Lecture 2

Association rules

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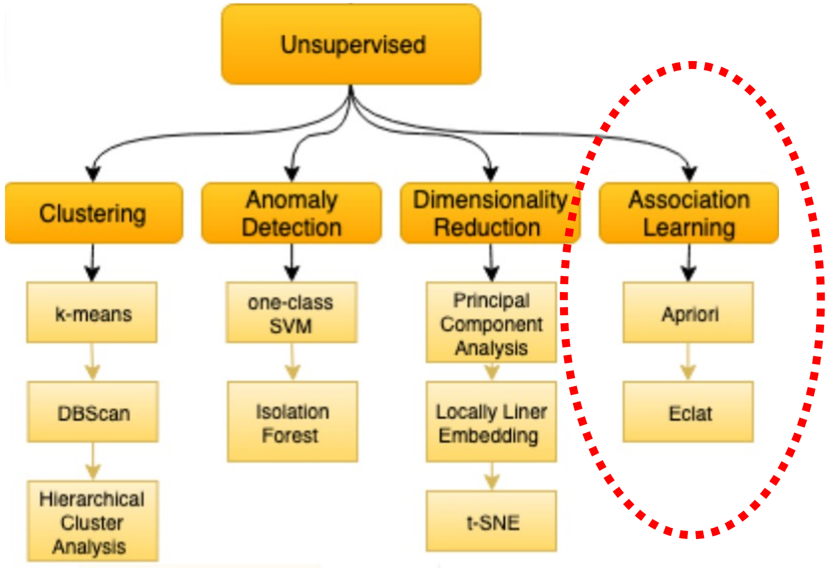
Unsupervised learning

<u>Experience:</u> objects for which **no class labels** have been given
<u>Performance:</u> typically concerns the ability to output useful **characterizations** (or groupings) of objects



Problem 1:
Mining
Frequent Itemsets

Problem 2:
Mining
Association Rules

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Problem 1: Mining frequent itemsets

- Given a set of transactions ***D***, find combinations of items that occur *frequently*

Market-Basket transactions

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Examples of *frequent* itemsets

{Diaper, Beer},
{Milk, Bread},
{Beer, Bread, Milk}

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Problem 2: Mining Association Rules

- Given a set of transactions ***D***, find rules that will predict the occurrence of an item (or a set of items) based on the occurrences of other items in the transaction

Market-Basket transactions

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Examples of *association rules*

{Diaper} $\rightarrow$ {Beer},
{Milk, Bread} $\rightarrow$ {Diaper, Coke},
{Beer, Bread} $\rightarrow$ {Milk}

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Why do we want to find frequent itemsets?

- Find all combinations of items that occur together

- They might be interesting
  - e.g., in placement of items in a store ☺

- Frequent itemsets are *only positive combinations* (we do not report combinations that do not occur frequently together)

- Frequent itemsets aim at providing a summary of the data

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Definitions

- Itemset
  - A set of one or more items
    - e.g.: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
- Absolute Support
  - Number of transactions in which an itemset appears
  - e.g., $\sigma(\{\text{Milk, Bread, Diaper}\}) = 2$
- Relative Support
  - Fraction of the transactions in which an itemset appears
  - e.g., $s(\{\text{Milk, Bread, Diaper}\}) = 2/5$
- Frequent Itemset
  - An itemset whose relative support is greater than or equal to a ***minsup*** threshold

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

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Mining frequent itemsets

- Task:

  Given a transaction database ***D*** and a ***minsup*** threshold, find all frequent itemsets and the frequency of each set in this collection

- Stated differently:

  Count the number of times combinations of attributes occur in the data. If the count of a combination is above ***minsup***, report it

- **Recall**: The input is a transaction database ***D*** where every transaction consists of a subset of items from some universe ***I***

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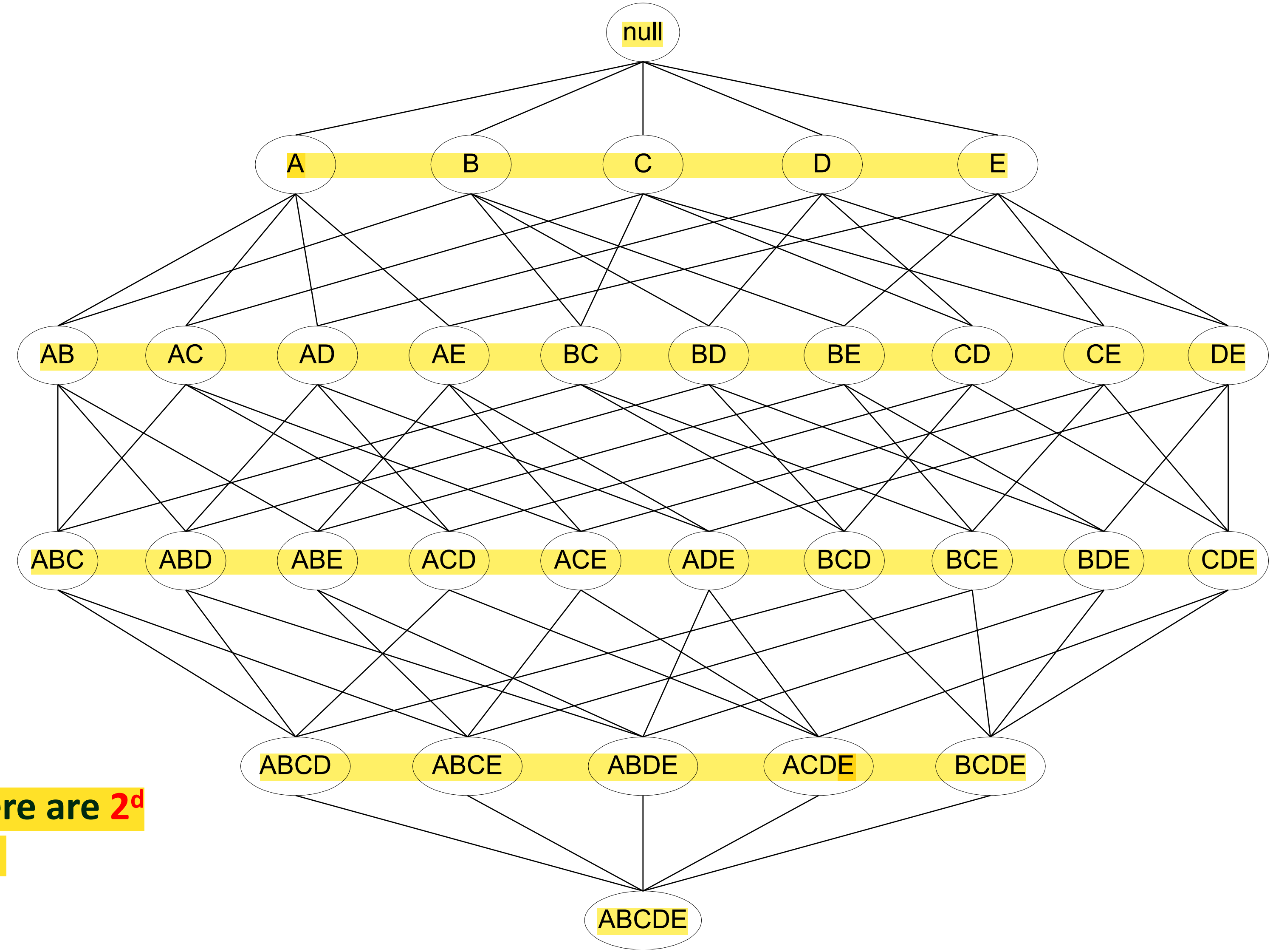
How many itemsets are there?



