

b) $f'(2) = 0$ (la recta tangente es horizontal, luego tiene pendiente 0).

c) $f'(4)$ es la pendiente de la recta que pasa por $(3, 3)$ y $(4, 0)$:

$$f'(4) = \frac{0-3}{4-3} = \frac{-3}{1} = -3$$

$$\textcircled{9} \quad f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{(a+h)^2 - a^2}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{a^2 + h^2 + 2ah - a^2}{h} = \frac{h^2 + 2ah}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{h(h+2a)}{h} = \lim_{h \rightarrow 0} (h+2a) = 2a$$

$$\textcircled{9} \quad f'(x) = \frac{2x}{5} - \frac{1}{4}$$

$$\textcircled{10} \quad f'(x) = \frac{1}{7} - \frac{\sqrt[7]{7}}{2\sqrt[7]{x}}$$

$$\textcircled{11} \quad f'(x) = \frac{-1}{x^2}$$

$$\textcircled{12} \quad f'(x) = \frac{-6}{x^3}$$

$$\textcircled{13} \quad f'(x) = \frac{-5}{x^4}$$

$$\textcircled{14} \quad f'(x) = \frac{4}{3}\sqrt[3]{x}$$

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$$\textcircled{1} \quad a) f'(x) = 3$$

$$b) f'(x) = \frac{1}{5}$$

$$c) f'(x) = \frac{-1}{2}$$

$$d) f'(x) = 1$$

$$e) f'(x) = 2x$$

$$f) f'(x) = 0$$

$$g) f'(x) = 0$$

$$h) f'(x) = 4x - 1$$

$$i) f'(x) = \frac{-1}{x^2}$$

$$j) f'(x) = \frac{-1}{(x+1)^2}$$

$$k) f'(x) = \frac{-6}{(3x-1)^2}$$

$$\textcircled{15} \quad f'(x) = \sqrt{3} \cdot x^{-3/2} \rightarrow f'(x) = \frac{-3\sqrt{3}}{2} \cdot \frac{1}{\sqrt{x^5}} =$$

$$= \frac{-3\sqrt{3}}{2\sqrt{x^5}}$$

$$\textcircled{16} \quad f'(x) = \frac{3}{2}x^{-5/2} \rightarrow f'(x) = \frac{-15}{4\sqrt{x^7}}$$

$$\textcircled{17} \quad f'(x) = \frac{-2}{x^2} + \frac{1}{2}$$

$$\textcircled{18} \quad f'(x) = \frac{1}{3\sqrt[3]{x}} - \frac{1}{3}$$

$$\textcircled{19} \quad f'(x) = x^{-3/4} \rightarrow f'(x) = \frac{-3}{4\sqrt[4]{x^7}}$$

$$\textcircled{20} \quad f'(x) = \frac{\sqrt{3}}{\sqrt{x^5}} = \sqrt{3} \cdot x^{-5/2} \rightarrow f'(x) = \frac{-5\sqrt{3}}{2\sqrt{x^7}}$$

$$\textcircled{21} \quad f'(x) = 2x^{-1/2} - 3x^{-2} + x^{-1} \rightarrow$$

$$\rightarrow f'(x) = \frac{-1}{\sqrt{x^3}} + \frac{6}{x^3} - \frac{1}{x^2}$$

$$\textcircled{22} \quad f'(x) = 1 - \frac{2}{x^3}$$

$$\textcircled{23} \quad f'(x) = \frac{2x}{3} + \frac{6}{x^3}$$

$$\textcircled{24} \quad f'(x) = \frac{x^3}{3} - 4x^{1/2} + 2x^{-3} + x^{2/5} \rightarrow$$

$$\rightarrow f'(x) = x^2 - \frac{2}{\sqrt{x}} - \frac{6}{x^4} + \frac{5\sqrt{x^3}}{2}$$

$$\textcircled{25} \quad f'(x) = 1 - \frac{1}{x^2}$$

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$$\textcircled{1} \quad f'(x) = 2$$

$$\textcircled{2} \quad f'(x) = \frac{3}{4}$$

$$\textcircled{3} \quad f'(x) = 0$$

$$\textcircled{4} \quad f'(x) = \frac{1}{2}$$

$$\textcircled{5} \quad f'(x) = 3x^2 - 6x$$

$$\textcircled{6} \quad f'(x) = 3x^4 - \frac{4}{3}$$

$$\textcircled{7} \quad f'(x) = 0$$

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$$\textcircled{8} \quad f'(x) = \frac{4}{3} \left(2x - \frac{4}{3} \right)$$

