

# EREMU GRABITATORIOA

• INDAR GRABITATORIOA:

$$|\vec{F}| = G \frac{M_1 \cdot m_2}{d^2} \quad (N)$$

$$\vec{F} = m_2 \cdot \vec{g}$$

$$|\vec{g}| = G \frac{M_1}{d^2}$$

$\left( \frac{N}{kg} \text{ edo } \frac{m}{s^2} \right)$

• EREMUAREN INTENTSITATE BEKTOREA EDO AZELERAZIOA

• EREMUKO PUNTU BATEKO ENERGIA POTENTZIALA:

$$E_p = -G \frac{M_1 \cdot m_2}{d} \quad (J)$$

$$V = -G \frac{M_1}{d} \quad \left( \frac{J}{kg} \right)$$

$$E_p = m_2 \cdot V$$

• EREMUKO PUNTU BATEKO POTENTZIAL GRABITATORIOA:

• EREMUAK EGINDAKO LANA:

$$W_{EREMUA} = -\Delta E_p$$

• ABIADURA ORBITALA: ...  $|\vec{F}_2| = |\vec{F}_G|$  ...

$$v_{orb} = \sqrt{G \frac{M_1}{d}} \quad \left( \frac{m}{s} \right)$$

• IHES ABIADURA: ...  $E_{ma} = E_{m\infty}$  ...

$$v_i = \sqrt{2G \frac{M_1}{d}} \quad \left( \frac{m}{s} \right)$$

• ORBITAN EGONDA:  $E_m$  ....

$$E_m = \frac{1}{2} E_p$$

$$E_z = \frac{1}{2} |E_p|$$

• KEPLERREN 3. LEGEA:

$$\frac{T^2}{R^3} = k \text{ te}$$

$$\frac{T_A^2}{R_A^3} = \frac{T_B^2}{R_B^3} = \dots = \frac{T_n^2}{R_n^3}$$

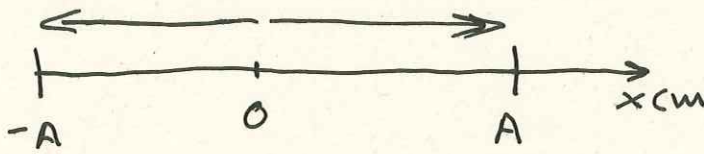
• ABIADURA ANGELUARRA

$$\omega = \frac{\text{angelua}}{\text{denbora}}$$

$$\omega = \frac{2\pi}{T} \quad \left( \frac{\text{rad}}{s} \right)$$

eta 
$$v = \omega \cdot R$$

# HHS (HIGIDURA HARMONIKO SINPLEA)



- ELONGAZIOA:  $x(t) = A \sin(\omega t + \phi_0)$  (m)
- ABIADURA:  $v(t) = \frac{dx(t)}{dt} = A\omega \cos(\omega t + \phi_0)$  (m/s)
- AZELERAZIOA:  $a(t) = \frac{dv(t)}{dt} = -A\omega^2 \sin(\omega t + \phi_0)$  (m/s<sup>2</sup>)  
 $\hookrightarrow$  edo  $\hookleftarrow a(t) = -\omega^2 x(t)$

- PULTSATIOA:  $\omega = \frac{2\pi}{T} = 2\pi \cdot f$  (rad/s)  $T = \frac{1}{f}$   
 PERIODOA: T (s) ; MAIZTASUNA EDO FREKUENTZIA: f (s<sup>-1</sup> edo Hz)

Hooke:  $F = -k \cdot x$   
 Nu. 2.1:  $F = m \cdot a$   $\xrightarrow{a = -\omega^2 x}$   $-m\omega^2 x \quad \} (=) \quad \boxed{K = m \cdot \omega^2} \rightarrow$

$\rightarrow$  OSZILADORE HARMONIKO SINPLEAREN PERIODOA  $\rightarrow$

$\rightarrow K = m \cdot \frac{2\pi^2}{T^2} \dots \rightarrow T = 2\pi \sqrt{\frac{m}{K}}$

