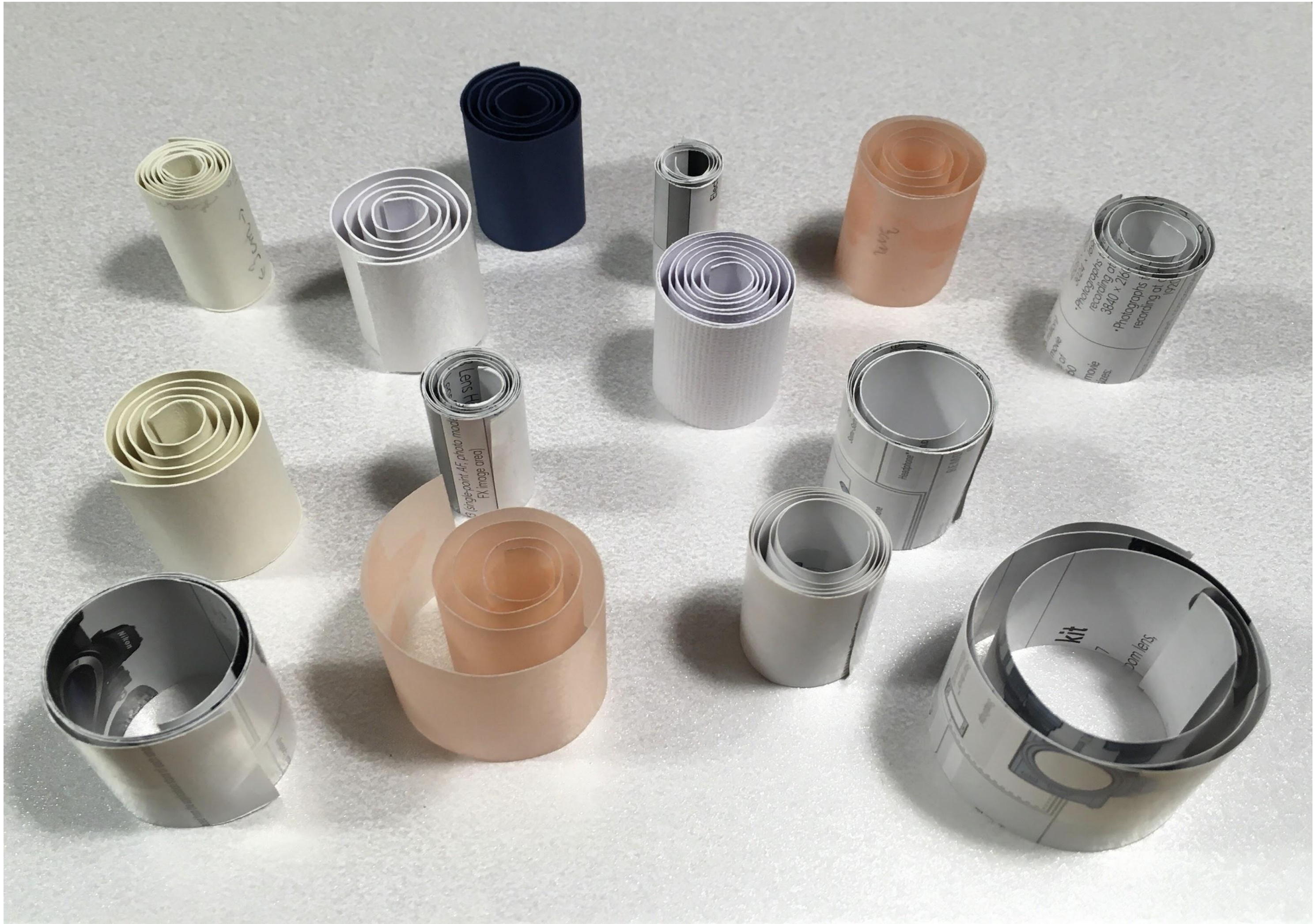
IPT #03 - Paper Tube

By: Aleksandar Ivanov

Official problem statement

Roll a long paper strip into a tight tube and put it vertically on a table. Why does it often unwind in jerks? What determines the period of the jerks?