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$$\textcircled{1} \quad f'(x) = 3 \cos x + 2 \sin x$$

$$2 \quad f'(x) = 4(1 + \operatorname{tg}^2 x) + e^x = 4 + 4 \operatorname{tg}^2 x + e^x$$

$$4 \quad f'(x) = 1 \cdot e^x + x \cdot e^x = e^x + x e^x$$

$$5 \quad f'(x) = 2x \cdot \operatorname{sen} x + (x^2 + 1) \cdot \cos x = \\ = 2x \operatorname{sen} x + x^2 \cos x + \cos x$$

$$6 \quad f'(x) = 2^x \cdot \ln 2 \cdot \operatorname{tg} x + 2^x \cdot (1 + \operatorname{tg}^2 x)$$

$$7 \quad f'(x) = 2x - \left(\frac{1}{3} \cdot e^x + \frac{x}{3} \cdot e^x \right) = \\ = 2x - \frac{1}{3} \cdot e^x - \frac{x}{3} \cdot e^x$$

$$8 \quad f'(x) = (3x^2 - 2) \cdot \cos x + (x^3 - 2x + 1) \cdot (-\operatorname{sen} x) = \\ = (3x^2 - 2) \cdot \cos x - (x^3 - 2x + 1) \cdot \operatorname{sen} x$$

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$$9 \quad f'(x) = 3^x \cdot \ln 3 + \frac{1}{x} - \frac{1}{x^2}$$

$$10 \quad f'(x) = 2^x \cdot \ln 2 + \frac{1}{x} - \frac{1}{\ln 2}$$

$$11 \quad f'(x) = 2x \cdot e^x + x^2 \cdot e^x + 2 \cdot \ln x + 2x \cdot \frac{1}{x} = \\ = 2x e^x + x^2 e^x + 2 \ln x + 2$$

$$12 \quad f'(x) = \frac{1}{2\sqrt{x}} \operatorname{sen} x + \sqrt{x} \cos x = \\ = \frac{\operatorname{sen} x}{2\sqrt{x}} + \sqrt{x} \cos x = \frac{\operatorname{sen} x + 2x \cos x}{2\sqrt{x}}$$

$$13 \quad f(x) = \frac{(x+1) \cdot (x-1)}{2(x+1)} = \frac{(x-1)}{2} \rightarrow$$

$$\rightarrow f'(x) = \frac{1}{2}$$

Otra forma:

$$f'(x) = \frac{2x(2x+2) - (x^2-1) \cdot 2}{(2x+2)^2} = \\ = \frac{2x^2 + 4x + 2}{(2x+2)^2} = \frac{2(x+1)^2}{4(x+1)^2} = \frac{1}{2}$$

$$14 \quad f'(x) = \frac{-3}{(x-2)^2}$$

$$15 \quad f'(x) = \frac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2}$$

$$16 \quad f'(x) = \frac{e^x - e^{-x}}{2}$$

$$17 \quad f'(x) = \frac{-2x}{(x^2+1)^2}$$

$$18 \quad f'(x) = \frac{3x^2(x+2) - x^3 \cdot 1}{(x+2)^2} = \frac{2x^3 + 6x^2}{(x+2)^2}$$

$$19 \quad f'(x) = \frac{2(3x+2) - (2x-1) \cdot 3}{(3x+2)^2} = \frac{7}{(3x+2)^2}$$

$$20 \quad f'(x) = \frac{2x(x^2-1) - x^2 \cdot 2x}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2}$$

