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Introduction to Signals and Systems HW-2

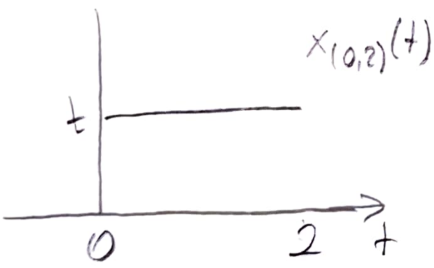
1)

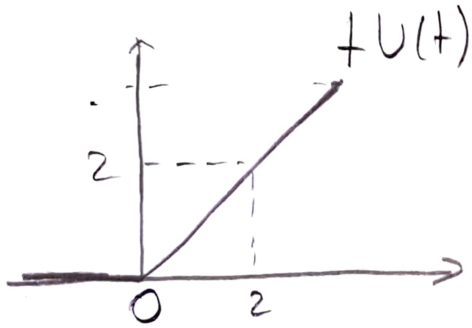
$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \qquad \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

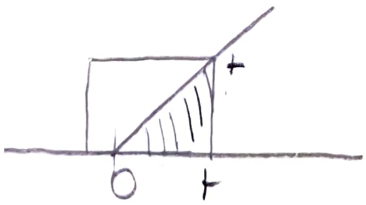
$$\sin(i) = \frac{\frac{1}{e} - e}{2i} \qquad \cos(i) = \left(\frac{1}{e} + e\right)\cdot\frac{1}{2} \qquad \tan(i) = \frac{\frac{e^{-1} - e}{2i}}{\frac{e^{-1}+e}{2}} = \frac{(e^{-1} - e)}{\cancel{2}i}\cdot\frac{\cancel{2}}{(e^{-1}+e)} = \boxed{\frac{e^{-1} - e}{(e^{-1}+e)\cdot i}}$$

2)



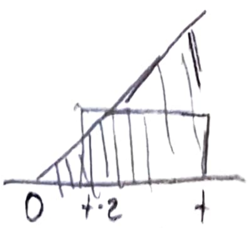


- $t < 0 \qquad y(t) = 0$
- $0 < t < 2$



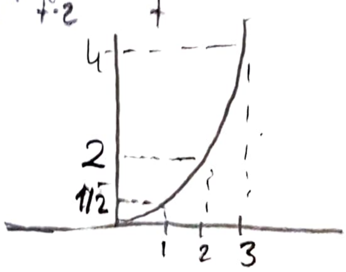
$$y(t) = \frac{t \cdot t}{2} = \frac{t^2}{2}$$

- $2 < t$



$$y(t) = \frac{t^2}{2} - \frac{(t-2)^2}{2} = 2t - 2$$

Result Plot =



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3) The result is zero between $-\infty, 0$

$$\int_0^{\infty} t e^{-(3+jw)t} \, dt$$

Integrated by parts $\quad f = t \quad g' = e^{-(jw+3)t}$

$$f' = 1 \quad g = \frac{e^{-(jw+3)t}}{jw+3}$$

$$= \frac{t e^{-(jw+3)t}}{jw+3} - \int - \frac{e^{(-jw-3)t}}{jw+3} \, dt$$

$$u = (-jw-3)t \quad \frac{du}{dt} = -jw-3 \quad dt = \frac{1}{-jw-3}$$

$$= \frac{1}{(jw+3)^2} \int e^u \, du = \frac{e^u}{(jw+3)^2} = \frac{e^{(-jw-3)t}}{(jw+3)^2}$$

$$\Rightarrow \frac{t e^{-(jw+3)t}}{jw+3} - \frac{e^{(-jw-3)t}}{(jw+3)^2} + C$$

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4) $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(w)\, e^{jwt}\, dw$

$$\frac{d\,x(t)}{dt} = \frac{1}{2\pi}\frac{d}{dt}\int_{-\infty}^{\infty} x(w)\, e^{jwt}\, dw$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} x(w)\, jw e^{jwt}\, dw$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} jw x(w)\, e^{jwt}\, dw$$

As a result the Fourier transform of $\frac{d}{dt} f(t)$ is $jw F(w)$. We can say that the Fourier Transfom has differentiation property.

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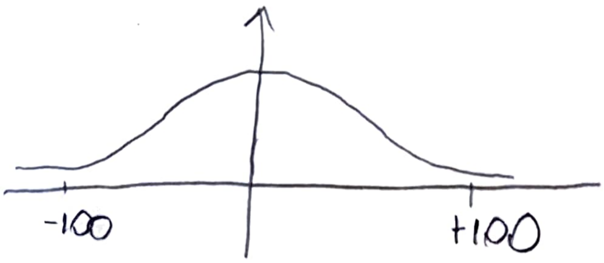
5)



$$\frac{1}{f_{samp}} \geq \frac{\pi}{2\pi f_{max}} \qquad f_{samp} \geq 2\underbrace{f_{max}}_{100}$$

Our lowest sampling rate without aliasing should be 200 because of the Nyquist Relation.

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6)

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 4 & 1 & 0 \\ 0 & 2 & 0 & 4 & 3 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \Longrightarrow \begin{matrix} \underline{2} & 10 & 11 & 6 \\ 7 & 7 & 13 & 11 \\ 2 & 13 & 7 & 10 \\ 3 & 2 & 10 & 5 \end{matrix}$$

<u>Reflection</u>

$$\begin{matrix} 1 & 0 & 1 \\ 0 & \underline{2} & 0 \\ 1 & 0 & 1 \end{matrix}$$

1 · 2
2 · 2+4+1=7
3 · 2
4 · 1+2=3
5 · 4+2+4=10
6 · 1+4+1+1=7
7 · 2+4+2+1+4=13
8 · 1+1=2
9 · 8+3=11
10 · 2+1+8+1+1=13
11 · 3+2+2=7
12 · 1+1+8=10
13 · 2+4=6
14 · 4+6+1=11
15 · 4+2+4=10
16 · 1+4=5