$\rightarrow$  Pendulo sinplean  $T = 2\pi \sqrt{\frac{L}{g}}$

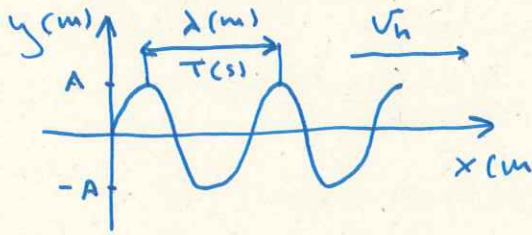
- ENERGIA MEKANIKOA (Kte)

$E_m = E_p + E_z$   $E_p = \frac{1}{2} K x^2$   $E_z = \frac{1}{2} m v^2$

Muturrean  $v=0$  eta  $x=A$ . Geldirik  $\Rightarrow E_m = E_{pA} = \frac{1}{2} K A^2$



# UHIN HARMONIKO SINPLEAK



## UHIN FUNTZIOA

$$y(x,t) = A \sin(\omega t - kx + \phi_0) \quad (\text{m edo cm})$$

↑ AURRERA (ATZERA +)

• PULTSAZIOA EDO MAIZTASUN ANGELARRA  $\omega = \frac{2\pi}{T}$  edo  $\omega = 2\pi \cdot f$  (rad/s)

• UHIN KOPURUA EDO UHIN ZENBAKIA  $k = \frac{2\pi}{\lambda}$  (rad/m)

• HEDAPEN ABIADURA:  $v_h = \frac{\lambda}{T}$  (m/s)

• BIBRAZIO ABIADURA:  $v(x,t) = \frac{dy(x,t)}{dt} = \dots = A\omega \cos(\quad)$

$v_{\max} \rightarrow \cos = \pm 1 \rightarrow v_{\max} = \pm A\omega$

• BIBRAZIO AZELERAZIOA:  $a(x,t) = \frac{dv(x,t)}{dt} = -A\omega^2 \sin(\quad)$

$a_{\max} \rightarrow \sin = \pm 1 \rightarrow a_{\max} = \pm A\omega^2$

— 0 —  
POTENTZIA  $\Rightarrow P = \frac{E}{t}$  (wat =  $\frac{J}{s}$ )

INTENSITATEA =  $\frac{\text{Potentzia}}{\text{azalera}} \rightarrow I = \frac{P}{S} \xrightarrow{\text{esferak}} I = \frac{P}{4\pi R^2}$  (wat/m<sup>2</sup>)

— 0 —  
**DESPASEAK** FASEA SIN BARRUKOA DA:  $(\omega t - kx + \phi_0)$

KASUAU:

→  $x_1 = 3\text{ m}$  eta  $x_2 = 5\text{ m}$ -ko partikulen arteko desfasea:

$$\Delta\phi_{3,5} = \phi_3 - \phi_5 = (\omega t - k \cdot 3 + \phi_0) - (\omega t - k \cdot 5 + \phi_0) = 2k \text{ (rad)}$$

→ Zeinturien artean desfasea  $\pi/3$  da?

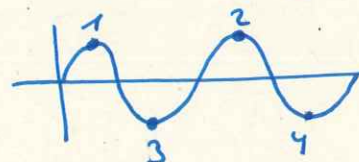
$$\phi_x - \phi_{x+d} = \pi/3 \rightarrow (\omega t - kx + \phi_0) - [\omega t - k(x+d) + \phi_0] = \pi/3 \rightarrow \textcircled{d}$$

→ Zeintuk fasean  $x = 2\text{ m}$ -koagert?  $\Rightarrow$  FASEAN:  $\Delta\phi = n \cdot 2\pi$  ( $n = \pm 1, \pm 2, \dots$ )

$$\phi_2 - \phi_{x_n} = n \cdot 2\pi \rightarrow (\omega t - k \cdot 2 + \phi_0) - [\omega t - k \cdot x_n + \phi_0] = n \cdot 2\pi \rightarrow \textcircled{x_n}$$