Given d items, there are 2^d possible itemsets, including *null*

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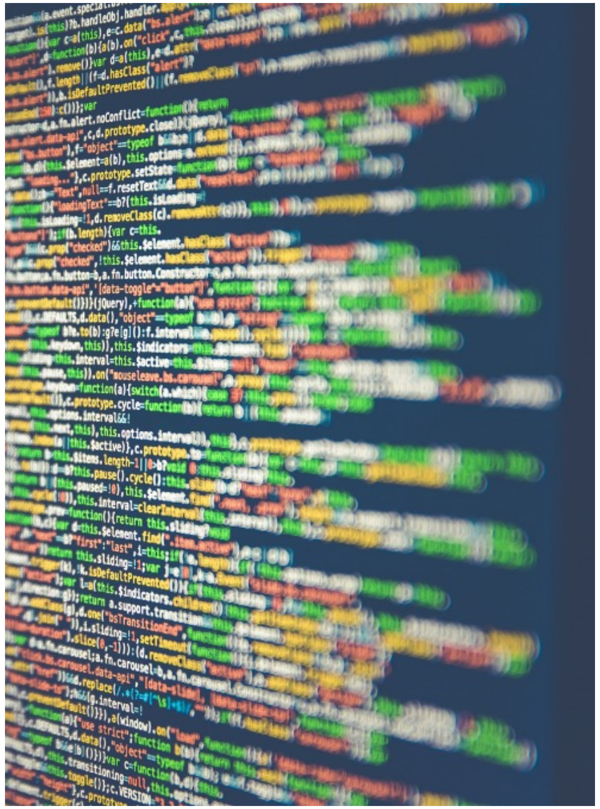
Brute-force algorithm for finding all frequent itemsets?

- Generate all possible itemsets (lattice of itemsets)
  - Start with 1-itemsets, 2-itemsets, ... , d-itemsets
- Compute the frequency of each itemset from the data
  - Count in how many transactions each itemset occurs
- If the support of an itemset is above ***minsup*** report it as a frequent itemset

Brute-force algorithm for finding all frequent itemsets?

- Complexity?
  - Match every candidate against each transaction
  - For M candidates and N transactions, the complexity is approximately **O(NM)**
    - Expensive since $M = 2^d$!!!



Speeding-up the brute-force algorithm

- Reduce the number of candidates (M)
  - Complete search: $M=2^d$
  - Use pruning techniques to reduce M
- Reduce the number of comparisons (NM)
  - Use efficient data structures to store the candidates or transactions
  - No need to match every candidate against every transaction
- Reduce the number of transactions (N)
  - Reduce the size of N as the size of itemsets increases
  - Use vertical-partitioning of the data to apply the mining algorithms

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Reduce the number of candidates

- **Apriori principle** (Main observation):
  - If an itemset is frequent, then all of its subsets must also be frequent

- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

– The support of an itemset ***never exceeds*** the support of its subsets
– This is known as the ***anti-monotone*** property of support

Example

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

s(Bread) ≥ s(Bread, Beer)
s(Milk) ≥ s(Bread, Milk)
s(Diaper, Beer) ≥ s(Diaper, Beer, Coke)

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

itemset infrequent all supersets also infrequent

- The support of an itemset ***never exceeds*** the support of its subsets
- This is known as the ***anti-monotone*** property of support

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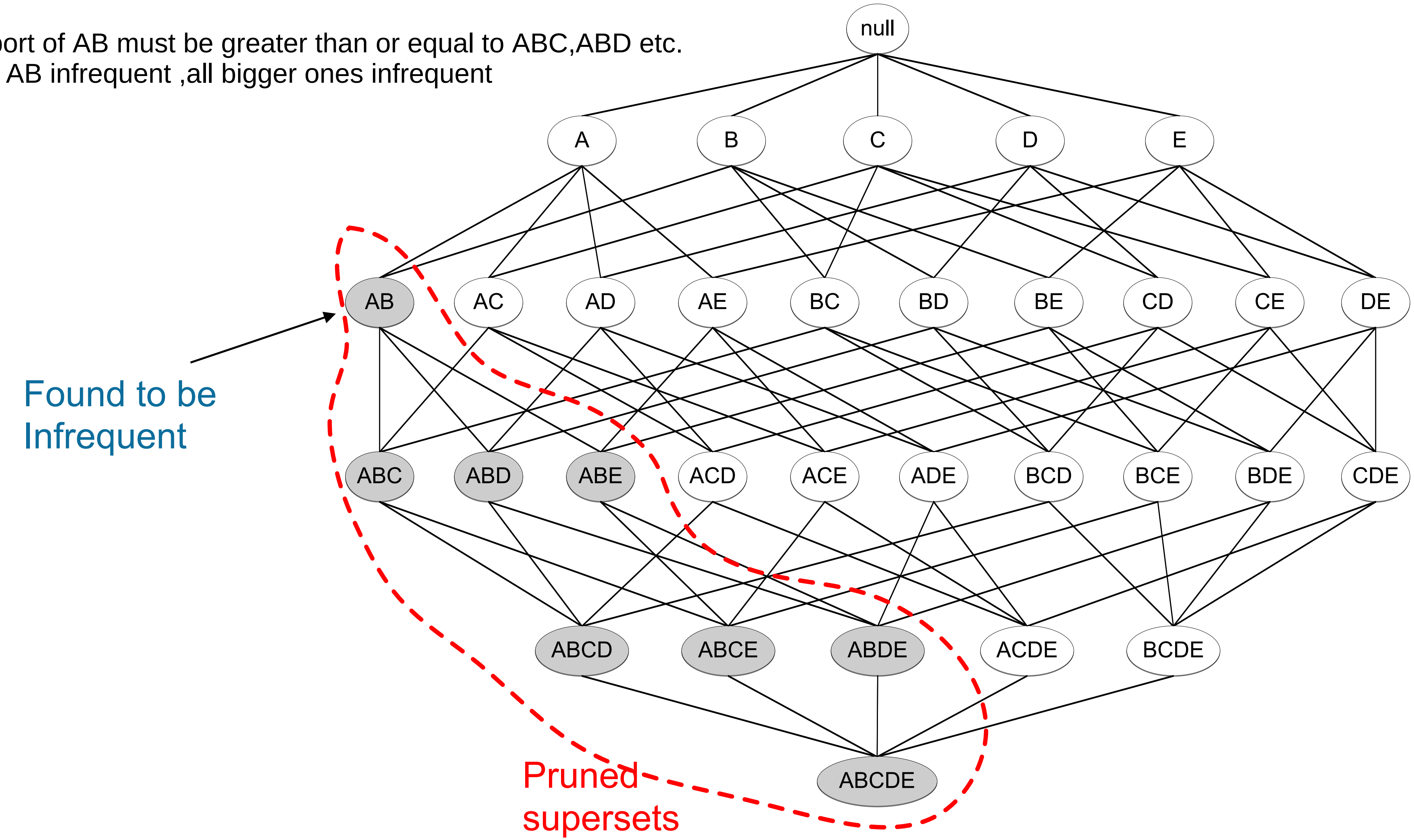
Illustrating the Apriori principle

support of AB must be greater than or equal to ABC,ABD etc.
So if AB infrequent ,all bigger ones infrequent



The Apriori algorithm [Agrawal&Srikant 1994]

C_k: Candidate itemsets of size k
L_k: Frequent itemsets of size k

L_1 = {frequent 1-itemsets};
for (k = 1; L_k != ∅; k++)
  C_{k+1} = GenerateCandidates(L_k)
  for each transaction t in the database do
    increment count of candidates in C_{k+1} that are contained in **t**
  endfor
  L_{k+1} = candidates in C_{k+1} with support ≥***minsup***
endfor
return $\cup_k L_k$;

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C_{k+1} = GenerateCandidates()

- Assume the items in L_k are listed in an order (e.g., alphabetical)
- **Step 1**: ***self-joining L_k*** *(IN SQL)*

  insert into C_{k+1}

  select $p.item_1, p.item_2, \ldots, p.item_k, q.item_k$

  from L_k p, L_k q

  where $p.item_1 = q.item_1, \ldots, p.item_{k-1} = q.item_{k-1}$ and $p.item_k < q.item_k$

$$p = \{\boxed{\mathbf{item_1, item_2, \ldots, item_{k-1}}}, \mathit{item_k}\}$$

$$= \qquad \neq$$

$$q = \{\boxed{\mathbf{item_1, item_2, \ldots, item_{k-1}}}, \mathit{item_k}\}$$

$$\text{candidate} = \{\mathbf{item_1, item_2, \ldots, item_{k-1}}, \mathit{item_k}, \mathit{item_k}\}$$

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Example of Candidate Generation

- $k=3$
- $L_3=\{abc, abd, acd, ace, bcd\}$
- $C_4 = ?$

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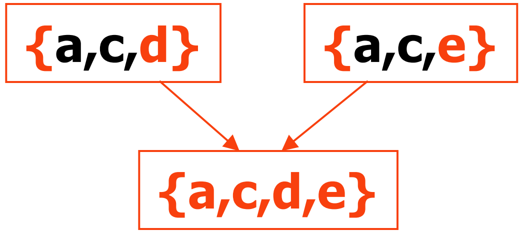
Example of Candidate Generation

