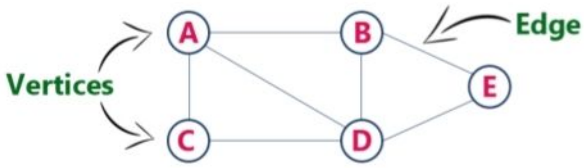
A graph contains a set of points known as nodes (or vertices) and set of links known as edges (or Arcs) which connects the vertices.

A graph is defined as Graph is a collection of vertices and arcs which connects vertices in the graph. A graph G is represented as G = (V , E), where V is set of vertices and E is set of edges.

Example: graph G can be defined as G = (V , E) Where V = {A,B,C,D,E} and

E = {(A,B),(A,C)(A,D),(B,D),(C,D),(B,E),(E,D)}. This is a graph with 5 vertices and 6 edges.



Graph Terminology

1.**Vertex** : An individual data element of a graph is called as Vertex. Vertex is also known as node. In above example graph, A, B, C, D & E are known as vertices.

2.**Edge** : An edge is a connecting link between two vertices. Edge is also known as Arc. An edge is represented as (starting Vertex, ending Vertex).

In above graph, the link between vertices A and B is represented as (A,B).

Edges are three types:

1.Undirected Edge - An undirected edge is a bidirectional edge. If there is an undirected edge between vertices A and B then edge (A , B) is equal to edge (B , A).

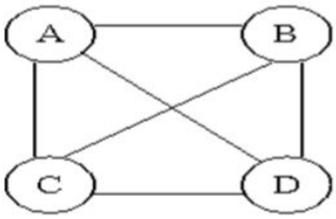
2.Directed Edge - A directed edge is a unidirectional edge. If there is a directed edge between vertices A and B then edge (A , B) is not equal to edge (B , A).

3.Weighted Edge - A weighted edge is an edge with cost on it.

Types of Graphs

1.Undirected Graph

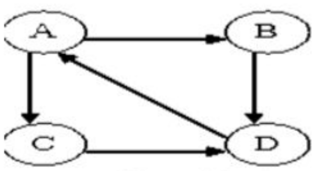
A graph with only undirected edges is said to be undirected graph.



Undirected Graph.

2.Directed Graph

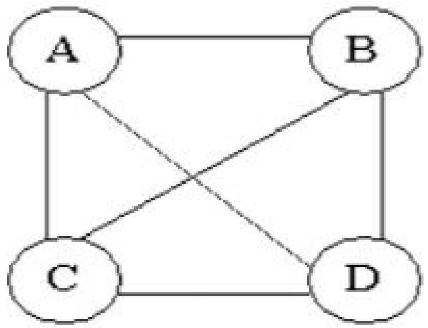
A graph with only directed edges is said to be directed graph.



Directed Graph.

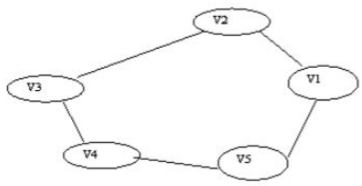
3.Complete Graph

A graph in which any V node is adjacent to all other nodes present in the graph is known as a complete graph. An undirected graph contains the edges that are equal to edges = n(n-1)/2 where n is the number of vertices present in the graph. The following figure shows a complete graph.



5.Cycle Graph

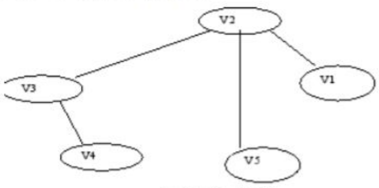
A graph having cycle is called cycle graph. In this case the first and last nodes are the same. A closed simple pat
is a cycle.



A cycle graph

6.Acyclic Graph

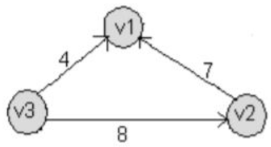
A graph without cycle is called acyclic graphs.



