

DERIVATNAK.

①

8. Oriibdea.

$$1.) f(x) = 2x + 1 \longrightarrow f'(x) = \boxed{2}$$

$$2.) f(x) = \frac{3x-2}{4} = \frac{3x}{4} - \frac{2}{4} = \frac{3x}{4} - \frac{1}{2} \longrightarrow f'(x) = \boxed{\frac{3}{4}}$$

$$3.) f(x) = \frac{3}{4} \longrightarrow f'(x) = \boxed{0}$$

$$4.) f(x) = \frac{x}{2} + 3 = \frac{1}{2}x + 3 \longrightarrow f'(x) = \boxed{\frac{1}{2}}$$

$$5.) f(x) = x^3 - 3x^2 + 2 \longrightarrow f'(x) = \boxed{3x^2 - 6x}$$

$$6.) f(x) = \frac{3x^5}{5} - \frac{4x}{3} + 5 = \frac{3}{5}x^5 - \frac{4}{3}x + 5 \longrightarrow f'(x) = \frac{3 \cdot 5}{5}x^{5-1} - \frac{4}{3} = \boxed{3x^4 - \frac{4}{3}}$$

$$7.) f(x) = \frac{4\pi - 2}{3} \longrightarrow f'(x) = \boxed{0}$$

$$8.) f(x) = \frac{4}{3}(x^2 - \frac{3}{4}x + 2) \longrightarrow f'(x) = \frac{4}{3}(2x - \frac{3}{4}) = \boxed{\frac{8x}{3} - 1}$$

$$9.) \frac{x^2}{5} - \frac{x}{4} + \sqrt{5} \longrightarrow f'(x) = \boxed{\frac{2x}{5} - \frac{1}{4}}$$

$$10.) \frac{x}{7} - \sqrt{7}x = \frac{1}{7}x - \sqrt{7} \cdot x^{\frac{1}{2}} \longrightarrow f'(x) = \frac{1}{7} - \frac{\sqrt{7}}{2}x^{\frac{1}{2}-1} = \frac{1}{7} - \frac{\sqrt{7}}{2\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} \\ = \frac{1}{7} - \frac{\sqrt{7}x}{2x} = \frac{2x - 7\sqrt{7}x}{7x}$$

$$11.) f(x) = \frac{1}{x} = x^{-1} \longrightarrow f'(x) = (-1)x^{-2} = \boxed{-\frac{1}{x^2}}$$

$$12.) f(x) = \frac{3}{x^2} = 3x^{-2} \longrightarrow f'(x) = 3(-2)x^{-3} = \boxed{-\frac{6}{x^3}}$$

$$13.) f(x) = \frac{5}{3x^3} = \frac{5}{3} \cdot x^{-3} \longrightarrow f'(x) = \frac{5(-3)}{3}x^{-4} = \boxed{-\frac{5}{x^4}}$$

$$14.) f(x) = \sqrt[3]{x^4} = x^{\frac{4}{3}} \longrightarrow f'(x) = \frac{4}{3}x^{\frac{4}{3}-1} = \frac{4}{3}x^{\frac{1}{3}} = \frac{4\sqrt[3]{x}}{3}$$

$$15.) f(x) = \frac{\sqrt{3x}}{x^2} = \frac{\sqrt{3} \cdot \sqrt{x}}{x^2} \longrightarrow f'(x) = \sqrt{3} \left(-\frac{3}{2}\right) \cdot x^{-3/2-1} =$$

$$= \sqrt{3}x^{\frac{1}{2}-2} = \sqrt{3} \cdot x^{-3/2}$$

$$= -\frac{3\sqrt{3}}{2}x^{-5/2} = -\frac{3\sqrt{3}}{2\sqrt{x^5}} = -\frac{3\sqrt{3}}{2x^2\sqrt{x}} = -\frac{3\sqrt{3x}}{2x^3}$$

16) $f(x) = \frac{3\sqrt{x^3}}{2x^4} = \frac{3}{2}x^{3/2-4} = \frac{3}{2}x^{-5/2} \rightarrow f'(x) = \frac{3}{2}\left(-\frac{5}{2}\right)x^{-5/2-1} =$
 $= \frac{-15}{4}x^{-7/2} = \frac{-15}{4\sqrt{x^7}} = \frac{-15}{4x^3\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} =$
 $= \boxed{\frac{-15\sqrt{x}}{4x^4}}$

17) $f(x) = \frac{2}{x} + \frac{x}{2} = 2x^{-1} + \frac{1}{2}x \rightarrow f'(x) = 2(-1)x^{-2} + \frac{1}{2} = \boxed{\frac{-2}{x^2} + \frac{1}{2}}$

18) $f(x) = \frac{\sqrt[3]{x^2}}{3} - \frac{x}{3} + \sqrt{5} = \frac{1}{3}x^{2/3} - \frac{1}{3}x + \sqrt{5}$
 $f'(x) = \frac{1}{3} \cdot \frac{2}{3}x^{2/3-1} - \frac{1}{3} = \frac{2}{9}x^{-1/3} - \frac{1}{3} = \frac{2}{9\sqrt[3]{x}} - \frac{1}{3} =$
 $= \frac{2\sqrt[3]{x^2}}{9x} - \frac{1}{3} = \boxed{\frac{2\sqrt[3]{x^2} - 3x}{9x}}$

19) $f(x) = \sqrt[4]{\frac{1}{x^3}} = x^{-3/4} \rightarrow f'(x) = -\frac{3}{4}x^{-3/4-1} = -\frac{3}{4}x^{-7/4} =$
 $= \frac{-3}{4\sqrt[4]{x^7}} = \frac{-3}{4x\sqrt[4]{x^3}} \cdot \frac{\sqrt[4]{x}}{\sqrt[4]{x}} = \boxed{\frac{-3\sqrt[4]{x}}{4x^2}}$

