Ordezkapen - metodoa

2) a) $\int \sqrt{x^3-3x^2+5}\,(x^2-2x)\,dx$

$x^3-3x^2+5 \longrightarrow 3x^2-6x$
$3(x^2-2x)$

$u = x^3-3x^2+5$

$du = 3(x^2-2x)\,dx$

$= \int \frac{\sqrt{u}}{3}\,du = \int \frac{1}{3}u^{1/2}\,du = \frac{1}{3}\cdot\frac{u^{1/2+1}}{\frac{1}{2}+1}+K =$

$= \frac{1}{3}\,\frac{u^{3/2}}{3/2}+K = \frac{2}{9}\sqrt{u^3} = \boxed{\frac{2}{9}\sqrt{(x^3-3x^2+5)^3}+K}$

b) $\int \frac{1}{\sqrt{1-e^{2\sqrt{x}}}}\,\frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

$2\,du$

$u = e^{\sqrt{x}}$

$du = \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$

$\int \frac{1}{\sqrt{1-u^2}}\cdot 2\,du = 2\arcsin u + K =$

$= 2\arcsin(e^{\sqrt{x}})+K$

c) $\int \frac{\cos^3 x\,dx}{\sin^4 x}$

$u = \sin x$

$du = \cos x\,dx$

$\int \frac{\cos^2 x\cdot \cos x\,dx}{\sin^4 x} = \int \frac{(1-\sin^2 x)\cos x\,dx}{\sin^4 x} = \int \frac{1-u^2}{u^4} =$!!

$\int u^{-4}du - \int u^{-2}du = -\frac{1}{3u^3}+\frac{1}{u}+K = \frac{1}{3\sin^3 x}+\frac{1}{\sin x}+K$

d) $\int (x^2+1)\ln(x^3+3x)\,dx$

$u = x^3+3x \qquad du = (3x^2+3)\,dx$

$du = 3(x^2+1)\,dx$

$\int \frac{\ln u}{3}\,du = \frac{1}{3}(u\cdot\ln u - u)+K =$

$= \frac{1}{3}\left[(x^3+3x)\cdot\ln(x^3+3x) - (x^3+3x)\right] + K$

e) $\int \frac{\sin x \cdot \cos x}{1+\sin^4 x} dx$ $u = \sin^2 x$

$du = \underbrace{2 \sin x \cos x \, dx}$

$\int \frac{du}{(1+u^2) \cdot 2} = \frac{1}{2} \cdot \operatorname{arctg} u + K$

$\frac{1}{2} \operatorname{arctg} (\sin^2 x) + K.$

f) $\int e^{x+\sqrt{x}} \left(\frac{6x+3\sqrt{x}}{x} \right) dx$ $u = x+\sqrt{x}$

$du = \left(1+\frac{1}{2\sqrt{x}}\right) dx.$

$\int e^{x+\sqrt{x}} \left(6+\frac{3}{\sqrt{x}}\right) dx =$ $\boxed{6du = \left(6+\frac{3}{\sqrt{x}}\right) dx}$

$= \int e^u \cdot 6 \, du = 6e^{x+\sqrt{x}} + K$

Beste aldagai aldaketak.

3) $\int \sqrt{x-4} \, (x+5) \, dx$ erroa kentzeko

$x-4 = t^2$ $dx = 2t \, dt$

$x = t^2+4.$

$\int \sqrt{t^2} \, (t^2+4+5) \cdot 2t \, dt = 2\int t^2 (t^2+9) \, dt =$

$= 2 \int (t^4+9t^2) \, dt = \frac{2t^5}{5} + 6t^3 + K$

$= \frac{2\sqrt{(x-4)^5}}{5} + 6\sqrt{(x-4)^3} + K$

$\boxed{4}$ $\displaystyle\int \frac{\sqrt[3]{x-1} + x - 1}{\sqrt{(x-1)^3}}\, dx$

~~Erroa kentzeko $x-1 = t^6$~~

$x - 1 = t^6 \longrightarrow x = t^6 + 1$

$dx = 6t^5\, dt$

$$\int \frac{\sqrt[3]{t^6} + t^6 + \cancel{1} - \cancel{1}}{\sqrt{(t^6)^3}}\, 6t^5\, dt = \int \frac{t^2 + t^6}{t^9}\, 6t^5 dt =$$

$$= 6\int \frac{t^7 + t^{11}}{t^9}\, dt =$$

$$6\int (t^{-2} + t^2)\, dt = 6\left(\frac{-1}{t} + \frac{t^3}{3}\right) =$$

$$= -\frac{6}{t} + 2t^3 + K = \frac{-6}{\sqrt[6]{x-1}} + 2\sqrt[6]{(x-1)^3} + K$$

$\boxed{5}$ $\displaystyle\int \sqrt{4 - x^2}\, dx = \int \sqrt{4\left(1 - \left(\frac{x}{2}\right)^2\right)}\, dx =$

$$= 2\int \sqrt{1 - \left(\frac{x}{2}\right)^2}\, dx = \qquad \frac{x}{2} = \sin\alpha \qquad \alpha = \arcsin\left(\frac{x}{2}\right)$$

$$= 2\int \underbrace{\sqrt{1 - \sin^2\alpha}}\; 2\cdot\cos\alpha\, d\alpha = \qquad \frac{1}{2}\, dx = \cos\alpha\, d\alpha$$

$$= 4\cdot\int \cos\alpha\cdot\cos\alpha\, d\alpha = 4\cdot\int \cos^2\alpha\, d\alpha =$$

$$= 4\int \frac{1 + \cos 2\alpha}{2}\, d\alpha = 2\alpha + \sin 2\alpha + K$$

$$= 2\arcsin\left(\frac{x}{2}\right) + \sin\left(2\arcsin\left(\frac{x}{2}\right)\right) + K$$