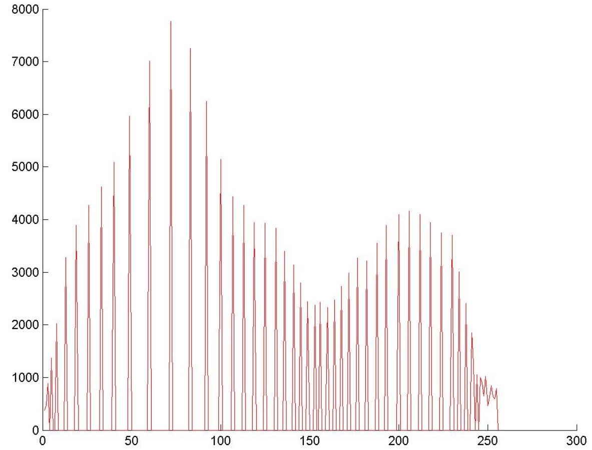
0	1	2	3	4	5	6	7	8	9
5	0	4	0	0	6	0	4	3	1



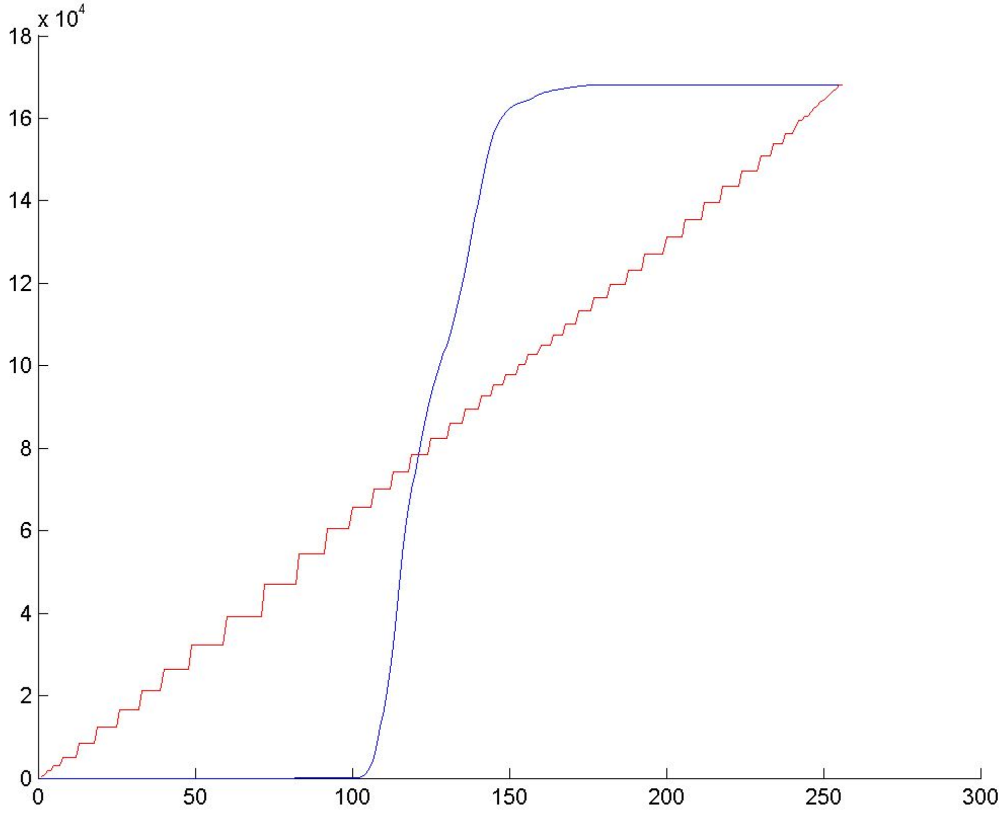


Histogram Equalization





Histogram Equalization



Histogram Equalization: Algorithm

1. Compute the image histogram (np.histogram)
2. Compute the cumulative histogram (np.cumsum)
3. Normalize the cumulative histogram (divide by the total number of pixels)
4. Multiply the normalized histogram by the maximal gray level value (Z-1)
5. Verify that the minimal value is 0 and that the maximal is Z-1, otherwise stretch the result linearly in the range [0,Z-1].
6. Round the values to get integers
7. Map the intensity values of the image using the result of step 6.

cumulative histogram C*(k)*
Let *m* be first grey level for which C*(m)≠0*

$$T(k) = round\left(255 * \frac{C(k) - C(m)}{C(255) - C(m)}\right)$$

Equalization Properties

- In most discrete images it is **impossible** to achieve a completely **uniform histogram** – only an approximation.

