Simulation Modelling: A Study for Hard-Workers in a Group Project

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ABSTRACT

This paper investigates the dynamics of group projects in educational settings, focusing on the varying levels of effort invested by students. The study develops a model using numerical and analytical solutions of a differential equation and an agent-based model implemented in Python. The model explores how strategies evolve within groups based on factors such as group size, cost of effort, and circle size.

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1 INTRODUCTION

In education settings, group projects play a crucial role in fostering collaboration and enhancing learning outcomes. One key challenge arises is the varying levels of efforts invested by students in the groups. Some students choose to work hard and strive for excellence while some choose a relaxed approach, preferring to ride on the efforts of peers. The aim of this project is to understand this dynamics of the groups and the strategies change over time period.

2 PROBLEM SETTING

Considering a population size of N i.e the number of students in the school. Each student is randomly assigned into groups of size n. Each student can adopt two strategies. Working Hard (S=1) or Working Lazy (S=0). The group is formed at the start of each semester and students adhere to their pre-determined strategy throughout the group work.

The total efforts exerted depends on the composition of the group. Hence for a group of size n, if there are h number of hard workers then there are l(n-h) number of lazy workers. The total effort of the group is given by $e = hH + lL$, where H=1 and L=0. To evaluate the group project, the lecturer assigns the same marks to all the students within the group. The mark is calculated by total efforts of the group divided by the number of students in the group, i.e m=e/n.

At the end of each semester, student will reassess their strategy for the upcoming semester. They do this by randomly selecting another student and comparing their payoffs. Payoffs are denoted by π, which is measured as $\pi = m - a * E$ (where E=1 for a hard worker and 0 for a lazy worker). Here a is the cost of effort .This performance measure prioritizes high marks achieved with minimal efforts. If the payoff of reference student is higher, then the student will imitate the reference student strategy in the next semester with a probability. This probability is proportional to the difference in their payoff scores.

3 ANALYTICAL SOLVING THE PROBLEM

In evolutionary game theory, the replicator's strategy is frequently employed when agents modify their strategiesin response to the comparative success or payoff of various strategies within a population. [4]. The students in the school of the problem statement also adapt their strategies based on the difference in payoff with respect to a random student. Therefore, We extend the use of differential equation from [4] to this problem.

Replicator equation for this problem is given by:

$$\dot{X}_H = X_H(\pi_H - \bar{\pi}) \tag{1}$$

$\dot{X}_H$ is the rate of change of concentration of hard working student in the population with respect to time. X_H is the concentration of hardworking students in the population, X_L is the concentration of lazy students in the population($X_L = 1 - X_H$). π_H is the expected pay off for a hardworking student, and $\bar{\pi}$ is average payoff of the two class i.e hardworking student and lazy workers. π_L is expected pay off for a lazy student

The average payoff the two classes is given by the following equation:

$$\bar{\pi} = (X_H \pi_H) + (X_L \pi_L)) \tag{2}$$

Substituting 2 in 1 yields:

$$\dot{X}_H = X_H(1 - X_H)(\pi_H - \pi_L) \tag{3}$$

The expected payoff of hardworking student π_H is the sum of the expected payoff for each interaction. Example, a hardworking student can have n-1 lazy students in his group. Therefore his expected payoff for this interaction will be $(1 - X_H)^{n-1}(\frac{1}{n} - a)$. $(1 - X_H)^{n-1}$ is the chance of interaction and $(\frac{1}{n} - a)$ is payoff of the interaction. The student can also have an 1 hardworking student and n-2 lazy students in his group. For this situation his expected payoff will be $X_H(1 - X_H)^{n-2}(\frac{2}{n} - a)$. Similarly, expected payoff for all interactions the student can have is calculated and aggregated to calculate π_H :

$$\begin{aligned}\pi_H =& (1 - X_4)^{n-1}(\frac{1}{n} - a) + X_H(1 - X_H)^{n-2} \\ &+ \ldots + X_H^{n-2}(1 - X_H)(\frac{n-1}{n} - a) + (X_H)^{n-1}(1 - a)\end{aligned} \tag{4}$$

