EJERCICIO RESUELTO

Calcula las siguientes integrales:

a) $\displaystyle\int \frac{x^4 - 3x^2 + 2x - 2}{x - 1}\,dx$ **b)** $\displaystyle\int \frac{\sqrt[3]{x} + \sqrt{2x^2}}{\sqrt[4]{x}}\,dx$ **c)** $\displaystyle\int \frac{2sen\,x + 3e^x}{5}\,dx$

RESOLUCIÓN

a) Efectuamos la división y expresamos el resultado de la forma $\dfrac{D}{d} = C + \dfrac{R}{d}$:

$$\int \frac{x^4 - 3x^2 + 2x - 2}{x - 1}\,dx = \int\left(x^3 + x^2 - 2x + \frac{-2}{x-1}\right)dx = \frac{x^4}{4} + \frac{x^3}{3} - x^2 - 2ln\,|x-1| + k$$

b) $$\int \frac{\sqrt[3]{x} + \sqrt{2x^2}}{\sqrt[4]{x}}\,dx = \int\left(\frac{\sqrt[3]{x}}{\sqrt[4]{x}} + \frac{\sqrt{2x^2}}{\sqrt[4]{x}}\right)dx = \int\left(x^{\frac{1}{3}-\frac{1}{4}} + \sqrt{2}\,x^{1-\frac{1}{4}}\right)dx =$$

$$= \int\left(x^{\frac{1}{12}} + \sqrt{2}\,x^{\frac{3}{4}}\right)dx = \frac{x^{13/12}}{13/12} + \sqrt{2}\,\frac{x^{7/4}}{7/4} + k = \frac{12\,\sqrt[12]{x^{13}}}{13} + \frac{4\sqrt{2}\cdot\sqrt[4]{x^7}}{7} + k$$

c) $$\int \frac{2sen\,x + 3e^x}{5}\,dx = \frac{-2cos\,x + 3e^x}{5} + k$$

Calcula las siguientes integrales:

1 a) $\displaystyle\int (x^4 - 3x^2 + 2x - 1)\,dx$ b) $\displaystyle\int (x^3 - 2x)\,dx$

2 a) $\displaystyle\int \left(\frac{3}{4}x^5 - \frac{2}{3}x^2 + \frac{1}{7}\right)dx$ b) $\displaystyle\int x^3\,(x+5)\,dx = \int (x^4 + 5x^3)\,dx$

3 a) $\displaystyle\int (x-2)(x^2+4x)\,dx$ b) $\displaystyle\int (x^2-3)^2\,dx = \int (x^4 - 6x^2 + 9)\,dx$

4 a) $\displaystyle\int (2x^2-3)^2\,dx$ b) $\displaystyle\int \frac{\sqrt[5]{x} - \sqrt[3]{x^2}}{\sqrt{x}}\,dx$

5 a) $\displaystyle\int \frac{x^2 - \sqrt{x}}{2x}\,dx$ b) $\displaystyle\int \left(\frac{1}{2}e^x - \frac{3}{4}cos\,x\right)dx$

6 a) $\displaystyle\int \frac{3}{cos^2 x}\,dx$ b) $\displaystyle\int \frac{3sen\,x + 2^x}{4}\,dx$

7 a) $\displaystyle\int \frac{3}{1+x^2}\,dx$ b) $\displaystyle\int \frac{e^x + tg\,x}{3}\,dx$

8 a) $\displaystyle\int \left(\frac{3}{x} + \frac{x^3}{3} + \frac{2}{x^4}\right)dx$ b) $\displaystyle\int \left(3^x + \frac{1}{x^3} - \frac{2}{x}\right)dx$

EJERCICIO RESUELTO

Calcula estas integrales:

a) $\int x\sqrt{1+x^2}\,dx$ ***b)*** $\int \frac{2\,dx}{x\ln x}$ ***c)*** $\int \frac{5x+3}{x^2+1}\,dx$

RESOLUCIÓN

a) $\int x\sqrt{1+x^2}\,dx = \frac{1}{2}\int 2x\,(1+x^2)^{1/2}\,dx = \frac{1}{2}\,\frac{(1+x^2)^{3/2}}{3/2} + k = \frac{\sqrt{(1+x^2)^3}}{3} + k$

b) $\int \frac{2\,dx}{x\ln x} = 2\int \frac{1/x}{\ln x}\,dx = 2\ln|\ln x| + k$

c) $\int \frac{5x+3}{x^2+1}\,dx = \int\left(\frac{5x}{x^2+1} + \frac{3}{x^2+1}\right)dx = \frac{5}{2}\int\frac{2x}{x^2+1}\,dx + 3\int\frac{1}{1+x^2}\,dx =$

$= \frac{5}{2}\ln|x^2+1|\ 3arc\,tg\,x + k$

Halla las siguientes integrales:

9 a) $\int 2x\,(x^2+1)^8\,dx$ b) $\int x\sqrt{x^2-1}\,dx$

10 a) $\int x\sqrt{1-3x^2}\,dx$ b) $\int \cos x\cdot(\operatorname{sen} x)^5\,dx$

11 a) $\int \operatorname{sen} x\cos^4 x\,dx$ b) $\int \frac{2x}{x^2+5}\,dx$

12 a) $\int \frac{x}{3x^2-2}\,dx$ b) $\int \frac{12x^2-4x}{4x^3-2x^2+1}\,dx$

13 a) $\int \frac{3x^2-x}{4x^3-2x^2+1}\,dx$ b) $\int \frac{\cos x}{\operatorname{sen}^6 x}\,dx$

14 a) $\int \frac{\cos x}{\operatorname{sen} x}\,dx$ b) $\int e^{6x+5}\,dx$

15 a) $\int 2x\,e^{x^2-3}\,dx$ b) $\int (x-1)\,e^{3x^2-6x}\,dx$

16 a) $\int \frac{e^{tg\,x}}{\cos^2 x}\,dx$ b) $\int (6x^3-1)\cos(3x^4-2x)\,dx$

17 a) $\int (2^{5x} - tg\,x)\,dx$ b) $\int 3\cos x\,e^{\operatorname{sen} x}\,dx$

18 a) $\int \frac{e^{2x}}{e^{2x}+3}\,dx$ b) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

BEREHALAKO INTEGRALAK

9 a) $\int 2x\cdot(x^2+1)^8\,dx = \frac{1}{9}\int 9\cdot 2x(x^2+1)^8\,dx$

$(x^2+1)^9 \longrightarrow 9(x^2+1)\cdot 2x$

$= \boxed{\frac{1}{9}\cdot(x^2+1)^9+k}$

b) $\int x\cdot\sqrt{x^2-1}\,dx = \frac{1}{3}\int 3x\cdot\sqrt{x^2-1}\,dx =$

$(x^2-1)^{1/2+1} \longrightarrow \frac{3}{\cancel{2}}(x^2-1)^{1/2}\cdot\cancel{2}x$

$= \frac{1}{3}(x^2-1)^{3/2}+k =$

$= \boxed{\frac{1}{3}\sqrt{(x^2-1)^3}+k}$

10 a) $\int x\cdot\sqrt{1-3x^2}\,dx = \frac{1}{(-9)}\int(-9)x\sqrt{1-3x^2}\,dx =$

$(1-3x^2)^{3/2} \longrightarrow \frac{3}{2}\cdot(1-3x^2)^{1/2}(-6x)$

$-9x$

$= -\frac{1}{9}(1-3x^2)^{3/2}+k =$

$= \boxed{-\frac{1}{9}\sqrt{(1-3x^2)^3}+k}$

b) $\int \cos x\cdot(\sin x)^5\,dx$

$(\sin x)^6 \longrightarrow 6(\sin x)^5\cdot\cos x$

$= \frac{1}{6}\int 6\cos x\cdot(\sin x)^5\cdot dx = \boxed{\frac{1}{6}\sin^6 x+k}$

11 a) $\int \sin x\cdot\cos^4 x\,dx.$

$\cos^5 x \longrightarrow -5\cdot\cos^4 x\cdot\sin x$

$= \frac{1}{5}\int -5\sin x\cdot\cos^4 x\cdot dx = \boxed{-\frac{1}{5}\cos^5 x+k}$

b) $\int \frac{2x}{x^2+5}\,dx = \boxed{\ln|x^2+5|+k}$

12) a) $\int \frac{x}{3x^2-2}\,dx =$ $\ln(3x^2-2) \rightarrow \frac{6x}{3x^2-2}$

$= \frac{1}{6}\int \frac{6x}{3x^2-2}\,dx = \boxed{\frac{1}{6}\ln|3x^2-2|+K}$

b) $\int \frac{12x^2-4x}{4x^3-2x^2+1}\,dx = \boxed{\ln|4x^3-2x^2+1|+K}$

13) a) $\frac{1}{4}\int \frac{4\cdot 3x^2-x}{4x^3-2x^2+1}\,dx$ $\ln(4x^3-2x^2+1) \rightarrow \frac{12x^2-4x}{4x^3-2x^2+1}$

