W3.

a) Union set is a constructive optimization (minimization) problem as it searches for the optimal subset B that is of minimum size and that satisfies the conditions.

b) Input: A finite set **A** of finite sets of numbers and an integer *b*.
   Output: A subset **B** of A such that the union of all sets in **B** equal to the union **U** of all the sets in **A** such that the size of **B** is less than or equal to the bound *b*.

c) Since the probability of choosing a set in A is random, it does not guarantee to give the correct answer, thus failing the Las Vegas algorithm property. If we compromise on correctness, this algorithm is guaranteed to run in polynomial time in the size of the input, however since the probability of getting a correct solution is not high, it does not satisfy the Monte Carlo algorithm property. Therefore, this algorithm is neither a Las Vegas nor Monte Carlo algorithm.

d) Yes, in some cases a randomized algorithm may be faster than a deterministic one. That is if you are willing to compromise on correctness or running time. The randomized algorithm may be simpler and faster.

e) If the optimal substructure property does not hold, then the heuristic hill climbing may be applied by slowly building up an optimal solution. To improve the correctness of this algorithm and to prevent the algorithm from only producing a local optimal solution, judiciously choose the initial solution that gives the steepest ascent.