$$21 \quad f'(x) = \frac{\frac{1}{2\sqrt{x}}(x+2) - \sqrt{x} \cdot 1}{(x+2)^2} = \frac{-x+2}{2\sqrt{x} \cdot (x+2)^2}$$

$$22 \quad f'(x) = 2x \cdot \sqrt{x} + (x^2-1) \cdot \frac{1}{2\sqrt{x}} = \\ = \frac{4x^2+x^2-1}{2\sqrt{x}} = \frac{5x^2-1}{2\sqrt{x}}$$

$$23 \quad f'(x) = \frac{3}{\sqrt{1-x^2}}$$

$$24 \quad f'(x) = \frac{-2}{\sqrt{1-x^2}} + e^x$$

$$25 \quad f'(x) = \frac{5}{1+x^2}$$

$$26 \quad f'(x) = \frac{1 \cdot e^x + x e^x - \frac{1}{x}}{2} = \frac{x e^x + x^2 e^x - 1}{2x}$$

$$27 \quad f'(x) = 3^x \cdot \ln 3 \cdot \operatorname{sen} x + 3^x \cdot \cos x - \left(\log_2 x + x \cdot \frac{1}{x} \cdot \frac{1}{\ln 2} \right) = 3^x \ln 3 \cdot \operatorname{sen} x + \\ + 3^x \cos x - \log_2 x - \frac{1}{\ln 2}$$

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$$1 \quad f'(x) = 6(x^2+5)^5(2x) = 12x(x^2+5)^5$$

$$2 \quad f'(x) = \cos(x^2-1) \cdot 2x = 2x \cos(x^2-1)$$

$$3 \quad f'(x) = -\operatorname{sen}(\ln x) \cdot \frac{1}{x} = \frac{-1}{x} \operatorname{sen}(\ln x)$$

$$4 \quad f'(x) = [1 + \operatorname{tg}^2(2x-3x^2)] \cdot (2-6x)$$

$$5 \quad f'(x) = e^{3x^2+1} \cdot 6x = 6x \cdot e^{3x^2+1}$$

$$6 \quad f'(x) = 2^{4x+1} \cdot \ln 2 \cdot 4 = (4 \cdot \ln 2) \cdot 2^{4x+1}$$

$$7 \quad f'(x) = 2 \cos x (-\operatorname{sen} x) = -2 \cos x \operatorname{sen} x$$

$$8 \quad f'(x) = e^{3x} \cdot 3 = 3 \cdot e^{3x}$$

$$9 \quad f'(x) = \frac{6x}{3x^2-6}$$

$$\textcircled{10} \quad f'(x) = \frac{3x}{\frac{3x^2 - 1}{2}} = \frac{6x}{3x^2 - 1}$$

$$\textcircled{11} \quad f'(x) = \frac{6x + 2}{1 + (3x^2 + 2x)^2} = \frac{6x + 2}{9x^4 + 12x^3 + 4x^2 + 1}$$

$$\textcircled{12} \quad f'(x) = \frac{2x}{\sqrt{1 - x^4}}$$

$$\textcircled{13} \quad f'(x) = \frac{-3x^2}{\sqrt{1 - (x^3 - 1)^2}} = \frac{-3x^2}{\sqrt{2x^3 - x^6}} = \\ = \frac{-3x^2}{x\sqrt{2x - x^4}} = \frac{-3x}{\sqrt{2x - x^4}}$$

$$\textcircled{14} \quad f'(x) = (\cos(3x^2 - 1)^2) \cdot 2(3x^2 - 1) \cdot 6x = \\ = 12x(\cos(3x^2 - 1)) \cos(3x^2 - 1)^2$$

$$\textcircled{15} \quad f'(x) = 2 \operatorname{sen}(3x^2 - 1) \cdot \cos(3x^2 - 1) \cdot 6x = \\ = 12x \operatorname{sen}(3x^2 - 1) \cos(3x^2 - 1)$$

$$\textcircled{16} \quad f'(x) = 3^{\cos x} \cdot \ln 3 \cdot (-\operatorname{sen} x) = -\operatorname{sen} x \cdot \ln 3 \cdot 3^{\cos x}$$