- *k=3*
- L_3={**abc**, **abd**, **acd**, **ace**, *bcd*}
- ***Self-joining***: $L_3 * L_3$
  - ***abcd*** from ***ab****c* and ***ab****d*
  - ***acde*** from ***ac****d* and ***ac****e*



C_{k+1} = GenerateCandidates()

- Assume the items in L_k are listed in an order (e.g., alphabetical)
- **Step 2**: ***pruning***

forall ***itemsets c in*** C_{k+1} do

forall ***k-subsets*** s ***of*** c do

if (s *is* ***not*** *in* L_k**) then delete** c **from** C_{k+1}

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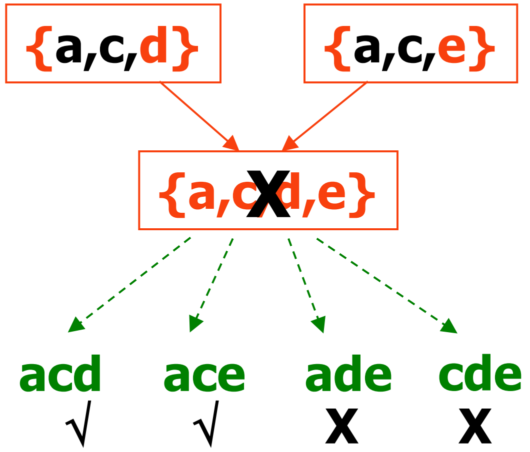
Example of Candidate Pruning

- L_3=*{abc, abd, acd, ace, bcd}*
- ***Self-joining***: $L_3 * L_3$
  - ***abcd*** from ***abc*** and ***abd***
  - ***acde*** from ***acd*** and ***ace***
- ***Pruning***
  - ***acde*** is removed because ***ade*** is not in L_3
- C_4=***{abcd}***



Discussion of the Apriori algorithm

- Much faster than the Brute-force algorithm
  - It avoids checking all elements in the lattice
- The running time is in the worst case **$O(2^d)$**
  - Pruning really prunes in practice
- It makes multiple passes over the dataset
  - One pass for every level **k**
- Multiple passes over the dataset are inefficient when we have thousands of candidates and millions of transactions

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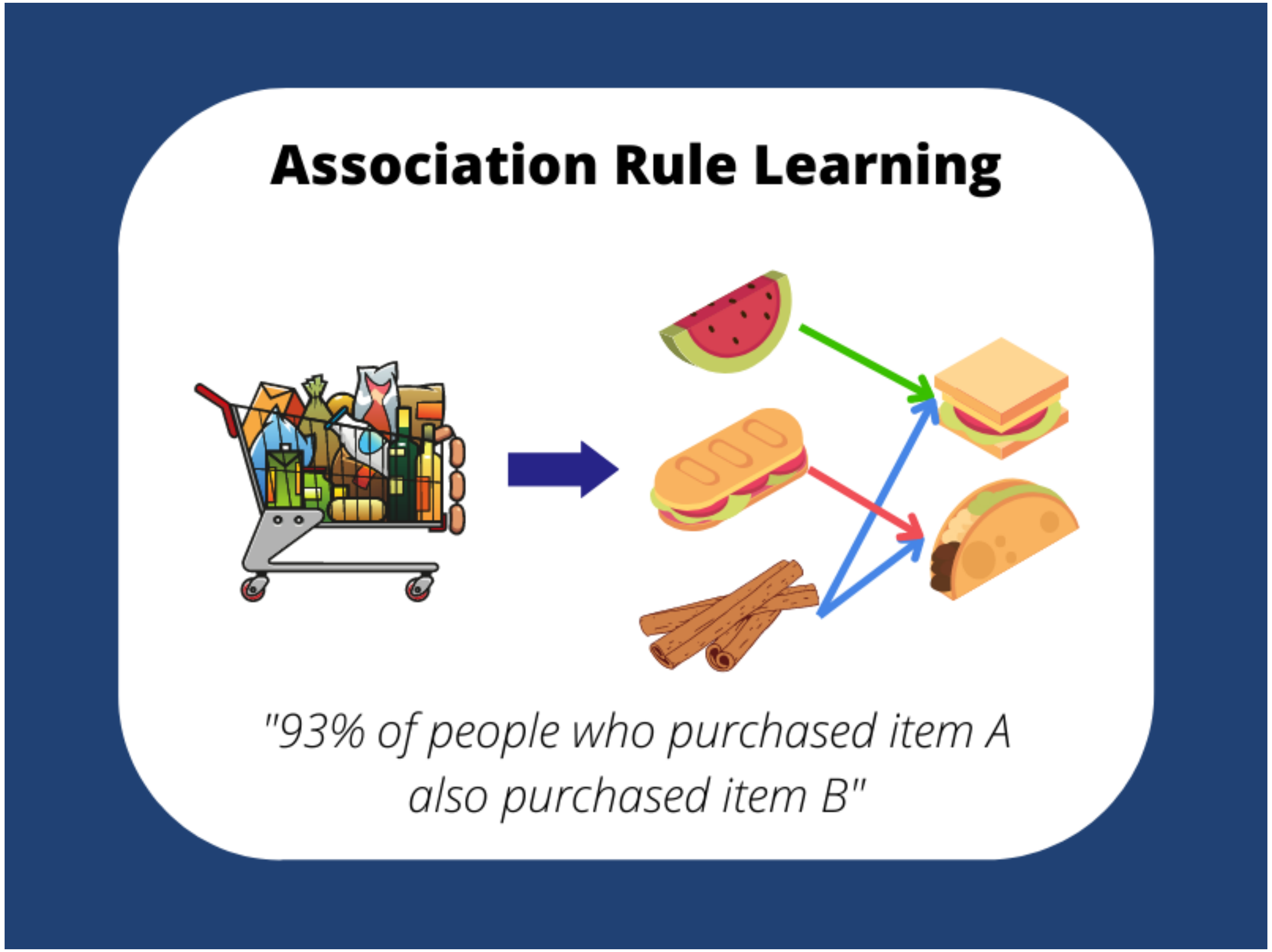
Implementations

- Lots of them around
- See, for example, the web page of Bart Goethals: http://www.adrem.ua.ac.be/~goethals/software/
- Typical input format: each row lists the items (using item id's) that appear in every row



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Association Rules

- Let **D** be a database of transactions, e.g.,

TID	*Items*
1	**A, B, C**
2	**A, C**
3	**A, D**
4	**B, E, F**

- Let ***I*** be the set of items that appear in the database, e.g., $I=\{A,B,C,D,E,F\}$
- A rule is defined by $X \rightarrow Y$, where $X \subset I$, $Y \subset I$, and $X \cap Y = \emptyset$
  - e.g., $\{B,C\} \rightarrow \{A\}$ is a rule

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Definitions

Association Rule

- An implication expression of the form **X** $\rightarrow$ **Y**, where **X** and **Y** are non-overlapping itemsets
  - Example:
    {Milk, Diaper} $\rightarrow$ *{Beer}*

Rule Evaluation Metrics

- **Support (s)**
  - Fraction of transactions that contain both **X** and **Y**
- **Confidence (c)**
  - Measures how often items in **Y** appear in transactions that also contain **X**

$$c = \frac{\sigma(X,Y)}{\sigma(X)}$$

Example:

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

$$\{\text{Milk}, \text{Diaper}\} \rightarrow \{\text{Beer}\}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|D|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

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Mining association rules

- **Task**:

  Given a set of transactions ***D***, the goal of association rule mining is to find all rules having:

  - support ≥ ***minsup*** threshold
  - confidence ≥ ***minconf*** threshold

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Brute-force algorithm for association-rule mining

- List all possible association rules
- Compute the support and confidence for each rule
- Prune rules that fail the ***minsup*** and ***minconf*** thresholds

$\Rightarrow$ Computationally prohibitive!

