345/ 3a) $I = \int \frac{5x-3}{x^3-x}\,dx$

Erroak
$x_1 = 0$
$x_2 = 1$
$x_3 = -1$

$x^3 - x = x(x^2-1) = x(x+1)(x-1)$

$$\frac{5x-3}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$$

$5x-3 = A(x+1)(x-1) + B\,x\cdot(x-1) + C(x+1)x.$

$x=0 \longrightarrow -3 = A(-1) + B\cdot 0 + C\cdot 0 \longrightarrow \boxed{A=3}$

$x=1 \longrightarrow 2 = A\cdot 0 + B\cdot 0 + C\,2 \longrightarrow \boxed{C=1}$

$x=-1 \longrightarrow -8 = A\cdot 0 + 2B + C\cdot 0 \longrightarrow \boxed{B=-4}$

$$I = \int \left(\frac{3}{x} + \frac{-4}{x+1} + \frac{1}{x-1} \right) dx =$$

$I = 3\ln|x| - 4\ln|x+1| + \ln|x-1| + k$

$= \ln\left| \frac{x^3(x-1)}{(x+1)^4} \right| + k.$

345 | 36.) $I = \int \frac{x^2-2x+6}{(x-1)^3}\,dx$ $\quad x-1=0$, $x=1$ hiruk.

$$\frac{x^2-2x+6}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$$

$$x^2-2x+6 = A(x-1)^2 + B(x-1) + C$$

$x=1 \longrightarrow 1-2+6 = \cancel{A\cdot 0} + \cancel{B\cdot 0} + C \longrightarrow \boxed{C=5}$

$x=0 \longrightarrow 6 = A + B(-1) + 5 \longrightarrow 1 = A - B$

$x=2 \longrightarrow 6 = A + B + 5 \longrightarrow 1 = A + B$

$2 = 2A$

$\boxed{A=1}$

$\boxed{B=0}$

$$I = \int \left(\frac{1}{x-1} + \frac{5}{(x-1)^3}\right) dx =$$

$$* \int \frac{1}{(x-1)^3}\,dx = \int (x-1)^{-3}\,dx = \frac{(x-1)^{-2}}{-2} = \frac{-1}{2(x-1)^2}$$

$$\boxed{I = \ln|x-1| - \frac{5}{2}\cdot\frac{1}{(x-1)^2} + k}$$