$$\textcircled{17} \quad f'(x) = \frac{1}{\frac{x+1}{x-2}} \cdot \frac{1 \cdot (x-2) - (x+1) \cdot 1}{(x-2)^2} = \\ = \frac{(x-2)}{(x+1)} \cdot \frac{-3}{(x-2)^2} = \frac{-3}{(x+1)(x-2)} = \\ = \frac{-3}{x^2 - x - 2}$$

$$\textcircled{18} \quad f'(x) = 2 \left(\frac{x^2 - 1}{x+2} \right) \cdot \frac{2x(x+2) - (x^2 - 1) \cdot 1}{(x+2)^2} = \\ = \frac{2(x^2 - 1)(x^2 + 4x + 1)}{(x+2)^3}$$

$$\textcircled{19} \quad f'(x) = \frac{2x - 4}{2\sqrt{x^2 - 4x}} = \frac{x - 2}{\sqrt{x^2 - 4x}}$$

$$\textcircled{20} \quad f'(x) = \frac{1 \cdot (x-2)^2 - (x+1) \cdot 2(x-2)}{(x-2)^4} = \\ = \frac{(x-2)[1 - 2(x+1)]}{(x-2)^4} = \frac{2x-1}{(x-2)^3}$$

$$\textcircled{21} \quad f'(x) = \frac{2(2x+1) \cdot 2(x-1) - (2x+1)^2 \cdot 1}{(x-1)^2} = \\ = \frac{(2x+1)(2x-5)}{(x-1)^2} = \frac{4x^2 - 8x - 5}{(x-1)^2}$$

$$\textcircled{22} \quad f'(x) = \frac{2(3x-1) \cdot 3(2x+1) - (3x-1)^2 \cdot 2}{(2x+1)^2} = \\ = \frac{(3x-1)(6x+8)}{(2x+1)^2} = \frac{18x^2 + 18x - 8}{(2x+1)^2}$$

$$\textcircled{23} \quad f'(x) = \frac{e^x(x-1)^2 - e^x \cdot 2(x-1)}{(x-1)^4} = \\ = \frac{e^x(x-1)(x-1-2)}{(x-1)^4} = \frac{e^x(x-3)}{(x-1)^3}$$

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$$\textcircled{1} \quad f'(x) = x^2 - \frac{x}{2}$$

$$\textcircled{2} \quad f'(x) = \frac{5x^4}{3} + \frac{4}{x^3}$$

$$\textcircled{3} \quad f'(x) = \frac{2x-2}{5}$$

$$\textcircled{4} \quad f'(x) = 3e^x + (3x-2)e^x = (3x+1)e^x$$

$$\textcircled{5} \quad f'(x) = \frac{1}{2\sqrt{x}} + \frac{6}{x^4}$$

$$\textcircled{6} \quad f'(x) = \frac{-1}{x^2} - \frac{1}{9\sqrt[3]{x^4}} + 4x$$

$$\textcircled{7} \quad f'(x) = \frac{-5}{3\sqrt[3]{x^8}} - \frac{2x}{3}$$

$$\textcircled{8} \quad f'(x) = x^2 - 3x^3 + 2 + \frac{1}{x}$$

$$f'(x) = 2x - 9x^2 - \frac{1}{x^2}$$

$$\textcircled{9} \quad f'(x) = \frac{-3}{x^3} - \frac{4x}{3}$$

$$\textcircled{10} \quad f'(x) = \frac{-3\sqrt[3]{2}}{2\sqrt[3]{x^5}} - \frac{2x}{3}$$

$$\textcircled{11} \quad f'(x) = \frac{3}{2x}$$

$$\textcircled{12} \quad f'(x) = \cos^2 x - \operatorname{sen}^2 x$$

$$\textcircled{13} \quad f'(x) = \frac{e^x(x^2 - 1) - e^x \cdot 2x}{(x^2 - 1)^2} = \frac{(x^2 - 2x - 1)e^x}{(x^2 - 1)^2}$$