A faster solution…

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

Example of Rules:

{Milk,Diaper} $\rightarrow$ {Beer} (s=0.4, c=0.67)
{Milk,Beer} $\rightarrow$ {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} $\rightarrow$ {Milk} (s=0.4, c=0.67)
{Beer} $\rightarrow$ {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} $\rightarrow$ {Milk,Beer} (s=0.4, c=0.5)
{Milk} $\rightarrow$ {Diaper,Beer} (s=0.4, c=0.5)

Observations:

- All the above rules are binary partitions of the same itemset:
  {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

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A faster solution…

- Two-step approach:
  - **Frequent Itemset Generation**

    Generate all itemsets whose support ≥ ***minsup***

    solved

  - **Rule Generation**

    Generate high-confidence rules from each frequent itemset, where each rule is a binary partition of a frequent itemset

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Rule Generation – Naive algorithm

- Given a frequent itemset **X**, find all non-empty subsets $\mathbf{y} \subset \mathbf{X}$ such that $\mathbf{y} \rightarrow \mathbf{X} - \mathbf{y}$ satisfies the minimum confidence requirement
- If **X = {A,B,C,D}** is a frequent itemset, the candidate rules are:

$$ABC \rightarrow D, \quad ABD \rightarrow C, \quad ACD \rightarrow B, \quad BCD \rightarrow A,$$
$$A \rightarrow BCD, \quad B \rightarrow ACD, \quad C \rightarrow ABD, \quad D \rightarrow ABC,$$
$$AB \rightarrow CD, \quad AC \rightarrow BD, \quad AD \rightarrow BC, \quad BC \rightarrow AD,$$
$$BD \rightarrow AC, \quad CD \rightarrow AB$$

- If $|\mathbf{X}| = \mathbf{k}$, then there are $\mathbf{2^k - 2}$ candidate association rules (ignoring $\mathbf{L} \rightarrow \varnothing$ and $\varnothing \rightarrow \mathbf{L}$)

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Efficient rule generation

- How to *efficiently* generate rules from frequent itemsets?
  - In general, confidence does not have an anti-monotone property
    - **c(ABC → D)** can be larger or smaller than **c(AB → D)**
  - ***But the confidence of rules generated from the same itemset has an anti-monotone property***
  - Example: X = {A,B,C,D}:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

  - **Why?**

$$c(ABC->D) = \frac{\sigma(\{A,B,C,D\})}{\sigma(\{A,B,C\})}$$

Confidence is anti-monotone w.r.t. number of items on the Right-Hand-Side of the rule

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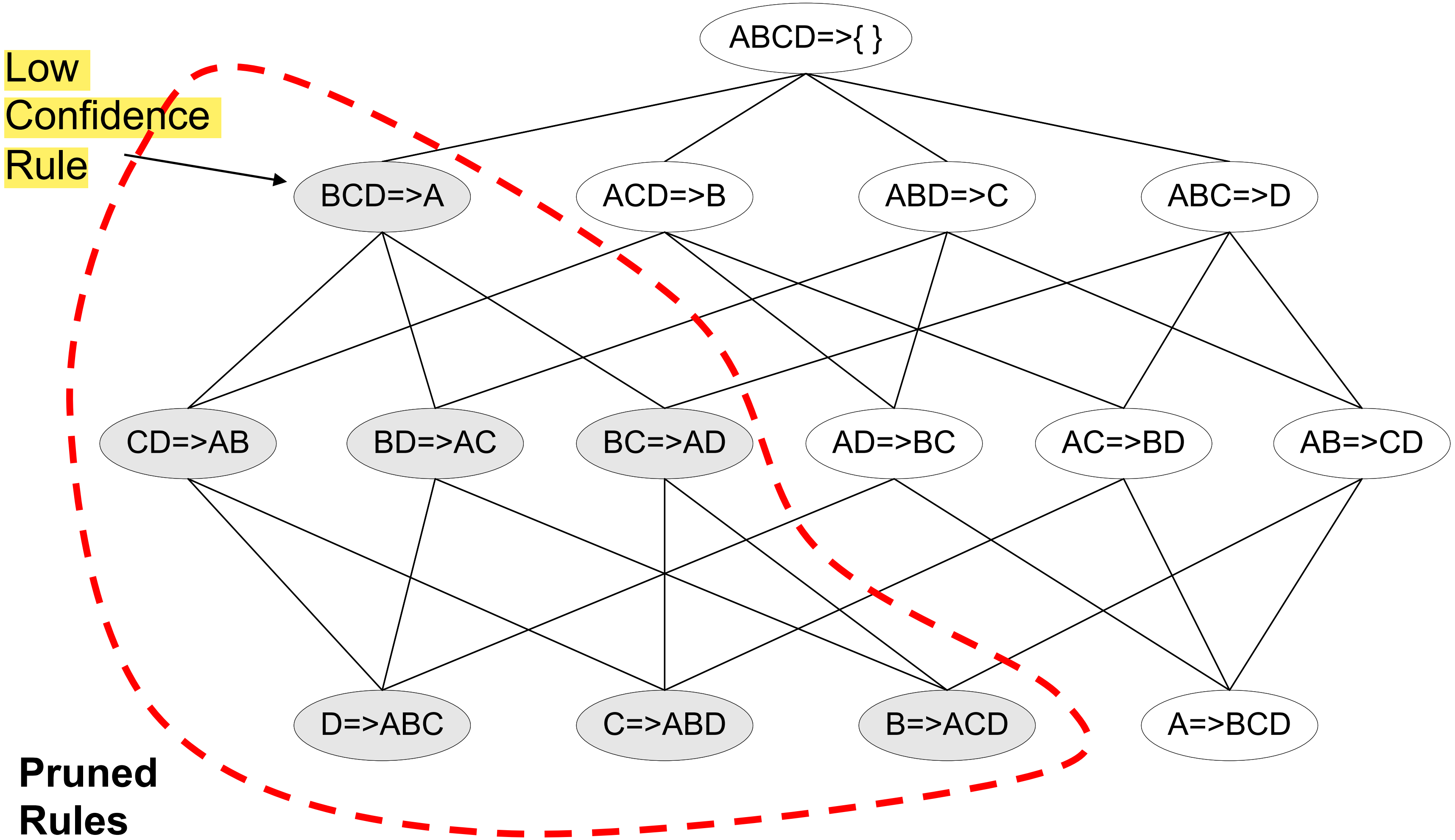
Rule Generation for Apriori Algorithm



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Other interestingness measures

- Lift
- Cosine
- All-confidence
- Leverage
- …

Drawback of Confidence

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Number of people that drink tea

Number of people that drink coffee and tea

Number of people that drink coffee

Number of people that drink coffee but not tea

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Drawback of Confidence

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea → Coffee

- **Confidence:** $P(Coffee|Tea) = \frac{15}{20} = 0.75$, but $P(Coffee) = \frac{90}{100} = 0.9$
  - Although confidence is high, the rule is misleading
    - someone buying tea is less likely to buy coffee than someone for whom we have no information
    - $P(Coffee|\overline{Tea}) = 0.9375$

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Lift of a Rule

Given association rule X → Y: $lift(X \rightarrow Y) = \dfrac{P(X,Y)}{P(X)P(Y)} = \dfrac{conf(X \rightarrow Y)}{P(Y)} = \dfrac{|X,Y|n}{|X||Y|}$

- If P(X,Y) = P(X) P(Y), then lift(X→Y) = 1
  - means that the occurrences of X and Y in the same transaction are independent events, i.e., X and Y are not correlated, and the rule is meaningless!
- lift(X→Y) = k > 1
  - means that X and Y are dependent so that if X occurs, then Y is k times more likely to occur than expected
- lift(X→Y) = k < 1
  - means that the occurrence of X prevents the occurrence of Y, i.e., Y is k times less likely to occur than expected

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Example: Lift/Interest

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea $\rightarrow$ Coffee

Confidence: $P(Coffee|Tea) = \frac{15}{20} = 0.75$, but $P(Coffee) = \frac{90}{100} = 0.9$

$\Rightarrow$ Lift = 0.75 / 0.9= 0.8333 (< 1, thus it is negatively associated)

$\Rightarrow$ therefore, *coffee occurs less than expected when tea also occurs*

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Too many frequent itemsets

- If $\{a_1, \ldots, a_{100}\}$ is a frequent itemset, then there are

$$\binom{100}{1} + \binom{100}{2} + \ldots + \binom{100}{100} = 2^{100} - 1$$

$1.27 \cdot 10^{30}$ frequent sub-patterns!

