355.orr) 17 ORDEZKAPEN METODOA

a) $\int \frac{dx}{x-\sqrt{x}} = \int \frac{2t\,dt}{t^2-\sqrt{t^2}} = \int \frac{2t\,dt}{t^2-t} =$

$t^2 = x$

$2t\,dt = dx$

$t = \sqrt{x}$

$= 2\int \frac{t}{t(t-1)}\,dt = 2\int \frac{1}{t-1}\,dt =$

$= 2 \cdot \ln|t-1| + K = \boxed{2\ln|\sqrt{x}-1| + K.}$

b) $\int x\sqrt[3]{x+2}\,dx = \int (t^3-2)\sqrt[3]{t^3}\,3t^2\,dt =$

$x+2 = t^3$

$dx = 3t^2\,dt$

$x = t^3-2$

$t = \sqrt[3]{x+2}$

$= \int (t^3-2)\,t \cdot 3t^2\,dt =$

$= \int (3t^6 - 6t^3)\,dt = \frac{3t^7}{7} - \frac{6t^4}{4} + K.$

$= \boxed{\frac{3}{7}\sqrt[3]{(x+2)^7} - \frac{3}{2}\sqrt[3]{(x+2)^4} + K.}$

c) $\int \frac{\sqrt{x}\,dx}{\sqrt[3]{x}-1} =$

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$mkt(2,3) = 6 \longrightarrow t^6$

$t^6 = x$

$6t^5\,dt = dx$

$= \int \frac{\sqrt{t^6}\;6t^5\,dt}{\sqrt[3]{t^6}-1} = \int \frac{t^3 \cdot 6t^5\,dt}{t^2-1} =$

$= \int \frac{6t^8\,dt}{t^2-1} = 6\int \left(t^6+t^4+t^2+1+\frac{1}{t^2-1}\right)dt$

$= 6\left(\frac{t^7}{7}+\frac{t^5}{5}+\frac{t^3}{3}+t\right) + 6\int \frac{1}{t^2-1}\,dt$

I_1 *

$t^8 : (t^2-1) = t^6+t^4+t^2+1$

$-t^8 + t^6$

t^6

$-t^6 + t^4$

t^4

$-t^4 + t^2$

t^2

$-t^2 + 1$

$I_1 = \int \frac{1}{t^2-1}\,dt$ INTEGRAL ARRAZIONAL

$$\frac{1}{t^2-1} = \frac{1}{(t+1)(t-1)} = \frac{A}{t+1} + \frac{B}{t-1}$$

$$1 = A(t-1) + B(t+1)$$

$t = 1 \longrightarrow 1 = B \cdot 2 \longrightarrow \boxed{B = 1/2}$

$t = -1 \longrightarrow 1 = A(-2) \longrightarrow \boxed{A = -1/2}$

$$\boxed{I_1 = \int \left(\frac{-1/2}{t+1} + \frac{1/2}{t-1} \right) dt}$$

$$= -\frac{1}{2} \ln|t+1| + \frac{1}{2} \ln|t-1|$$

Berat:

$$\int \frac{\sqrt{x}\,dx}{\sqrt[3]{x}-1} = 6\left(\frac{t^7}{7} + \frac{t^5}{5} + \frac{t^3}{3} + t\right) + 6\left(-\frac{1}{2}\ln|t+1| + \frac{1}{2}\ln|t-1|\right) = +K$$

$t = \sqrt[6]{x} = x^{1/6}$

$$= 6\left(\frac{x^{7/6}}{7} + \frac{x^{5/6}}{5} + \frac{x^{3/6}}{3} + x^{1/6}\right) + \left(-3\ln|\sqrt[6]{x}+1| + 3\ln|\sqrt[6]{x}-1|\right) + K$$

$$= \boxed{\frac{6}{7}\sqrt[6]{x^7} + \frac{6}{5}\sqrt[6]{x^5} + 2\sqrt{x} + \sqrt[6]{x} - 3\ln|\sqrt[6]{x}+1| + 3\ln|\sqrt[6]{x}-1| + K}$$

d) $\int \frac{dx}{(3-x)\sqrt{2-x}} = \int \frac{-2t\,dt}{(3-(2-t^2))\sqrt{t^2}} =$

$2-x = t^2$

$-dx = 2t\,dt$

$x = 2-t^2$

$t = \sqrt{2-x}$

$= \int \frac{-2t\,dt}{(1+t^2)\cdot t} =$

$= \int \frac{-2\,dt}{1+t^2} = -2\operatorname{arctg} t + k.$

$= \boxed{-2\operatorname{arctg}(\sqrt{2-x}) + k.}$