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a) $\int x^4 \cdot e^{x^5} dx = \frac{1}{5}\int 5x^4 e^{x^5} dx = \boxed{\frac{1}{5} e^{x^5} + k}$ Berehalakoa

b) $\int x \cdot \sin x^2 dx = \frac{1}{2}\int 2x \cdot \sin(x^2) dx = \boxed{-\frac{1}{2}\cos(x^2) + k}$

c) $\int \underset{P}{x} \cdot \underset{E}{2^{-x}} dx =$ ZATIKAKO METODOA

$$\begin{cases} u = x \longrightarrow du = dx \\ dv = 2^{-x} dx \longrightarrow v = \frac{-1}{\ln 2}\int \ln 2 \, 2^{-x} \cdot dx = -\frac{1}{\ln 2} 2^{-x} \end{cases}$$

$2^{-x} \longrightarrow -2^{-x} \cdot \ln 2$

$$\int u \, dv = u \cdot v - \int v \cdot du$$

$$I = \int x \cdot 2^{-x} dx = x \cdot \frac{-1}{\ln 2} \cdot 2^{-x} - \int -\frac{1}{\ln 2} 2^{-x} dx =$$

$$= -\frac{1}{\ln 2} \cdot x \cdot 2^{-x} + \frac{1}{\ln 2} \frac{-1}{\ln 2} \int 2^{-x} \ln 2 \, dx =$$

$$= \boxed{-\frac{1}{\ln 2} x \cdot 2^{-x} - \frac{1}{(\ln 2)^2} \cdot 2^{-x} + k}$$

d) $\int \underset{P}{x^3} \underset{S.}{\sin x} \, dx$ ZATIKAKO METODOA $\begin{cases} u = x^3 \longrightarrow du = 3x^2 dx \\ dv = \sin x \longrightarrow v = -\cos x \end{cases}$

$$I = \overset{uv}{x^3 \cdot (-\cos x)} - \overset{\int v \, du}{\int -\cos x \cdot 3x^2 dx} =$$

$$= -x^3 \cdot \cos x + 3\underbrace{\int \underset{S}{\cos x} \cdot \underset{P}{x^2} dx}_{I_1}$$

$$I_1 \begin{cases} u = x^2 \longrightarrow du = 2x dx \\ dv = \cos x dx \longrightarrow v = \sin x \end{cases}$$

$$I_1 = x^2 \cdot \sin x - \int \sin x \cdot 2x \, dx$$

$$\boxed{I_1 = x^2 \cdot \sin x - 2\underbrace{\int \sin x \cdot x \, dx}_{I_2}}$$

$$I_2 = \begin{cases} u = x \longrightarrow du = dx \\ dv = \sin x \, dx \longrightarrow v = -\cos x \end{cases}$$

$$I_2 = x \cdot (-\cos x) - \int -\cos x \cdot dx = \boxed{-x\cos x + \sin x}$$

$$I = -x^3 \cdot \cos x + 3 I_1$$
$$= -x^3 \cos x + 3 \cdot (x^2 \sin x - 2 I_2)$$
$$= -x^3 \cos x + 3 \left[x^2 \sin x - 2 \cdot (-x \cos x + \sin x) \right]$$

$$\boxed{I = -x^3 \cdot \cos x + 3x^2 \cdot \sin x + 6 x \cdot \cos x - 6 \sin x + k}$$

e) $\int \sqrt{(x+3)^5} \, dx = \int (x+3)^{5/2} dx = \frac{(x+3)^{5/2+1}}{5/2+1} + k$

$$= \frac{(x+3)^{7/2}}{7/2} + k = \frac{2}{7} \sqrt{(x+3)^7} + k.$$

f) $\int \frac{-3x}{2-6x^2} dx = \frac{1}{4} \int \frac{4(-3x)}{2-6x^2} dx = \frac{1}{4} \ln |2-6x^2| + k$

g) atzean

h) $\int \underset{P}{x^5} \cdot \underset{E}{e^{-x^3}} dx = \int \underbrace{x^3}_{du} \cdot \underbrace{x^2 e^{-x^3} dx}_{dv} =$

$u = x^3 \quad du = 3x^2 dx$

$dv = x^2 e^{-x^3} dx \longrightarrow v = \frac{1}{-3} \int -3 x^2 e^{-x^3} dx = -\frac{1}{3} e^{-x^3}$

$$I = -\frac{x^3}{3} e^{-x^3} - \int \left(-\frac{1}{3} e^{-x^3}\right) \cdot 3x^2 dx$$
$$= -\frac{x^3}{3} e^{-x^3} - \frac{1}{3} \int -3x^2 e^{-x^3} dx =$$

$$\boxed{I = -\frac{x^3}{3} e^{-x^3} - \frac{1}{3} e^{-x^3} + k}$$

$e^{-x^3} \longrightarrow -3x^2 \cdot e^{-x^3}$

g) $I = \int \underbrace{e^{2x+1}}_{E} \cdot \underbrace{\cos x}_{S}\, dx.$

$$\begin{cases} u = e^{2x+1} \longrightarrow du = 2 \cdot e^{2x+1} \\ dv = \cos x\, dx \longrightarrow v = \sin x \end{cases}$$

$$I = e^{2x+1} \cdot \sin x - \int \sin x \cdot 2 \cdot e^{2x+1} dx$$

$$I = e^{2x+1} \sin x - 2 \cdot \underbrace{\int \underset{S}{\sin x}\ \underset{E}{e^{2x+1}} dx}_{I_1} =$$

$$I_1 \begin{cases} u = e^{2x+1} \longrightarrow du = 2 \cdot e^{2x+1} \\ dv = \sin x\, dx \longrightarrow v = -\cos x \end{cases}$$

$$I_1 = e^{2x+1} \cdot (-\cos x) - \int -\cos x \cdot 2 \cdot e^{2x+1} dx.$$

$$I = e^{2x+1} \cdot \sin x - 2 (I_1)$$

$$I = e^{2x+1} \cdot \sin x - 2 \cdot \left[e^{2x+1} \cdot (-\cos x) + 2 \underbrace{\int \cos x \cdot e^{2x+1} dx}_{I} \right]$$

$$\boxed{I = e^{2x+1} \cdot \sin x + 2 \cdot e^{2x+1} \cdot \cos x - 4 I}$$

$$5I = e^{2x+1} \sin x + 2 e^{2x+1} \cdot \cos x$$

$$\boxed{I = \frac{e^{2x+1} (\sin x + 2 \cos x)}{5}}$$