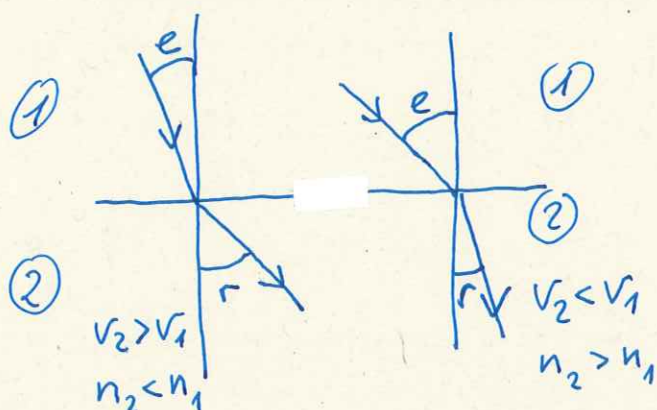
→ Zeintuk kontrafasean?  $\Rightarrow$  KONTRAFASEA  $\Delta\phi = (2n+1)\pi$  ( $n = 0, \pm 1, \pm 2, \dots$ )

(Garaipena Sardin)



1-2  $\Rightarrow$  FASEAN  
1-3, 4  $\Rightarrow$  KONTRAFASEAN

# ERREFRAKZIOA



$f$  ez da aldatzen

$$v_i = \lambda_i \cdot f$$

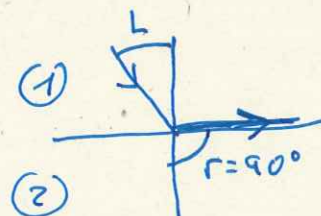
$$c = 3 \cdot 10^8 \text{ m/s}$$

- ERREFRAKZIO-INDIZEAK:
  - ABSOLUTUA  $n_i = c/v_i$
  - ERLATIBOA  $n_{21} = v_1/v_2$

• SNELL-EN ERREFRAKZIOAREN LEGEA

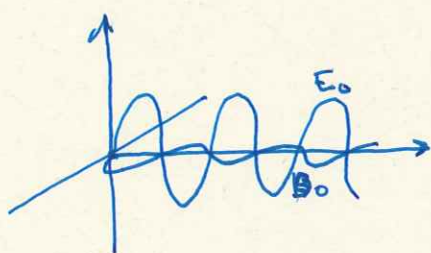
$$\frac{\sin e}{\sin r} = \frac{v_1}{v_2} = \frac{n_2}{n_1} = n_{21}$$

• ISLAPEN OSOA  $\rightarrow$  BALDINTZA  $v_2 > v_1$



$$\frac{\sin e}{\sin r} = \frac{v_1}{v_2} \xrightarrow[r=90^\circ]{e=L} \sin L = \frac{v_1}{v_2} \rightarrow L = \arcsin(v_1/v_2) = \arcsin(n_2/n_1)$$

## UHIN ELEKTROMAGNETIKOAK



$$E(x,t) = E_0 \sin(\omega t - kx + \phi_0)$$

$$B(x,t) = B_0 \sin(\omega t - kx + \phi_0)$$

Eta beti:  $\boxed{\frac{E}{B} = c}$

Uhin arruntak berata laurten dira. Biak erlatibonaketo edozein momentutan  $\frac{E}{B} = c$  setekun da.

(Hau selektibitateko. Kurtsoan ez dago galdetzen)



# EREMU ELEKTRIKOA

- INDAR ELEKTRIKOA:

$$|\vec{F}| = k \frac{|Q_1| \cdot |Q_2|}{d^2} \text{ (N)}$$

$$\vec{F} = \vec{E} \cdot Q_2$$

- EREMU ELEKTRIKOAREN

INTENSITATE BEKTOREA:

$$|\vec{E}| = k \frac{|Q_1|}{d^2} \left( \frac{\text{N}}{\text{C}} \right)$$

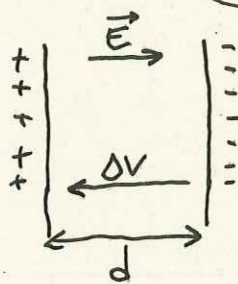
- ENERGIA POTENTIALA:

$$E_p = k \frac{Q_1 \cdot Q_2}{d} \text{ (J)}$$

$$E_p = V \cdot Q_2$$

- POTENTIAL ELEKTROSTATIKOA

$$V = k \frac{Q_1}{d} \left( \frac{\text{J}}{\text{C}} \text{ edo } \text{V} \right)$$



$$|\vec{E}| \cdot d = \Delta V$$

$\hookrightarrow$  ( $\vec{E}$  bebai  $\frac{\text{V}}{\text{m}}$ )

- EREMU ELEKTRIKOAK EGINDAKO LANA:

$$W_{\text{EREM}} = -\Delta E_p$$

- ENERGIEN GALANTZAK:

- Eremua zentrala da, serot kontsektakorra, holan  $E_m = k \frac{q}{r^2} \rightarrow$

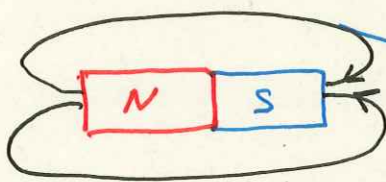
$$\rightarrow E_{mA} = E_{mB} \rightarrow E_{zA} + E_{pA} = E_{zB} + E_{pB} \rightarrow$$

$$\rightarrow \frac{1}{2} m_2 v_A^2 + Q_2 \cdot V_A = \frac{1}{2} m_2 v_B^2 + Q_2 \cdot V_B \rightarrow$$

$$\rightarrow \frac{1}{2} m_2 v_A^2 - \frac{1}{2} m_2 v_B^2 = Q_2 \cdot \Delta V$$

$\nwarrow$  Hortik joango gara,  
er  $E_p$  kontsektuak kalkulatu (orkarrean)

# EREMU MAGNETIKOA

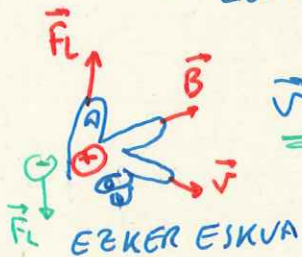


$\vec{B}$  : indukzio bektorea (T, Tesla)

## - KARGA BATEN GAINEKO INDAR MAGNETIKOA

• LORENTZEN INDARRA:

$$\vec{F}_L = q \cdot (\vec{v} \wedge \vec{B}) = q \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix}$$



$\vec{v} \perp \vec{B} \Rightarrow$  ibilbide zirkularra  $\Rightarrow \vec{F}_L \equiv \vec{F}_z \rightarrow$

$$\rightarrow q v B \sin \alpha = m \frac{v^2}{R} \rightarrow \textcircled{R} \text{ lortu} \rightarrow \text{denpora} \rightarrow$$

$\uparrow_{90^\circ}$

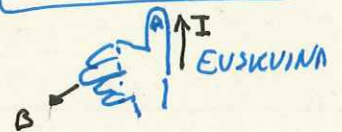
$$\rightarrow v = \frac{2\pi}{T} \cdot R \rightarrow \textcircled{T} \text{ lortu}$$

## - KORRONTE ELEMENTU BATEN GAINEKO INDAR MAGNETIKOA:

$$\vec{F}_L = q (\vec{v} \times \vec{B}) = q \left( \frac{\vec{\ell}}{\Delta t} \times \vec{B} \right) = \frac{q}{\Delta t} (\vec{\ell} \times \vec{B}) \Rightarrow \boxed{\vec{F}_L = I \cdot (\vec{\ell} \times \vec{B})}$$

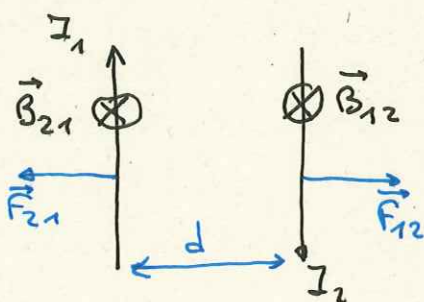
## - KORRONTE BATEK SORTURIKO EREMU MAGNETIKOA

BIOT-SAVART LEGEA



ESPIRA BATEN ERDIAN	KORRONTE ZUZEN INFINITIVAK R DISTANTZIARA
$B = \frac{\mu_0 I}{2R}$	$B = \frac{\mu_0 I}{2\pi R}$

## - BI KABLEEN ARTEKO ALKARRERAKINTZA



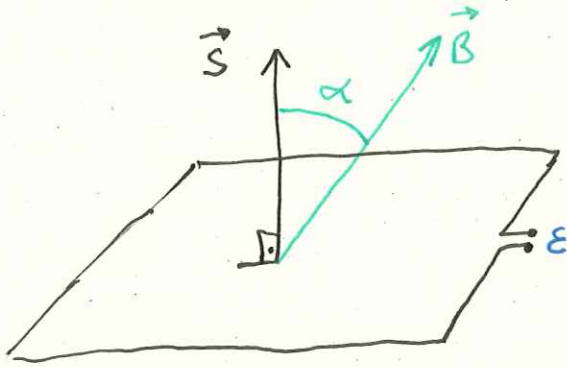
$$B_{12} = \frac{\mu_0 I_1}{2\pi d} \rightarrow F_{12} = I_2 \cdot l_2 \cdot B_{12} = \frac{I_2 l_2 \mu_0 I_1}{2\pi d}$$

