335

4.

a) $\int \sin^2 x \, dx =$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$-\left(1 = \cos^2 x + \sin^2 x\right)$$
$$\cos 2x - 1 = -2\sin^2 x \Rightarrow \boxed{\sin^2 x = \frac{1-\cos 2x}{2}}$$

$$\int \sin^2 x \, dx = \int \frac{1-\cos 2x}{2} dx = \frac{1}{2}x - \frac{1}{2}\frac{1}{2}\sin 2x + K$$
$$= \frac{1}{2}x - \frac{1}{4}\sin 2x + K.$$

b) $\displaystyle\int \frac{dx}{1+9x^2} = \int \frac{dx}{1+(3x)^2} = \frac{1}{3}\,\mathrm{arctg}(3x) + K$

c) $\displaystyle\int \frac{dx}{1+8x^2} = \int \frac{dx}{1+(\sqrt{8}x)^2} = \frac{1}{\sqrt{8}}\,\mathrm{arctg}(\sqrt{8}x) + K$

d) $\displaystyle\int \frac{dx}{25+9x^2} = \int \frac{dx}{25\left(1+\left(\frac{3x}{5}\right)^2\right)} = \frac{1}{25}\cdot\frac{5}{3}\,\mathrm{arctg}\,\frac{3x}{5} + K$

$$= \frac{1}{15}\,\mathrm{arctg}\left(\frac{3x}{5}\right) + K$$

e) $\displaystyle\int \frac{dx}{3+2x^2} = \int \frac{dx}{3\left(1+\frac{2x^2}{3}\right)} = \int \frac{dx}{3\left(1+\left(\sqrt{\frac{2}{3}}x\right)^2\right)} =$

$$= \frac{1}{3}\frac{1}{\sqrt{\frac{2}{3}}}\,\mathrm{arctg}\,\sqrt{\frac{2}{3}}x + K = \frac{1}{\sqrt{6}}\,\mathrm{arctg}\sqrt{\frac{2}{3}}x + K$$

$$\frac{\sqrt{3}}{3\sqrt{2}}$$

f) $\displaystyle\int \frac{dx}{\sqrt{1-9x^2}} = \int \frac{dx}{\sqrt{1-(3x)^2}} = \frac{1}{3} \arcsin(3x) + K$

g) $\displaystyle\int \frac{dx}{\sqrt{1-8x^2}} = \int \frac{dx}{1-(\sqrt{8}x)^2} = \frac{1}{\sqrt{8}} \arcsin(\sqrt{8}x) + K$

h) $\displaystyle\int \frac{dx}{\sqrt{25-9x^2}} = \int \frac{dx}{5\sqrt{1-(\frac{3}{5}x)^2}} = \frac{1}{5} \cdot \frac{1}{\frac{3}{5}} \arcsin\left(\frac{3}{5}x\right) + K$

$\sqrt{25-9x^2} = \sqrt{25(1-\frac{9}{25}x^2)} = 5\sqrt{1-(\frac{3}{5}x)^2}$

$\displaystyle = \frac{1}{3} \arcsin\left(\frac{3x}{5}\right) + K.$

i) $\displaystyle\int \frac{dx}{\sqrt{3-2x^2}} = \int \frac{1}{\sqrt{3}} \cdot \frac{dx}{1-(\sqrt{\frac{2}{3}}x)^2} = \frac{1}{\cancel{\sqrt{3}}} \frac{1}{\frac{\sqrt{2}}{\cancel{\sqrt{3}}}} \arcsin \sqrt{\frac{2}{3}}x + K$

$\sqrt{3(1-\frac{2}{3}x^2} = \sqrt{3}\,\sqrt{1-(\sqrt{\frac{2}{3}}x)^2}$

$\displaystyle = \frac{1}{\sqrt{2}} \arcsin\left(\sqrt{\frac{2}{3}}x\right) + K.$

d) $\displaystyle\int e^{5x-2}\,dx = \frac{1}{5} e^{5x-2} + K$