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7. Essentially proof 5 except replace environment 2 with environment 3 (x is a string)

6. $\dfrac{}{\Gamma_2 \vdash x : \text{int}} \quad \dfrac{}{\Gamma_2 \vdash (::) : \text{int} \to \text{int}\ list \to \text{int}\ list}$

5. $\dfrac{\dfrac{}{\Gamma_2 \vdash x : \text{int}} \quad \dfrac{}{\Gamma_2 \vdash f : \text{int} \to \text{int} \to \text{int}\ list}}{\Gamma_2 \vdash (x ::) : \text{int} \to \text{int}\ list \to \text{int}\ list}\ 6. \quad \dfrac{\dfrac{}{\Gamma_2 \vdash (::) : \text{int}\ list \to \text{int}\ list \to \text{int}\ list} \quad \dfrac{}{\Gamma_2 \vdash f : \text{int} \to \text{int} \to \text{int}\ list}}{\Gamma_2 \vdash (::(f\ x\ (n-1))) : \text{int}\ list \to \text{int}\ list \to \text{int}\ list}$

4. $\dfrac{}{\Gamma_2 \vdash x : \text{int}} \quad \dfrac{}{\Gamma_2 \vdash n : \text{int}} \quad \dfrac{}{\Gamma_2 \vdash f : \text{int} \to \text{int} \to \text{int}\ list} \quad \dfrac{\dfrac{}{\Gamma_2 \vdash n : \text{int}} \quad \dfrac{}{\Gamma_2 \vdash 0 : \text{int}} \quad \dfrac{}{\Gamma_2 \vdash <= : \text{int} * \text{int} \to bool} \quad \dfrac{}{\Gamma_2 \vdash x :: (f\ x\ (n-1)) : \text{int}\ list}\ 5.}{\Gamma_2 \vdash if\ n <= 0\ then [] \ else\ x :: (f\ x\ (n-1)) : \text{int}\ list}$

3. $\dfrac{}{\Gamma_1 \vdash f : string \to \text{int} \to string\ list} \quad \dfrac{}{\Gamma_1 \vdash \text{"}a\text{"} : string} \quad \dfrac{}{\Gamma_1 \vdash 4 : \text{int}}$

2. $\dfrac{}{\Gamma_1 \vdash 3 : \text{int}} \quad \dfrac{}{\Gamma_1 \vdash 2 : \text{int}} \quad \dfrac{}{\Gamma_1 \vdash f : \text{int} \to \text{int} \to \text{int}\ list}$

$\Gamma_2 = \Gamma_1 + \{x : \text{int}; n : \text{int}\}$
$\Gamma_1 = \{f : 'a \to \text{int} \to 'a\ list\}$
$\Gamma_3 = \Gamma_1 + \{x : string; n : \text{int}\}$

$\dfrac{\dfrac{}{\Gamma_1 \vdash f\ 3\ 2 : \text{int}\ list} \quad \dfrac{}{\Gamma_1 \vdash f\ \text{"}a\text{"}\ 4 : string\ list}\ 3.}{\{\} \vdash (f\ 3\ 2, f\ \text{"}a\text{"}\ 4) : \text{int}\ list * string\ list}\ 2. \quad 4. \dfrac{}{\Gamma_1 \vdash let\ rec\ f = fun\ x \to fun\ n \to if\ n <= 0\ then [] \ else\ x :: (f\ x\ (n-1)) : 'a\ list}\ 7.$

$$\{\} \vdash \mathit{let\ rec}\ f = \mathit{fun}\ x \to \mathit{fun}\ n \to \mathit{if}\ n <= 0\ \mathit{then}\ []\ \mathit{else}\ x :: (f\ x\ (n-1))\ \mathit{in}\ (f\ 3\ 2, f\ "a"\ 4) : \mathrm{int}\ \mathit{list} * \mathit{string}\ \mathit{list}$$