W6.

a) Since P is a subset of the NP complexity class, any problem in P can be reduced to any problem in NP by transforming the solution in at most polynomial time. Thus, problem A is reducible to B since it can be solved using the algorithm in problem B at most polynomial times.

b) NP-complete problems are the hardest problem in NP. Thus a problem in NP can't be harder than an NP-complete problem, therefore proving that any problem in B can be reduced to C.

c) If we can reduce an NP-complete problem in P, it would imply that the NP-complete problem can be solved in polynomial time, which would imply $NP \subseteq P$. For $NP \subseteq P$ to be true, NP must equal P since we know that $P \subseteq NP$. However, $P \neq NP$, thus disproving the statement that C is reducible to A.

d) Since B is in NP, it is unsure whether it can be solved in $O(n^3)$ polynomial time. We would only be sure that this is true if P = NP, meaning every problem in NP is also in P. However, since $P \neq NP$, this statement is only true for some problems B such that they are also in P.

e) For some NP-complete problems, if k is small, it can be solved using a fixed-parameter algorithm.

f) By definition of an NP-complete problem, C must be in NP and is NP-hard. Since C is in NP, then there exists a polynomial-time verification algorithm for it.