- There should be some more **condensed** way to describe the data

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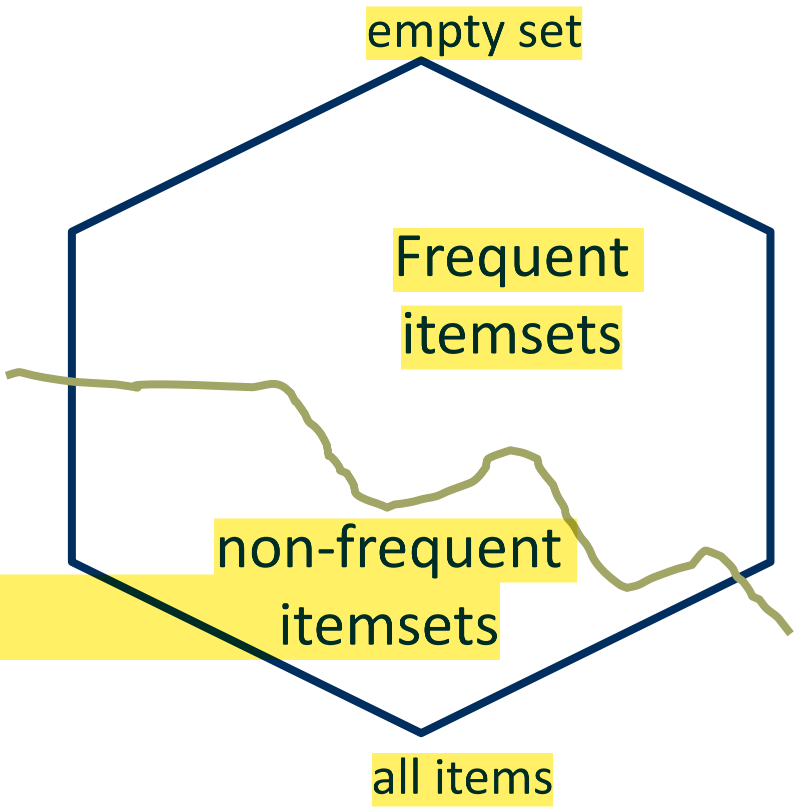
Frequent itemsets maybe too many to be helpful

- If there are many & large frequent itemsets, enumerating all of them is costly!
- We may be interested in finding the ***boundary*** frequent patterns

Question: Is there a good definition of such boundary?



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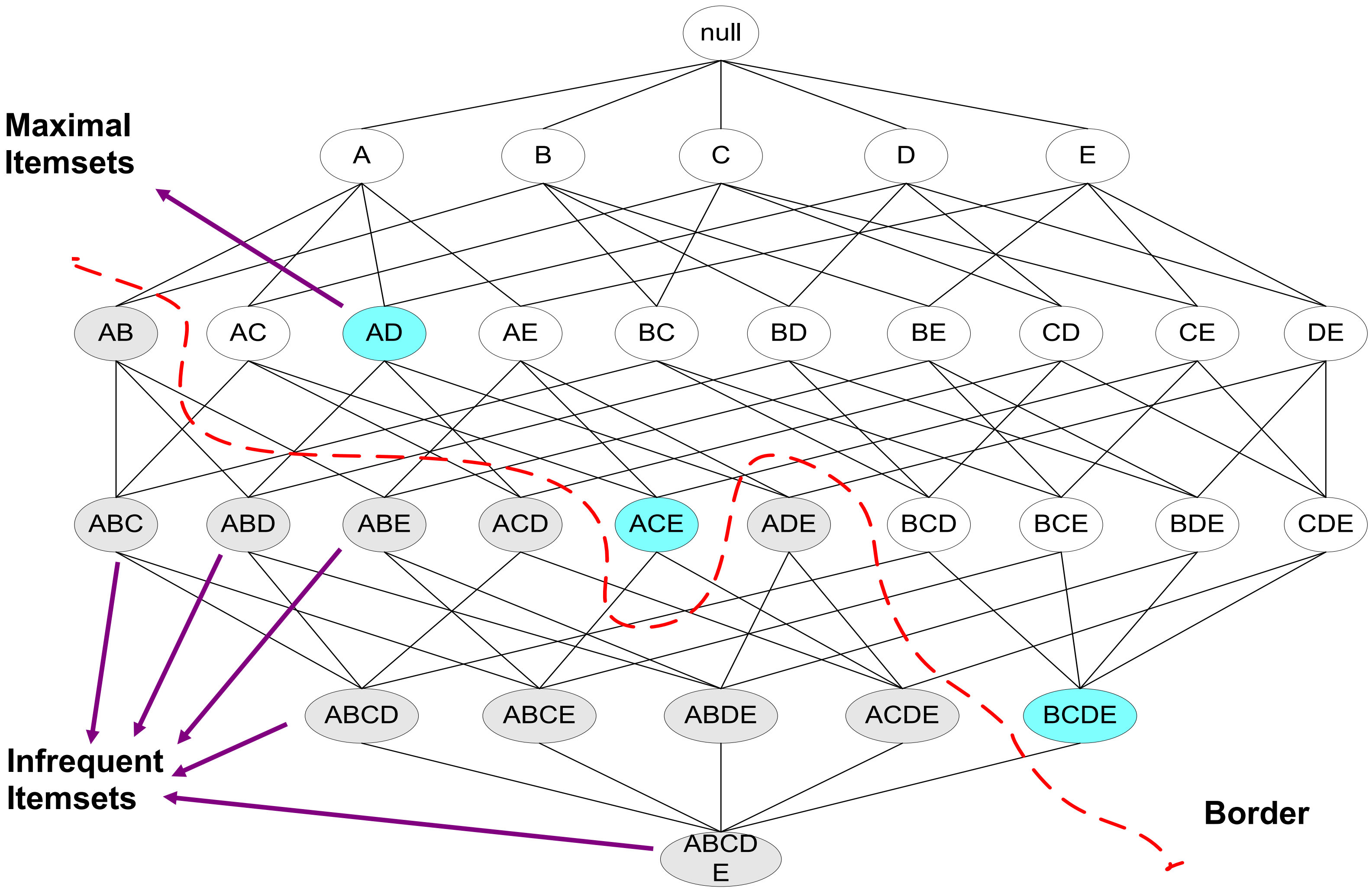
Maximal frequent itemsets

An itemset is **maximal frequent** if none of its immediate supersets is frequent



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Descriptive power of maximal patterns

- Knowing the set of all maximal patterns allows us to reconstruct the set of all frequent itemsets!!

- We can only reconstruct the set, not the actual frequencies

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Closed itemsets

An itemset is **closed** if **none of its immediate supersets** has the same support as the itemset

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	3
{A,B,C,D}	2

Closed itemsets: green color

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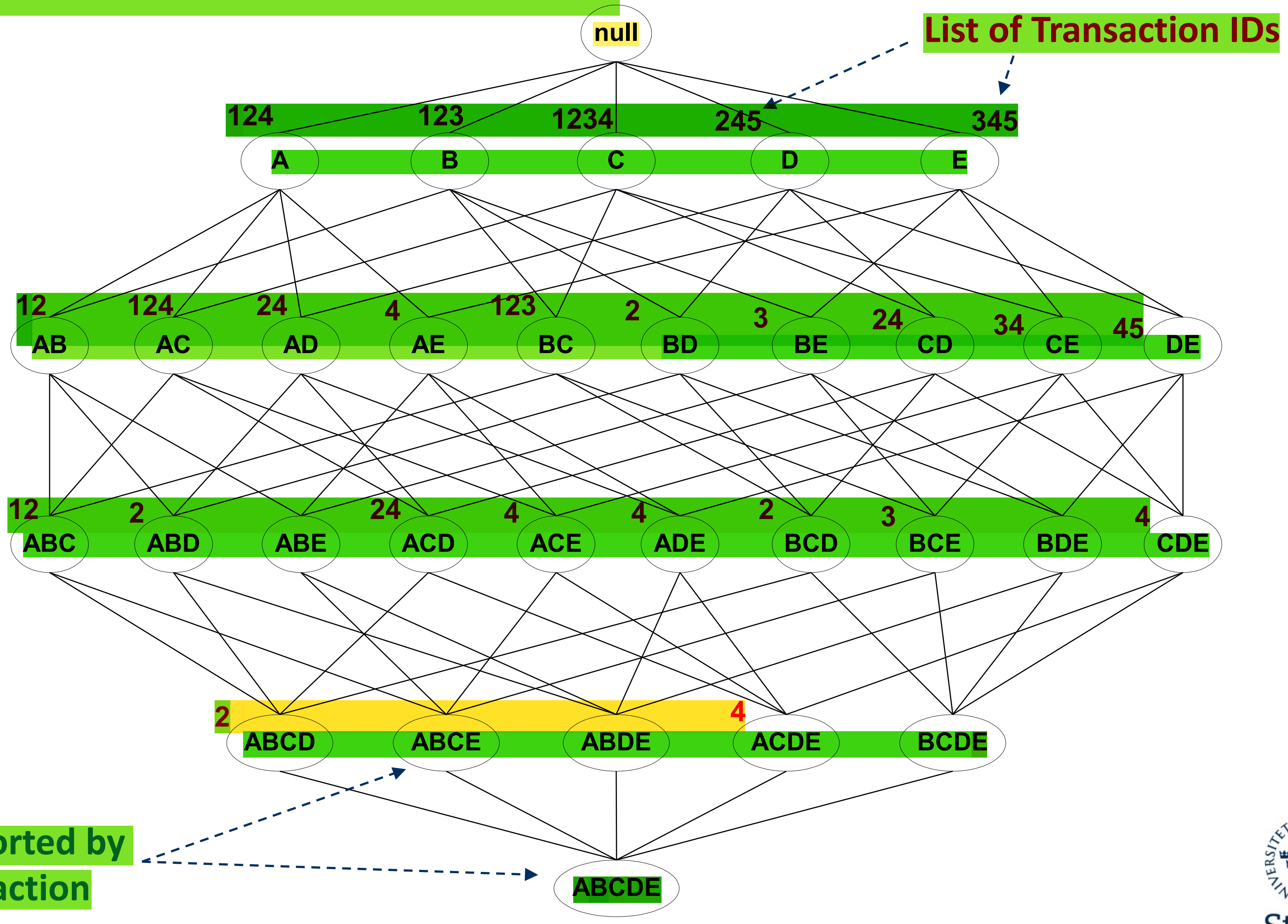
Maximal vs Closed itemsets