- The histogram equalization transformation is **monotonic** (as the process involves the cumulative sum), and therefore the relative brightness of a pixel is preserved.

- The number of different pixel values (=full bins in the histogram) can only decrease. We might **merge bins**, but we can't split them.

When will this fail ?

- When the desired gray level distribution is not uniform (e.g. text)
- When content from different sources is equalized together (e.g. two non related images)

OFFICIAL NEWSLETTER FOR THE LEONARD PELTIER DEFENSE COMMITTEE

Spirit of Crazy Horse

VOL. 7 NO. 2 | MARCH/APRIL | $2.00

Statement of Leonard Peltier

By Leonard Peltier

Greetings Friends and Supporters.

All too often, we fall into the habit of assuming the future brings us progress and enlightenment, while we often see the past as a more brutal and ignorant time. We kept hearing about how this new millennium would bring so much change, but from my perspective it sure seems like the racist attitudes and bigotry that existed one hundred years ago, are still prevailing today. In this issue of our paper, I want my message to be a reminder that we can never stop living in the present and uniting ourselves against injustice. The year 2000

Pine Ridge still do not have their basic needs met as is guaranteed under international law. I say that because I know that once the people are awake and in realization of their commonality, we can make the kinds of changes happen that need to happen. And, I know that it is you who can define what this new millennium really will bring.

A broad and united coalition must be built to stop police and FBI brutality, corruption, and obstruction of justice. This coalition must also focus on ending the prison industrial complex and the war against the people so often referred to as the war on

with a story to tell.

When I am finally released from prison, I do not plan to close the LPDC office. Instead, I plan to use our office as another resource to continue the struggle for Native rights, to free all political prisoners, and to stop FBI and police abuses. I want to thank Rigoberta Menchu Tum for her recent visit to the United States on my behalf. And, I want to thank everyone who has been sacrificing and working on the behalf of my freedom. I know it is you who defines my future and you make me hopeful enough to dream of soon being free.

Equalization Results

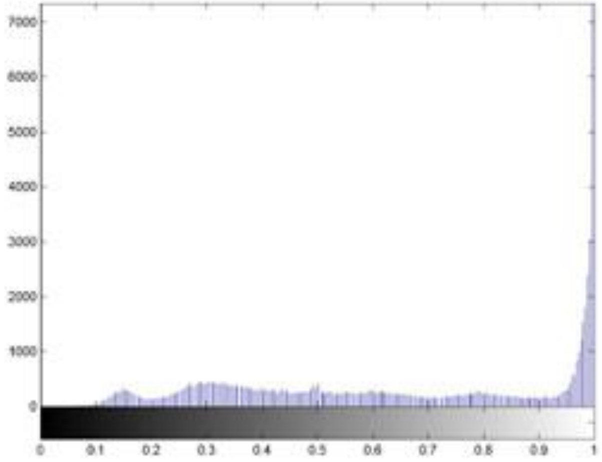

OFFICIAL NEWSLETTER FOR THE LEONARD PELTIER DEFENSE COMMITTEE

Spirit of Crazy Horse

VOL. 7 NO. 2 MARCH/APRIL $2.00

Statement of Leonard Peltier

By Leonard Peltier

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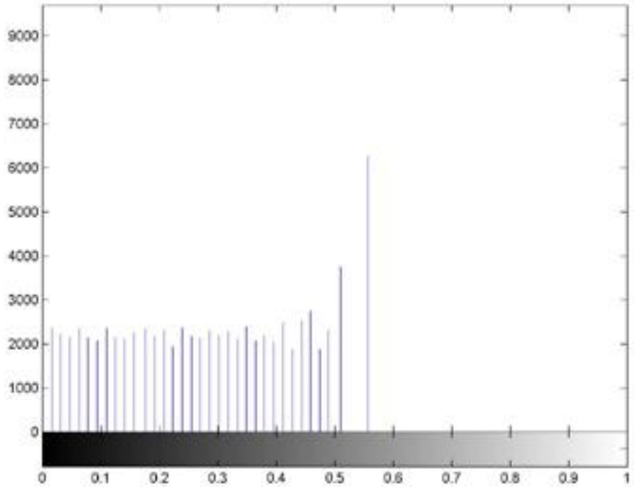
Spirit of Crazy Horse