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Similarly expected payoff for a lazy lazy worker π_L is given by:

$$\begin{aligned}\pi_L =& \frac{1}{n}(X_H)(1-X_H)^{n-2} + (X_H)^2(1-X_H)^{n-3}(\frac{2}{n}) + \dots \\ &+ (X_H)^{n-2}(1-X_H)\frac{n-2}{n} + (X_H)^{n-1}\frac{n-1}{n}\end{aligned} \tag{5}$$

Difference of 4 and 5 is given by:

$$\begin{aligned}\pi_H - \pi_L =& (\frac{1}{n} - a)((1-X_H)^{n-1} + X_H(1-X_H)^{n-2} + \dots \\ &+ (X_H)^{n-2}(1-X_H) + X_H^{n-1})\end{aligned} \tag{6}$$

Substituting 6 in 3 will give:

$$\dot{X}_H = X_H(1-X_n)(\frac{1}{n} - a)[\sum_{i=0}^{n-1}(X_H)^i(1-X_H)^{n-1-i}] \tag{7}$$

The last term is the sum of products of all the chances, therefore it is equal to 1.

$$\sum_{i=0}^{n-1}(X_H)^i(1-X_H)^{n-1-i} = 1 \tag{8}$$

When substituted in 7 our final differential equation is:

$$\dot{X}_H = X_H(1-X_n)(\frac{1}{n} - a)] \tag{9}$$

Upon solving the above differential equation we get the following solution:

$$X_H = \frac{1}{1 \pm e^{t(a-\frac{1}{n})+c}} \tag{10}$$

c is a constant. We can hence find the concentration of the population using the above function, by substituting the values of t.

3.1 Identification of the stable and unstable equilibrium

To identify the equilibrium points of the differential equation 7, the left hand side of the equation is equated to 0. Values of X_H obtained are 0 and 1, which are the equilibrium points of this equation.

The stability of these equilibrium points are obtained by differentiating the left hand side of equation with respect to X_H 7 . Upon differentiating the left hand side we get:

$$\frac{\partial f}{\partial X_H}\Big|_{X^*} = (1-X_H)(\frac{1}{n} - a) - X_H(\frac{1}{n} - a) \tag{11}$$

If $\frac{1}{n}$ > a, then the differential equation is unstable at the equilibrium point 0 and stable at the equilibrium point 1. If $\frac{1}{n}$ < a, then the differential equation is unstable at the equilibrium point 1 and stable at the equilibrium point 0.

The above stability analysis can also be verified by substituting the values of a and n in the equation 10. If a > $\frac{1}{n}$ and $t \rightarrow \infty$, the value of $X_H \rightarrow 0$. Whereas If a < $\frac{1}{n}$ and $t \rightarrow \infty$, the value of $X_H \rightarrow 1$.

The above solution resonates with the theory that if the group sizes increases, then the students would bank on their peers' efforts and will tend to adapt a lazy strategy.

4 SIMULATION OF MODEL

[3] successfully simulate COVID-19 outbreak model called SEIR-NDC using Euler Scheme and Improved Euler Scheme. Thus, Euler Scheme and Improved Euler Scheme was chosen to simulate the differential equation 7.

Developed by Swiss mathematician Leonhard Euler, this method numerically approximates a one dimensional differential equation. The main idea behind this method is dividing the ODE's interval into small sub-intervals. Starting from an initial value, the method calculates the slope of the ODE at that point and uses it to estimate the value of the function at the next point. This process is iterated until the desired interval is covered. This paper studies, how different size intervals(i.e. step_size) affects in converging the solution [1]. The pseudo code for the algorithm used is given below:

Algorithm 1 Euler's method of solving differential equation

Require: X_H^0, n_iter, group_size , a ,step_size
function EQUATION(X_H)
  return $_H * (1-X_H) * (1/group\_size - a)$
end function
$H\_concentration \leftarrow []$
$L\_concentration \leftarrow []$
$X_H\_prev \leftarrow X_H^0$
for $i \leftarrow 1$ **to** n_iter **do**
  $X_H\_next \leftarrow X_H\_prev + step\_size * EQUATION(X_H\_prev)$
  $X_H\_prev \leftarrow X_H\_next$
  $H\_concentration$.append(X_H_next)
end for=0

