

2ΑΤΙΚΑΚΩΔΑΚ

$$\left\{ \begin{array}{l} \int u \cdot dv = u \cdot v - \int v \cdot du \\ \text{ALPES} \end{array} \right.$$

350) 12.

$$a) \int_P^E x e^{2x} dx \quad \left\{ \begin{array}{l} u = x \rightarrow du = dx \\ dv = e^{2x} dx \rightarrow v = \frac{1}{2} \int 2e^{2x} dx = \frac{1}{2} e^{2x} \end{array} \right.$$

$$\begin{aligned} I &= x \cdot \frac{1}{2} e^{2x} - \int \frac{1}{2} e^{2x} dx = \frac{1}{2} x \cdot e^{2x} - \frac{1}{2} \int 2e^{2x} dx \\ &= \frac{x \cdot e^{2x}}{2} - \frac{1}{4} e^{2x} + k = \boxed{\frac{e^{2x}}{2} \left( x - \frac{1}{2} \right) + k} \end{aligned}$$

$$b) \int_P^L x^2 \ln x dx \quad \left\{ \begin{array}{l} u = \ln x \rightarrow du = \frac{1}{x} dx \\ dv = x^2 dx \rightarrow v = \frac{x^3}{3} \end{array} \right.$$

$$\begin{aligned} I &= \ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} dx = \ln x \cdot \frac{x^3}{3} - \frac{1}{3} \int x^2 dx = \\ &= \frac{x^3 \cdot \ln x}{3} - \frac{1}{3} \frac{x^3}{3} + k = \boxed{\frac{x^3}{3} \left( \ln x - \frac{1}{3} \right) + k} \end{aligned}$$

$$c) \int 3x \cdot \cos x dx = 3 \underbrace{\int_P^B x \cdot \cos x dx}_I =$$

$$\begin{aligned} I &= x \cdot \sin x - \int \sin x \cdot dx \\ &= x \cdot \sin x - (-\cos x) + k \\ &= x \cdot \sin x + \cos x + k. \end{aligned}$$

$$\boxed{\int 3x \cdot \cos x dx = 3 \cdot (x \sin x + \cos x) + k.}$$

$$d) \int \underbrace{\ln(2x-1)}_L \underbrace{dx}_P \quad \left\{ \begin{array}{l} u = \ln(2x-1) \rightarrow du = \frac{2}{2x-1} dx \\ dv = dx \rightarrow v = x \end{array} \right.$$

$$\begin{aligned} I &= \ln(2x-1) \cdot x - \int x \cdot \frac{2}{2x-1} dx = \\ &= \ln(2x-1) \cdot x - \int \left( 1 + \frac{2}{2x-1} \right) dx \quad \frac{2x}{-2x+1} \quad \frac{2x-1}{1} \\ &= \boxed{x \cdot \ln(2x-1) - x - \frac{\ln|2x-1|}{2} + K} \end{aligned}$$

$$e) \int \frac{\overset{P}{x}}{\underset{E}{e^x}} dx \quad \left\{ \begin{array}{l} u = x \rightarrow du = dx \\ dv = \frac{1}{e^x} dx \rightarrow v = \int \frac{1}{e^x} dx = \frac{e^{-x}}{-1} = -e^{-x} \end{array} \right.$$

$$\begin{aligned} I &= x \cdot (-e^{-x}) - \int (-e^{-x}) dx = -x e^{-x} + \int e^{-x} dx = \\ &= -x e^{-x} - e^{-x} + K = \boxed{e^{-x}(-x-1) + K} \end{aligned}$$

$$f) \int \underbrace{\arccos x}_A dx \quad \left\{ \begin{array}{l} u = \arccos x \rightarrow du = \frac{-1}{\sqrt{1-x^2}} dx \\ dv = dx \rightarrow v = x \end{array} \right.$$

$$I = \arccos x \cdot x - \int x \cdot \frac{-1}{\sqrt{1-x^2}} dx = \arccos x \cdot x + \int \frac{x}{\sqrt{1-x^2}} dx$$

*orderkper*  
 $\frac{x}{\sqrt{1-x^2}} \xrightarrow{I_2}$

$$I_2 = \int \frac{x}{\sqrt{1-x^2}} dx = \int \frac{1}{\sqrt{t}} \cdot \frac{dt}{(-2)} = \int \frac{t^{-1/2}}{-2} =$$

*orderkper*  $t = 1-x^2 \quad dt = -2x dx \rightarrow \frac{dt}{-2} = x dx$

$$I_2 = -\frac{1}{2} \cdot \frac{t^{-1/2+1}}{-1/2+1} = -\frac{2}{2} \sqrt{t} = -\sqrt{t} = -\sqrt{1-x^2}$$

$$I = \arccos x \cdot x + (-\sqrt{1-x^2}) + K = \boxed{x \cdot \arccos x - \sqrt{1-x^2} + K}$$

Bei orderkper:  
 $1-x^2 = t$   
 $0 \dots 1$