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



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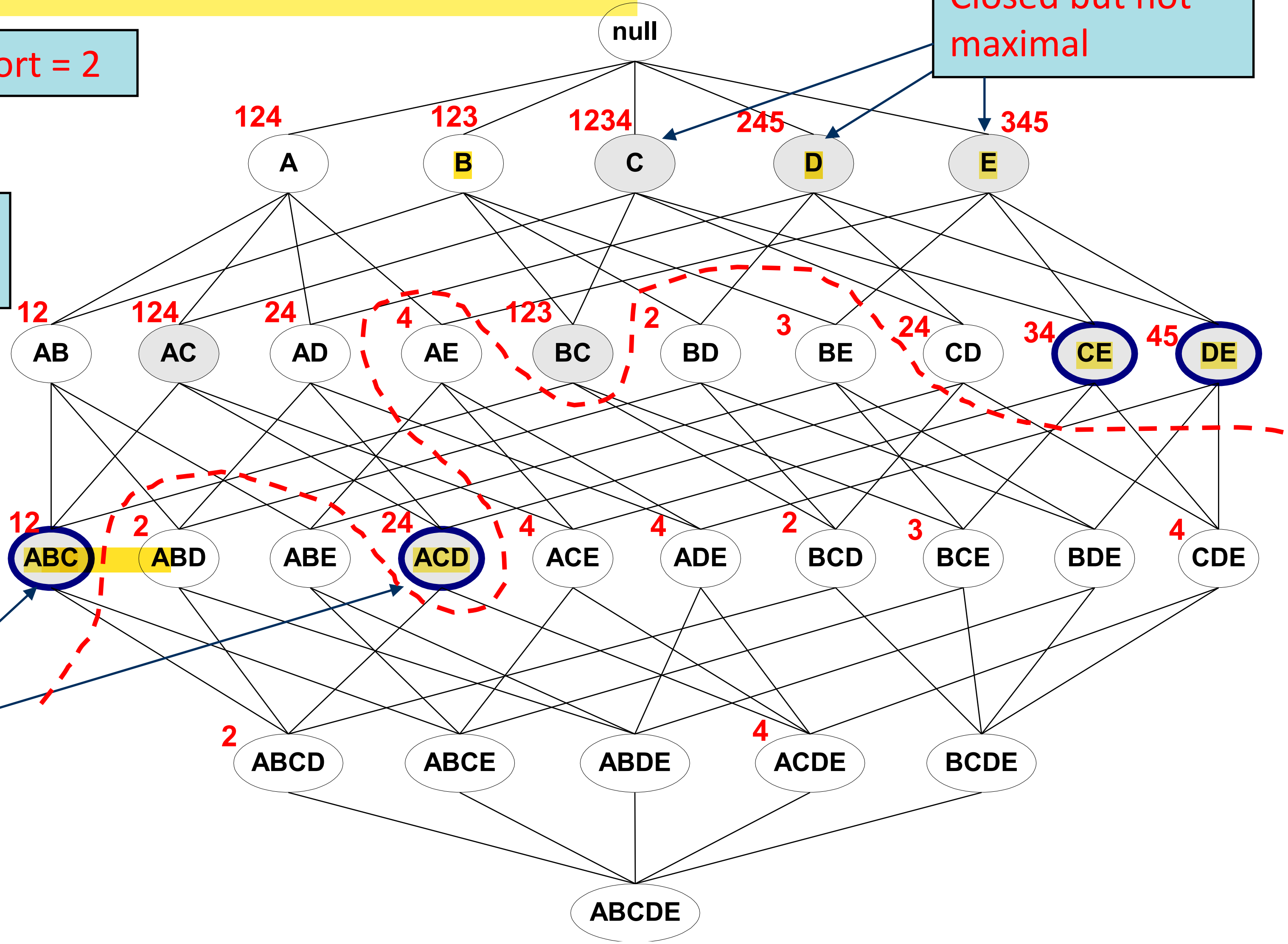
Maximal vs Closed itemsets

Minimum absolute support = 2

Closed = 9
Maximal = 4

Closed but not maximal

Closed and maximal



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Why are closed itemsets interesting?

- Closed patterns and their frequencies alone are a sufficient representation of all the frequencies of all frequent patterns

- **Proof:** Assume a frequent itemset ***X***:
    - ***X*** is closed → ***s(X)*** is known
    - ***X*** is not closed →

      s(X) = max {s(Y) | Y is closed and X subset of Y}

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Closed itemsets

An itemset is **closed** if **none of its immediate supersets** has the same support as the itemset

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Itemset	Support
{A}	?
{B}	5
{C}	?
{D}	?
{A,B}	4
{A,C}	?
{A,D}	?
{B,C}	3
{B,D}	4
{C,D}	3

Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	3
{A,B,C,D}	2

s(X) = max {s(Y) | Y is closed and X subset of Y}

Closed itemsets

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Itemset	Support
{A}	?
{B}	5
{C}	?
{D}	?
{A,B}	4
{A,C}	?
{A,D}	?
{B,C}	3
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{C,D}	3

Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
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Closed itemsets

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s(X) = max {s(Y) | Y is closed and X subset of Y}

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{A}	?
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	?
{A,D}	?
{B,C}	3
{B,D}	4
{C,D}	3

Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	3
{A,B,C,D}	2

s(X) = max {s(Y) | Y is closed and X subset of Y}

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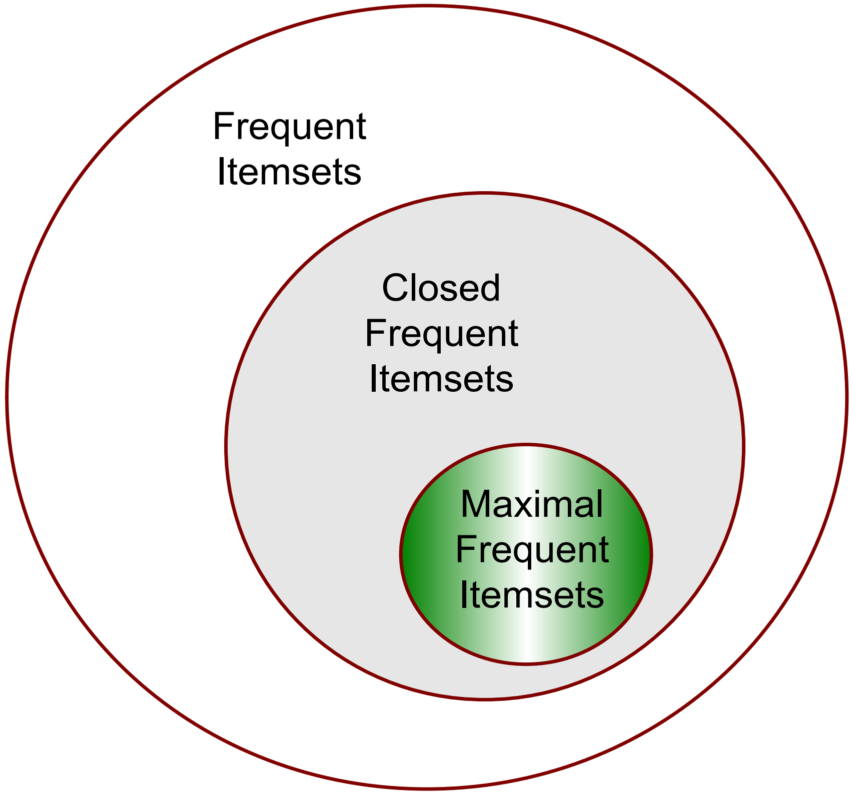
Maximal vs Closed itemsets

- Knowing all maximal patterns (and their frequencies) allows us to reconstruct the set of frequent itemsets
- Knowing all closed patterns and their frequencies allows us to reconstruct the set of all frequent itemsets and their frequencies
- The problems of finding the set of frequent itemsets, maximal itemsets, and closed itemsets are all **NP-hard**

Proof?