The second method of integration implemented was Heun's method, also known as improved euler's method. It is an enhancement over Euler's method and provides a more accurate estimate of the solution by taking into account the average slope over a step. Like the previous method, the ODE's interval is divided into small sub-intervals. At each step it then calculates the slope of ODE at the current point using Euler's method. Then, instead of directly using this slope to estimate the value of the function at the next time step, Heun's method takes into account the average slope over the step. Then it uses a average slope to obtained a refined prediction. This average slop is calculated by evaluating ODE at current point and predicted point using euler's method. Finally, the average of both the result (i.e Euler's method and refined prediction) is used as an estimate of of time step. This process is iterated over the desired interval. This paper studies, how different size intervals(i.e. step_size) affects in converging the solution [1]. The pseudo code for the algorithm used is given below:

Algorithm 2 Heun's method of solving differential equation

Require: X_H^0, n_iter, group_size , a ,step_size
function EQUATION(X_H)
  return $_H * (1 - X_H) * (1/group\_size - a)$
end function
$H\_concentration \leftarrow []$
$L\_concentration \leftarrow []$
$X_H\_prev \leftarrow X_H^0$
for $i \leftarrow 1$ **to** n_iter **do**
  $K\_1 \leftarrow step\_size * EQUATION(X_H\_prev)$
  $K\_2 \leftarrow step\_size * EQUATION(X_H\_prev + K\_1)$
  $X_H\_next \leftarrow 0.5 * (K\_1 + K\_1)$
  $X_H\_prev \leftarrow X_H\_next$
  $H\_concentration$.append(X_H_next)
end for=0

5 AGENT BASED MODEL

[2] implemets and Agent-Based Model to simulate the different behaviou of occupants in a commercial building and it's impact on the energy consumption. Inspired by it, this paper implements an agent-based model to the problem statement given in Section 2 to examine the dynamics of the group with different strategies. Using the agent based model, the concentration of the population was observed for different values of a and different values of group_size, over a period of time. The population size (i.e the number of students in the school) was set to 10000 and the change in concentration was evaluated for 20 years (i.e 40 semesters). In the first run, group size was set to 5 and was tested against different values of a starting from 0 to 1 with a step size of 0.1. In the second run, a was set to 0.5 and the concentration was tested for different group size starting from 2 to 100, with a step size of 1. The code was implemented in Python, using pandas,numpy and random packages.

6 RESULTS AND EVALUATION

For Both Euler's and Heun's method, the concentration of population was evaluated with different step_sizes. The equation were tested against different initial concentration of the population (0.2 and 0.5 respectively). The group_size is set to 7, a (Cost of Effort) is set to 0.2 and number of iterations is set to 500. The values of step_size tested were [50,25,1,0.1,0.01].

6.1 Euler's Method

The following graph shows the concentration for Euler's method:

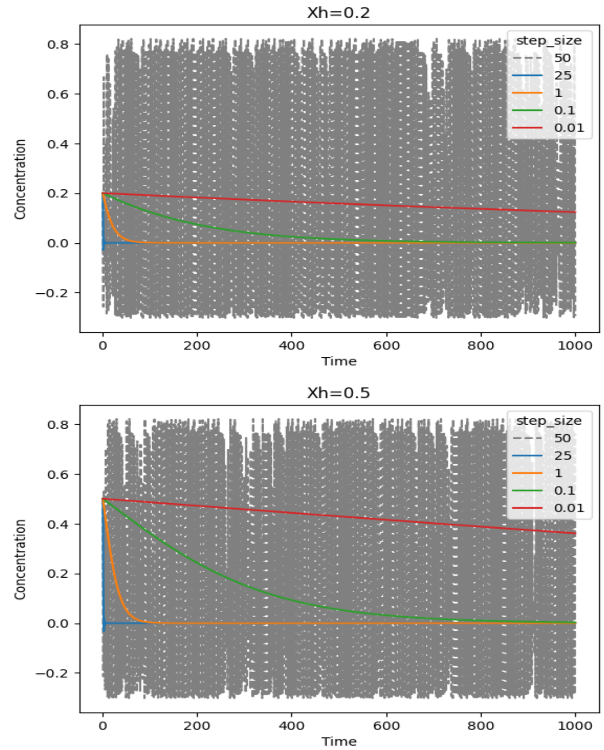

Figure 1: Euler's Scheme

Based on the graph above, it can be observed that the solution remains consistent despite the change in initial concentration. When the step size is set to 0.1, 1, and 25, the equation converges towards 0. Notably, the convergence is faster with a step size of 0.25 compared to 1, and even faster than with a step size of 0.1. While the equation also converges for a step size of 0.01, the convergence is linear and takes a longer duration. However, when the step size is set to 50, the equation fluctuates between 0.821 and -0.301, and does not converge to any solution. The fluctuation is also visible when step_size is 25, but later it converges This convergence to 0 agrees with the analytical solution of Section 3.1. Since a > $\frac{1}{7}$, the result should converge to 0.

Euler method of approximation can lead to two type of error, Truncation error and rounding-off error. Euler's method, truncates higher order terms of Taylor series to make an estimation. The round-off error is caused by the limited precision of numerical calculations is an effect of the device at use.

6.2 Heun's Method

The following graph shows the concentration for Heun's method:

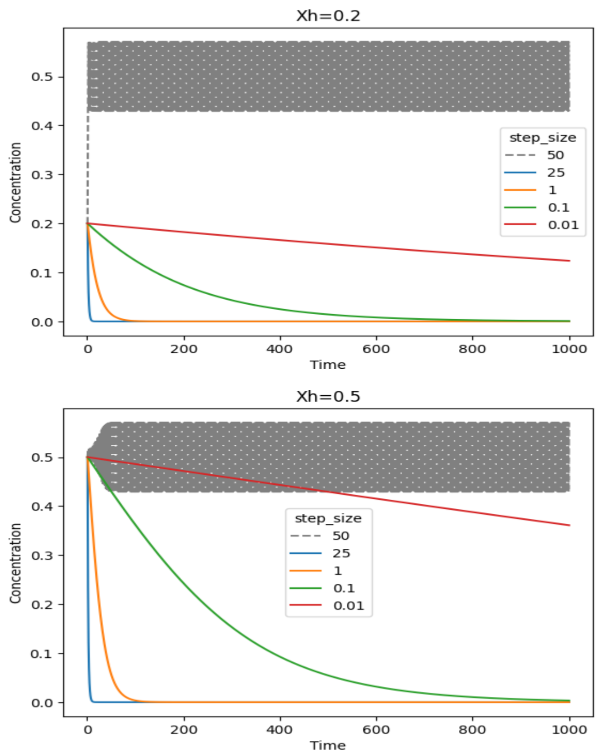

Figure 2: Heun's Scheme

Based on the observed graph, it is evident that the solution remains consistent regardless of the initial concentration. When the step sizes are 0.1, 1, and 25, the equation converges towards 0. Notably, for a step size of 0.25, the convergence is initially rapid and then stabilizes at 0. The equation converges faster with a step size of 1 compared to 0.1. The equation with a step size of 0.01 linearly decrease with time. When the step size is set to 50, the equation exhibits fluctuations around $X_H = 0.5$. This convergence to 0 agrees with the analytical solution of Section 3.1. Since a > $\frac{1}{7}$, the result should converge to 0.

Unlike Euler's method, this is a second order method, hence the truncation error cause by this relatively less than the eurler's method. The round-off error is caused by the limited precision of numerical calculations is an effect of the device at use.

6.3 Agent Based Model

The graph below is the observation of concentration with respect to semesters for different values of a(Cost of Effort) for a population of size 10000 and group_size 5. The initial concentration of the population was set 0.5.