Statement of Leonard Peltier



Images
Intensity Transformation
Histogram Equalization

Histogram Matching

Ex 1

What is Histogram Matching?

A process to adjust the histogram of one image to resemble the histogram of another (target) image.

Used to transfer the appearance characteristics (such as contrast and brightness) of one image to another.

Steps in Histogram Matching

1. **Compute histograms** of both the source and target images.
2. **Calculate the Cumulative Distribution Function** (CDF) for both histograms.
3. **Match intensities** by mapping pixel values from the source image to corresponding values in the target histogram using the CDFs.
4. **Transform the image** based on the mapping.

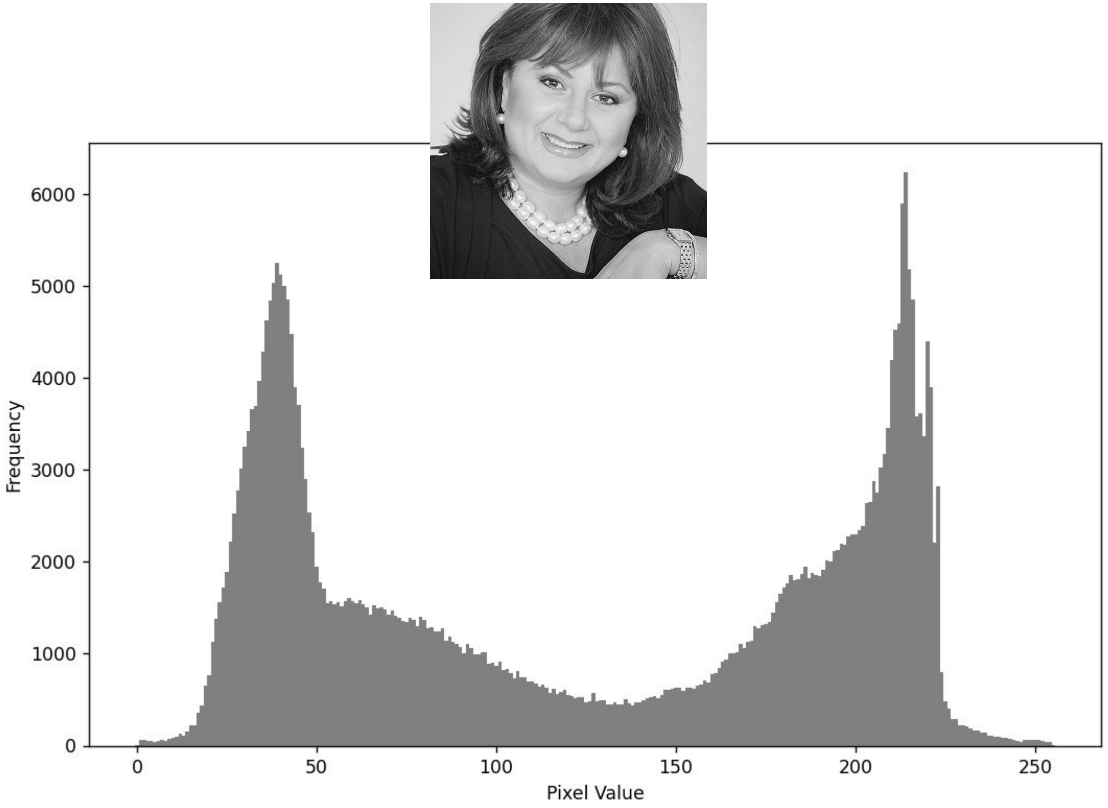
Example

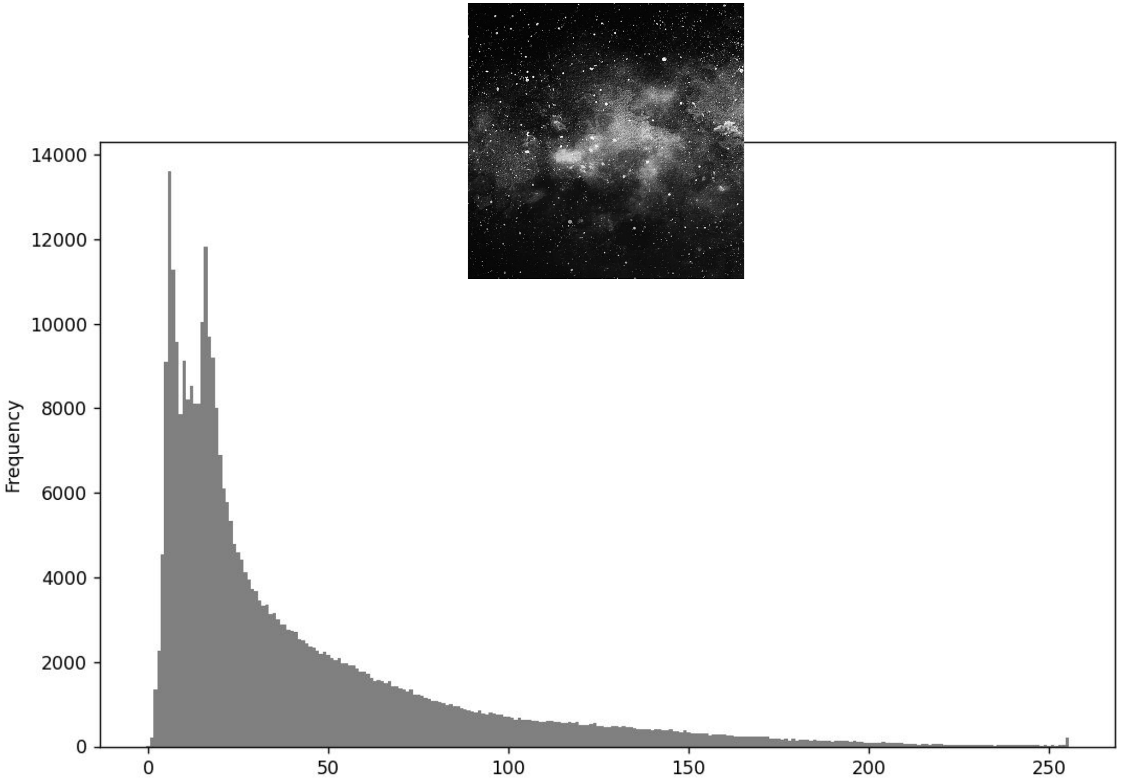
Source