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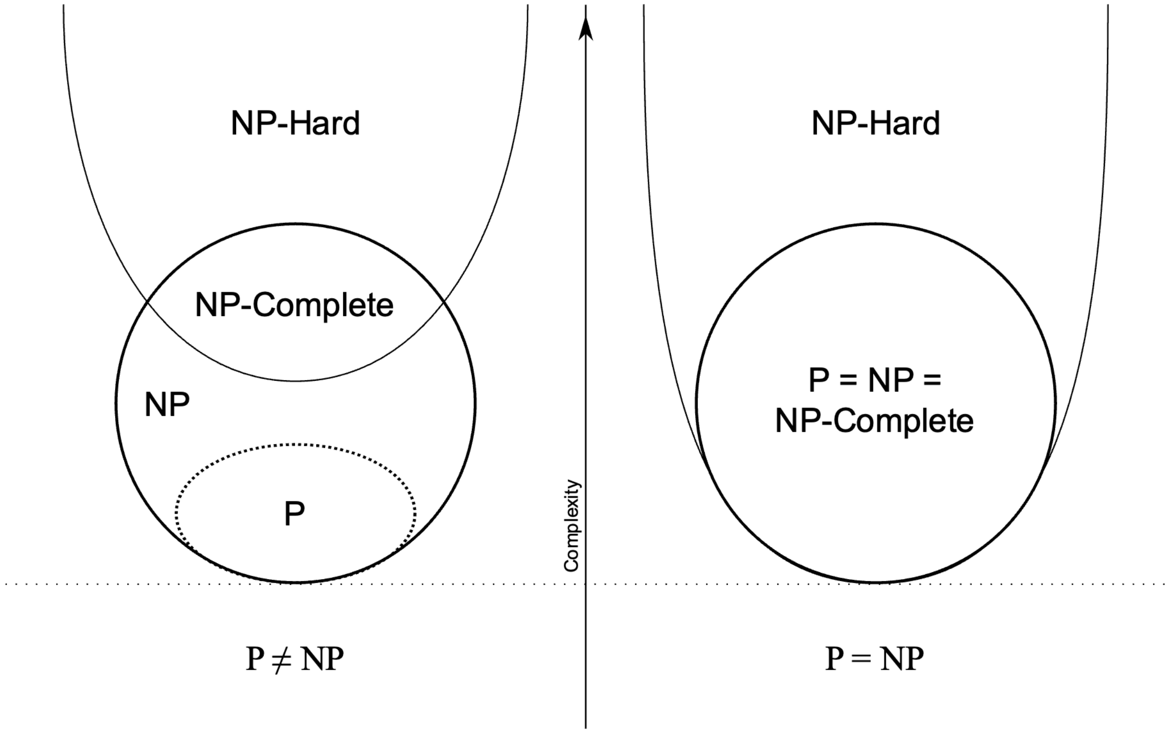
Back to the theory of complexity



NP: a solution can be verified in polynomial time

NP-hard: every NP problem can be reduced to it in polynomial time (the problem can be used as a subroutine to solve any NP problem)

NP-complete: the problem is in NP and it is NP-hard

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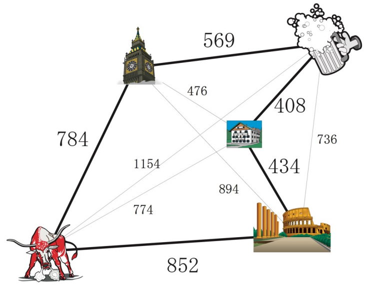
Examples of NP-hard problems

- Subset sum: given a set of integers is there a non-empty set of those integers that adds up to zero?

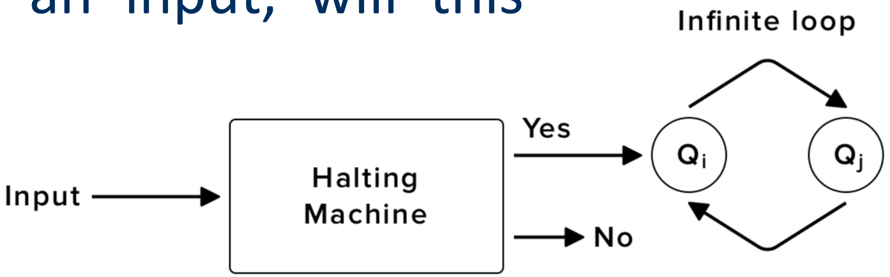
{-7, -3, -2, 5, 8}

{ -3, -2, 5 }

- Traveling salesman: given a set of cities and the distances between the cities, what is the possible shortest route visiting each and every city (once) and returning to the starting city?



- Halting problem: given a program and an input, will this program run for ever?



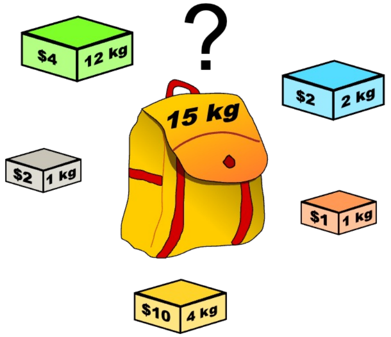
Examples of NP-complete problems

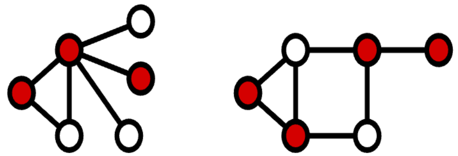
- Boolean satisfiability (SAT): given a boolean formula, is there a configuration of the variables that satisfies it?
- Knapsack problem: given a set of items, each having a value and a weight, find the subset of items so that the weight is **at most W** (minimized) and the total value is **at least V** (maximized)
- Vertex cover: given a graph, find a set of **at most k vertices** so that each and every edge in the graph is incident to at least one of these vertices
- Maximum Clique: given a graph, find the largest complete subgraph of **size at most k**

$(x \text{ OR } y \text{ OR } z) \text{ AND } (x \text{ OR } \bar{y} \text{ OR } z) \text{ AND}$

$(x \text{ OR } y \text{ OR } \bar{z}) \text{ AND } (x \text{ OR } \bar{y} \text{ OR } \bar{z}) \text{ AND}$

$(\bar{x} \text{ OR } y \text{ OR } z) \text{ AND } (\bar{x} \text{ OR } \bar{y} \text{ OR } \bar{z})$





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Proving NP-hardness

- Not always easy
- But sometimes easier than you think
- To prove that problem **T** is NP-hard:
  - **find** a known NP-hard problem **L**
  - **reduce** **L** to **T** in polynomial time: show that all instances of **L** can be solved by solving **T**, which is done by mapping **L** to **T** in polynomial time
  - since **L** is NP-hard, **T** cannot be solved faster than that…hence it is also NP-hard
- If we can also **verify** any solution to **T** in polynomial time, then **T** is in NP; hence **T** is also NP-complete

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Maximal frequent itemset mining

- *Use the set-cover problem formulation!*
  - universe of **m** elements **$U = \{U_1,\ldots,U_m\}$:** each **U_i** is a frequent itemset
  - set of **n** sets **$S = \{S_1,\ldots,S_n\}$:** each **S_i** contains all itemsets covered by a given frequent itemset X, i.e., all subsets of X
- **Formulation:**
  - **Problem A:** find the **smallest collection** of sets in **S** such that **all elements in U** are **covered** (set cover)
  - **equivalent to Problem B:** find the **smallest collection** of itemsets each covering a set of **frequent itemsets**, so that ***all frequent itemsets*** are **covered** (maximal itemsets)
  - so…if we can solve **Problem B** we can also solve all instances of **Problem A**
  - since A is known to be NP-hard, then B is also NP-hard