$$B_{21} = \frac{\mu_0 I_2}{2\pi d} \rightarrow F_{21} = I_1 \cdot l_1 \cdot B_{21} = \frac{I_1 l_1 \mu_0 I_2}{2\pi d}$$

Edo  $l_1$  eta  $l_2$  erosten deuskuve, edo bestela 1m jarri eta indararen emaitza, adibidez:  $8 \frac{N}{m}$



# INDUKZIO ELEKTROMAGNETIKOA



• FLUXU MAGNETIKOA:  $\Phi$

$$\Phi = \vec{B} \cdot \vec{S} = B \cdot S \cdot \cos \alpha \quad (\text{Wb})$$

$S = \text{azalera}$

$$S = a \times b \quad (\text{m}^2)$$
$$S = \pi R^2 \quad (\text{m}^2)$$

• INDAR ELEKTROERAGILE INDUZITIBA:  $\mathcal{E}$  (V)

$\Rightarrow$  FARADAY-REN LEGEA

TARTEKA:

$$\mathcal{E} = - \frac{\Delta \Phi}{\Delta t} \quad (\text{V})$$

JARRAIA:

$$\mathcal{E}(t) = - \frac{d\Phi(t)}{dt} \quad (\text{V})$$

MINUSA  
 $\Leftarrow$  EZINBESTEKOA  
LENZ-ERI JARRAITUZ

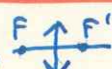

• KANPOKO ZIRKUITO BATERI  $\mathcal{E}$ -ek HORNITUTAKO KORRONTEA (I)  
ZIRKUITUAN,  $R$  ( $\Omega$ ) ERRESISTENTZIA EGONDA  $\Rightarrow$

$\Rightarrow$  OHM-EN LEGEA

$$I = \frac{\mathcal{E}}{R} \quad (\text{A})$$

$\vec{B}$ : induzio bektorea (T; Tesla)  
 $\Phi$ : fluxu magnetikoa (Wb; Weber)  
 $\vec{S}$ : azalera bektorea ( $\text{m}^2$ )  
 $\mathcal{E}$ : indarelektroeragile induzitiba (V; volt)  
 $R$ : erresistentzia ( $\Omega$ ; ohm)  
 $I$ : intentsitatea (A; ampere)

# OPTIKA GEOMETRIKOA

LEIAR MEHEAK (LENTE MEHEAK): KONBERGENTEAK   
DIBERGENTEAK 

$$\frac{1}{s'} - \frac{1}{s} = \frac{1}{f} = \frac{-1}{f}$$

Emendazioa:

$$\frac{y'}{y} = \frac{s'}{s}$$

Potentzia (dioptriak):

$$P = \frac{1}{|f|} \text{ dioptriak}$$

s: objektu-distantzia

s': irudi-distantzia

f: objektu distantzia fokala

f': irudi distantzia fokala

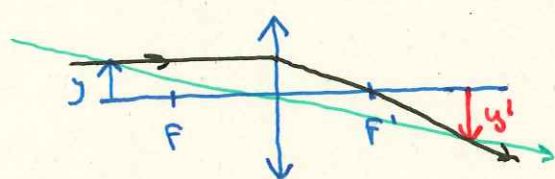
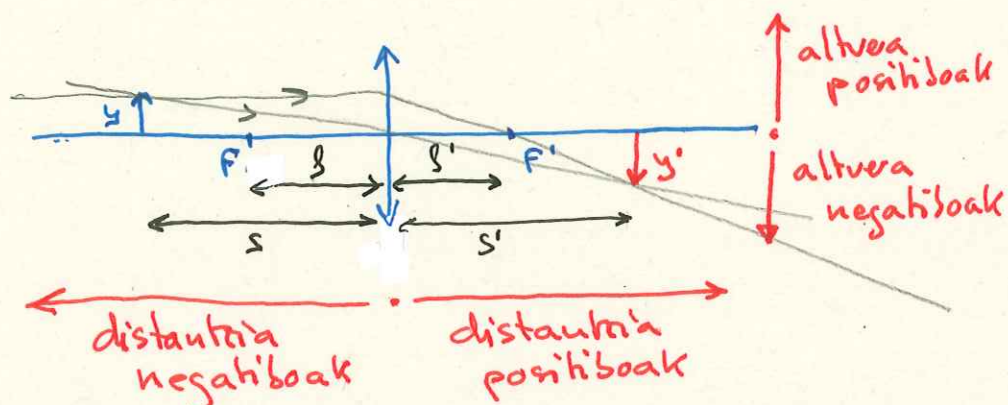
$$|f| = |f'| = \frac{R}{2} \quad (R: lentearen kurbadura erradioa)$$