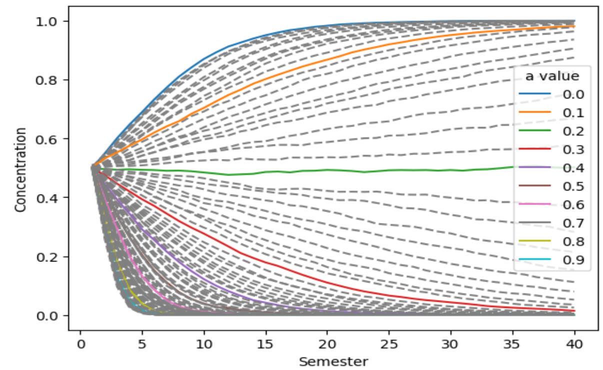

Figure 3: Agent Based Model against different values of Cost of Effort

From the above graph it is observed that for a<0.2, the concentration of the population is converging to 1, whereas when a>0.2, the concentration is converging to 0. The speed of convergence increases as a's value moves away from 0.2. At a=0.2, the concentration of the population is approximately constant at $X_H = 0.5$. This agrees with the analytical solution of Section 3.1, where if $a < \frac{1}{5}$, it will converge to 1 and for $a > \frac{1}{5}$, it will converge to 0.

The graph below is the observation of concentration with respect to semesters for different values of group_size for a population of size 10000 and a is set to 0.5. The initial concentration of the population was set 0.5.

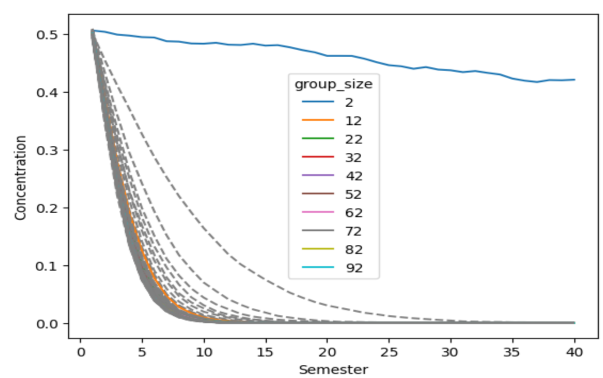

Figure 4: Agent Based Model against different values of a

In the above graph, it is evident that as the group size increases, the concentration decreases. This observation aligns with the theoretical understanding that in larger groups, individuals tend to rely on the collective efforts of their peers. This is further supported by the analytical solution of Section 3.1, as a=0.5>$\frac{1}{group\_size}$, concentration will converge towards 0. Notably, for a group size of 2, the concentration remains approximately the same as the initial condition.

7 EXTENSION OF THE PROBLEM

In real life, individuals are significantly influenced by their social circles, making it a crucial factor in shaping their behaviors and decisions. Therefore, the study aims to expand its investigation to explore the impact of social circle on the strategy and how it changes the population concentration. To achieve this, an agent-based model with a different update strategy is created, deviating from the previous model.

In the first semester, each student is randomly assigned to a friend circle, which remains unchanged throughout the study. In contrast to the model described in Section 5, where students were randomly selected from the population, this new model involves students randomly selecting a fellow student from their friend circle for comparison. They then compare their own payoff with that of the selected friend and imitate the friend's strategy with a certain probability if the friend's payoff is higher. This probability is proportional to the difference in payoffs.

The graph below is the concentration of the population with respect to time for different values of a(Cost of effort). The population size is set = 10000, group_size is set to 5, circle size is set to 10 and the initial concentration of population is set to 0.5. The change in concentration is observed for 20 years, i.e 40 semesters.

