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a) $\int x\sqrt{x+1}\,dx = \int (t^2-1)\sqrt{t^2}\,2t\,dt =$

$= \int (t^2-1)\cdot t\cdot 2t\,dt = \int (2t^4-2t^2)\,dt$

$= \frac{2}{5}t^5 - \frac{2}{3}t^3 + K.$

$= \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + K$

$\boxed{= \frac{2}{5}\sqrt{(x+1)^5} - \frac{2}{3}\sqrt{(x+1)^3} + K.}$

Erroo kentzeko

$x+1 = t^2$

$dx = 2t\,dt$

$x = t^2-1$

$t = \sqrt{x+1}$

$t = (x+1)^{1/2}$

b) $\int \frac{dx}{x-\sqrt[4]{x}} = \int \frac{4t^3\,dt}{t^4-t} = \int \frac{4t^3\,dt}{t(t^3-1)} = \int \frac{4t^2\,dt}{t^3-1}$

$= 4\frac{1}{3}\int \frac{3t^2\,dt}{t^3-1} = \frac{4}{3}\ln|t^3-1| + K$

$= \frac{4}{3}\ln|x^{3/4}-1| + K$

$\boxed{I = \frac{4}{3}\ln|\sqrt[4]{x^3}-1| + K}$

Erroa kentzeko

$x = t^4$

$dx = 4t^3\,dt.$

$t = x^{1/4}$

c) $\int \frac{x}{\sqrt{x+1}}\,dx = \int \frac{(t^2-1)\,2t\,dt}{\sqrt{t^2}} = \int \frac{2\cancel{t}(t^2-1)\,dt}{\cancel{t}}$

$= 2\int (t^2-1)\,dt = 2\left(\frac{t^3}{3} - t\right) + K.$

$= 2\left(\frac{(x+1)^{3/2}}{3} - \sqrt{x+1}\right) + K.$

$\boxed{I = \frac{2}{3}\sqrt{(x+1)^3} - 2\sqrt{x+1} + K}$

Erroo kentzeko.

$x+1 = t^2$

$dx = 2t\,dt.$

$x = t^2-1.$

$t = \sqrt{x+1} = (x+1)^{1/2}$

d) $\int \frac{1}{x\sqrt{x+1}}\,dx = \int \frac{1}{(t^2-1)\sqrt{t^2}}\,2t\,dt =$

Erroa kentzeko:

$x+1=t^2$

$dx = 2t\,dt$

$x = t^2-1$

$$\int \frac{2\cancel{t}\,dt}{(t^2-1)\cdot\cancel{t}} = 2\underbrace{\int \frac{dt}{t^2-1}}_{I_1}$$

$$I_1 = \frac{1}{t^2-1} = \frac{A}{t+1} + \frac{B}{t-1} = \frac{A(t-1)+B(t+1)}{(t+1)(t-1)}$$

$1 = A(t-1) + B(t+1)$

$t=1 \longrightarrow 1 = 2B \longrightarrow B = 1/2$

$t=-1 \longrightarrow 1 = -2A \longrightarrow A = -1/2$

$$I = 2\cdot\int\left(\frac{1/2}{t+1} - \frac{1/2}{t-1}\right)dt = 2\left[\frac{1}{2}\ln|t+1| - \frac{1}{2}\ln|t-1|\right] + K$$

$$= \ln|t+1| - \ln|t-1| + K = \ln\left|\frac{t+1}{t-1}\right| + K$$

$$= \boxed{\ln\left|\frac{\sqrt{x+1}+1}{\sqrt{x+1}-1}\right| + K}$$

e) $\int \frac{1}{x+\sqrt{x}}\,dx = \int \frac{1}{t^2+t}\,2t\,dt = 2\int \frac{\cancel{t}}{\cancel{t}(t+1)}\,dt$

Erroa kentzeko

$\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

$$= 2\int \frac{1}{t+1}\,dt = 2\ln|t+1| + K$$

$$= \boxed{2\ln|\sqrt{x}+1| + K}$$

f) $\int \frac{\sqrt{x}}{1+x}\,dx = \int \frac{t}{1+t^2}\,2t\,dt =$

Errso kentzeko

$\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

$= 2\int \frac{t^2}{1+t^2}\,dt = 2\int \left(1 - \frac{1}{1+t^2}\right)dt$

$$\begin{array}{r|l} t^2 & t^2+1 \\ \hline -t^2-1 & 1 \\ \hline -1 & \end{array}$$

$= 2t - 2\,\mathrm{arctg}(t) + k =$

$= \boxed{2\sqrt{x} - 2\,\mathrm{arctg}(\sqrt{x}) + k}$