20) $f(x) = \sqrt{\frac{3}{x^5}} = \sqrt{3}x^{-5/2} \rightarrow f'(x) = \sqrt{3}\left(-\frac{5}{2}\right)x^{-5/2-1} = \frac{-5\sqrt{3}}{2\sqrt{x^7}} =$
 $= \frac{-5\sqrt{3} \cdot \sqrt{x}}{2x^3\sqrt{x}} = \boxed{\frac{-5\sqrt{3x}}{2x^4}}$

21) $f(x) = \frac{2\sqrt{x}}{x} - \frac{3}{x^2} + \frac{1}{x} = 2x^{1/2-1} - 3x^{-2} + x^{-1} = 2x^{-1/2} - 3x^{-2} + x^{-1}$
 $f'(x) = 2\left(-\frac{1}{2}\right)x^{-1/2-1} - 3(-2)x^{-2-1} + (-1)x^{-1-1} =$
 $= -x^{-3/2} + 6x^{-3} - x^{-2} = \frac{-1}{\sqrt{x^3}} + \frac{6}{x^3} - \frac{1}{x^2} \leftarrow$
 $= \frac{-1 \cdot \sqrt{x}}{x\sqrt{x}} + \frac{6}{x^3} - \frac{1}{x^2} = \boxed{\frac{-\sqrt{x}}{x^2} + \frac{6}{x^3} - \frac{1}{x^2}}$

$$22.) f(x) = x - \frac{3\sqrt{5}}{4} + \frac{1}{x^2} = x - \frac{3\sqrt{5}}{4} + x^{-2}$$

$$f'(x) = 1 - 0 + (-2)x^{-2-1} = 1 - 2x^{-3} = \left(1 - \frac{2}{x^3}\right)$$

$$23.) f(x) = \frac{x^2}{3} - \frac{3}{x^2} + \frac{3\sqrt{5}}{2} = \frac{1}{3}x^2 - 3 \cdot x^{-2} + \frac{3\sqrt{5}}{2}$$

$$f'(x) = \frac{2}{3}x - 3(-2)x^{-3} + 0 = \left[\frac{2x}{3} + \frac{6}{x^3}\right]$$

$$24.) f(x) = \frac{x^3}{3} - 4\sqrt{x} - \frac{2}{x^3} - \underbrace{x^2\sqrt{x}}_{x^{5/2}} = \frac{1}{3}x^3 - 4x^{1/2} - 2x^{-3} - x^{2+1/2}$$

$$f'(x) = \frac{1}{3} \cdot 3 \cdot x^2 - 4 \cdot \frac{1}{2} x^{-1/2} - 2(-3)x^{-4} - \frac{5}{2} x^{3/2} = \left[x^2 - \frac{2}{\sqrt{x}} + \frac{6}{x^4} - \frac{5}{2}\sqrt{x}^3\right]$$

$$25.) f(x) = \frac{x^2 - 3x + 1}{x} = \frac{x^2}{x} - \frac{3x}{x} + \frac{1}{x} = x - 3 + \frac{1}{x} = x - 3 + x^{-1}$$

$$f'(x) = 1 - 0 + (-1)x^{-1-1} = 1 - x^{-2} = 1 - \frac{1}{x^2} = \left[\frac{x^2 - 1}{x^2}\right]$$

10. omialdea

$$1.) f(x) = 3 \sin x - 2 \cos x \longrightarrow f'(x) = 3 \cos x + 2 \sin x$$

$$2.) f(x) = 4 \tan x + e^x \longrightarrow f'(x) = \frac{4}{\cos^2 x} + e^x$$

$$3.) f(x) = x \cdot \ln x. \xrightarrow{\text{Pielikto}} f'(x) = \underbrace{f'g}_{1 \cdot \ln x} + \underbrace{f g'}_{x \cdot \frac{1}{x}} = \ln x + 1$$

$$4.) f(x) = x \cdot e^x \longrightarrow f'(x) = 1 \cdot e^x + x \cdot e^x = \boxed{e^x(1+x)}$$

$$5.) f(x) = (x^2 + 1) \cdot \sin x \longrightarrow f'(x) = 2x \cdot \sin x + (x^2 + 1) \cdot \cos x$$

$$6.) f(x) = 2^x \cdot \tan x \longrightarrow f'(x) = 2^x \cdot \ln 2 \cdot \tan x + \frac{2^x}{\cos^2 x}$$

$$7.) f(x) = x^2 - \underbrace{\frac{x}{3}}_{\text{BIDERK}} e^x \rightarrow f'(x) = 2x - \left(\frac{1}{3} e^x + \frac{x}{3} e^x \right) \quad (4)$$

$$\boxed{f'(x) = 2x - \frac{1}{3} e^x - \frac{x}{3} e^x}$$

$$8.) f(x) = \underbrace{(x^3 - 2x + 1)}_f \cdot \underbrace{\cos x}_g \rightarrow f'(x) = \underbrace{(3x^2 - 2)}_{f'} \cdot \cos x + \underbrace{(x^3 - 2x + 1)}_f \cdot \underbrace{(-\sin x)}_{g'}$$

$$f'(x) = (3x^2 - 2) \cdot \cos x - (x^3 - 2x + 1) \cdot \sin x.$$

$$9.) f(x) = 3^x + \ln x - \left(\frac{1}{x} \right)^{x^{-1}}$$

$$f'(x) = 3^x \ln 3 + \frac{1}{x} - (-1) x^{-2} = \boxed{3^x \ln 3 + \frac{1}{x} + \frac{1}{x^2}}$$

$$10.) f(x) = 2^x + \log_2 x$$

$$\boxed{f'(x) = 2^x \cdot \ln 2 + \frac{1}{x \ln 2}}$$

$$11.) f(x) = \underbrace{x^2 \cdot e^x}_f + \underbrace{2x \cdot \ln x}_g$$

$$f'(x) = \underbrace{2x \cdot e^x + x^2 \cdot e^x}_f + \underbrace{2 \cdot \ln x + 2 \cdot \frac{1}{x}}_g =$$

$$= \boxed{2x \cdot e^x + x^2 \cdot e^x + 2 \ln x + 2}$$

$$12.) f(x) = \sqrt{x} \cdot \sin x - \log_3 5 = x^{1/2} \cdot \sin x - \log_3 5.$$

$$f'(x) = \underbrace{\frac{1}{2} x^{-1/2}}_{f'} \cdot \underbrace{\sin x}_g + \underbrace{\sqrt{x}}_f \cdot \underbrace{\cos x}_{g'} - 0$$

$$= \frac{1}{2\sqrt{x}} \sin x + \sqrt{x} \cdot \cos x = \frac{\sin x + \overbrace{2\sqrt{x} \sqrt{x} \cdot \cos x}^x}{2\sqrt{x}}$$

$$= \boxed{\frac{\sin x + 2x \cos x}{2\sqrt{x}}}$$

$$13.) f(x) = \frac{4x}{x+1} = \frac{F(x)}{G(x)}$$

$$f'(x) = \frac{F'G - FG'}{G^2}$$

$$f'(x) = \frac{4 \cdot (x+1) - 4x \cdot 1}{(x+1)^2} = \frac{\cancel{4x} + 4 - \cancel{4x}}{(x+1)^2} = \frac{4}{(x+1)^2}$$

$$14.) f(x) = \frac{x^2-1}{2x+2} \rightarrow f'(x) = \frac{2x(2x+2) - (x^2-1) \cdot 2}{(2x+2)^2} \dots$$

Erwartung! Faktorisieren etc. simplifizieren.

$$f(x) = \frac{x^2-1}{2x+2} = \frac{\cancel{(x+1)}(x-1)}{2\cancel{(x+1)}} = \frac{x-1}{2} = \frac{1}{2}(x-1)$$

$$f'(x) = \frac{1}{2}$$

$$15.) f(x) = \frac{x+1}{x-2} \rightarrow f'(x) = \frac{1 \cdot (x-2) - (x+1) \cdot 1}{(x-2)^2} = \frac{\cancel{x} - 2 - \cancel{x} - 1}{(x-2)^2}$$

$$f'(x) = \frac{-3}{(x-2)^2}$$

$$16.) f(x) = \frac{\ln x}{x} \rightarrow f'(x) = \frac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2}$$

$$17.) f(x) = \frac{e^x + e^{-x}}{2} = \frac{1}{2}(e^x + e^{-x}) \rightarrow \text{Kettenregel.}$$

$$f'(x) = \frac{1}{2}(e^x + (-1) \cdot e^{-x})$$

$$18.) f(x) = \frac{1}{x^4+1} \rightarrow f'(x) = \frac{0 \cdot (x^4+1) - 1 \cdot 2x}{(x^4+1)^2} = \frac{-2x}{(x^4+1)^2}$$

$$19.) f(x) = \frac{x^3}{x+2} \rightarrow f'(x) = \frac{3x^2(x+2) - x^3 \cdot 1}{(x+2)^2}$$

$$= \frac{3x^3 + 6x^2 - x^3}{(x+2)^2} = \frac{2x^3 + 6x^2}{(x+2)^2}$$

$$20.) f(x) = \frac{2x-1}{3x+2} \rightarrow f'(x) = \frac{2(3x+2) - (2x-1) \cdot 3}{(3x+2)^2} = \textcircled{6}$$

$$= \frac{\cancel{6x} + 4 - \cancel{6x} + 3}{(3x+2)^2} = \frac{7}{(3x+2)^2}$$

$$21.) f(x) = \frac{x^2}{x^2-1} \rightarrow f'(x) = \frac{2x(x^2-1) - x^2(2x)}{(x^2-1)^2} =$$

$$= \frac{\cancel{2x^3} - 2x - \cancel{2x^3}}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2}$$

$$22.) f(x) = \frac{\sqrt{x}}{x+2} \rightarrow f' = \frac{\frac{1}{2\sqrt{x}}(x+2) - \sqrt{x} \cdot 1}{(x+2)^2} =$$

$$= \frac{\frac{x+2}{2\sqrt{x}} - \frac{\sqrt{x} \cdot 2\sqrt{x}}{2\sqrt{x}}}{(x+2)^2} = \frac{\frac{x+2-2x}{2\sqrt{x}}}{(x+2)^2} = \frac{2-x}{(x+2)^2 \cdot 2\sqrt{x}}$$

$$23.) f(x) = (x^2-1) \cdot \sqrt{x}$$

Produkt $\rightarrow f'(x) = 2x \cdot \sqrt{x} + (x^2-1) \cdot \frac{1}{2\sqrt{x}} = \frac{2x\sqrt{x} \cdot 2\sqrt{x}}{2\sqrt{x}} + \frac{x^2-1}{2\sqrt{x}} =$

$$= \frac{4x^2 + x^2 - 1}{2\sqrt{x}} = \frac{5x^2 - 1}{2\sqrt{x}}$$

$$24.) f(x) = 3 \arcsin x \rightarrow f'(x) = \frac{3}{\sqrt{1-x^2}}$$

$$25.) f(x) = 2 \cdot \arccos x + e^x \rightarrow f'(x) = \frac{-2}{\sqrt{1-x^2}} + e^x$$

$$26.) y = 5 \operatorname{arctg} x \rightarrow f'(x) = \frac{5}{1+x^2}$$

$$27.) y = \frac{x \cdot e^x - \ln x}{2} \rightarrow f'(x) = \frac{1}{2} \cdot \left(1 \cdot e^x + x \cdot e^x - \frac{1}{x} \right) = \frac{x e^x + x^2 e^x - 1}{2x}$$

28) $f(x) = 3^x \cdot \sin x - \log_2 x$

$$f'(x) = 3^x \cdot \ln 3 \cdot \sin x + 3^x \cdot \cos x - \frac{1}{x \ln 2}$$

KATEGORIEN ERREGECLA: FN KONSTRUKTIONEN

13. onaldea

1.) $f(x) = (x^2 + 5)^6$

$$f'(x) = 6 \cdot (x^2 + 5)^5 \cdot (x^2 + 5)' = 6(x^2 + 5)^5 \cdot 2x = \boxed{12x(x^2 + 5)^5}$$

2.) $f(x) = \sin(x^2 - 1)$

$$f'(x) = \cos(x^2 - 1) \cdot (x^2 - 1)' = \boxed{2x \cdot \cos(x^2 - 1)}$$

3.) $f(x) = \cos(\ln x)$

$$f'(x) = -\sin(\ln x) \cdot (\ln x)' = \boxed{-\frac{\sin(\ln x)}{x}}$$

4.) $f(x) = \tan(2x - 3x^2)$

$$f'(x) = [1 + \tan^2(2x - 3x^2)] \cdot (2x - 3x^2)' = \boxed{(-6x + 2) \cdot [1 + \tan^2(2x - 3x^2)]}$$

5.) $f(x) = e^{3x^2 + 1}$

$$f'(x) = e^{3x^2 + 1} \cdot (3x^2 + 1)' = \boxed{6x \cdot e^{3x^2 + 1}}$$

6.) $f(x) = 2^{4x + 1}$

$$f'(x) = 2^{4x + 1} \ln 2 \cdot (4x + 1)' = 2^{4x + 1} \cdot \ln 2 \cdot 4 = \boxed{4 \cdot \ln 2 \cdot 2^{4x + 1}}$$

$$7.) f(x) = \cos^2 x$$

8.

$$f'(x) = 2 \cdot \cos x \cdot (\cos x)' = 2 \cdot \cos x \cdot (-\sin x) = -2 \cos x \cdot \sin x = -\sin(2x)$$

$$\sin 2x = 2 \cdot \sin x \cdot \cos x$$

$$8.) f(x) = e^{3x}$$

$$f'(x) = e^{3x} \cdot (3x)' = 3 \cdot e^{3x}$$

$$9.) f(x) = \ln(3x^2 - 6)$$

$$f'(x) = \frac{1}{3x^2 - 6} \cdot (3x^2 - 6)' = \frac{6x}{3x^2 - 6} = \frac{6x}{3(x^2 - 2)} = \frac{2x}{x^2 - 2}$$

$$10.) f(x) = \ln\left(\frac{3x^2 - 1}{2}\right) = \ln(3x^2 - 1) - \ln 2$$

LOGARIT. PROP !!

$$\ln \frac{F(x)}{G(x)} = \ln F(x) - \ln G(x)$$

$$f'(x) = \frac{1}{3x^2 - 1} \cdot (3x^2 - 1)' - 0 = \frac{6x}{3x^2 - 1}$$

$$11.) f(x) = \arctg(3x^2 + 2x)$$

$$f'(x) = \frac{1}{1 + (3x^2 + 2x)^2} \cdot (3x^2 + 2x)' = \frac{6x + 2}{1 + (3x^2 + 2x)^2} = \frac{6x + 2}{9x^4 + 12x^3 + 4x^2 + 1}$$

$$12.) f(x) = \arcsin(x^2)$$

$$f'(x) = \frac{1}{\sqrt{1 - (x^2)^2}} \cdot (x^2)' = \frac{2x}{\sqrt{1 - x^4}}$$

$$13.) f(x) = \arccos(x^3 - 1)$$

$$f'(x) = \frac{-(x^3 - 1)'}{\sqrt{1 - (x^3 - 1)^2}} = \frac{-3x^2}{\sqrt{1 - (x^3 - 1)^2}}$$

$$= \frac{-3x^2}{\sqrt{x^6 + 2x^3}}$$

$$18.) f(x) = \left(\frac{x^2-1}{x+2} \right)^2$$

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$$f'(x) = 2 \cdot \frac{x^2-1}{x+2} \cdot \left(\frac{x^2-1}{x+2} \right)' = 2 \frac{x^2-1}{x+2} \cdot \frac{2x \cdot (x+2) - (x^2-1) \cdot 1}{(x+2)^2}$$

$$= 2 \frac{x^2-1}{x+2} \cdot \frac{2x^2+4x-x^2+1}{(x+2)^2} = \frac{2(x^2-1)(x^2+4x+1)}{(x+2)^3}$$

$$19.) f(x) = \sqrt{x^2-4x} = (x^2-4x)^{1/2}$$

$$f'(x) = \frac{1}{2} (x^2-4x)^{\frac{1}{2}-1} \cdot (x^2-4x)' = \frac{1}{2\sqrt{x^2-4x}} \cdot 2x-4 =$$

$$= \frac{x-2}{\sqrt{x^2-4x}}$$

$$20.) f(x) = \frac{x+1}{(x-2)^2}$$

$$f'(x) = \frac{1 \cdot (x-2)^2 - 2(x-2) \cdot (x+1)}{(x-2)^4} =$$

$$= \frac{(x-2)[(x-2)-2(x+1)]}{(x-2)^4} = \frac{x-2-2x-2}{(x-2)^3} = \frac{x-4}{(x-2)^3}$$

$$21.) f(x) = \frac{(2x+1)^2}{x-1}$$

$$f'(x) = \frac{2(2x+1) \cdot (2x+1)' \cdot (x-1) - (2x+1)^2 \cdot 1}{(x-1)^2} =$$

$$= \frac{2(2x+1) \cdot 2 \cdot (x-1) - (2x+1)^2}{(x-1)^2} = \frac{4(2x^2-2x+x-1) - 4x^2-4x-1}{(x-1)^2}$$

$$= \frac{8x^2-4x-4-4x^2-4x-1}{(x-1)^2} = \frac{4x^2-8x-5}{(x-1)^2}$$

$$14.) f(x) = \sin(3x^2 - 1)^2$$

$$\begin{aligned} f'(x) &= \cos(3x^2 - 1)^2 \cdot (3x^2 - 1)^2' \\ &= \cos(3x^2 - 1)^2 \cdot 2 \cdot (3x^2 - 1) \cdot (3x^2 - 1)' \\ &= \cos(3x^2 - 1)^2 \cdot 2 \cdot (3x^2 - 1) \cdot 6x \\ &= 12x(3x^2 - 1) \cdot \cos(3x^2 - 1)^2 \end{aligned}$$

$$15.) f(x) = \sin^2(3x^2 - 1) = [\sin(3x^2 - 1)]^2$$

$$\begin{aligned} f'(x) &= 2 \cdot \sin(3x^2 - 1) \cdot (\sin(3x^2 - 1))' \\ &= 2 \cdot \sin(3x^2 - 1) \cdot \cos(3x^2 - 1) \cdot (3x^2 - 1)' \\ &= 2 \cdot \sin(3x^2 - 1) \cdot \cos(3x^2 - 1) \cdot 6x \end{aligned}$$

$$= 6x \cdot \sin 2(3x^2 - 1)$$

$$= \boxed{6x \cdot \sin(6x^2 - 2)}$$

$$\sin 2x = 2 \sin x \cdot \cos x$$

$$16.) f(x) = 3^{\cos x}$$

$$f'(x) = 3^{\cos x} \cdot \ln 3 \cdot (\cos x)' = \ln 3 \cdot (-\sin x) \cdot 3^{\cos x}$$

$$17.) f(x) = \ln\left(\frac{x+1}{x-2}\right) \quad \text{Log. Prop. ET!!} \quad \log \frac{f(x)}{g(x)} = \log f(x) - \log g(x)$$

$$f(x) = \ln(x+1) - \ln(x-2)$$

$$f'(x) = \frac{1}{x+1} - \frac{1}{x-2} = \frac{x-2-(x+1)}{(x+1)(x-2)} = \frac{-3}{(x+1)(x-2)}$$

$$22) f(x) = \frac{(3x-1)^2}{2x+1} \quad \text{Kata}$$

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$$f'(x) = \frac{2(3x-1) \cdot (3x-1)' \cdot (2x+1) - (3x-1)^2 \cdot 2}{(2x+1)^2}$$

$$= \frac{2(3x-1) \cdot 3 \cdot (2x+1) - 2(3x-1)^2}{(2x+1)^2} =$$

$$= \frac{6(6x^2 + 3x - 2x - 1) - 2(9x^2 - 6x + 1)}{(2x+1)^2} =$$

$$= \frac{36x^2 + 18x - 12x - 6 - 18x^2 + 12x - 2}{(2x+1)^2} =$$

$$= \frac{18x^2 + 18x - 8}{(2x+1)^2}$$

$$23) f(x) = \frac{e^x}{(x-1)^2}$$

$$f'(x) = \frac{e^x(x-1)^2 - e^x \cdot 2(x-1) \cdot 1}{(x-1)^4} =$$

$$= \frac{e^x [x^2 - 2x + 1 - 2x + 2]}{(x-1)^4} = \frac{e^x (x^2 - 4x + 3)}{(x-1)^4} =$$

$$= \frac{e^x (x-1)(x-3)}{(x-1)^4} =$$

$$= \boxed{\frac{e^x (x-3)}{(x-1)^3}}$$

$$\begin{array}{r|rrr} & x^2 & -4x & +3 \\ 1 & 1 & -4 & 3 \\ & 1 & -3 & 0 \end{array}$$

14. omaldea

12

$$1.) y = \frac{x^3}{3} - \frac{x^2}{4} + \frac{2}{3} \rightarrow y' = \frac{3x^2}{3} - \frac{2x}{4} = \boxed{x^2 - \frac{1}{2}x}$$

$$2.) y = \frac{x^5}{3} - \frac{2}{x^2} + 3 \rightarrow y' = \frac{5x^4}{3} - 2(-2)x^{-3} = \boxed{\frac{5x^4}{3} + \frac{4}{x^3}}$$

$$3.) y = \frac{x^2 - 2x + 1}{5} \rightarrow y' = \boxed{\frac{2x - 2}{5}}$$

$$4.) y = (3x - 2) \cdot e^x \rightarrow y' = 3e^x + (3x - 2) \cdot e^x = \boxed{(3x + 1) \cdot e^x}$$

$$5.) y = \sqrt{x} - \frac{2}{x^3} + \sqrt{5} \rightarrow y' = \frac{1}{2}x^{\frac{1}{2}-1} - 2(-3)x^{-4} = \frac{\sqrt{x}}{2x} + \frac{6}{x^4} = \boxed{\frac{\sqrt{x}}{2x} + \frac{6}{x^4}}$$

$$6.) y = \frac{1}{x} - \frac{\sqrt[3]{x}}{3} + 2x^2 = x^{-1} - \frac{x^{1/3}}{3} + 2x^2$$
$$y' = (-1) \cdot x^{-2} - \frac{1}{3} \cdot \frac{x^{1/3-1}}{3} + 2 \cdot 2 \cdot x = -\frac{1}{x^2} - \frac{1}{9\sqrt[3]{x}^2} + 4x$$
$$= -\frac{1}{x^2} - \frac{1}{9x} + 4x = \boxed{\frac{-9 - x + 36x^3}{9x^2}}$$
$$\frac{1}{9\sqrt[3]{x}^2} \cdot \frac{\sqrt[3]{x}}{\sqrt[3]{x}} = \frac{\sqrt[3]{x}}{9x}$$

$$7.) y = \frac{\sqrt[3]{x}}{x^2} - \frac{x^2-1}{3} = x^{1/3-2} - \frac{x^2-1}{3} = x^{-5/3} - \frac{x^2-1}{3} =$$

$$y' = -\frac{5}{3}x^{-5/3-1} - \frac{1}{3}2x = -\frac{5}{3}x^{-8/3} - \frac{2}{3}x =$$

$$= \frac{-5\sqrt[3]{x}}{3x} - \frac{2}{3}x =$$

$$= \boxed{\frac{-5\sqrt[3]{x} - 2x^2}{3}}$$

$$\frac{-5}{3x} = \frac{1}{\sqrt[3]{x^3}} = \frac{1}{x\sqrt[3]{x^2}} \cdot \frac{\sqrt[3]{x}}{\sqrt[3]{x}} = \frac{\sqrt[3]{x}}{x^3}$$

8.) $y = \frac{x^3 - 3x^4 + 2x + 1}{x} = x^2 - 3x^3 + 2 + x^{-1}$ 13

$$y' = 2x - 9x^2 - 1x^{-2} = \boxed{-9x^2 + 2x - \frac{1}{x^2}}$$

9.) $y = \frac{3}{2x^4} - \frac{2x^2}{3} + \ln 5 = \frac{3}{2}x^{-2} - \frac{2}{3}x^2 + \ln 5$

$$y' = \frac{3}{2} \cdot (-2) \cdot x^{-3} - \frac{2}{3} \cdot 2x + 0 = \frac{-3}{x^3} - \frac{4x}{3} = \boxed{\frac{-9 - 4x^4}{3x^3}}$$

10.) $y = \sqrt{\frac{2}{x^3}} - \frac{x^2}{3} + \sqrt{2} = \sqrt{2} \cdot x^{-3/2} - \frac{1}{3}x^2 + \sqrt{2}$

$$y' = \sqrt{2} \cdot \frac{-3/2}{2} x^{-3/2-1} - \frac{1}{3} \cdot 2x + 0 = \frac{-3\sqrt{2}}{2} \cdot \frac{1}{\sqrt{x^5}} - \frac{2x}{3} = \boxed{\frac{-3\sqrt{2}x}{x^3} - \frac{2x}{3}}$$

11.) $y = \frac{2\sqrt{3}}{4} + \frac{3\ln x}{2} \rightarrow y' = \frac{3}{2x}$ $\frac{1}{\sqrt{x^5}} = \frac{1}{x^2\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{x^3}$

12.) $y = \sin x \cdot \cos x \rightarrow y' = \cos x \cdot \cos x + \sin x \cdot (-\sin x)$
 $= \boxed{\cos^2 x - \sin^2 x = \cos(2x)}$!!!

13.) $y = \frac{e^x}{x^2 - 1}$

$$y' = \frac{e^x(x^2 - 1) - e^x \cdot 2x}{(x^2 - 1)^2} = \frac{e^x(x^2 - 2x - 1)}{(x^2 - 1)^2}$$

14.) $y = \frac{x^2 - 1}{2x + 1} \rightarrow y' = \frac{2x(2x + 1) - (x^2 - 1) \cdot 2}{(2x + 1)^2} =$

$$= \frac{4x^2 + 2x - 2x^2 + 2}{(2x + 1)^2} = \frac{2x^2 + 2x + 2}{(2x + 1)^2}$$

14.) $y = \frac{x^2 - 1}{2x + 1} \rightarrow y' = \frac{2x(2x + 1) - (x^2 - 1) \cdot 2}{(2x + 1)^2} =$

$$= \frac{4x^2 + 2x - 2x^2 + 2}{(2x + 1)^2} = \frac{2x^2 + 2x + 2}{(2x + 1)^2}$$

15.) $y = (x^2 - 1)e^x - \ln x$

$y' = 2x \cdot e^x + (x^2 - 1)e^x - \frac{1}{x} = e^x(x^2 + 2x - 1) - \frac{1}{x}$

16.) $y = 2^x - 3 + \log x \rightarrow y' = 2^x \ln 2 - 3(1 + \log^2 x)$

17.) $y = x^3 \cdot e^x + x^2 \sin x \rightarrow$

$y' = 3x^2 \cdot e^x + x^3 \cdot e^x + 2x \cdot \sin x + x^2 \cos x$

18.) $y = \frac{x-1}{3x-2} \rightarrow y' = \frac{1 \cdot (3x-2) - (x-1) \cdot 3}{(3x-2)^2} =$

$= \frac{3x-2-3x+3}{(3x-2)^2} = \frac{1}{(3x-2)^2}$

19.) $y = \frac{\sqrt{x}}{\sin x} \rightarrow y' = \frac{\frac{1}{2}x^{-1/2} \cdot \sin x - \sqrt{x} \cdot \cos x}{\sin^2 x} =$

$= \frac{\frac{\sin x}{2\sqrt{x}} - \sqrt{x} \cos x}{\sin^2 x} = \frac{\frac{\sin x - 2x \cos x}{2\sqrt{x}}}{\sin^2 x} =$

$= \frac{\sin x - 2x \cos x}{2\sqrt{x} \cdot \sin^2 x}$

20.) $y = (x^2 - 1)^4 \rightarrow y' = 4(x^2 - 1)^3 \cdot 2x = 8x(x^2 - 1)^3$

21.) $y = \left(\frac{x-1}{x+2}\right)^3 \rightarrow y' = 3\left(\frac{x-1}{x+2}\right)^2 \cdot \left(\frac{x-1}{x+2}\right)' =$

$y' = 3\left(\frac{x-1}{x+2}\right)^2 \cdot \frac{1(x+2) - (x-1) \cdot 1}{(x+2)^2} = \frac{3(x-1)^2}{(x+2)^2} \cdot \frac{3}{(x+2)^2} =$

$= \frac{9(x-1)^2}{(x+2)^4}$

$$\begin{aligned}
 22.) \quad y &= \frac{2x-1}{(x+1)^2} \rightarrow y' = \frac{2(x+1)^2 - (2x-1) \cdot 2(x+1)}{(x+1)^4} = \\
 &= \frac{(x+1) [2(x+1) - (2x-1) \cdot 2]}{(x+1)^4} = \frac{2x+2 - 4x+2}{(x+1)^3} = \\
 &= \boxed{\frac{-2x+4}{(x+1)^3}}
 \end{aligned}$$

$$\begin{aligned}
 23.) \quad y &= \frac{x+1}{(x-1)^3} \rightarrow y' = \frac{1 \cdot (x-1)^3 - (x+1) \cdot 3(x-1)^2}{(x-1)^6} = \\
 y' &= \frac{(x-1)^2 [(x-1) - (x+1) \cdot 3]}{(x-1)^6} = \frac{(x-1) - 3x-3}{(x-1)^4} = \boxed{\frac{-2x-4}{(x-1)^4}}
 \end{aligned}$$

$$24.) \quad y = \ln\left(\frac{x-1}{x+4}\right) \quad y = \ln(x-1) - \ln(x+4)$$

$$y' = \frac{1}{x-1} - \frac{1}{x+4} = \frac{(x+4) - (x-1)}{(x-1)(x+4)} = \frac{5}{(x-1)(x+4)}$$

$$25.) \quad y = \cos^2(3x-2) = [\cos(3x-2)]^2$$

$$\begin{aligned}
 y' &= 2 \cdot \cos(3x-2) \cdot (\cos(3x-2))' = \\
 &= 2 \cdot \cos(3x-2) \cdot (-\sin(3x-2)) \cdot (3x-2)' = \\
 &= -2 \sin(3x-2) \cdot \cos(3x-2) \cdot (3x-2)' = \\
 &= - (3x-2) \sin(6x-4)
 \end{aligned}$$

$$\boxed{\sin 2x = 2 \cdot \sin x \cdot \cos x}$$

$$26.) y = \sqrt{\sin x} = (\sin x)^{1/2}$$

$$y' = \frac{1}{2} (\sin x)^{1/2-1} \cdot (\sin x)' = \frac{1}{2} (\sin x)^{-1/2} \cos x =$$

$$= \frac{1}{2} \frac{\cos x}{\sqrt{\sin x}} \cdot \frac{\sqrt{\sin x}}{\sqrt{\sin x}} = \boxed{\frac{\sqrt{\sin x} \cdot \cos x}{2 \sin x}}$$

$$27.) y = \ln(\sin x^2)$$

$$y' = \frac{1}{\sin x^2} \cdot (\sin x^2)' = \frac{1}{\sin x^2} \cos x^2 \cdot (x^2)' =$$

$$= \frac{\cos x^2}{\sin x^2} 2x = \boxed{\frac{2x}{\tan x^2}}$$

$$28.) y = e^{4x-1} \sin(3x^2) \quad f'g + fg'$$

$$y' = (e^{4x-1})' \sin(3x^2) + e^{4x-1} (\sin 3x^2)' =$$

$$= \underbrace{e^{4x-1}}_f \cdot \underbrace{4}_f \cdot \underbrace{\sin(3x^2)}_g + \underbrace{e^{4x-1}}_f \cdot \underbrace{\cos 3x^2 \cdot 6x}_{g'} =$$

$$= e^{4x-1} [4 \cdot \sin(3x^2) + 6x \cdot \cos(3x^2)]$$

$$29.) y = 2^{4x^2-1} \ln(8x)$$

$$y' = (2^{4x^2-1})' \ln(8x) + 2^{4x^2-1} (\ln(8x))'$$

$$= \underbrace{2^{4x^2-1} \ln 2 \cdot 8x}_f \cdot \underbrace{\ln(8x)}_g + \underbrace{2^{4x^2-1}}_f \cdot \underbrace{\frac{1}{8x} \cdot 8}_{g'} =$$

$$= \ln 2 \cdot 8x \cdot 2^{4x^2-1} \ln(8x) + \frac{2^{4x^2-1}}{x}$$

$$30.) y = \frac{(2x+3)^2}{1-x} \rightarrow y' = \frac{2(2x+3)(1-x) - (2x+3)^2(-1)}{(1-x)^2} = \textcircled{17}$$

