14 a) $\int \frac{dx}{(x+1)(x+2)}$ $\quad (x+1)(x+2)=0 \Rightarrow \begin{matrix} x=-1 \\ x=-2 \end{matrix}$

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} \Rightarrow \frac{1}{(x+1)(x+2)} = \frac{A(x+2)}{(x+1)(x+2)} + \frac{B(x+1)}{(x+1)(x+2)}$$

$$1 = A(x+2) + B(x+1)$$

$x=-2 \longrightarrow 1 = \cancel{A(-2+2)} + B(-2+1) \Rightarrow \boxed{B=-1}$.

$x=-1 \longrightarrow 1 = A(-1+2) + \cancel{B(-1+1)} \longrightarrow \boxed{A=1}$

$$I = \int \left(\frac{1}{x+1} - \frac{1}{x+2} \right) dx = \ln|x+1| - \ln|x+2| + K$$

$$I = \ln\left|\frac{x+1}{x+2}\right| + K.$$

b) $\int \frac{2x+1}{(x+1)(x-3)}\,dx$ $\quad x=-1$, $x=3$.

$$\frac{2x+1}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$2x+1 = A(x-3) + B(x+1)$$

$x=3 \longrightarrow 2\cdot 3+1 = \cancel{A\cdot 0} + 4B \longrightarrow B = 7/4.$

$x=-1 \longrightarrow 2(-1)+1 = A(-4) + \cancel{B\cdot 0} \longrightarrow A = 1/4.$

$$I = \int \left(\frac{1/4}{x+1} + \frac{7/4}{x-3} \right) dx = \frac{1}{4}\ln|x+1| + \frac{7}{4}\ln|x-3| + K$$

c) $\int \frac{3\,dx}{x(x+4)}$ $\quad \frac{3}{x(x+4)} = \frac{A}{x} + \frac{B}{x+4}$

$$3 = A(x+4) + Bx.$$

$x=0 \longrightarrow 3 = 4A + \cancel{B\cdot 0} \longrightarrow A = 3/4$

$x=-4 \longrightarrow 3 = \cancel{A\cdot 0} - 4B \longrightarrow B = -3/4$

$$I = \int \left(\frac{3/4}{x} - \frac{3/4}{x+4} \right) dx = \boxed{\frac{3}{4}\ln|x| - \frac{3}{4}\ln|x+4| + K}$$

d) $\int \frac{dx}{x^2(x-1)}$

$$\frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

$$\frac{1}{x^2(x-1)} = \frac{Ax(x-1)}{x^2(x-1)} + \frac{B(x-1)}{x^2(x-1)} + \frac{Cx^2}{x^2(x-1)}$$

$$1 = Ax(x-1) + B(x-1) + Cx^2$$

$x=0 \longrightarrow 1 = A\cdot 0 + B(-1) + C\cdot 0 \longrightarrow \boxed{B=-1}$

$x=1 \longrightarrow 1 = A\cdot 0 + B\cdot 0 + C \longrightarrow \boxed{C=1}$.

$$1 = A\cdot x(x-1) - (x-1) + x^2$$

$x=2 \longrightarrow 1 = 2A - 1 + 4 \longrightarrow \boxed{A=-1}$.

$$I = \int \left(\frac{-1}{x} - \frac{1}{x^2} + \frac{1}{x-1}\right) dx = -\ln|x| - \frac{x^{-2+1}}{-2+1} + \ln|x-1| + k$$

$$= \boxed{\ln|x-1| - \ln|x| + \frac{1}{x} + k.}$$