

W5.

- a) Using a 4D array of size $(m + 1) \times (n + 1) \times (m + 1) \times (n + 1)$ allows us to account for each of the dependencies. Such dependencies are $l-1$, $k-1$, $j-1$, and $i-1$.
- b) Initialize the table by filling in all entries of the base case. Then increasing order by the fourth dimension; within entries of the same value in the fourth dimension, by increasing values of the third dimension; within entries of the same value in the third and fourth dimension, by increasing values of the second dimension; within entries of the same value in second, third, and fourth dimension, by increasing order of the first dimension.
- c) The answer can be extracted from the m^{th} entry in the first and third dimension, and the n^{th} entry in the second and fourth dimension of the 4D array.
- d) Given that the dimensions of the 4D table is $m + 1 \times (n + 1) \times (m + 1) \times (n + 1)$, the running time of the iterations through this table is in $\theta((m + 1) \times (n + 1) \times (m + 1) \times (n + 1)) = \theta((n + 1)^2 \times (m + 1)^2)$. From this running time, we can see that the dominant terms are n^2 and m^2 , thus further simplifying this function would give us a running time of $\theta(n^2 m^2)$.