

# BIR PASS ARIKETAK

(1)

$$\int \frac{\sin 3x}{e^x} dx$$

16c, 20d, 21a, 23c, 22a,  
19b, 17d., 19g

$$I = \int \frac{\sin 3x}{e^x} dx = \int \underset{S}{\sin(3x)} \cdot \underset{E}{e^{-x}} dx$$

2ANIKAKO METODOA

$$I = \begin{cases} u = e^{-x} \rightarrow du = -e^{-x} dx \\ dv = \sin 3x dx \rightarrow v = \frac{1}{3} \int \sin(3x) dx = -\frac{1}{3} \cos(3x) \end{cases}$$

$$I = u \cdot v - \int v \cdot du$$

$$I = e^{-x} \cdot \left(-\frac{1}{3}\right) \cos(3x) - \int -\frac{1}{3} \cos(3x) \cdot (-e^{-x}) dx =$$

$$I = -\frac{1}{3} e^{-x} \cos(3x) - \frac{1}{3} \int \underset{S}{\cos(3x)} \cdot \underset{E}{e^{-x}} dx =$$

$$I_1 = \begin{cases} u = e^{-x} \rightarrow du = -e^{-x} dx \\ dv = \cos 3x dx \rightarrow v = \frac{1}{3} \int \cos 3x dx = \frac{1}{3} \sin(3x) \end{cases}$$

$$I_1 = e^{-x} \cdot \frac{1}{3} \sin(3x) - \int \frac{1}{3} \sin(3x) \cdot (-e^{-x}) dx$$

$$I_1 = \frac{1}{3} e^{-x} \sin(3x) + \frac{1}{3} \int \sin(3x) \cdot e^{-x} dx$$

I.

GULTURA:

$$I = -\frac{1}{3} e^{-x} \cos(3x) - \frac{1}{3} I_1$$

$$I = -\frac{1}{3} e^{-x} \cos(3x) - \frac{1}{3} \left[ \frac{1}{3} e^{-x} \sin(3x) + \frac{1}{3} \int \sin(3x) \cdot e^{-x} dx \right]$$

I

(2)

$$J = -\frac{1}{3} e^{-x} \cos(3x) - \frac{1}{9} e^{-x} \sin(3x) - \frac{1}{9} I$$

$$J + \frac{1}{9} I = -\frac{1}{3} e^{-x} \cos(3x) - \frac{1}{9} e^{-x} \sin(3x)$$

$$\frac{10}{9} I = -\frac{e^{-x}}{9} (3 \cos 3x + \sin 3x)$$

$$J = -\frac{e^{-x}}{10} (3 \cos 3x + \sin 3x)$$

22a]  $\int \frac{x+4}{\sqrt{1-x^2}} dx = \int \underbrace{\frac{x}{\sqrt{1-x^2}}}_{I_1} + \underbrace{\frac{4}{\sqrt{1-x^2}}}_{I_2} dx$

$$I_1 = \int \frac{x}{\sqrt{1-x^2}} dx = \int -\frac{dt}{2\sqrt{t}} = -\frac{1}{2} \int t^{-1/2} dt =$$

Ordekt  
met

$$t = 1-x^2$$

$$dt = -2x dx$$

$$-\frac{dt}{2} = x dx$$

$$= -\frac{1}{2} \frac{t^{1/2}}{1/2} = -\sqrt{t} + K =$$

$$= -\sqrt{1-x^2}$$

$$I_2 = \int \frac{4}{\sqrt{1-x^2}} dx = 4 \arcsin x$$

$$I = I_1 + I_2 = \boxed{-\sqrt{1-x^2} + 4 \arcsin x + K}$$

$$21g) \int \frac{e^x}{1-\sqrt{e^x}} dx = \int \frac{2t dt}{1-t} = \int \frac{2t dt}{1-t} \quad (3)$$

Ordetkopen  
metodisc

$$\left. \begin{array}{l} e^x = t^2 \\ e^x dx = 2t dt \end{array} \right\} \begin{aligned} &= 2 \int \frac{t dt}{1-t} = 2 \int_{-1}^{1} \frac{1}{1-t} dt \\ &= 2 \left[ -t + \ln|1-t| \right] + K. \end{aligned}$$

$\boxed{I = 2 \left[ -\sqrt{e^x} - \ln|\sqrt{e^x} - 1| \right] + K.}$

$$20d) \int \frac{\sin x}{\cos^4 x} dx = \int \frac{-dt}{t^4} = -\frac{t^{-4+1}}{-4+1} + K$$

Ordetkopen

$$t = \cos x$$

$$dt = -\sin x dx$$

$$= \frac{1}{+3t^3} + K$$

$$= \boxed{\frac{1}{3 \cos^3 x} + K}$$

$$19g) \int \frac{\ln \sqrt{x}}{\sqrt{x}} dx = \int \frac{x^{-1/2}}{P} \cdot \ln \sqrt{x} L dx.$$

Zatikoks metodisc

$$u = \ln \sqrt{x} \rightarrow du = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2x} dx$$

$$dv = \frac{-1/2}{x} dx \rightarrow v = \frac{x^{-1/2+1}}{-1/2+1} = \frac{1}{1/2} \sqrt{x}$$

$$u \cdot v - \int v \cdot du = 2\sqrt{x}$$

$$\int \frac{\ln \sqrt{x}}{\sqrt{x}} dx = \ln \sqrt{x} \cdot 2\sqrt{x} - \int 2\sqrt{x} \cdot \frac{1}{2x} dx =$$

$$= 2\sqrt{x} \cdot \ln \sqrt{x} - \int \frac{x^{-1/2}}{x} dx = 2\sqrt{x} \cdot \ln \sqrt{x} - \underline{2\sqrt{x} + K}$$

$$\downarrow \frac{x^{-1/2+1}}{-1/2+1} = \frac{\sqrt{x}}{1/2} = 2\sqrt{x} \quad \boxed{2\sqrt{x} \cdot (\ln \sqrt{x} - 1) + K}$$