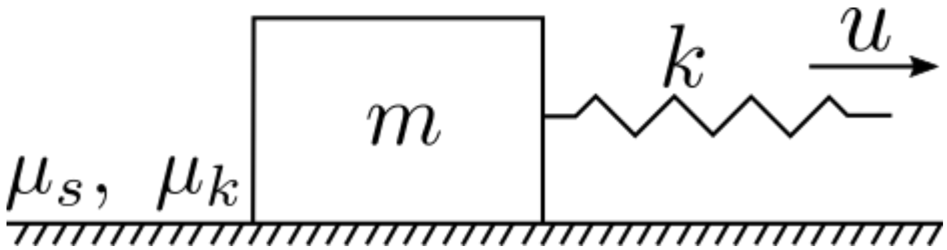
Relevant phenomena

-Stick and slip motion

-Stresses in the paper are drivers

-Surface and layer friction

Theoretical description



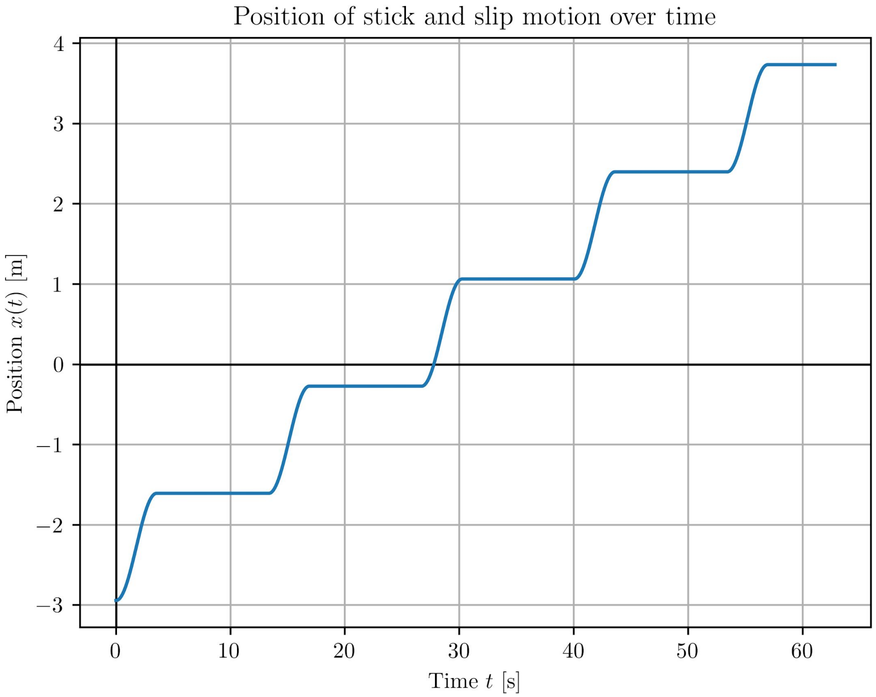
The differential equation for the elongation of the effective spring driven by a constantly moving end is

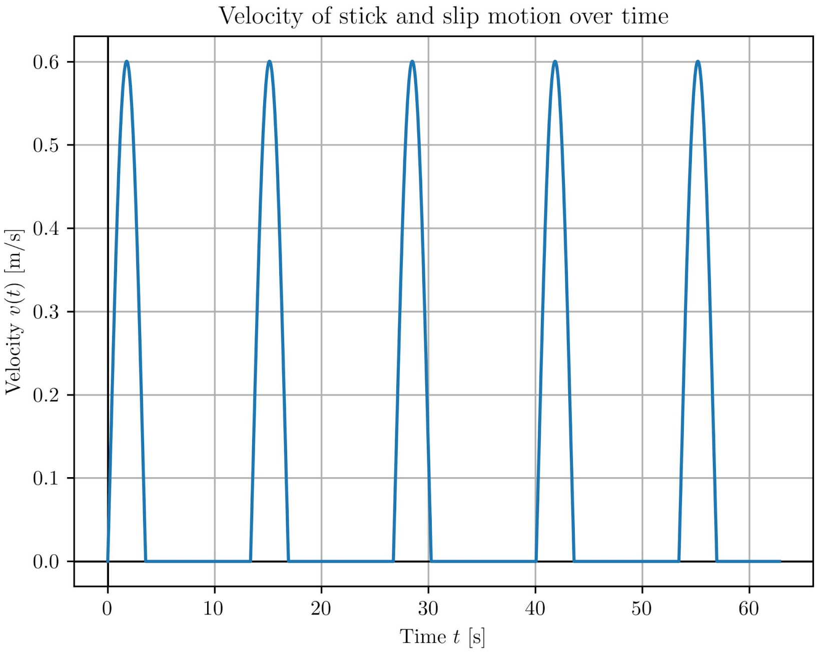
$$\ddot{\delta} = -\omega_0^2 \delta - \mu_k g \,\mathrm{sgn}(\dot{\delta} - u)$$

with effective initial conditions

$$\delta(0) = \frac{\mu_s g}{\omega_0^2}, \qquad \dot{\delta}(0) = u$$

Model motion





Theoretical description

This gives us the durations of the stick and slip phases

$$T = \underbrace{\frac{2(\mu_s - \mu_k)g}{u\omega_0^2}}_{\text{stick } T_1} + \underbrace{\frac{\pi}{\omega_0} + \frac{2}{\omega_0}\arctan\left(\frac{u\omega_0}{(\mu_s - \mu_k)g}\right)}_{\text{slip } T_2}$$