Target

Example

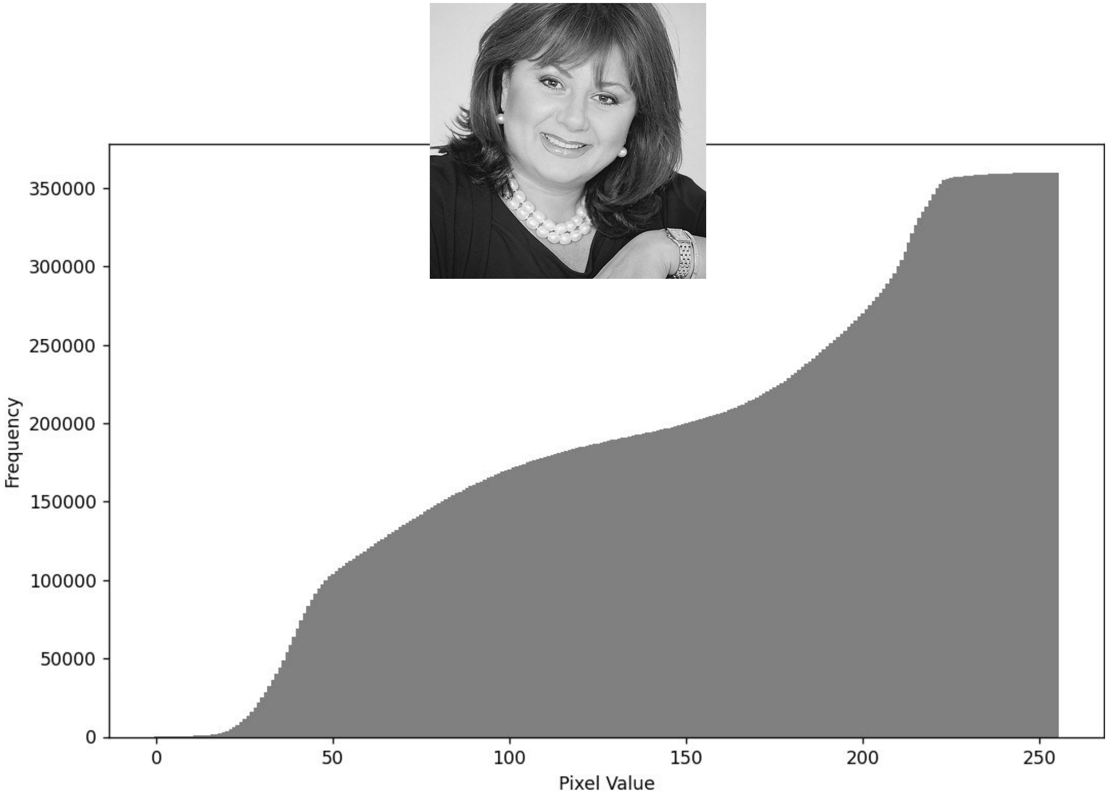
1. **Compute histograms** of both the source and target images.

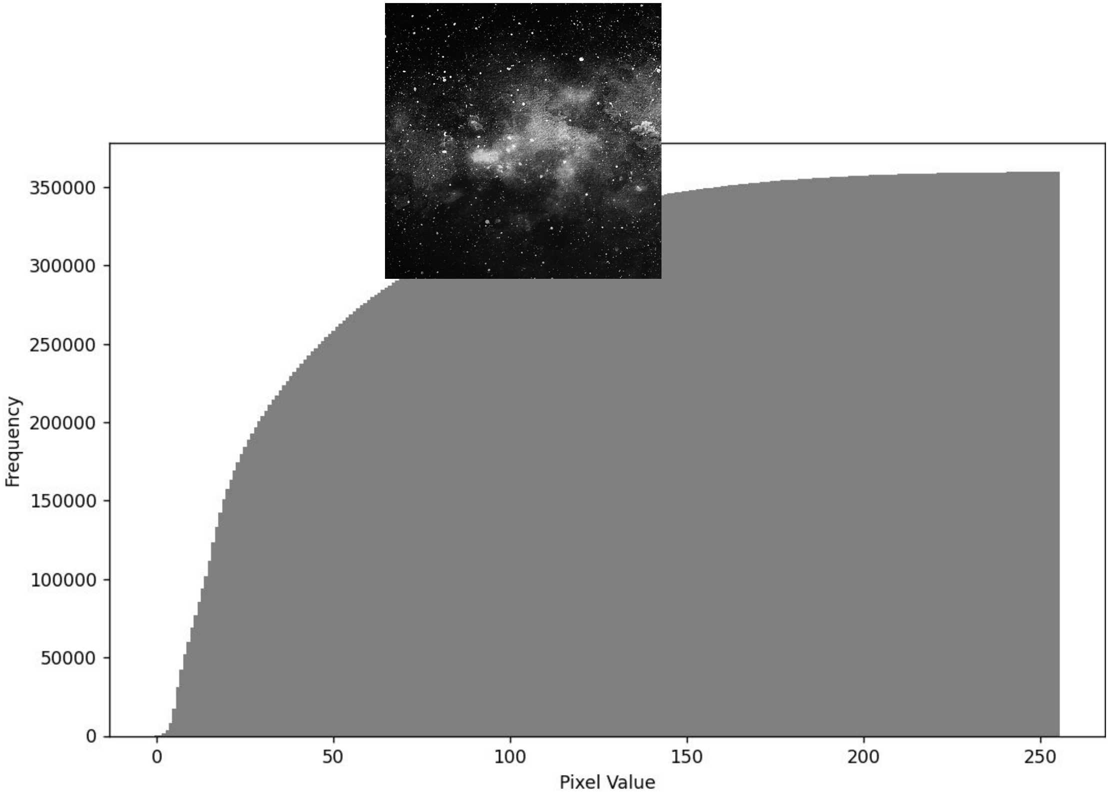




Example

2. Calculate the Cumulative Distribution Function (CDF) for both histograms.





Example

3. **Match intensities** by mapping pixel values from the source image to corresponding values in the target histogram using the CDFs.