A acyclic graph

7. Weighted Graph

A graph is said to be weighted if there are some non negative value assigned to each edges of the graph. The value is equal to the length between two vertices. Weighted graph is also called a network.



A weighted graph

Outgoing Edge

A directed edge is said to be outgoing edge on its orign vertex.

Incoming Edge

A directed edge is said to be incoming edge on its destination vertex.

Degree

Total number of edges connected to a vertex is said to be degree of that vertex.

Indegree

Total number of incoming edges connected to a vertex is said to be indegree of that vertex.

Outdegree

Total number of outgoing edges connected to a vertex is said to be outdegree of that vertex.

Parallel edges or Multiple edges

If there are two undirected edges to have the same end vertices, and for two directed edges to have the same origin and the same destination. Such edges are called parallel edges or multiple edges.

Self-loop

An edge (undirected or directed) is a self-loop if its two endpoints coincide.

Simple Graph

A graph is said to be simple if there are no parallel and self-loop edges.

Adjacent nodes

When there is an edge from one node to another then these nodes are called adjacent nodes.

Incidence

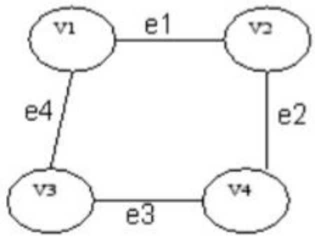
In an undirected graph the edge between v1 and v2 is incident on node v1 and v2.

Walk

A walk is defined as a finite alternating sequence of vertices and edges, beginning and ending with vertices, such that each edge is incident with the vertices preceding and following it.

Closed walk

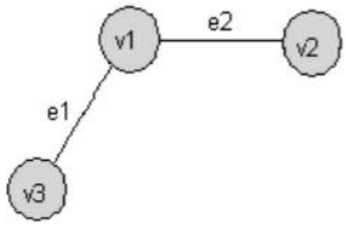
A walk which is to begin and end at the same vertex is called close walk. Otherwise it is an open walk.



If e1,e2,e3,and e4 be the edges of pair of vertices (v1,v2),(v2,v4),(v4,v3) and (v3,v1) respectively ,then v1 e1 v2 e2 v4 e3 v3 e4 v1 be its closed walk or circuit.

Path

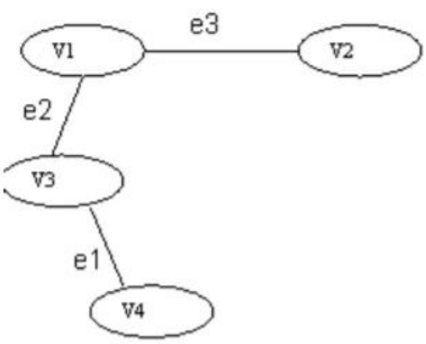
A open walk in which no vertex appears more than once is called a path.



If e1 and e2 be the two edges between the pair of vertices (v1,v3) and (v1,v2) respectively, then v3 e1 v1 e2 v2 be its path.

Length of a path

The number of edges in a path is called the length of that path. In the following, the length of the path is 3.



An open walk Graph

Circuit

A closed walk in which no vertex (except the initial and the final vertex) appears more than once is called a circuit.

A circuit having three vertices and three edges.

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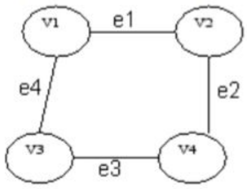

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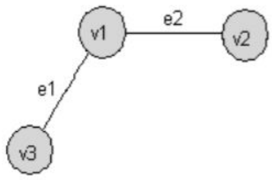

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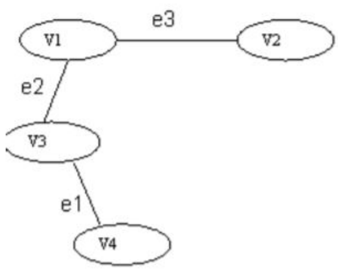