For generic parameters: $T_1 \gg T_2$

In the rotational case

$$\boxed{\begin{aligned} E &= E_0 - n\Delta E \\ &\approx E_0 \exp(-n\tfrac{\Delta E}{E_0}) \end{aligned}}$$

-Driving u is internal, affected by friction

-At every jerk we lose energy $\Delta E \propto \Delta x = uT$

-Energy loss causes $u \propto \exp(-\alpha n)$

-Period grows like $T \propto \exp(\alpha n)$

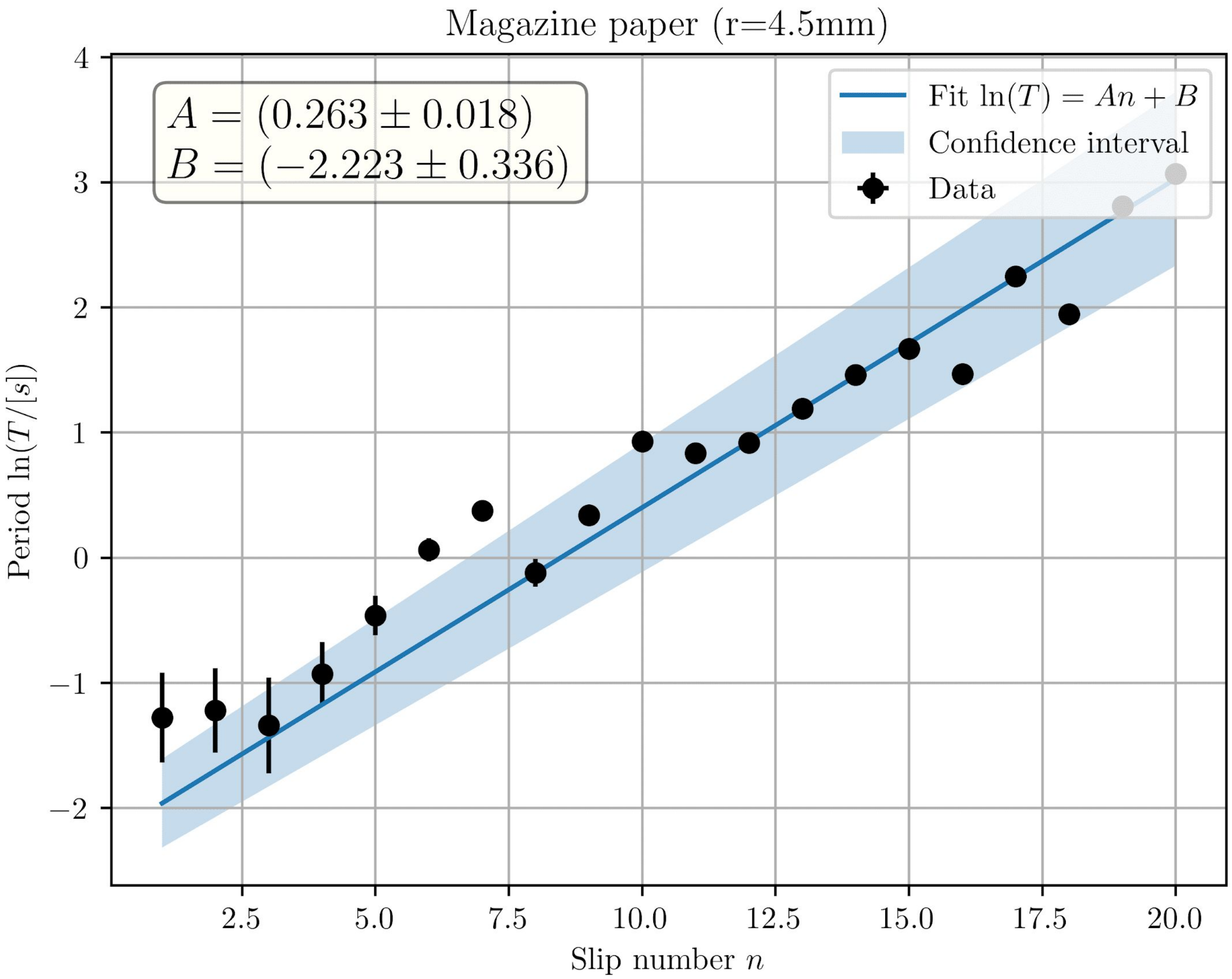
$$\boxed{E \propto u^2}$$

Setup and measurements

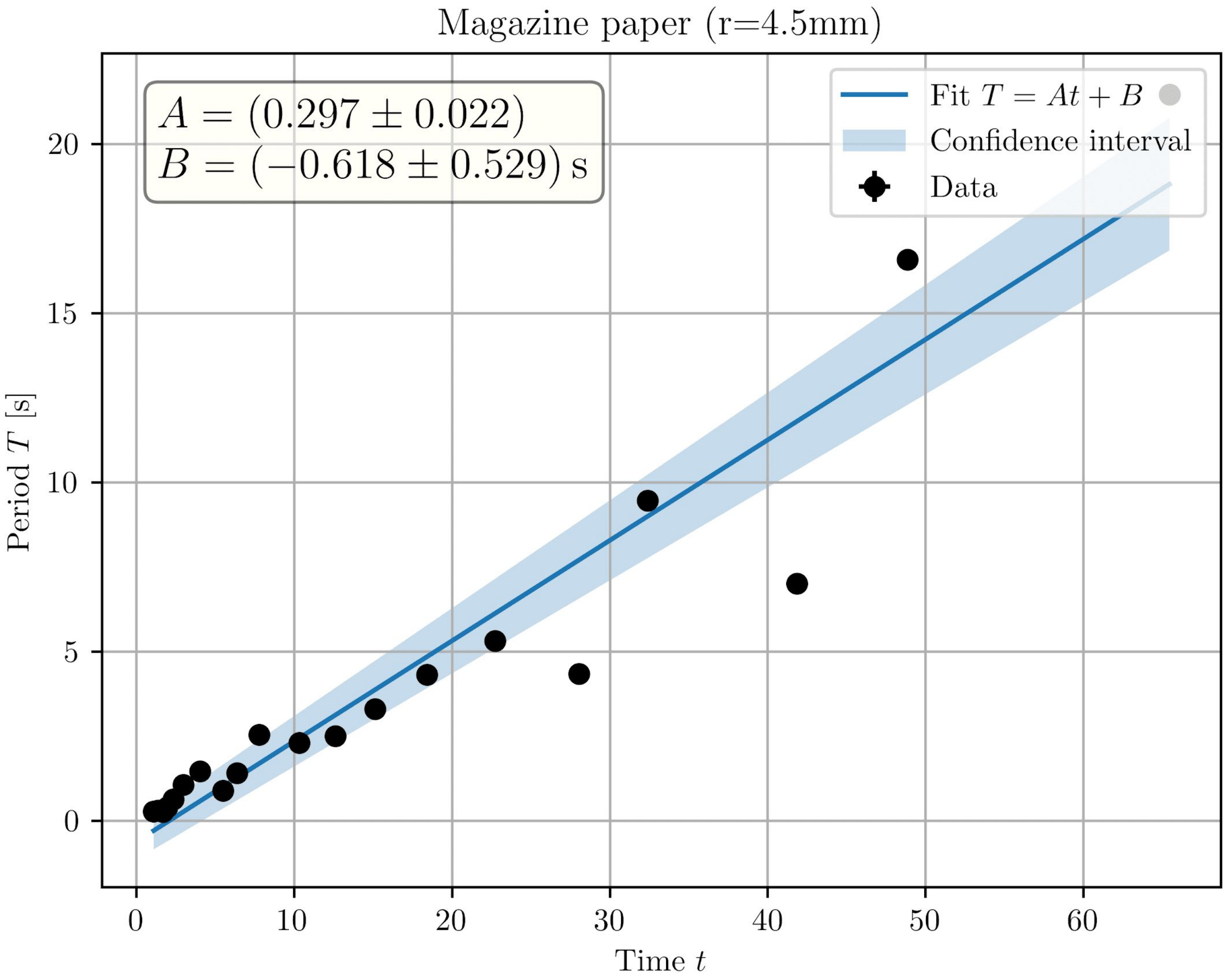
-Top down video of the paper roll

-Recorded in slow motion (iPhone camera)

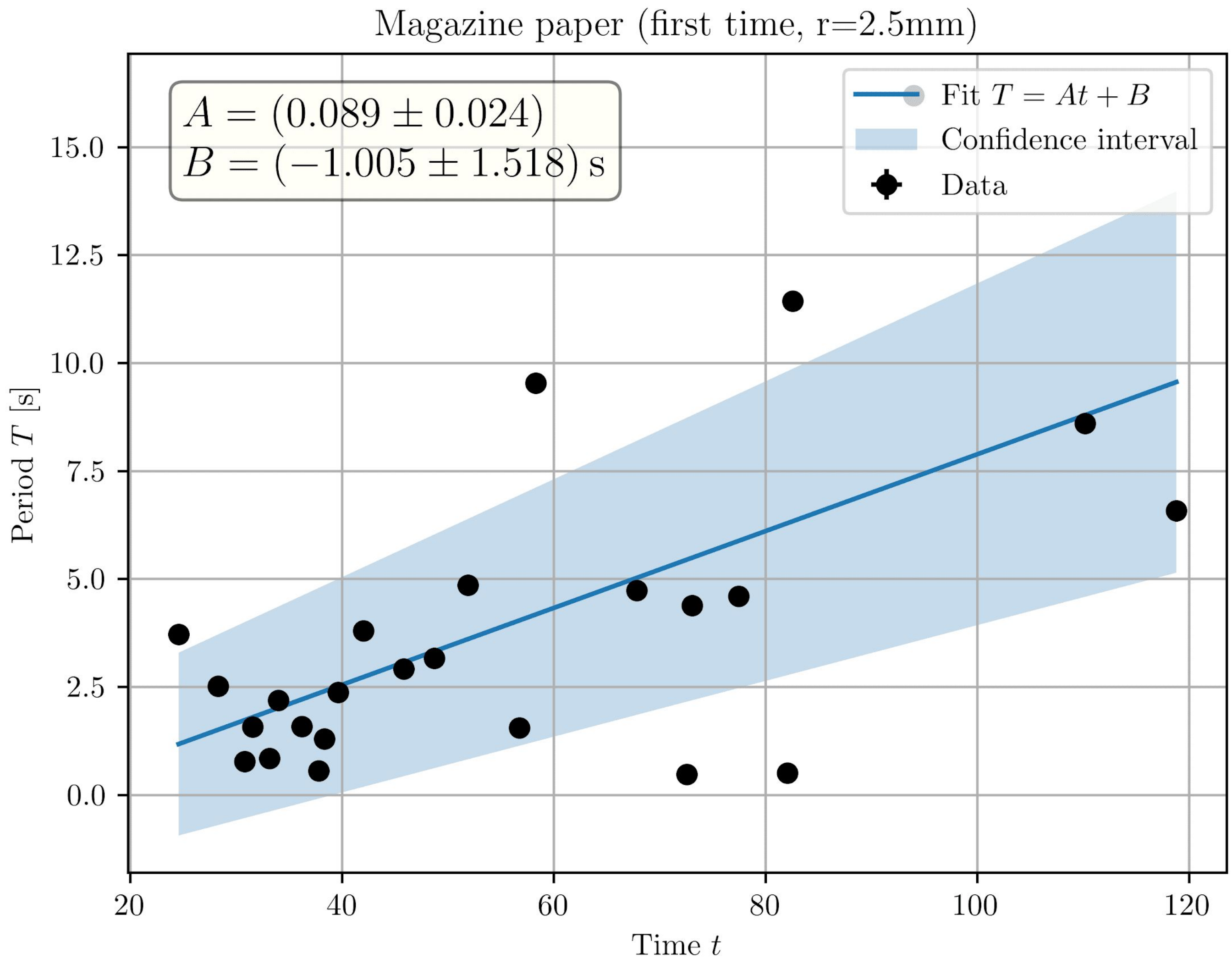
-Times of jerks extracted to frame precision with video software (mpv)



