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1) $\cdot\ 3x_{[4,5]}(t) = 3x_{[0,1]}(t+4)$

$\cdot\ 7x_{[8,9]}(t) = 7x_{[0,1]}(t+8)$

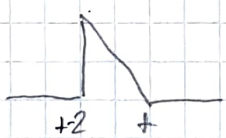
$x_{[0,1]}(t) \rightarrow \boxed{T} \longrightarrow e^{-2t}u(t)$

$\cdot\ 3\left(e^{-2(t+4)}\right)u(t+4) + 7\left(e^{-2(t+8)}\right)u(t+8)$

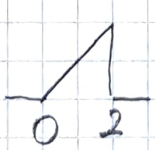
if $\begin{cases} t \leq -8 & 0 \\ -8 < t \leq -4 & 7\left(e^{-2(t+8)}\right) \\ -4 < t & 3e^{-2(t+4)} + 7\left(e^{-2(t+8)}\right) \end{cases}$

2) $tx_{[0,2]} * tx_{[0,2]}$

$x(t-\tau)$:

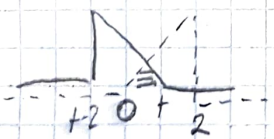


$x(\tau)$:

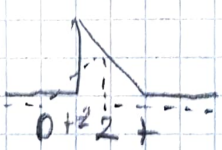


$\cdot\ t \leq 0$

$\cdot\ 0 < t < 2$



$\cdot\ 2 < t < 4$



$\cdot\ t > 4 \Rightarrow 0$

$0 < t < 2$

$$\int_0^t (t\tau - \tau^2)\,d\tau = \left(\frac{t\tau^2}{2} - \frac{\tau^3}{3}\right)\Big]_0^t$$

$$\Rightarrow \frac{t^3}{2} - \frac{t^3}{3}$$

$2 < t < 4$

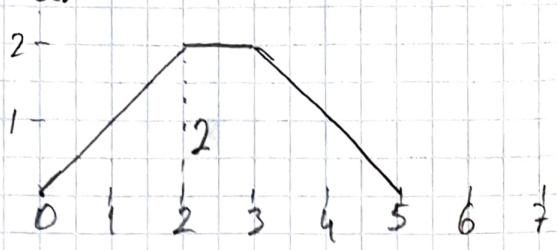
$$\int_2^t (t\tau - \tau^2)\,d\tau = \left(\frac{t\tau^2}{2} - \frac{\tau^3}{3}\right)\Big]_2^t$$

$$\left(\frac{t^3}{2} - \frac{t^3}{3}\right) - \left(2t - \frac{8}{3}\right)$$

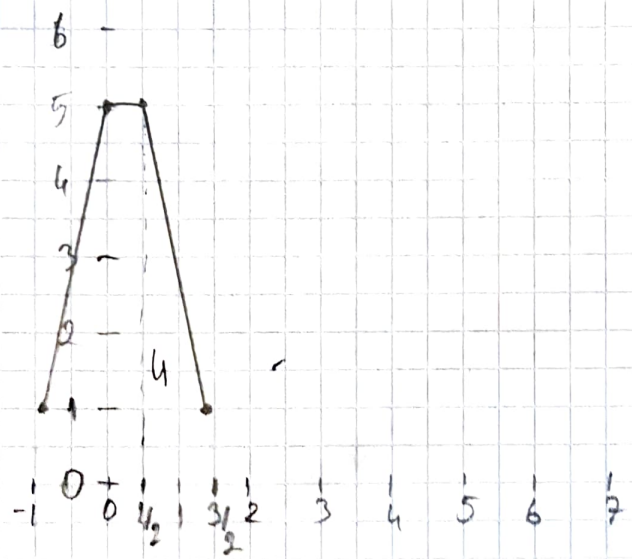
$$= \frac{3t - 16}{6}$$

3) —

4) a.



b.



5) $x[k]$

a. $h[-k]$

0 1 2 3 4 5 4 3 2 1 0

0 0 0 0 0 1 0 0 0 0 0

1 0 0 0 0 0 0 0 0 0 1 $\quad h[-5-k] = 0$

1 0 0 0 0 0 0 0 0 0 0 $\quad h[-4-k] = 1$

1 0 0 0 0 0 0 0 0 0 $\quad h[-3-k] = 2$

1 0 0 0 0 0 0 0 0 $\quad h[-2-k] = 3$

1 0 0 0 0 0 0 0 $\quad h[-1-k] = 4$

1 0 0 0 0 0 0 $\quad h[-k] = 5$

1 0 0 0 0 0 $\quad h[1-k] = 4$

1 0 0 0 0 $\quad h[2-k] = 3$

1 0 0 0 $\quad h[3-k] = 2$

1 0 0 $\quad h[4-k] = 1$

1 0 $\quad h[5-k] = 0$

$y[n] = 0\ 1\ 2\ 3\ 4\ 5\ 4\ 3\ 2\ 1$

b.

