CSE 348 Homework. Due: June 7, 2020

Dear Students,

Solving the questions below will prepare you well for the coming exams.. None of the questions can be considered difficult, with the exception of Question 3. Even in that case, you are only required to remember the formulas for the summation of geometric series and the complex representation of $\sin(x)$, ie,

$$\sin(x) = \frac{e^x - e^{-x}}{2i} \tag{1}$$

1) Let us have an LTI system. I enter $\chi_{[0,1]}(t)$ to the system as input and I receive $e^{-2t}U(t)$ as output. If I enter $3\chi_{[4,5]}(t) - 7\chi_{[8,9]}(t)$ as input, what will be the output?

2) Convolve $t\chi_{[0,2]}(t)$ with itself.

3) Prove that

$$\sum_{k=-N}^{N} e^{2\pi ikt} = \frac{\sin[(2N+1)\pi t]}{\sin(\pi t)} \tag{2}$$

4) Let $f(t) = t\chi_{[0,2]}(t) + 2\chi_{[2,3]}(t) + (5-t)\chi_{[3,5]}(t)$.
...a) Plot $f(t)$
...b) Plot $2f(2t+1)+1$

5) Convolve $(t+5)\chi_{[-5,0]}(t) + (5-t)\chi_{[0,5]}(t)$ with
...a)

$$\sum_{k=-\infty}^{\infty} \delta(t-11k) \tag{3}$$

...b)

$$\sum_{k=-\infty}^{\infty} \delta(t-8k) \tag{4}$$

Plot your results.

6) Consider a LTI system whose impulse response is $e^{-5t}U(t)$. If I enter $\chi_{[2,5]}(t)$ to this system as input, what will be its output?

7) Find the Fourier transform of $t\chi_{[0,2]}(t) + U(t)$. Show all your work.

8) Find the fourier transform of $2 + \delta(t-3)$. Show all your work.

9) Consider a signal $f(t)$ whose Fourier Transform is $f(\omega) = \chi_{[-20,20]}(\omega)$. Let us modulate this signal with $\cos(2\pi 30 t)$.
...a) Draw the modulated signal in frequency domain.
...b) What would you do to demodulate this signal?

10) Let us sample $f(t)$ given in Question 9 with
...a) 5 Hz.
...b) 30 Hz.
In both cases, draw the sampled signal in frequency domain. In which case(s) we can recover the original signal from the sampled signal?
Note: $\omega = 2\pi f$, where f is frequency, measured in Hertz.

11) Convolve 2 0 5 $\underline{3}$ 1 -4 6 with itself.

12) Filter the signal

$\underline{1}$ 2 4 1
2 0 4 3
1 1 1 1
1 0 4 2

with the filter

1 0 1
0 $\underline{2}$ 0
1 0 1

Use periodic boundary conditions.