

BIRPASSARIKETAK

1

$$\int \frac{\sin 3x}{e^x} dx$$

16c, 20d, 21a, 23c, 22a,
19b, 17d, 19g

$$I = \int \frac{\sin 3x}{e^x} dx = \int \underbrace{\sin(3x)}_S \cdot \underbrace{e^{-x}}_E dx \quad \text{2Arikako Method}$$

$$I = \begin{cases} u = e^{-x} \rightarrow du = -e^{-x} dx \\ dv = \sin 3x dx \rightarrow v = \frac{1}{3} \int \sin(3x) dx = -\frac{1}{3} \cos(3x) \end{cases}$$

$$I = \underbrace{u \cdot v}_{e^{-x} \cdot (-\frac{1}{3}) \cdot \cos 3x} - \int \underbrace{v \cdot du}_{-\frac{1}{3} \cos(3x) \cdot (-e^{-x})} dx =$$

$$I = -\frac{1}{3} e^{-x} \cos(3x) - \frac{1}{3} \int \underbrace{\cos(3x)}_S \cdot \underbrace{e^{-x}}_E dx =$$

$$I_1 = \begin{cases} u = e^{-x} \rightarrow du = -e^{-x} dx \\ dv = \cos 3x dx \rightarrow v = \frac{1}{3} \int \cos 3x dx = \frac{1}{3} \sin(3x) \end{cases} \quad \text{I}_1$$

$$I_1 = \underbrace{u \cdot v}_{e^{-x} \cdot \frac{1}{3} \sin(3x)} - \int \underbrace{v \cdot du}_{\frac{1}{3} \sin(3x) \cdot (-e^{-x})} dx$$

$$I_1 = \frac{1}{3} e^{-x} \sin 3x + \frac{1}{3} \int \sin 3x \cdot e^{-x} dx \quad \text{I}$$

Quarta:

$$I = -\frac{1}{3} e^{-x} \cos(3x) - \frac{1}{3} I_1$$

$$I = -\frac{1}{3} e^{-x} \cos(3x) - \frac{1}{3} \left[\frac{1}{3} e^{-x} \sin(3x) + \frac{1}{3} \int \sin 3x \cdot e^{-x} dx \right] \quad \text{I}$$

(2)

$$I = -\frac{1}{3} e^{-x} \cos(3x) - \frac{1}{9} e^{-x} \sin(3x) - \frac{1}{9} I$$

$$I + \frac{1}{9} I = -\frac{1}{3} e^{-x} \cos(3x) - \frac{1}{9} e^{-x} \sin(3x)$$

$$\frac{10}{9} I = -\frac{e^{-x}}{9} (3 \cos 3x + \sin 3x)$$

$$I = -\frac{e^{-x}}{10} (3 \cos 3x + \sin 3x)$$

$$\underline{22a)} \int \frac{x+4}{\sqrt{1-x^2}} dx = \underbrace{\int \frac{x}{\sqrt{1-x^2}} dx}_{I_1} + \underbrace{\int \frac{4}{\sqrt{1-x^2}} dx}_{I_2}$$

$$I_1 = \int \frac{x}{\sqrt{1-x^2}} dx = \int \frac{-\frac{dt}{2\sqrt{t}}}{2\sqrt{t}} = -\frac{1}{2} \int t^{-1/2} dt =$$

Orderk met

$$t = 1-x^2$$

$$dt = -2x dx$$

$$-\frac{dt}{2} = x dx$$

$$= -\frac{1}{2} \frac{t^{1/2}}{1/2} = -\sqrt{t} + k =$$

$$= \boxed{-\sqrt{1-x^2}}$$

$$I_2 = \int \frac{4}{\sqrt{1-x^2}} dx = 4 \arcsin x$$

$$I = I_1 + I_2 = \boxed{-\sqrt{1-x^2} + 4 \arcsin x + k}$$

$$21g) \int \frac{e^x}{1-\sqrt{e^x}} dx = \int \frac{2t dt}{1-\sqrt{t^2}} = \int \frac{2t dt}{1-t} \quad (3)$$

Ordetkoppen
metoden

$$\left\{ \begin{array}{l} e^x = t^2 \\ e^x dx = 2t dt \end{array} \right. \quad = 2 \int \frac{t dt}{1-t} = 2 \int -1 + \frac{1}{1-t} dt$$

$$= 2 \left[-t + \ln|1-t| \right] + k$$

$\frac{t}{-t} \quad \frac{1-t}{-1}$
 $\frac{0}{0+1}$

$$\boxed{I = 2 \left[-\sqrt{e^x} - \ln|1-\sqrt{e^x}| \right] + k}$$

$$20d) \int \frac{\sin x}{\cos^4 x} dx = \int \frac{-dt}{t^4} = -\frac{t^{-4+1}}{-4+1} + k$$

Ordetkoppen

$$t = \cos x$$

$$dt = -\sin x dx$$

$$= \frac{+1}{+3t^3} + k$$

$$= \boxed{\frac{1}{3 \cos^3 x} + k}$$

$$19g) \int \frac{\ln \sqrt{x}}{\sqrt{x}} dx = \int \underbrace{x^{-1/2}}_P \cdot \underbrace{\ln \sqrt{x}}_L dx$$

Zatukoko metoden

$$\left\{ \begin{array}{l} u = \ln \sqrt{x} \rightarrow du = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2x} dx \\ dv = x^{-1/2} dx \rightarrow v = \frac{x^{-1/2+1}}{-1/2+1} = \frac{1}{1/2} \sqrt{x} = 2\sqrt{x} \end{array} \right.$$

$$\int \frac{\ln \sqrt{x}}{\sqrt{x}} dx = \ln \sqrt{x} \cdot 2\sqrt{x} - \int 2\sqrt{x} \cdot \frac{1}{2x} dx =$$

$$= 2\sqrt{x} \cdot \ln \sqrt{x} - \int x^{-1/2} dx = 2\sqrt{x} \cdot \ln \sqrt{x} - 2\sqrt{x} + k$$

$$\downarrow \frac{x^{-1/2+1}}{-1/2+1} = \frac{\sqrt{x}}{1/2} = 2\sqrt{x}$$

$$= \boxed{2\sqrt{x} \cdot (\ln \sqrt{x} - 1) + k}$$

19b) $\int \frac{1 - \sin x}{x + \cos x} dx$

Bereubloa $\rightarrow \boxed{\ln |x + \cos x| + K}$

23c) $I = \int \frac{x^4 - 2x - 6}{x^3 + x^2 - 2x} dx$

$$\begin{array}{r} x^4 - 2x - 6 \\ -x^4 - x^3 + 2x^2 \\ \hline \end{array} \quad \begin{array}{r} | x^3 + x^2 - 2x \\ x - 1 \end{array}$$

$$\begin{array}{r} / -x^3 + 2x^2 - 2x - 6 \\ +x^3 + x^2 - 2x \\ \hline / 3x^2 - 4x - 6 \end{array}$$

$$I = \int x - 1 + \underbrace{\frac{3x^2 - 4x - 6}{x^3 + x^2 - 2x}}_{I_1} dx.$$

$$I_1 = \int \frac{3x^2 - 4x - 6}{x^3 + x^2 - 2x} dx = \int \frac{3x^2 - 4x - 6}{x(x+2)(x-1)} dx.$$

$$\frac{3x^2 - 4x - 6}{x(x+2)(x-1)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-1}$$

$$3x^2 - 4x - 6 = A(x+2)(x-1) + B \cdot x(x-1) + C(x+2)x.$$

$$x=0 \rightarrow -6 = -2A + \cancel{B \cdot 0} + \cancel{C \cdot 0} \rightarrow \boxed{A=3}$$

$$x=1 \rightarrow -7 = \cancel{A \cdot 0} + \cancel{B \cdot 0} + 3C \rightarrow \boxed{C=-7/3}$$