0 1 2 3 4 5 4 3 2 1 0

0 1 2 3 4 5 4 3 2 1 0

0 1 2 3 4 5 4 3 2 1 0

0 1 2 3 4 5 4 3 2 1 0

0 1 2 3 4 5 4 3 2 1 0

∞

$h[-k]$

$h[-5-k] = 2$

$h[-4-k] = 2$

$h[-3-k] = 2$

$h[-2-k] = 3$

$h[-1-k] = 4$

$h[-k] = 5$

$h[1-k] = 4$

$h[2-k] = 3$

$h[3-k] = 2$

$h[4-k] = 2$

$h[5-k] = 2$

$y[n] = -\infty \ldots 2\ 2\ 2\ 3\ 4\ 5\ 4\ 3\ 2\ 2\ 2 \ldots \infty$

6) $y(t) = x_{[2,5]}(t) * e^{-5t}u(t)$

Case 1: $t < 2$

$y(t) = 0$

Case 2: $2 < t < 5$

$$y(t) = \int_0^{t-2} e^{-5t}dt \Rightarrow \frac{e^{-5t}}{-5}\Big]_0^{t-2} = \frac{e^{-5t+10}}{-5} + \frac{1}{5}$$

Case 3: $5 < t$

$$y(t) = \int_{t-5}^{t-2} e^{-5t}dt \Rightarrow \frac{e^{-5t}}{-5}\Big]_{t-5}^{t-2} = \left(\frac{e^{-5t+10}}{-5}\right) - \left(\frac{e^{-5t+25}}{-5}\right)$$

7)

$$y(t) = t\,x_{[0,2]}(t) + u(t)$$

$$y(t) = \begin{cases} 0 & \text{if } t<2 \\ t+1 & \text{if } 0 \le t \le 2 \\ 1 & \text{if } 2 < t \end{cases}$$

Fourier Transform:

$$f(w) = \int_{-\infty}^{\infty} y(t)\, e^{-iwt}\, dt$$

$$= \underbrace{\int_{-\infty}^{0} 0 \cdot e^{-iwt}\, dt}_{0} + \underbrace{\int_{0}^{2} (t+1) e^{-iwt}\, dt}_{} + \int_{2}^{\infty} e^{-iwt}\, dt$$

$$= 0 + (t+1)\left(\frac{e^{-iwt}}{-iw}\right) - \left(\frac{e^{-iwt}}{-w^2}\right)\Bigg]_0^2 + \left(\frac{e^{-iwt}}{-iw}\right)\Bigg]_2^{\infty}$$

$$f(w) = \left(\frac{-2}{iw}\right) e^{-2iw} + \frac{e^{-2iw}}{w^2} + \frac{1}{iw} - \frac{1}{w^2}$$

8) $\int_{-\infty}^{\infty} (2 + \delta(t+3))\, e^{-j\omega t}\, dt$

$= \underbrace{\int_{-\infty}^{\infty} 2 \cdot e^{-j\omega t}\, dt}_{\lim_{T\to\infty} \int_{-T}^{T} 2 \cdot e^{-j\omega t}\, dt} + \underbrace{\int_{-\infty}^{\infty} \delta(t+3)\, e^{-j\omega t}\, dt}_{e^{-3\omega}}$

$= \lim_{T\to\infty} \left(\frac{2 \cdot e^{-j\omega t}}{j\omega} \right) \Big|_{-T}^{T}$

$= \lim_{T\to\infty} 2\left(\frac{2(e^{j\omega t} - e^{-j\omega t})}{2j\omega} \right)$

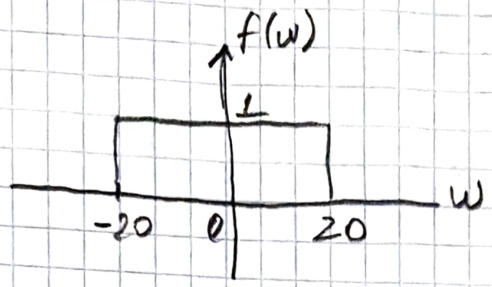
$\sin \omega t$

$= 4 \underbrace{\lim_{T\to\infty} \frac{\sin \omega t}{\omega}}_{\pi\, \delta(\omega)}$

$= 4\pi\, \delta(\omega)$

$FT = 4\pi\, \delta(\omega) + e^{-3j\omega}$

9) a.

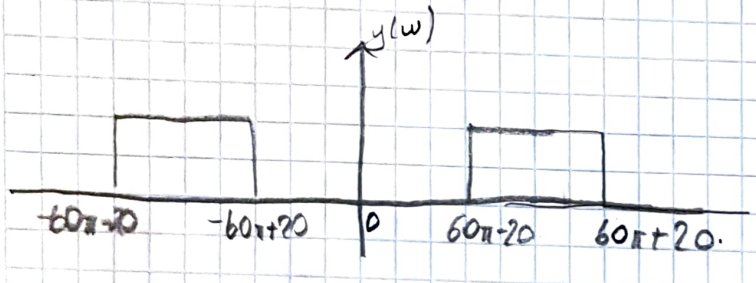


$x(t) = \cos(2\pi 30t)$

Fourier Transform : $x(w) = \pi[\delta(w-60\pi) + \delta(w+60\pi)]$

$y(w) = x(w) * f(w)$

$= \pi[f(w-60\pi) + f(w+60\pi)]$



b. —

10) —

11)

2	0	5	3	1	-4	6							
2	0	5	3	1	-4	6							
4	0	10	6	2	-8	12							
	0	0	0	0	0	0	0						
		10	0	25	15	5	-20	30					
			6	0	15	9	3	-12	18				
				2	0	5	3	1	-4	6			
					-8	0	-20	-12	-4	16	-24		
+						12	0	30	18	6	-24	36	

$y[n] = 4\ 0\ 20\ 12\ 29\ 14\ 43\ -34\ 37\ 28\ 28\ -48\ 36$

12)

2	1	0	4	2
1	1	2	4	1
3	2	0	4	3
1	1	1	1	1
2	1	0	4	2

$\Rightarrow$

7	15	13
9	7	13
7	13	7

1	0	1
0	2	0
1	0	1

$\Rightarrow$

1	0	1
0	2	0
1	0	1

- $2+2+3=7$
- $1+4+4+2+4=15$
- $2+8+3=13$
- $1+2+4+1+1=9$
- $1+4+1+1=7$
- $2+1+8+1+1=13$
- $3+2+2=7$