# Izendatzaileak ERRO IRUDIKARIAK ditu

346.orr

**5**

$$\int \frac{dx}{3x^2+3} = \frac{1}{3}\int \frac{dx}{x^2+1} = \boxed{\frac{1}{3}\,\text{arctg}\,x + k}$$

b) $$\int \frac{dx}{9x^2+3} = \frac{1}{3}\int \frac{dx}{3x^2+1} = \frac{1}{3}\frac{1}{\sqrt{3}}\int \frac{\sqrt{3}\,dx}{(\sqrt{3}x)^2+1} =$$

$$\boxed{\frac{1}{3\sqrt{3}}\,\text{arcTg}\,(\sqrt{3}x) + k}$$

c) $$\int \frac{dx}{6x^2+3} = \int \frac{dx}{3(2x^2+1)} = \frac{1}{3}\frac{1}{\sqrt{2}}\int \frac{\sqrt{2}\,dx}{(\sqrt{2}x)^2+1}$$

$$= \boxed{\frac{1}{3\sqrt{2}}\,\text{arctg}\,(\sqrt{2}x) + k}$$

d) $$\int \frac{dx}{7x^2+11} = \int \frac{dx}{11\left(\frac{7x^2}{11}+1\right)} = \frac{1}{11}\int \frac{dx}{\left(\sqrt{\frac{7}{11}}x\right)^2+1} =$$

$$= \frac{1}{11}\cdot\frac{1}{\sqrt{7/11}}\int \frac{\sqrt{7/11}\,dx}{\left(\sqrt{\frac{7}{11}}x\right)^2+1} = \boxed{\frac{1}{\sqrt{77}}\,\text{arctg}\left(\sqrt{\frac{7}{11}}x\right) + k}$$

**6)** $$\int \frac{dx}{x^2-4x+5} = \int \frac{dx}{x^2-4x+4+1} = \int \frac{dx}{(x-2)^2+1} =$$

$$\underbrace{x^2-4x+4}_{(x-2)^2}+1$$

$$= \boxed{\text{arctg}\,(x-2) + k}$$

b) $$\int \frac{dx}{x^2-4x+10} = \int \frac{dx}{x^2-4x+4+6} = \int \frac{dx}{(x-2)^2+6} =$$

$$= \frac{1}{1/\sqrt{6}}\int \frac{1/\sqrt{6}\,dx}{6\left(\frac{x-2}{\sqrt{6}}\right)^2+1} = \boxed{\frac{1}{\sqrt{6}}\,\text{arctg}\left(\frac{x-2}{\sqrt{6}}\right) + k}$$

# IZENDATZAILEAK ERRO IRUDIKARIAK DITU.

· Hirugarren kasua: $\int \frac{Mx+N}{ax^2+bx+c}$.

7 a) $\int \frac{\overset{2x-4}{x-2}}{x^2-4x+10}\,dx = \frac{1}{2}\int \frac{2(x-2)}{x^2-4x+10}\,dx =$

$= \frac{1}{2}\ln|x^2-4x+10| + K.$

b) $\int \frac{\overset{2x-4}{x-11}}{x^2-4x+10}\,dx = \frac{1}{2}\int \frac{\overset{2x-22}{2(x-11)}}{x^2-4x+10}\,dx$

$= \frac{1}{2}\int \frac{\boxed{2x-22+18}-18}{x^2-4x+10} = \frac{1}{2}\int\left(\frac{2x-4}{x^2-4x+10} - \underbrace{\frac{18}{x^2-4x+10}}_{I_2}\right)dx$

$x^2-4x+10=0$

$x = \frac{4\pm\sqrt{4^2-4\cdot1\cdot10}}{2} = \frac{4\pm\sqrt{\ominus}}{2}$ Ez dauko erro errealik.