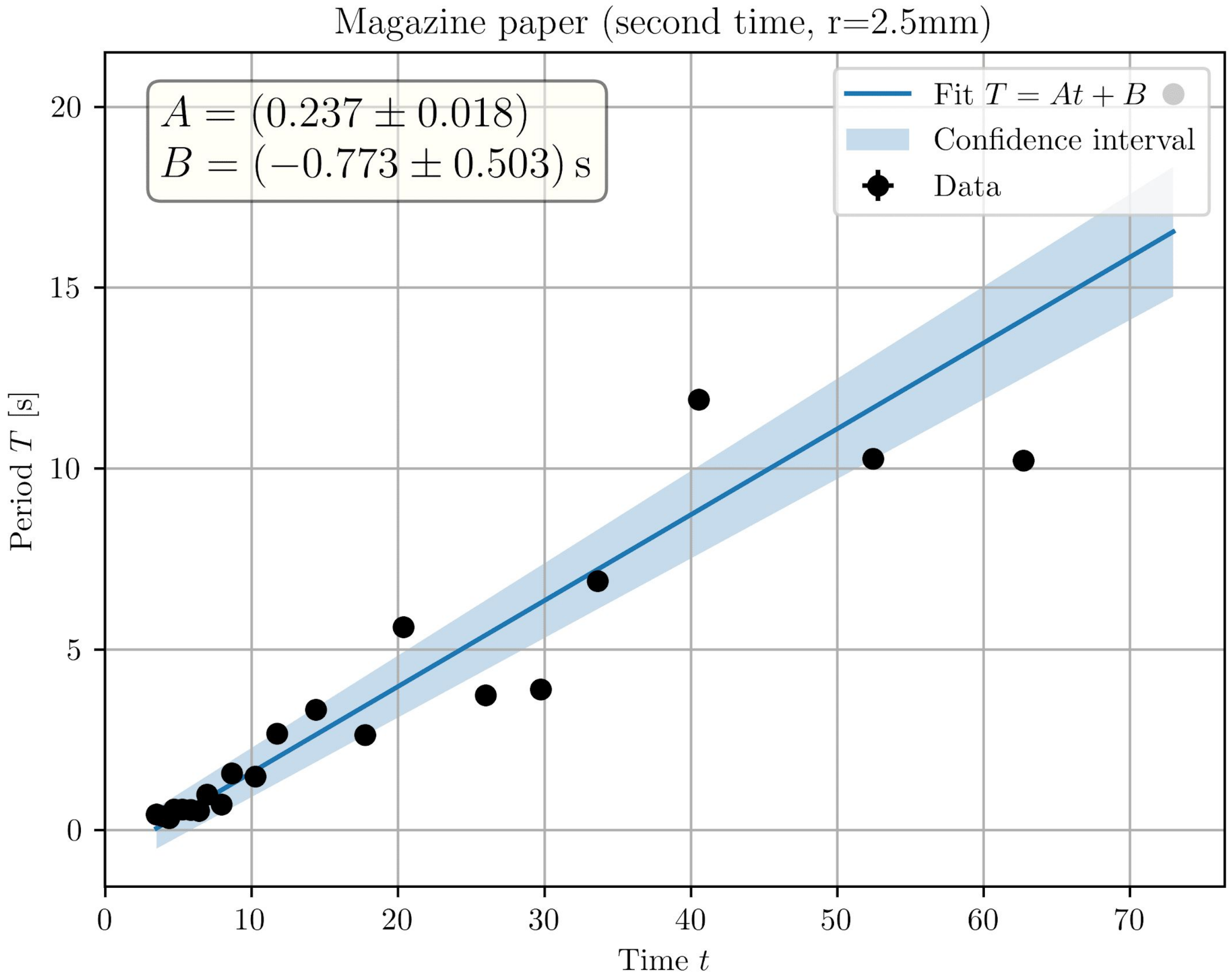
Period grows exponentially with number of slips, as predicted