An open walk Graph

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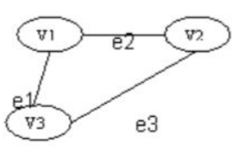
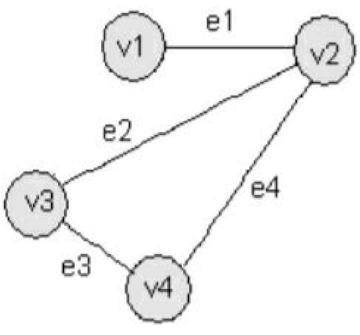
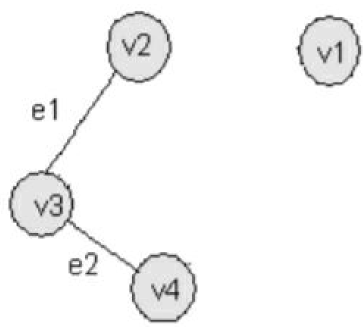
A circuit having three vertices and three edges.



Connected Graph

A graph G is said to be connected if there is at least one path between every pair of vertices in G. Otherwise,G is disconnected.





A connected graph G   A disconnected graph G

This graph is disconnected because the vertex v1 is not connected with the other vertices of the graph.

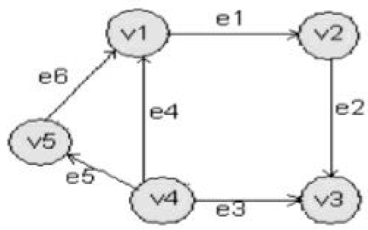
Degree

In an undirected graph, the number of edges connected to a node is called the degree of that node or the degree of a node is the number of edges incident on it.

In the above graph, degree of vertex v1 is 1, degree of vertex v2 is 3, degree of v3 and v4 is 2 in a connected graph.

Indegree

The indegree of a node is the number of edges connecting to that node or in other words edges incident to it.



In the above graph,the indegree of vertices v1, v3 is 2, indegree of vertices v2, v5 is 1 and indegree of v4 is zero.

Outdegree

The outdegree of a node (or vertex) is the number of edges going outside from that node or in other words the

1.Adjacency Matrix

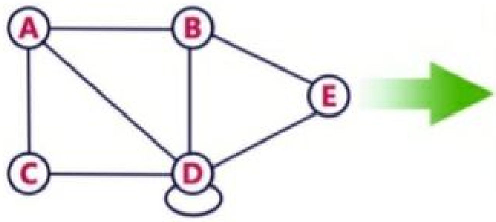
In this representation, graph can be represented using a matrix of size total number of vertices by vertices; means if a graph with 4 vertices can be represented using a matrix of 4X4 size.

In this matrix, rows and columns both represent vertices.

This matrix is filled with either 1 or 0. Here, 1 represents there is an edge from row vertex to colu represents there is no edge from row vertex to column vertex.

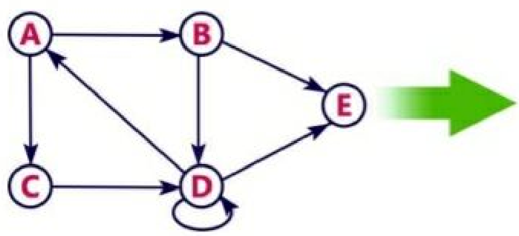
Adjacency Matrix : let $G = (V, E)$ with n vertices, $n \geq 1$. The adjacency matrix of G is a 2-dimens matrix, A, $A(i, j) = 1$ iff $(v_i, v_j) \in E(G)$ ($\langle v_i, v_j \rangle$ for a diagraph), $A(i, j) = 0$ otherwise.

example : for undirected graph



$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

For a Directed graph



$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

2. Adjacency List

In this representation, every vertex of graph contains list of its adjacent vertices. The n rows of the adjacency matrix are represented as n chains. The nodes in chain I represent the vertices that are adjacent to vertex i.

It can be represented in two forms. In one form, array is used to store n vertices and chain is used to store its adjacencies. Example: