INTEGRAL ARRAZIONALAK

355.orr) 15 a) $\int \dfrac{1}{x^2+x-6}\,dx = \int \dfrac{1}{(x+3)(x-2)}\,dx$

$\dfrac{1}{x^2+x-6} = \dfrac{A}{x+3} + \dfrac{B}{x-2}$

$\dfrac{1}{x^2+x-6} = \dfrac{A(x-2)+B(x+3)}{(x+3)(x-2)}$

$1 = A(x-2) + B(x+3)$

$x=2 \qquad 1 = B\cdot 5 \qquad \boxed{B = 1/5}$

$x=-3 \qquad 1 = A(-5) \qquad \boxed{A = -1/5}$

$I = \int \dfrac{-1/5}{x+3}\,dx + \int \dfrac{1/5}{x-2}\,dx = \boxed{-\dfrac{1}{5}\,\text{Lu}\,|x+3| + \dfrac{1}{5}\,\text{Lu}\,|x-2| + k}$

b) $\int \dfrac{3x^3}{x^2-4}\,dx = \int \left(3x + \dfrac{12x}{x^2-4}\right)dx = \dfrac{3x^2}{2} +$

$\begin{array}{l} 3x^3 \\ \underline{-3x^3+12x} \\ 12x \end{array} \quad \dfrac{|x^2-4}{3x}$

$= \int 3x\,dx + 12\cdot\dfrac{1}{2}\int \dfrac{2x}{x^2-4}\,dx =$

$\boxed{I = \dfrac{3x^2}{2} + 6\cdot \text{Lu}\,|x^2-4| + k.}$

c) $\int \dfrac{dx}{(x^2-25)(x-4)} = \int \dfrac{dx}{(x+5)(x-5)(x-4)} =$

$\dfrac{1}{(x+5)(x-5)(x-4)} = \dfrac{A}{x+5} + \dfrac{B}{x-5} + \dfrac{C}{x-4}$

$1 = A(x-5)(x-4) + B(x+5)(x-4) + C(x+5)(x-5)$

$x=5 \qquad 1 = B\cdot 10 \cdot \longrightarrow \boxed{B=1/10}$

$x=4 \qquad 1 = C\cdot 9\cdot(-1) \longrightarrow \boxed{C = -1/9}$

$x=-5 \qquad 1 = A(-10)(-9) \qquad \boxed{A = 1/90}$

$I = \int \dfrac{1/90}{x+5}\,dx + \int \dfrac{1/10}{x-5}\,dx - \int \dfrac{1/9}{x-4}\,dx$

$\boxed{I = \dfrac{1}{90}\,\text{Lu}\,|x+5| + \dfrac{1}{10}\,\text{Lu}\,|x-5| - \dfrac{1}{9}\,\text{Lu}\,|x-4| + k}$

351) 14d.) $\int \dfrac{x^2+1}{x^2+x}\,dx$

$$\begin{array}{r}x^2+1 \\ -x^2-x \\ \hline -x+1\end{array} \quad \begin{array}{|l}x^2+x \\ \hline 1\end{array}$$

$I = \int \left(1 + \dfrac{-x+1}{x^2+x}\right)dx.$

$-x+1 = A(x+1) + Bx$

$\dfrac{-x+1}{x^2+x} = \dfrac{A}{x} + \dfrac{B}{x+1}$

$x=0 \quad \boxed{1 = A}$

$x=-1 \quad 1+1 = B(-1)$

$\boxed{B=-2}$

$\dfrac{-x+1}{x^2+x} = \dfrac{A(x+1) + Bx}{x(x+1)}$

$I = \int \left(1 + \dfrac{1}{x} - \dfrac{2}{x+1}\right)dx = \boxed{x + \ln|x| - 2\ln|x+1| + k}$

e/ $\int \dfrac{4}{x^2+x-2}\,dx = \int \dfrac{4}{(x+2)(x-1)}\,dx$

$\dfrac{4}{x^2+x-2} = \dfrac{A}{x+2} + \dfrac{B}{x-1}$

$\rightarrow \quad 4 = A(x-1) + B(x+2)$

$x=1 \rightarrow \quad 4 = 3B \quad \boxed{B = 4/3}$

$x=-2 \rightarrow \quad 4 = -3A \quad \boxed{A = -4/3}$

$I = \int \left(\dfrac{-4/3}{x+2} + \dfrac{4/3}{x-1}\right)dx$

$\boxed{I = -\dfrac{4}{3}\ln|x+2| + \dfrac{4}{3}\ln|x-1| + k}$

d/ $\int \dfrac{x^2}{x^2+4x+3}\,dx.$

$$\begin{array}{r}x^2 \\ -4x-3 \\ \hline \end{array} \quad \begin{array}{|l}x^2+4x+3 \\ \hline 1.\end{array}$$

$\dfrac{x^2}{x^2+4x+3} = 1 + \dfrac{-4x-3}{x^2+4x+3} = 1 + \dfrac{-4x-3}{(x+3)(x+1)}$

$\ast \; \dfrac{-4x-3}{(x+3)(x+1)} = \dfrac{A}{x+3} + \dfrac{B}{x+1}$

$\rightarrow x=-1 \quad 1 = 2B \quad \boxed{B = 1/2}$

$x=-3 \quad 9 = -2A \quad \boxed{A = -9/2}$

$-4x-3 = A(x+1) + B(x+3)$

$I = \int \left(1 + \dfrac{-9/2}{x+3} + \dfrac{1/2}{x+1}\right)dx = \boxed{x - \dfrac{9}{2}\ln|x+3| + \dfrac{1}{2}\ln|x+1| + k}$