355.orr) 16

a) $\int \frac{2x^2-5x+3}{x^2-3x+2}\,dx = \int \left(2 + \frac{x-1}{x^2-3x+2}\right)dx$

1) Zatikato epiu:

$2x^2-5x+3 \quad | x^2-3x+2$

$-2x^2\ 6x\ -4 \quad 2.$

$/\ x-1.$

$x^2-3x+2=0$

$x = \frac{3 \pm \sqrt{3^2-4\cdot 2}}{2} = \frac{3\pm 1}{2} \begin{matrix} 2 \\ 1 \end{matrix}$

$= \int \left(2 + \frac{x-1}{(x-1)(x-2)}\right)dx = \int \left(2 + \frac{1}{x-2}\right)dx =$

$= \boxed{2x + \ln|x-2| + k}$

b) $\int \frac{-16}{x^2-2x-15}\,dx = \int \frac{-16}{(x-5)(x+3)}\,dx.$

$x^2-2x-15=0$

$x = \frac{2 \pm \sqrt{2^2-4\cdot(-15)}}{2} = \begin{matrix} 5 \\ -3 \end{matrix}$

$\frac{-16}{(x-5)(x+3)} = \frac{A}{x-5} + \frac{B}{x+3} = \frac{A(x+3)+B(x-5)}{(x-5)(x+3)}$

$-16 = A(x+3) + B(x-5)$

$x=-3 \rightarrow -16 = B(-3-5) \rightarrow \boxed{B=2}$

$x=5 \rightarrow -16 = A(5+3) \rightarrow \boxed{A=-2}$

$I = \int \left(\frac{2}{x-5} - \frac{2}{x+3}\right)dx = \boxed{2\cdot\ln|x-5| - 2\cdot\ln|x+3| + k}$

c) $\int \frac{2x-4}{(x-1)^2(x+3)}\,dx \qquad \frac{2x-4}{(x-1)^2(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3}$

$2x-4 = A(x-1)(x+3) + B(x+3) + C(x-1)^2$

$x=1 \rightarrow 2-4 = B(4) \rightarrow \boxed{B=-1/2.}$

$x=-3 \rightarrow -6-4 = C(-4)^2 \rightarrow C = -10/16 \Rightarrow \boxed{-5/8 = C}$

$x=0 \rightarrow -4 = A(-1)(3) + -1/2\cdot 3 + -5/8 \rightarrow \boxed{A=5/8}$

$$I = \int \left(\frac{5/8}{x-1} + \frac{-1/2}{(x-1)^2} + \frac{-5/8}{x+3} \right) dx$$

$$= \frac{5}{8} \ln|x-1| - \frac{1}{2} \frac{(x-1)^{-2+1}}{-2+1} - \frac{5}{8} \ln|x+3| + k$$

$$= \frac{5}{8} \ln\left|\frac{x-1}{x+3}\right| + \frac{1}{2} (x-1)^{-1} + k$$

$$\boxed{I = \frac{5}{8} \ln\left|\frac{x-1}{x+3}\right| + \frac{1}{2} \frac{1}{(x-1)} + k}$$

d) $$\int \frac{2x+3}{(x-2)(x+5)} dx$$

$$\frac{2x+3}{(x-2)(x+5)} = \frac{A}{x-2} + \frac{B}{x+5} = \frac{A(x+5) + B(x-2)}{(x-2)(x+5)}$$

$x = -5 \longrightarrow 2(-5)+3 = B(-5-2) \Rightarrow -7 = B(-7)$ $\boxed{B=1}$

$x = 2 \longrightarrow 2 \cdot 2 + 3 = A(2+5) \longrightarrow \boxed{A=1}$

$$\int \frac{2x+3}{(x-2)(x+5)} dx = \int \frac{1}{x-2} dx + \int \frac{1}{x+5} dx =$$

$$= \boxed{\ln|x-2| + \ln|x+5| + k}$$

$$\Rightarrow \boxed{\ln|(x-2)\cdot(x+5)| + k}$$

e) $$\int \frac{1}{(x-1)(x+3)^2} dx \Rightarrow \frac{1}{(x-1)(x+3)^2} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$$

$$\frac{1}{(x-1)(x+3)^2} = \frac{A(x+3)^2 + B(x+3) + C(x-1)}{(x-1)(x+3)^2}$$

$x = -3 \longrightarrow 1 = -4C \longrightarrow \boxed{C = -1/4}$

$x = 1 \longrightarrow 1 = 16A + 4B$

$x = 0 \longrightarrow 1 = 9A + 3B + 1/4$

$$\left.\begin{array}{l} 16A + 4B = 1 \\ 9A + 3B = 3/4 \end{array}\right\} \Rightarrow \boxed{\begin{array}{l} A = 1/16 \\ B = -1/16 \end{array}}$$

$$I = \int \frac{1/16}{x-1}\,dx - \int \frac{1/16}{x+3}\,dx - \int \frac{1/4}{(x+3)^2}\,dx \qquad (x+3)^{-2}$$

$$= \frac{1}{16}\ln|x-1| - \frac{1}{16}\ln|x+3| - \frac{1}{4}\frac{(x+3)^{-2+1}}{-2+1} + k$$

$$-1$$

$$= \boxed{\frac{1}{16}\ln\left|\frac{x-1}{x+3}\right| + \frac{1}{4(x+3)} + k}$$

f) $\int \frac{3x-2}{x^2-4}\,dx$

$$\frac{3x-2}{x^2-4} = \frac{A}{x+2} + \frac{B}{x-2} = \frac{A(x-2) + B(x+2)}{(x+2)(x-2)}$$

$$3x-2 = A(x-2) + B(x+2)$$

$x=2 \rightarrow \quad 4 = 4B \rightarrow \boxed{B=1}$

$x=-2 \rightarrow \quad -8 = -4A \rightarrow \boxed{A=2}$

$$\int \frac{3x-2}{x^2-4}\,dx = \int \frac{2}{x+2}\,dx + \int \frac{1}{x-2}\,dx =$$

$$= 2\ln|x+2| + \ln|x-2| + k$$

$$= \boxed{\ln|(x+2)^2(x-2)| + k}$$