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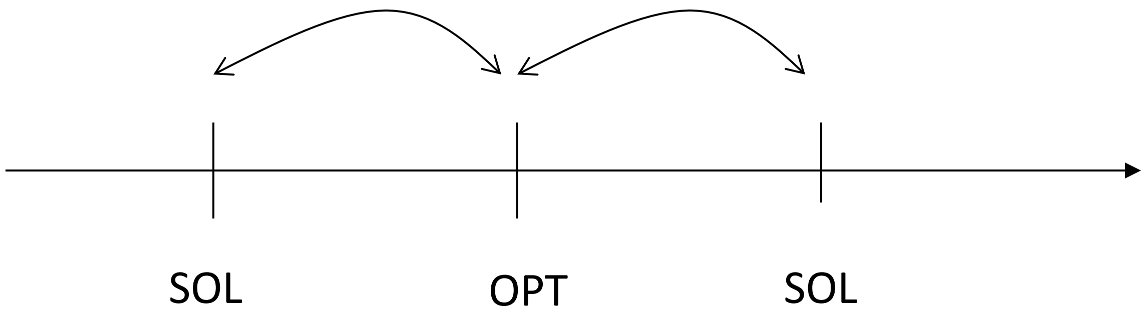
Approximation algorithms

For an NP-hard problem, we cannot compute an optimal solution in polynomial time

The key of designing a polynomial time approximation algorithm is to obtain a good (lower or upper) bound to the optimal solution

- **OPT:** value of an optimal solution
- **SOL:** value of the solution that our algorithm returns

The general strategy (for an optimization problem) is:



$f \cdot \text{OPT} \leq \text{SOL} \leq \text{OPT}, \quad \text{if } f < 1$

Maximization Problem:
maximize some cost function

$\text{OPT} \leq \text{SOL} \leq f \cdot \text{OPT}, \quad \text{if } f > 1$

Minimization Problem:
minimize some cost function

Approximation for set cover

The greedy algorithm for set cover has an approximation factor of:

$f = |s_{max}|$

s_{max}: the size of the largest set

Proof: From CLR "Introduction to Algorithms"

The set cover cannot be approximated with a factor better than **$O(\log |s_{max}|)$**

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Best-collection problem

- Universe of **m** elements **U = {U_1,…,U_m}**
- A set of **n** sets **S = {S_1,…,S_n}** such that they cover the universe
- **Question:** Find a collection **C** consisting of **k** sets from **S** such that

$f(C) = |\bigcup_{c \in C} c|$ is ***maximized***

f: the number of elements in **U** that are covered by **C**

- The best-collection problem is NP-hard

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Greedy approximation algorithm for the best-collection problem

- **C = {}**
- **for every** set **s** in **S** and ***not*** in **C** compute the gain of **s**:

  g(s) = f(C U {s}) – f(C)

- Select the set **s** with the ***maximum*** gain
- **C = C U {s}**
- **Repeat until C** has **k** elements

The ***greedy*** algorithm for the best-collection problem has the following approximation factor:

F = (e-1)/e = 0.6321

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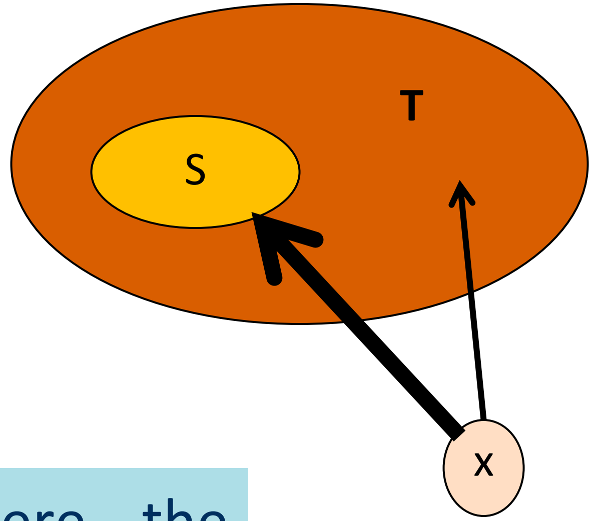
Submodular functions

- A function **f** (defined on sets of some universe) is **submodular** if
  - ***for all*** sets **S**, **T,** such that **S** is ***subset*** of **T** and **x** ***any*** element in the universe
  - **f(S U {x}) – f(S) ≥ f(T U {x}) – f(T)**

adding an element to a smaller subset will have a higher gain than adding it to a superset

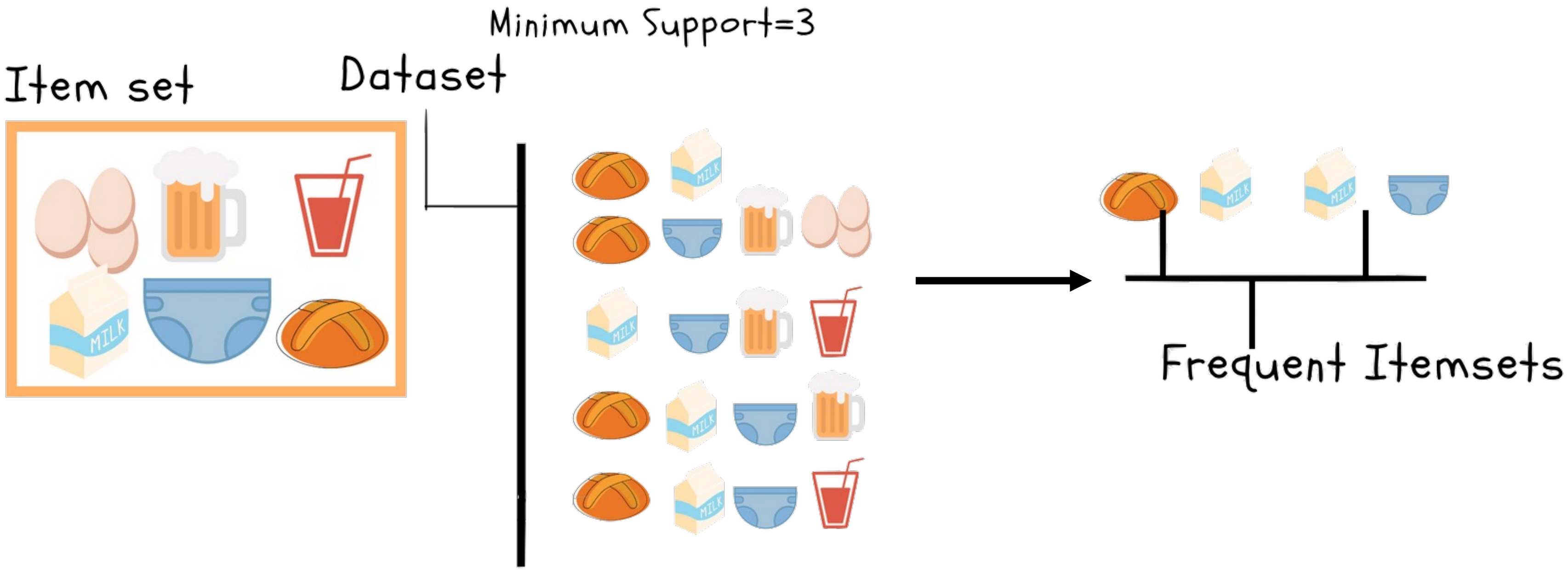


Theorem: For **all maximization** problems where the optimization function is **submodular**, the ***greedy*** algorithm has the following approximation factor

F = (e-1)/e = 0.6321

Today…

- What is **Itemset Mining**?
- What is the **Apriori Principle**?
- How do we use the **Apriori Algorithm**?
- The importance of **Maximal** and **Closed** Itemsets
- The **set cover problem** and **Approximation Algorithms**



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TODOs

Reading:

Main course book: Chapter 3.5

Extra Material

Lab 0

Recommended to complete the lab before the end of the week

Quiz 1

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Coming up next week

Monday

Lecture 3 – Dimensionality Reduction

Wednesday

Lab 1 – Data Exploration

Thursday

Lecture 4 – Clustering I