337) ① a) $\int \underbrace{\cos^5 x}_{f(x)} \underbrace{(-\sin x)}_{f'} dx = \int t^5 dt = \frac{t^6}{6} + k =$

$t = \cos x$
$dt = -\sin x \, dx$

$$= \frac{\cos^6 x}{6} + k.$$

b) $\int \sqrt[3]{\cos^2 x} \, (-\sin x) dx = \int \underbrace{(\cos x)^{2/3}}_{t} \underbrace{(-\sin x)}_{f'} dx =$

$t = \cos x$
$dt = -\sin x \, dx$

$$= \int t^{2/3} dt = \frac{t^{2/3+1}}{2/3+1} + k = \frac{t^{5/3}}{5/3} + k =$$

$$= \frac{3 t^{5/3}}{5} + k = \frac{3 (\cos x)^{5/3}}{5} = \frac{3}{5} \sqrt[3]{\cos^5 x} + k$$

c) $\int e^{\underset{t}{\cos x}} \underbrace{\sin x}_{f'} dx = \int e^t (-dt) = -\int e^t dt =$

$t = \cos x$
$dt = -\sin x \, dx$
$-dt = \sin x \cdot dx$

$$= -e^t + k = \boxed{-e^{\cos x} + k.}$$

d) $\int e^{\underbrace{\overset{t}{x^3+x^2}}} \underbrace{(3x^2+2x)}_{f'} dx = \int e^t dt = e^t + k =$

$t = x^3 + x^2$
$dt = (3x^2 + 2x) \cdot dx$

$$= e^{x^3+x^2} + k$$

e) $\int tg \underbrace{x^2}_{f} \cdot \underbrace{2x}_{f'} dx = \int tg \, t \cdot dt = \int \frac{\sin t}{\cos t} dt =$

$t = x^2$
$dt = 2x \cdot dx$

$$= \ln|\cos t| + k = \boxed{\ln|\cos x^2| + k}$$

f) $\int \frac{\overset{f'}{\boxed{3x^2}}}{\underset{f=x^3}{\boxed{1+x^6}}} dx = \int \frac{dt}{1+t^2} = arctg \, t + k =$

$t =$
$dt = \quad 3x^2 \, dx$

$$= arctg \, x^3 + k$$

g) $\int \dfrac{-e^{-x}}{\sqrt{1-e^{-2x}}}\, dx = \int \dfrac{-e^{-x}}{\sqrt{1-(e^{-x})^2}}\, dx = \int \dfrac{dt}{\sqrt{1-t^2}}$

$t = e^{-x}$
$dt = -e^{-x}dx.$

$= \arcsin t + k = \arcsin(e^{-x}) + k$

* TAULA

h) $\int \ln(x^2+1)\cdot \overbrace{2x}^{f'}\, dx = \int \ln t \cdot dt =$

$\underbrace{}_{f}$

$t = x^2 + 1$
$dt = 2x\, dx$

$= (x^2+1)\cdot \ln(x^2+1) - (x^2+1) + k$

Ariketa hau aurrerago ebatziko
do **ZATIKAKO METODDA** aplikatuz

7) $\int \sqrt[3]{(x^4+5x)^2}\; (4x^3+5)\, dx = \int \sqrt[3]{t^2}\, dt = \int t^{2/3}\, dt$

$\underbrace{}_{t}\quad \underbrace{}_{f'\;dt}$

$t = x^4 + 5x$
$dt = (4x^3+5)\cdot dx$

$= \dfrac{t^{2/3+1}}{2/3+1} + k = \dfrac{t^{5/3}}{5/3} + k =$

$= \dfrac{3}{5}\sqrt[3]{(x^4+5x)^5} + k.$

ORDEZKAPEN METODOA

338 2.ariketa

a) $\int \sqrt{x^3-3x^2+5} \ (x^2-2x) \, dx$.

$\qquad$ $\begin{cases} t = x^3-3x^2+5 \\ dt = (3x^2-6x)\,dx. \\ \dfrac{dt}{3} = (x^2-2x)\,dx \end{cases}$

$= \int \sqrt{t} \cdot \dfrac{dt}{3} = \dfrac{1}{3}\int t^{1/2} dt = \dfrac{1}{3}\dfrac{t^{1/2+1}}{1/2+1}+k$

$= \boxed{\dfrac{2}{9}\sqrt{(x^3-3x^2+5)^3}+k.}$

b) $\int \dfrac{1}{\sqrt{1-e^{2\sqrt{x}}}} \ \dfrac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$

$\underbrace{\quad}_{(e^{\sqrt{x}})^2}$

$\qquad$ $\begin{cases} t = e^{\sqrt{x}} \\ dt = e^{\sqrt{x}} \cdot \dfrac{1}{2\sqrt{x}} \, dx \\ 2\,dt = e^{\sqrt{x}} \cdot \dfrac{1}{\sqrt{x}} \, dx \end{cases}$

$= \int \dfrac{1\ \textcircled{2}}{\sqrt{1-t^2}} \, dt = 2 \, \mathrm{arcsin}\,t + k = \boxed{2 \cdot \mathrm{arcsin}\,e^{\sqrt{x}}+k.}$

Bereko c) $\int \dfrac{\cos^3 x \, dx}{\sin^4 x} =$

$\qquad$ $\begin{cases} t = \sin x \\ dt = \cos x \, dx \end{cases}$

$\int \dfrac{\cos x (\cos^2 x)\, dx}{\sin^4 dx} = \int \dfrac{(1-\sin^2 x) \cdot \cos x \, dx}{\sin^4 dx} =$