$I_2 = \int \frac{1}{x^2-4x+10}\,dx = \int \frac{1}{\underbrace{x^2-4x+4}+6}\,dx = \int \frac{dx}{(x-2)^2+6} =$

$= \frac{1}{6}\sqrt{6}\int \frac{1/\sqrt{6}\,dx}{\left(\frac{x-2}{\sqrt{6}}\right)^2+1} = \frac{1}{\sqrt{6}}\operatorname{arctg}\left(\frac{x-2}{\sqrt{6}}\right)$

$I = \frac{1}{2}\left[\ln|x^2-4x+10| - 18\frac{1}{\sqrt{6}}\operatorname{arctg}\frac{x-2}{\sqrt{6}}\right] + K$

$= \frac{1}{2}\ln|x^2-4x+10| - \underbrace{\frac{9}{\sqrt{6}}}_{\frac{9}{\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}} = \frac{9\sqrt{6}}{6} = \frac{3\sqrt{6}}{2}}\operatorname{arctg}\left(\frac{x-2}{\sqrt{6}}\right) + K$

d) $\frac{1}{2/5}\int \frac{2/5\,(5x+12)}{x^2+3x+10}\,dx = \frac{5}{2}\int \frac{2x+24/5}{x^2+3x+10} =$

$$= \frac{5}{2}\int \frac{2x+3\overbrace{-3+24/5}^{9/5}}{x^2+3x+10} = \frac{5}{2}\underbrace{\int \frac{2x+3}{x^2+3x+10}}_{I_1} - \frac{\cancel{5}\,9}{2\,\cancel{5}}\underbrace{\int \frac{1}{x^2+3x+10}\,dx}_{I_2}$$

$$I_1 = \int \frac{2x+3}{x^2+3x+10}\,dx = \ln|x^2+3x+10| + k$$

$$I_2 = \int \frac{1\,dx}{x^2+3x+10} = \int \frac{dx}{\underbrace{x^2+2\cdot\frac{3x}{2}+\frac{9}{4}}_{(x+3/2)^2}\underbrace{-\frac{9}{4}+10}_{31/4}} =$$

$$= \int \frac{dx}{(x+3/2)^2+31/4} =$$

$$\frac{2(x+3/2)}{\sqrt{31}} = \frac{2x+3}{\sqrt{31}}$$

$$= \frac{1}{\frac{2}{\sqrt{31}}\cdot 31/4}\int \frac{\frac{2}{\sqrt{31}}\,dx}{1+\left(\frac{x+3/2}{\sqrt{31/4}}\right)^2}$$

$$= \frac{2}{\sqrt{31}}\,\text{arctg}\,\frac{2x+3}{\sqrt{31}} + k$$

$$\boxed{I = \frac{5}{2}\ln|x^2+3x+10| - \frac{9}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{31}}\,\text{arctg}\,\frac{2x+3}{\sqrt{31}} + k}$$

$$I_2 = \frac{1}{(-2)}\int \frac{\overset{2x+2}{(-2)(-x+1)}}{x^2+2x+3}\,dx = -\frac{1}{2}\int \frac{2x-2+2-2}{x^2+2x+3}\,dx$$

$$= -\frac{1}{2}\int \left(\frac{2x+2}{x^2+2x+3} - \frac{4}{x^2+2x+3}\right)dx =$$

$$= -\frac{1}{2}\ln|x^2+2x+3| - 2\underbrace{\int \frac{1}{x^2+2x+3}\,dx}_{I_3}$$

$$I_3 = \int \frac{1}{x^2+2x+3}\,dx = \int \frac{1}{\underbrace{(x^2+2x+1)}_{(x+1)^2}+2}\,dx =$$

$$= \frac{1}{1/\sqrt{2}}\int \frac{1}{2}\,\frac{1/\sqrt{2}\,dx}{\left(\frac{x+1}{\sqrt{2}}\right)^2+1} = \frac{1}{\sqrt{2}}\operatorname{arctg}\left(\frac{x+1}{\sqrt{2}}\right)$$

$$\boxed{I = \frac{x^2}{2} + \ln|x| - \frac{1}{2}\ln|x^2+2x+3| - \frac{2}{\sqrt{2}}\operatorname{arctg}\left(\frac{x+1}{\sqrt{2}}\right) + k}$$