$$y' = \frac{2(2x - 2x^2 + 3 - 3x) - (4x^2 + 12x + 9)}{(1-x)^2} =$$

$$= \frac{4x - 4x^2 + 6 - 6x - 4x^2 - 12x - 9}{(1-x)^2} = \frac{-8x^2 - 14x - 3}{(1-x)^2}$$

$$31.) y = \operatorname{tg}\left(\frac{2}{x-3}\right) = \operatorname{tg}[2 \cdot (x-3)^{-1}]$$

$$y' = \left[1 + \operatorname{tg}^2\left(\frac{2}{x-3}\right)\right] \cdot (2(x-3)^{-1})' =$$

$$= \left[1 + \operatorname{tg}^2\left(\frac{2}{x-3}\right)\right] \cdot 2 \cdot (-1) \cdot (x-3)^{-2} =$$

$$= \frac{-2}{(x-3)^2} \left[1 + \operatorname{tg}^2 \frac{2}{x-3}\right]$$

$$32.) y = \frac{e^{5x+1}}{x+2}$$

$$y' = \frac{(e^{5x+1})'(x+2) - e^{5x+1}(x+2)'}{(x+2)^2} =$$

$$= \frac{e^{5x+1} \cdot 5 \cdot (x+2) - e^{5x+1} \cdot 1}{(x+2)^2} = \frac{e^{5x+1}(5x+9)}{(x+2)^2}$$

$$33.) y = \frac{\ln^2 x}{x} = \frac{(\ln x)^2}{x}$$

$$y' = \frac{2 \ln x (\ln x)' \cdot x - (\ln x)^2 \cdot x'}{x^2}$$

$$y' = \frac{2 \ln x \cdot \frac{1}{x} \cdot x - (\ln x)^2}{x^2} = \frac{2 \ln x - (\ln x)^2}{x^2} = \frac{\ln x (2 - \ln x)}{x^2}$$

34.) $y = \frac{x \cdot e^x}{x+2}$

$$\begin{aligned} y' &= \frac{(x \cdot e^x)' \cdot (x+2) - x \cdot e^x \cdot (x+2)'}{(x+2)^2} = \\ &= \frac{(1 \cdot e^x + x \cdot e^x) \cdot (x+2) - x \cdot e^x \cdot 1}{(x+2)^2} = \\ &= \frac{\cancel{x \cdot e^x} + 2 \cdot e^x + \cancel{x^2 e^x} + 2x e^x - \cancel{x e^x}}{(x+2)^2} = \\ &= \frac{e^x (x^2 + 2x + 2)}{(x+2)^2} \end{aligned}$$

35.) $y = \frac{\sqrt{x-1}}{3x+4}$

$$\begin{aligned} y' &= \frac{\frac{1}{2\sqrt{x-1}} \cdot (3x+4) - \sqrt{x-1} \cdot 3}{(3x+4)^2} = \frac{\frac{(x-1) \cdot 6}{3x+4 - \sqrt{x-1} \cdot 3 \cdot (2 \cdot \sqrt{x-1})}}{(3x+4)^2} = \\ &= \frac{3x+4 - 6(x-1)}{2\sqrt{x-1} \cdot (3x+4)^2} = \frac{-3x+10}{2\sqrt{x-1} (3x+4)^2} \end{aligned}$$

36.) $y = \sqrt{\frac{3x+1}{x+2}} = \left(\frac{3x+1}{x+2} \right)^{1/2}$

$$\begin{aligned} y' &= \frac{1}{2} \cdot \frac{3x+1}{x+2} \cdot \left(\frac{3x+1}{x+2} \right)' = \frac{1}{2} \cdot \frac{3x+1}{x+2} \cdot \frac{3(x+2) - (3x+1) \cdot 1}{(x+2)^2} = \\ &= \frac{3x+1}{2(x+2)} \cdot \frac{\cancel{3x+6} - \cancel{3x} - 1}{(x+2)^2} = \frac{5(3x+1)}{2(x+2)^3} \end{aligned}$$

37.) $y = \operatorname{arctg}(x^2+2)$

$$y' = \frac{1}{1 + (x^2+2)^2} \cdot (x^2+2)' = \frac{2x}{1 + x^4 + 4x^2 + 4} =$$

$$= \frac{2x}{x^4 + 4x^2 + 5}$$

38.) $y = \sqrt{\operatorname{arctg} x} = (\operatorname{arctg} x)^{1/2}$

$$y' = \frac{1}{2} (\operatorname{arctg} x)^{-1/2} \cdot (\operatorname{arctg} x)' = \frac{1}{2\sqrt{\operatorname{arctg} x}} \cdot \frac{1}{1+x^2}$$

39.) $y = \frac{3 \cdot \operatorname{arcsin}(2x-1)}{4}$

$$y' = \frac{3}{4} \cdot \frac{1}{\sqrt{1-(2x-1)^2}} \cdot (2x-1)' = \frac{3 \cdot 2}{4 \sqrt{1-(2x-1)^2}} =$$

$$= \frac{3}{2\sqrt{-4x^2+4x}}$$

40.) $f(x) = \arccos(\sqrt{x})$

$$y' = \frac{-1}{\sqrt{1-(\sqrt{x})^2}} \cdot (\sqrt{x})' = \frac{-1}{2\sqrt{x} \sqrt{1-x}} =$$

$$= \frac{-1}{2\sqrt{x(1-x)}} = \frac{-1}{2\sqrt{x-x^2}}$$