F: objektu fokua

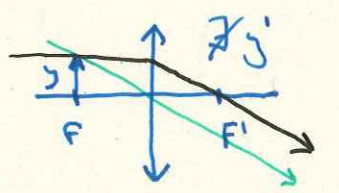
F': irudi fokua

y: objektuaren tamaina

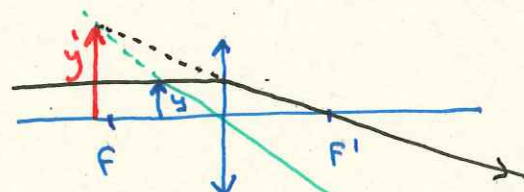
y': irudiaren tamaina



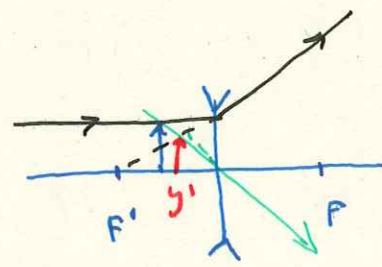
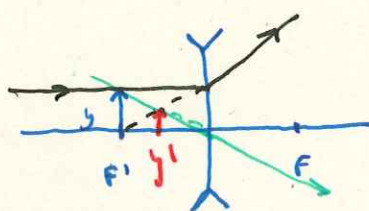
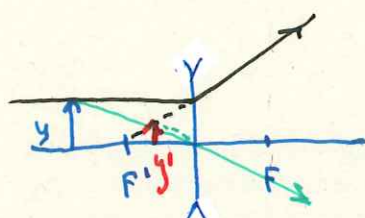
IRUDI ERREALA ALDERNTZIT



IRUDIRIK



IRUDI BIRTUALA  
ETA ZUZENA



DIBERGENTEETAN IRUDIA BETI: BIRTUALA

ZUZENA

OBJEKTUA BAINO TXIKIAGOA

Dioptrio esferikoaren oinarriko ekuazioa:

$$\frac{n'}{s'} - \frac{n}{s} = \frac{n' - n}{R} \quad M_L = \frac{y'}{y} = \frac{n s'}{n' s} \quad f = \frac{R}{2}$$

Ispiluetan:  $n' = -n$  ; laketan  $R = \infty$



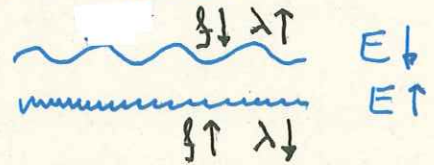
# EFEKTU FOTOELEKTRIKOA

- PLANCK: Uhinak energia paketeetan datur.

Pakete, fotoi edo kuantum bakoitzaren

energia:

$$E = h \cdot f$$



- EINSTEINEN INTERPRETATIOA:

$$E = W_0 + E_{zmax}$$

$W_0$  (erauzketa lana)

- ATARI MAIZTASUNA  $f_u \Rightarrow W_0$  (lotura apurkeko energia datur)

(edo  $\lambda < \lambda_u$ )  $\rightarrow f < f_u \Rightarrow \nexists$  fotoelektroiak

(edo  $\lambda > \lambda_u$ )  $\rightarrow f > f_u \Rightarrow \exists$  fotoelektroiak.

- BALAZTATZE POTENTIALA.

Fotoelektroien solako  $E_z$  anulaheko potentziala, beraz:

$$E_z = q V_B \rightarrow \frac{1}{2} m v^2 = q \cdot V_B \quad (\text{edo } V_{stop})$$