$$\textcircled{14} \quad f'(x) = \frac{2x(2x+1) - (x^2 - 1) \cdot 2}{(2x+1)^2} = \frac{2x^2 + 2x + 2}{(2x+1)^2}$$

$$\textcircled{15} \quad f'(x) = 2x e^x + (x^2 - 1)e^x - \frac{1}{x} = \\ = (x^2 + 2x - 1)e^x - \frac{1}{x}$$

$$\textcircled{16} \quad f'(x) = 2^x \cdot \ln 2 - 3(1 + \operatorname{tg}^2 x) = \\ = 2^x \cdot \ln 2 - 3 - 3 \operatorname{tg}^2 x$$

$$\textcircled{17} \quad f'(x) = 3x^2 e^x + x^3 e^x + 2x \operatorname{sen} x + x^2 \cos x$$

$$\textcircled{18} \quad f'(x) = \frac{1 \cdot (3x-2) - (x-1) \cdot 3}{(3x-2)^2} = \frac{1}{(3x-2)^2}$$

$$\begin{aligned} \textcolor{red}{\diamond} f'(x) &= \frac{\frac{1}{2\sqrt{x}} \sin x - \sqrt{x} \cos x}{\sin^2 x} = \\ &= \frac{\sin x - 2x \cos x}{2\sqrt{x} \sin^2 x} \end{aligned}$$

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$$\textcolor{red}{\diamond} f'(x) = 4(x^2 - 1)^3 \cdot 2x = 8x(x^2 - 1)^3$$

$$\textcolor{red}{\diamond} f'(x) = 3\left(\frac{x-1}{x+2}\right)^2 \cdot \frac{3}{(x+2)^2} = \frac{9(x-1)^2}{(x+2)^4}$$

$$\begin{aligned} \textcolor{red}{\diamond} f'(x) &= \frac{2(x+1)^2 - (2x-1) \cdot 2(x+1)}{(x+1)^4} = \\ &= \frac{(x+1) \cdot (2x+2 - 4x+2)}{(x+1)^4} = \frac{-2x+4}{(x+1)^3} \end{aligned}$$

$$\begin{aligned} \textcolor{red}{\diamond} f'(x) &= \frac{(x-1)^3 - (x+1) \cdot 3(x-1)^2}{(x-1)^6} = \\ &= \frac{(x-1)^2(x-1-3x-3)}{(x-1)^6} = \frac{-2x-4}{(x-1)^4} \end{aligned}$$

$$\begin{aligned} \textcolor{red}{\diamond} f'(x) &= \frac{1}{\frac{x-1}{x+4}} \cdot \frac{5}{(x+4)^2} = \frac{(x+4)}{(x-1)} \cdot \frac{5}{(x+4)^2} = \\ &= \frac{5}{(x-1)(x+4)} = \frac{5}{x^2 + 3x - 4} \end{aligned}$$

$$\begin{aligned} \textcolor{red}{\diamond} f'(x) &= 2 \cos(3x-2) \cdot (-\sin(3x-2)) \cdot 3 = \\ &= -6 \cos(3x-2) \sin(3x-2) \end{aligned}$$

$$\textcolor{red}{\diamond} f'(x) = \frac{\cos x}{2\sqrt{\sin x}}$$

$$\textcolor{red}{\diamond} f'(x) = \frac{2x \cos x^2}{\sin x^2}$$

$$\begin{aligned} \textcolor{red}{\diamond} f'(x) &= e^{4x-1} \cdot 4 \sin(3x^2) + e^{4x-1} \cdot \cos(3x^2) \cdot 6x = \\ &= 4e^{4x-1} \sin(3x^2) + 6x e^{4x-1} \cos(3x^2) \end{aligned}$$

$$\begin{aligned} \textcolor{red}{\diamond} f'(x) &= 2^{4x^2-1} \cdot \ln 2 \cdot 8x \cdot \ln(8x) + 2^{4x^2-1} \cdot \frac{8}{8x} = \\ &= 2^{4x^2-1} \cdot \ln 2 \cdot 8x \cdot \ln(8x) + \frac{2^{4x^2-1}}{x} \end{aligned}$$