$$x=-2 \rightarrow 14 = \cancel{A \cdot 0} + 6B + \cancel{C \cdot 0} \rightarrow \boxed{B=7/3}$$

$$I_1 = \int \frac{3}{x} + \frac{7/3}{x+2} + \frac{-7/3}{x-1} dx$$

$$I_1 = 3 \cdot \ln|x| + \frac{7}{3} \ln|x+2| - \frac{7}{3} \ln|x-1|$$

$$I_1 = \ln \left| \frac{x^3(x+2)}{(x-1)} \right|$$

$$I = \int \left(x-1 + \frac{3}{x} + \frac{7/3}{x+2} + \frac{-7/3}{x-1} \right) dx =$$

$$= \left[\frac{x^2}{2} - x + \ln \left| \frac{x^3(x+2)}{(x-1)} \right| + k \right]$$

edo

$$\left[\frac{x^2}{2} - x + 3 \ln|x| + \frac{7}{3} \ln|x+2| - \frac{7}{3} \ln|x-1| + k \right]$$

16c $\int \frac{\sqrt{x} dx}{\sqrt[3]{x}-1}$

ORDEN KAPEN METODJA
Aldopois bilotzuko
erabileren mkt \rightarrow **6**

$$\int \frac{\sqrt{u^6} \cdot 6 \cdot u^5 du}{\sqrt[3]{u^6} - 1} = \quad \left\{ \begin{array}{l} x = u^6 \\ dx = 6u^5 du \end{array} \right.$$

$$\int \frac{u^3 \cdot 6u^5 du}{u^2 - 1} = \int \frac{6u^8 du}{u^2 - 1} \rightarrow \text{Fu araziora le
p(x)ren maila > Q(x)ren
maila.$$

$$\begin{array}{r} u^8 \\ u^8 \quad + \quad u^6 \\ \hline u^6 \\ -u^6 \quad + u^4 \\ \hline u^4 \\ -u^4 \quad + u^2 \\ \hline u^2 \\ -u^2 \quad + 1 \\ \hline 1 \end{array}$$

$$I = 6 \int u^6 + u^4 + u^2 + 1 + \frac{1}{u^2 - 1} dx.$$

\hookrightarrow Fu araziora le
oro baturak

$$I_1 = \frac{1}{u^2 - 1} = \frac{1}{(u+1)(u-1)} = \frac{A}{u+1} + \frac{B}{u-1}$$

$$1 = A(u-1) + B(u+1)$$

$$u=1 \rightarrow 1 = A \cdot 0 + B \cdot 2 \rightarrow \boxed{B = 1/2}$$

$$u=-1 \rightarrow 1 = -2A + B \cdot 0 \rightarrow \boxed{A = -1/2}$$

$$I_1 = \int \frac{-1/2}{u+1} + \frac{1/2}{u-1} du$$

$$I = 6 \int \left(u^6 + u^4 + u^2 + 1 + \frac{-1/2}{u+1} + \frac{1/2}{u-1} \right) du.$$

$$= 6 \cdot \frac{u^7}{7} + \frac{u^5}{5} + \frac{u^3}{3} + u - \frac{1}{2} \ln|u+1| + \frac{1}{2} \ln|u-1| + k$$

Desegin aldaketa

$$x = u^6 \rightarrow u = \sqrt[6]{x} = x^{1/6}$$

$$I = 6 \left[\frac{x^{7/6}}{7} + \frac{x^{5/6}}{5} + \frac{x^{3/6}}{3} + \frac{x^{1/6}}{1} - \frac{1}{2} \ln|\sqrt[6]{x}+1| + \frac{1}{2} \ln|\sqrt[6]{x}-1| \right] + k$$

$$= 6 \left[\frac{x \sqrt[6]{x}}{7} + \frac{\sqrt[6]{x^5}}{5} + \frac{\sqrt{x}}{3} + \sqrt[6]{x} + \frac{1}{2} \ln \left| \frac{\sqrt[6]{x}-1}{\sqrt[6]{x}+1} \right| \right] + k$$