$= \boxed{\ln|4x^3-2x^2+1|+K}$

b) $\int \frac{\cos x}{\sin^6 x}\,dx = \int \frac{\cancel{\cos x}}{t^6}\cdot\frac{dt}{\cancel{\cos x}}$!! $\cancel{\ln(\sin^6 x) \rightarrow \frac{6\sin^5 x\cos x}{\sin^6 x}}$

!! ORDEZKAPEN METODOA

$\sin x = t$

$\cos x\,dx = dt$

$dx = dt/\cos x.$

$= \int \frac{dt}{t^6} = \int t^{-6}\,dt = \frac{t^{-5}}{-5}+K$

$= \boxed{\frac{-1}{5\sin^5 x}+K}$

14) a) $\int \frac{\cos x}{\sin x}\,dx = \boxed{\ln|\sin x|+K}$

b) $\int e^{6x+5}\,dx = \frac{1}{6}\int 6 e^{6x+5}\,dx =$ $e^{6x+5} \rightarrow 6\cdot e^{6x+5}$

$= \boxed{\frac{1}{6}e^{6x+5}+K}$

15) a) $\int 2x\,e^{x^2-3}\,dx = \boxed{e^{x^2-3}+K}$ $e^{x^2-3} \rightarrow 2x\,e^{x^2-3}$

b) $\int (x-1)\cdot e^{3x^2-6x}\,dx =$ $e^{3x^2-6x} \rightarrow (6x-6)e^{3x^2-6x}$

$6\cdot(x-1)\,e^{3x^2-6x}$

$= \frac{1}{6}\int 6(x-1)\cdot e^{3x^2-6x}\,dx = \boxed{\frac{1}{6}e^{3x^2-6x}+K}$

16) a) $\int \frac{e^{tg x}}{\cos^2 x}\,dx = \boxed{e^{tg x} + k}$

$e^{tg x} \longrightarrow \frac{e^{tg x}}{\cos^2 x}$

b) $\int (6x^3-1)\cos(3x^4-2x)\,dx$

$\sin(3x^4-2x) \longrightarrow \cos(3x^4-2x)(12x^3-2)$
$2(6x^3-1)$

$= \frac{1}{2}\int 2(6x^3-1)\cdot\cos(3x^4-2x)\,dx = \boxed{\frac{1}{2}\sin(3x^4-2x) + k}$

17) a) $\int (2^{5x} - tg x)\,dx$

$2^{5x} \longrightarrow 2^{5x}\ln 5.$
$\ln\cos x \longrightarrow \frac{-\sin x}{\cos x}$

$= \frac{1}{\ln 5}\int \ln 5\, 2^{5x}\,dx - (-)\int (-)\frac{\sin x}{\cos x}\,dx = \boxed{\frac{1}{\ln 5} + \ln(\cos x) + k.}$

b) $\int 3\cdot\cos x\cdot e^{\sin x}\,dx$

$e^{\sin x} \longrightarrow e^{\sin x}\cdot\cos x.$

$= 3\int \cos x\, e^{\sin x}\,dx = \boxed{3\cdot e^{\sin x} + k.}$

$\ln(e^{2x}+3) \Longrightarrow \frac{2\cdot e^{2x}}{e^{2x}+3}$

18) a) $\int \frac{e^{2x}}{e^{2x}+3}\,dx =$

$= \frac{1}{2}\int \frac{2e^{2x}}{e^{2x}+3}\,dx = \boxed{\frac{1}{2}\ln(e^{2x}+3) + k.}$

b) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx =$

$e^{\sqrt{x}} \longrightarrow \frac{e^{\sqrt{x}}}{2\sqrt{x}}.$

$= 2\int \frac{e^{\sqrt{x}}}{2\cdot\sqrt{x}} = \boxed{2\cdot e^{\sqrt{x}} + k.}$