$$C_{source}(i) = \frac{1}{N_s} \sum_{j=0}^{i} H_s(j)$$

$$C_{target}(i) = \frac{1}{N_t} \sum_{j=0}^{i} H_t(j)$$

For matching the intensities from the source to the target, find the intensity j in the target image such that:

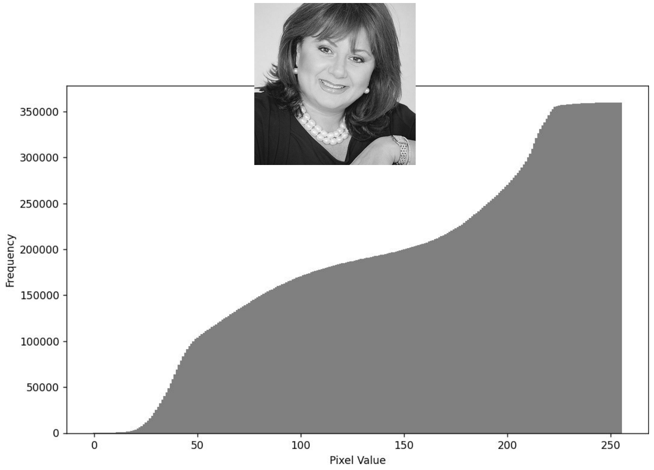
$$C_{source}(i) \approx C_{target}(j)$$

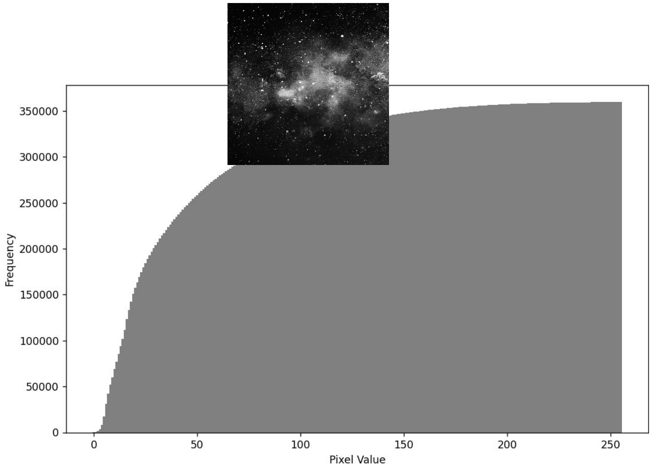
Transformation Function

$$T(i) = arg \min_j |C_{source}(i) - C_{target}(j)|$$

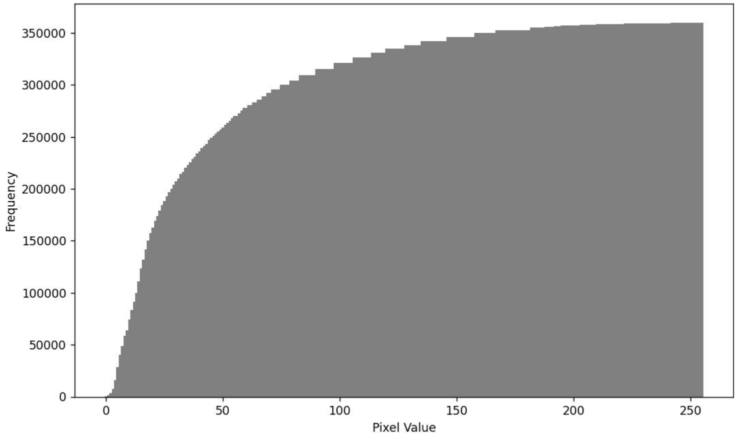
Example

