

W1.

- a) Since the running time of A is in  $\theta(n^3 + m^2 + mn^2)$ , it is bounded both above and below by multiplicative constants of  $n^3 + m^2 + mn^2$ . Thus, since it is bounded below by the value of at least  $n^3$ , it must also be bounded below by  $n^2$ . Therefore the worst-case running time of A is in  $\Omega(n^2)$ .
- b) Since the running time of A is in  $\theta(n^3 + m^2 + mn^2)$ , it must be bounded below by the multiplicative constant of  $n^3 + m^2 + mn^2$ . However, since the dominant term of n is  $n^3$ , it cannot be bounded below by  $n^4$ . Thus, given that the values of n are of lower polynomial order, the worst-case running time of A cannot be in  $\Omega(n^4)$ .
- c) Since the running time of A is in  $\theta(n^3 + m^2 + mn^2)$ , both  $\Omega(n^3 + m^2 + mn^2)$  and  $O(n^3 + m^2 + mn^2)$  must be true. Thus even in the best case, the running time would be bounded below by  $\Omega(n^3 + m^2 + mn^2)$ . Therefore, the statement is true.
- d) There is not enough information to prove whether this statement is true or false since the computation of the average-case running time of an algorithm requires knowing the instances it takes. However, since the worst-case running time of this algorithm of all instances is bounded above by  $O(n^3 + m^2 + mn^2)$ , then it must also be true for the average-case running time.
- e) Since n is categorized using  $\theta(m)$ , we can replace n by m. Thus  $\theta(n^3 + m^2 + mn^2)$  becomes  $\theta(m^3 + m^2 + m^3)$  where the dominant term of this complexity is  $m^3$ . Given that n is proportional to m, the dominant term of this complexity is also  $n^3$ . Thus the worst-case running time of A can be described as  $\theta(n^3)$ .
- f) Since n is categorized as  $\theta(m^2)$ , we can replace instances of n by  $m^2$ , giving  $\theta(m^6 + m^2 + m^5)$ . Since  $m^6$  dominates the other polynomial terms of m, the complexity can be simplified to  $\theta(m^6)$ . Since this is true, then  $O(m^6)$  and  $\Omega(m^6)$  must also be true, thus proving  $O(m^6)$ .