19b)  $\int \frac{1 - \sin x}{x + \cos x} dx$

Berechnung →

$$\boxed{\ln |x + \cos x| + C}$$

(4)

23c)  $I = \int \frac{x^4 - 2x - 6}{x^3 + x^2 - 2x} dx$

$$\begin{array}{r} x^4 - 2x - 6 \\ -x^4 - x^3 + 2x^2 \\ \hline -x^3 + 2x^2 - 2x \\ +x^3 + x^2 - 2x \\ \hline 3x^2 - 4x - 6 \end{array}$$

$$\begin{array}{r} x^3 + x^2 - 2x \\ x - 1 \end{array}$$

$$I = \int x - 1 + \frac{3x^2 - 4x - 6}{x^3 + x^2 - 2x} dx.$$

$I_1$

$$I_1 = \int \frac{3x^2 - 4x - 6}{x^3 + x^2 - 2x} dx = \int \frac{3x^2 - 4x - 6}{x(x+2)(x-1)} dx.$$

$$\frac{3x^2 - 4x - 6}{x(x+2)(x-1)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-1}$$

$$3x^2 - 4x - 6 = A(x+2)(x-1) + B \cdot x(x-1) + C(x+2)x$$

$$x=0 \rightarrow -6 = -2A + \cancel{B \cdot 0} + \cancel{C \cdot 0} \rightarrow \boxed{A = 3}$$

$$x=1 \rightarrow -7 = \cancel{A \cdot 0} + \cancel{B \cdot 0} + 3C \rightarrow \boxed{C = -7/3}$$

$$x=-2 \rightarrow 14 = \cancel{A \cdot 0} + 6B + \cancel{C \cdot 0} \rightarrow \boxed{B = 7/3}$$

$$I_1 = \int \frac{3}{x} + \frac{7/3}{x+2} + \frac{-7/3}{x-1} dx$$

$$I_1 = 3 \ln|x| + \frac{7}{3} \ln|x+2| - \frac{7}{3} \ln|x-1|$$

$$I_1 = \ln \left| \frac{x^3(x+2)}{(x-1)} \right|$$

$$I = \int \left( x-1 + \frac{3}{x} + \frac{7/3}{x+2} + \frac{-7/3}{x-1} \right) dx =$$

$$= \boxed{\frac{x^2}{2} - x + \ln \left| \frac{x^3(x+2)}{(x-1)} \right| + C}$$

$$\text{eds} \quad \boxed{\frac{x^2}{2} - x + 3 \ln|x| + \frac{7}{3} \ln|x+2| - \frac{7}{3} \ln|x-1| + C}$$

16c

$$\int \frac{\sqrt{x} dx}{\sqrt[3]{x-1}}$$

ORDEZKAPEN METODUA

Aldapoen bilatzeko  
erabakien mkt  $\rightarrow$  6)

$$\int \frac{\sqrt{u^6} \cdot 6u^5 du}{\sqrt[3]{u^6 - 1}} =$$

$$\left\{ \begin{array}{l} x=u^6 \\ dx=6u^5 du \end{array} \right.$$

$$\int \frac{u^3 \cdot 6u^5 du}{u^2 - 1} = \int \frac{6u^8 du}{u^2 - 1} \rightarrow \text{Tu arazizunak le}  
P(x)ren mslib > Q(x)ren  
baile.$$

$$\begin{array}{r} u^8 \\ u^8 + u^6 \\ \hline -u^6 + u^4 \\ \hline -u^4 + u^2 \\ \hline -u^2 + 1 \\ \hline 1 \end{array}$$

$$I = 6 \int u^6 + u^4 + u^2 + 1 + \frac{1}{u^2 - 1} dx.$$

 $I_1$ Fn arazizunak  
era bakoizan

$$I_1 = \frac{1}{u^2 - 1} = \frac{1}{(u+1)(u-1)} = \frac{A}{u+1} + \frac{B}{u-1}$$

$$1 = A(u-1) + B(u+1)$$

$$u=1 \rightarrow 1 = A \cdot \cancel{0} + B \cdot 2 \rightarrow$$

$$B = 1/2$$

$$u=-1 \rightarrow 1 = -2A + B \cdot \cancel{0} \rightarrow$$

$$A = -1/2$$

$$J_1 = \int -\frac{1/2}{u+1} + \frac{1/2}{u-1} \, du$$

$$I = 6 \int \left( u^6 + u^4 + u^2 + 1 + \frac{-1/2}{u+1} + \frac{1/2}{u-1} \right) du.$$

$$= 6 \cdot \frac{u^7}{7} + \frac{u^5}{5} + \frac{u^3}{3} + u - \frac{1}{2} \ln |u+1| + \frac{1}{2} \ln |u-1| + K$$

Deseğin aldağıtis

$$x = u^6 \implies u = \sqrt[6]{x} = x^{1/6}$$

$$J = 6 \cdot \left[ \frac{x^{7/6}}{7} + \frac{x^{5/6}}{5} + \frac{x^{3/6}}{3} + x - \frac{1}{2} \ln |\sqrt[6]{x+1}| + \frac{1}{2} \ln |\sqrt[6]{x-1}| + K \right]$$

$$= 6 \left[ \frac{x^{6\sqrt[6]{x}}}{7} + \frac{\sqrt[6]{x^5}}{5} + \frac{\sqrt[6]{x^3}}{3} + \sqrt[6]{x} + \frac{1}{2} \ln \left| \frac{\sqrt[6]{x-1}}{\sqrt[6]{x+1}} \right| + K \right]$$