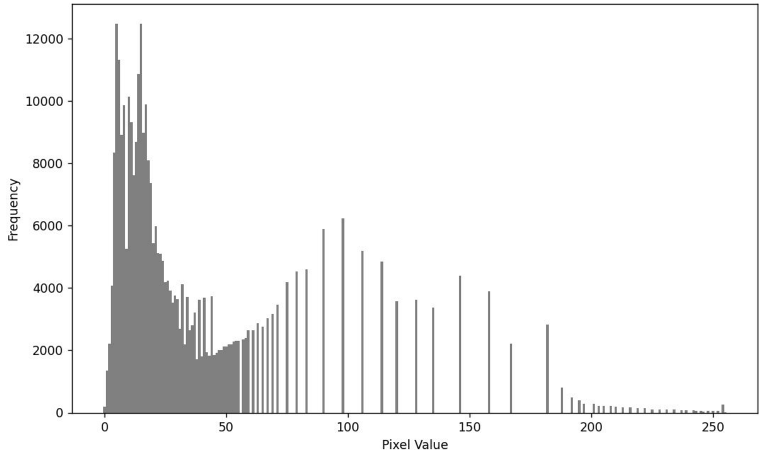
4. **Transform the image** based on the mapping.





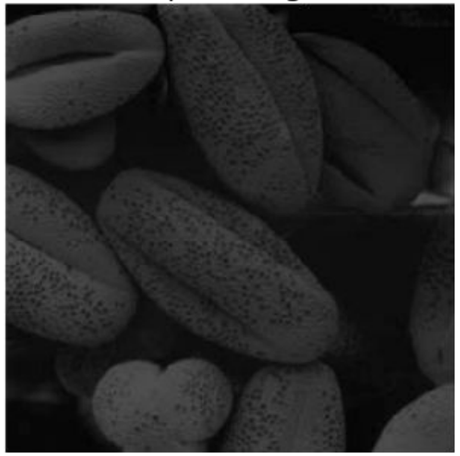
Matching





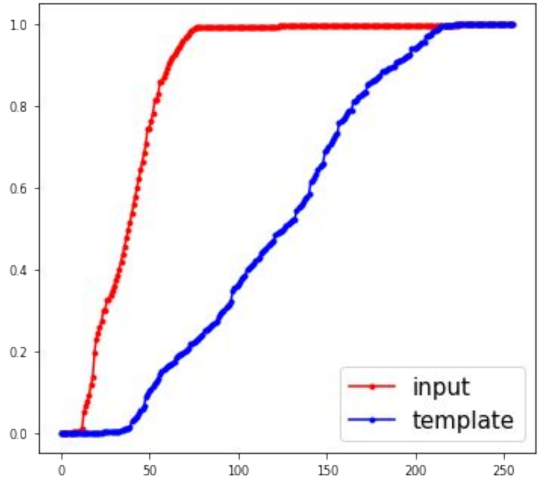
Example

Input image



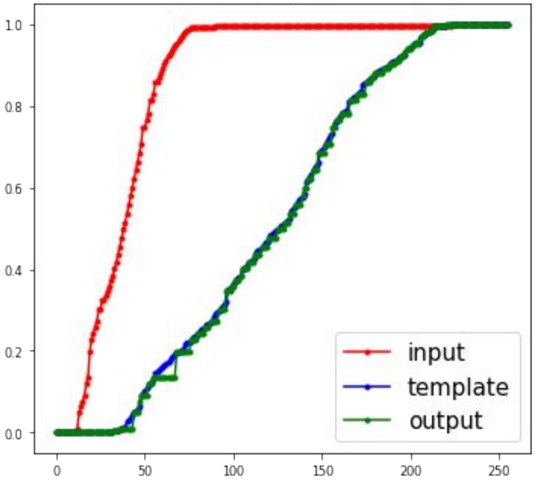
Template image

CDF



Output image with Hist. Matching

CDF



Intensity Transformation

Histogram equalization

Histogram Matching

Ex 1

Ex1

- COLAB Bootcamp - IMPORTANT
- cut detection

Good Luck!

Next week:

1D Fourier