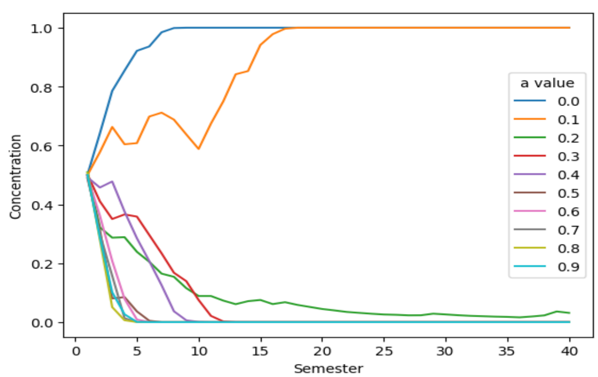

Figure 5: Agent Based Model against different values of a

In the above graph, the concentration of the population converges to 1 when a is less than $\frac{1}{5}$, and to 0 when a is higher than $\frac{1}{5}$. Notably, when value of a=0.2, the concentration is still converging to 0 unlike the model from Section 5 where it was consistent at 0.5. It is also observed that, the graphs lines are not smooth like previous model.

The following graph is the concentration of the population with respect to time for different values of 'group_size'. The population size is set = 10000, a is set to 0.5, circle size is set to 10 and the initial concentration of population is set to 0.5. The change in concentration is observed for 20 years, i.e 40 semesters.

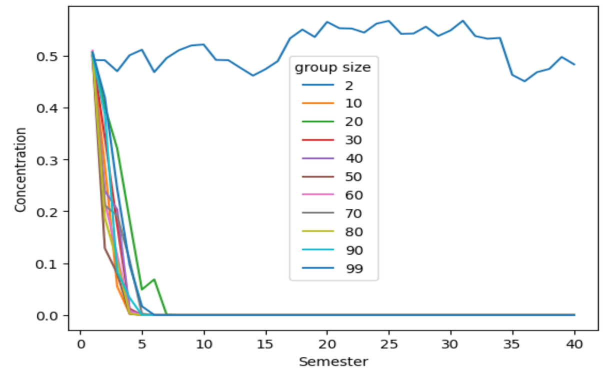

Figure 6: Agent Based Model against different values of a

For group_size =2, the concentration fluctuates around $X_H = 0.5$. Whereas for the remaining group_sizes, the concentration of the group reduces to 0.

The following graph is the concentration of the population with respect to time for different values of 'circle_size'. The population size is set = 10000, a is set to 0.5, group_size is set to 10 and the initial concentration of population is set to 0.5. The change in concentration is observed for 20 years, i.e 40 semesters.

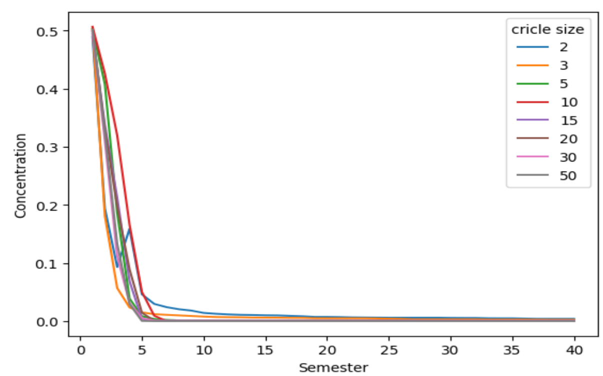

Figure 7: Agent Based Model against different values of a

In general, the population concentration tends to approach 0 regardless of the group size. However, it is worth noting that when the group size is small, the graph does not reach an absolute zero but rather converges near it.This observation can be attributed to certain groups where all the students are initially assigned a lazy strategy and are unable to change their strategy due to the absence of any hard workers in their circle. This aligns with real-life situations where individuals tend to be influenced by their social circles, and if the circle primarily consists of lazy workers, people are more likely to follow suit and remain lazy.

8 CONCLUSION

In conclusion, the paper presents a model which successfully understand the dynamics of group projects in the education setting. It investigates how the strategies evolve within the group depending on the group_size, a value and the circle_size using solutions of the differential equation solved numerically and analytically as well as using an agent-based model implemented in Python. The model has the potential for further improvement by incorporating different environments. For instance, it can explore the concentration of the population with varied levels of strategy (e.g., S = 1, 0.5, 0), where the first individual neither exhibits a hardworking nor a lazy behavior.Additionally, intervention strategies like mentoring programs can be implemented to payoffs to see it's effect on group's concentration.

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