$$\begin{aligned} \textcolor{red}{\diamond} f'(x) &= \frac{2(2x+3) \cdot 2(1-x) - (2x+3)^2 \cdot (-1)}{(1-x)^2} = \\ &= \frac{(2x+3)(-2x+7)}{(1-x)^2} = \frac{-4x^2 + 8x + 21}{(1-x)^2} \end{aligned}$$

$$\textcolor{red}{\diamond} f'(x) = \left[1 + \tan^2\left(\frac{2}{x-3}\right)\right] \cdot \frac{-2}{(x-3)^2}$$

$$\begin{aligned} \textcolor{red}{\diamond} f'(x) &= \frac{e^{5x+1} \cdot 5 \cdot (x+2) - e^{5x+1} \cdot 1}{(x+2)^2} = \\ &= \frac{e^{5x+1}(5x+9)}{(x+2)^2} \end{aligned}$$

$$\textcolor{red}{\diamond} f'(x) = \frac{2 \ln x \cdot \frac{1}{x} \cdot x - \ln^2 x \cdot 1}{x^2} = \frac{2 \ln x - \ln^2 x}{x^2}$$

$$\textcolor{red}{\diamond} f'(x) = \frac{(e^x + x e^x)(x+2) - x e^x}{(x+2)^2} =$$

$$= \frac{(x^2 + 2x + 2) e^x}{(x+2)^2}$$

$$\begin{aligned} \textcolor{red}{\diamond} f'(x) &= \frac{\frac{1}{2\sqrt{x-1}} \cdot (3x+4) - \sqrt{x-1} \cdot 3}{(3x+4)^2} = \\ &= \frac{-3x+2}{2\sqrt{x-1}(3x+4)^2} \end{aligned}$$

$$\begin{aligned} \textcolor{red}{\diamond} f'(x) &= \frac{1}{2\sqrt{\frac{3x+1}{x+2}}} \cdot \frac{3(x+2) - (3x+1) \cdot 1}{(x+2)^2} = \\ &= \frac{\sqrt{x+2}}{2\sqrt{3x+1}} \cdot \frac{5}{(x+2)^2} \end{aligned}$$

$$\textcolor{red}{\diamond} f'(x) = \frac{2x}{1 + (x^2 + 2)^2} = \frac{2x}{x^4 + 4x^2 + 5}$$

$$\textcolor{red}{\diamond} f'(x) = \frac{1}{2\sqrt{\arctg x}} \cdot \frac{1}{1+x^2} = \frac{1}{2(1+x^2)\sqrt{\arctg x}}$$

$$\textcolor{red}{\diamond} f'(x) = \frac{3}{4} \cdot \frac{2}{\sqrt{1-(2x-1)^2}} = \frac{3}{2\sqrt{-4x^2 + 4x}}$$

$$\begin{aligned} \textcolor{red}{\diamond} f'(x) &= \frac{-1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}} = \frac{-1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} = \\ &= \frac{-1}{2\sqrt{x-x^2}} \end{aligned}$$

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$$\begin{aligned} \textcolor{red}{1} \quad f'(x) &= \frac{-x^2 + 4x - 10}{(x-2)^2}, \quad f'(1) = \frac{-15}{9}; \\ f'(3) &= -7; \quad f'(5) = \frac{-15}{9} \end{aligned}$$

$$\begin{aligned} \textcolor{red}{2} \quad f'(x) &= \frac{4}{3} \left(\frac{x}{3} + 1\right)^3, \quad f'(-4) = \frac{-4}{81}; \quad f'(-3) = 0; \\ f'(0) &= \frac{4}{3}; \quad f'(1) = \frac{256}{81} \end{aligned}$$

$$\begin{aligned} \textcolor{red}{3} \quad f'(x) &= \frac{e^x - e^{-x}}{2}, \quad f'(-2) \approx 3,63; \quad f'(0) = 0; \\ f'(3) &= 10,02 \end{aligned}$$