$= \int \dfrac{(1-t^2) \cdot dt}{t^4} = \int \left(\dfrac{1}{t^4} - \dfrac{1}{t^2}\right) dt = \int (t^{-4} - t^{-2}) \, dt$

$= \dfrac{t^{-4+1}}{-4+1} - \dfrac{t^{-2+1}}{-2+1}+k = \boxed{\dfrac{-1}{3\sin^3 x} + \dfrac{1}{\sin x} +k}$

d) Hau hobeto ZATIKAKO METODOA6

$\int (x^2+1) \ln(x^3+3x)\, dx = \dfrac{1}{3}\int \ln t \, dt =$

$\qquad$ $\begin{cases} t = x^3+3x \\ dt = (3x^2+3)\,dx. \\ dt/3 = (x^2+1)\,dx \end{cases}$

$= \dfrac{1}{3} \boxed{(t \ln t - t)}+k = \longrightarrow \text{TAULAN'K}$

$= \dfrac{1}{3}\left[(x^3+3x)\cdot \ln(x^3+3x) - (x^3+3x)\right]+k$

e) $\displaystyle\int \frac{\sin x \cdot \cos x}{1 + \sin^4 x}\, dx$

$\quad\quad\quad\quad\quad\quad\quad \begin{vmatrix} t = \sin^2 x \\ dt = 2 \cdot \sin x \cdot \cos x\, dx \\ \frac{dt}{2} = \sin x \cdot \cos x \cdot dx \end{vmatrix}$

$\dfrac{1}{2}\displaystyle\int \frac{dt}{1 + t^2} = \frac{1}{2} \operatorname{arctg} t + k =$

$\quad = \dfrac{1}{2} \operatorname{arctg}(\sin^2 x) + k$

f) $\displaystyle\int e^{x + \sqrt{x}}\left(\frac{6x + 3\sqrt{x}}{x}\right) dx \qquad \begin{array}{l} t = x + \sqrt{x} \\ dt = 1 + \dfrac{1}{2\sqrt{x}}\, dx \\ dt = \dfrac{2\sqrt{x} + 1}{2\sqrt{x}}\, dx \end{array}$

$\displaystyle\int e^{x+\sqrt{x}}\; 3 \underbrace{\frac{(2x + \sqrt{x})}{x}}\, dx =$

$\quad \dfrac{2\sqrt{x}+1}{2\sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{2x + \sqrt{x}}{2x}$

$= \displaystyle\int e^t \cdot 3 \cdot 2\, dt = 6\int e^t\, dt =$

$\quad\quad = \dfrac{1}{2}\dfrac{2x + \sqrt{x}}{x}$

$= \boxed{6 \cdot e^{x + \sqrt{x}} + k.}$

$\quad dt = \dfrac{1}{2} \cdot \dfrac{2x + \sqrt{x}}{x}\, dx$

$\quad 2 \cdot dt = \boxed{\dfrac{2x + \sqrt{x}}{x}\, dx}$

ORDEZKAPEN METODOA – BESTE ALDAGAI BATZUK.
(erroak kentzeko)

Erroa kentzeko:

③ $\displaystyle\int \sqrt{x - 4} \cdot (x + 5)\, dx.$

$\quad\quad\quad \begin{array}{l} \sqrt{x-4} = t \\ x - 4 = t^2 \\ dx = 2t\, dt \\ \hline x = t^2 + 4 \\ x + 5 = t^2 + 4 + 5 \end{array}$

$= \displaystyle\int t \cdot \underbrace{(t^2 + 4 + 5)}_{x}\; \underbrace{2t\, dt}_{dx} =$

$= \displaystyle\int t(t^2 + 9) \cdot 2t\, dt =$

$= \displaystyle\int (2t^4 + 18t^2)\, dt =$

$= \dfrac{2t^5}{5} + \dfrac{18 t^3}{3} + k = \dfrac{2\sqrt{(x-4)^5}}{5} + \dfrac{18\sqrt{(x-4)^3}}{3} + k$

$= \boxed{\dfrac{2\sqrt{(x-4)^5}}{5} + 6\sqrt{(x-4)^3} + k}$

④ $\displaystyle\int \sqrt{\dfrac{\sqrt[3]{x-1} + x - 1}{\sqrt{(x-1)^3}}}\, dx$

$x-1 = t^{⑥}$ ← 3 eto 2-reu artiko mkt

$\sqrt[3]{x-1} = \sqrt[3]{t^6} = t^2$

$\sqrt{(x-1)^3} = \sqrt{(t^6)^3} = t^9$

$x-1 = t^6$
$dx = 6t^5\, dt$
$x = t^6 + 1$

$\displaystyle\int \dfrac{\sqrt[3]{t^6} + \overbrace{(t^6+1)}^{x} - 1}{\sqrt{(t^6)^3}}\underbrace{6t^5\, dt}_{dx} = \int \dfrac{t^2 + t^6}{t^9}\cdot 6t^5\, dt =$

$= \displaystyle\int \dfrac{6t^7 + 6t^{11}}{t^9}\, dt = 6\int \left(\dfrac{1}{t^2} + t^2\right)dt = 6\int \left(t^{-2} + t^2\right)dt$

$= 6\left(\dfrac{t^{-2+1}}{-2+1} + \dfrac{t^3}{3}\right) + k = 6\left(\dfrac{-1}{t} + \dfrac{t^3}{3}\right) + k =$

$x-1 = t^6 \longrightarrow t = \sqrt[6]{x-1} = (x-1)^{1/6}$

$= 6\left(\dfrac{-1}{\sqrt[6]{x-1}} + \dfrac{\sqrt[6]{(x-1)^3}}{3}\right) = \boxed{\dfrac{-6}{\sqrt[6]{x-1}} + 2\sqrt{x-1} + k}$