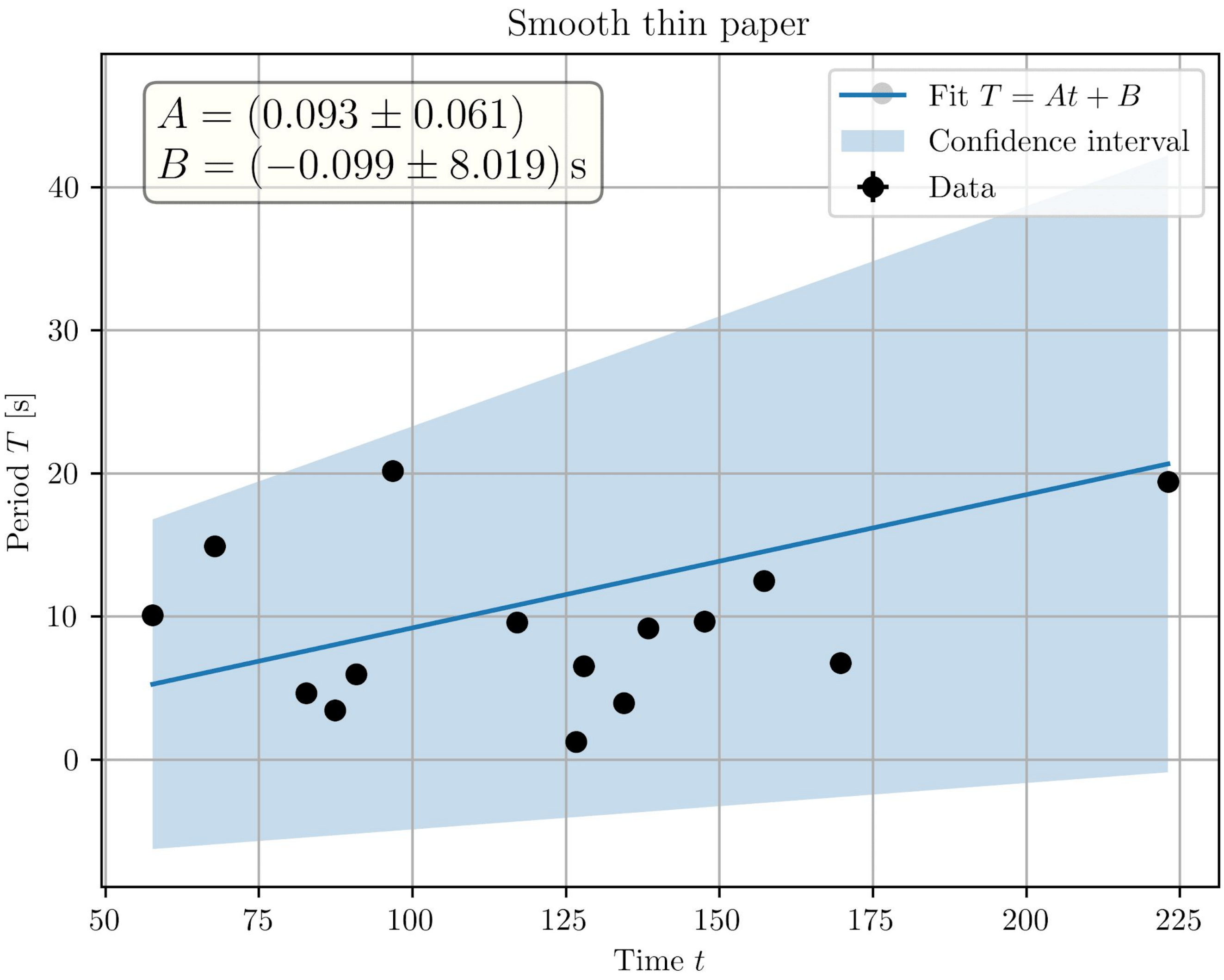
Deviations large, but infrequent



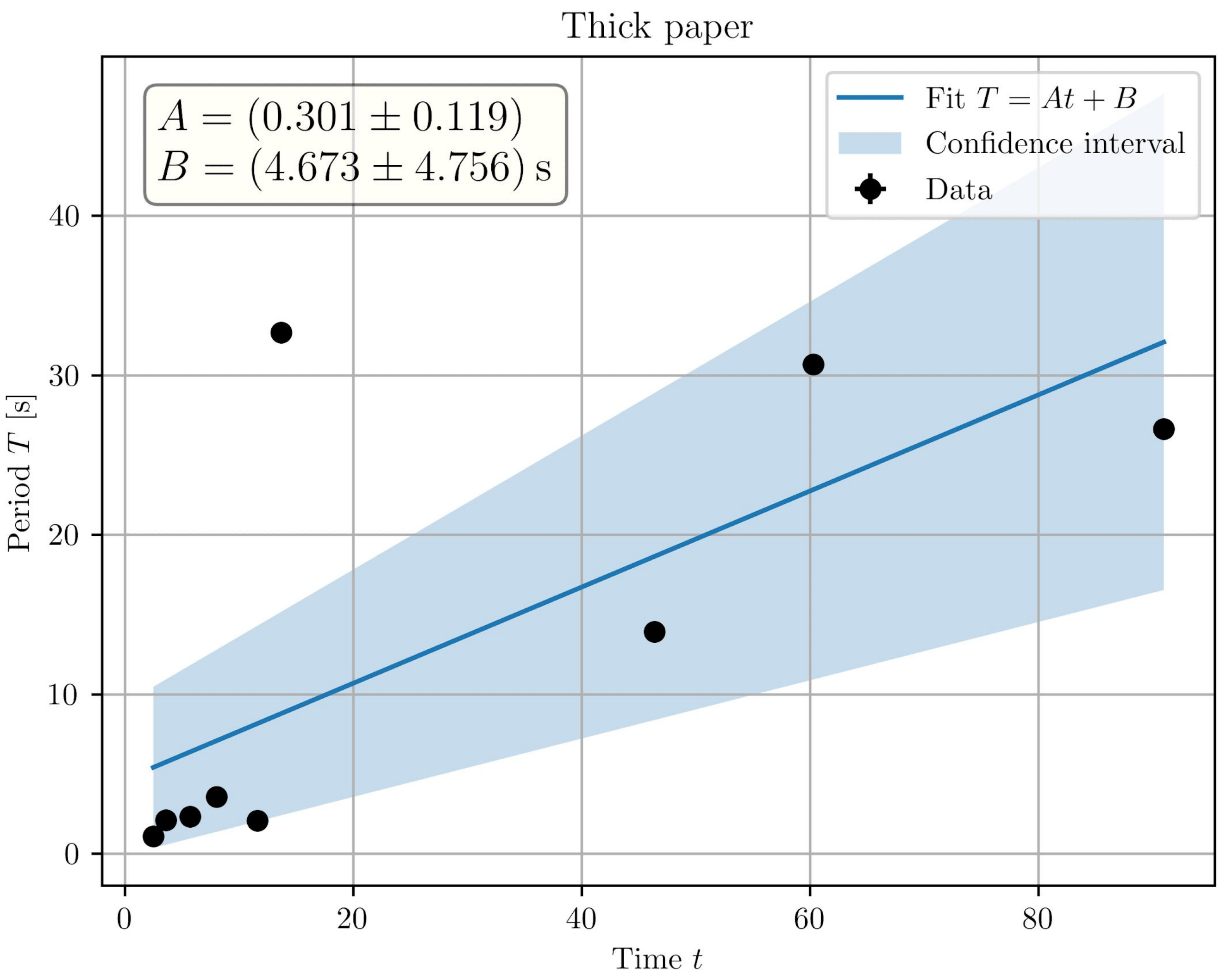
Trend still present but less clear because of deviations



Less deviation when paper stays compressed

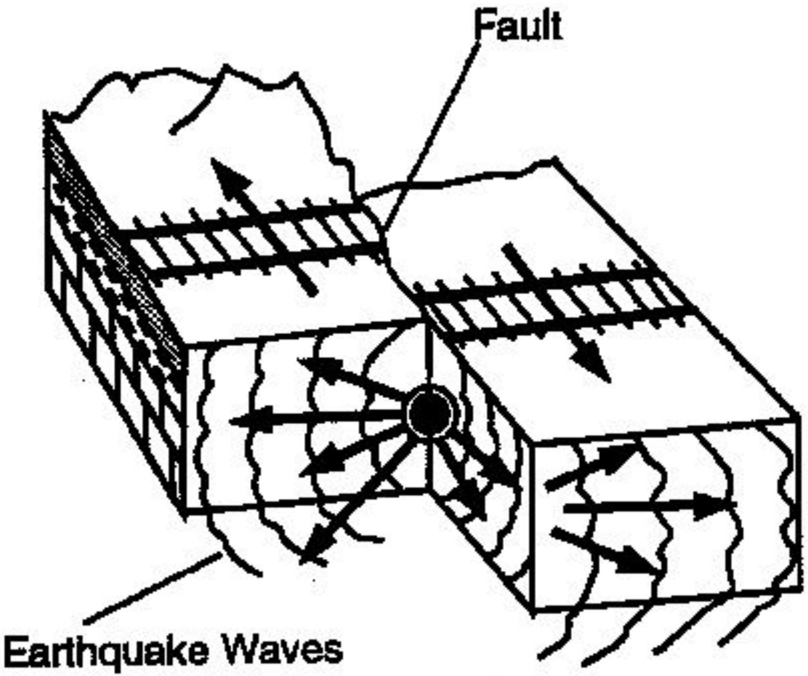


If the paper becomes less tight, it starts moving disjointly



A single displaced jerk moves two neighboring points -- one up, the other one down

An analogy



By FEMA-USA

-Motion is similar to **earthquakes**

-Plates build up tension in the stick phase and release it in the slip phase

-Bigger earthquakes induce smaller earthquakes

Conclusions and Discussion

-Jerks of the paper are caused by **stick and slip**
-Adiabatically changing parameters give exponential period
-Deviations from trend are because of induced slip

-Better model of the internal stresses
-Way to get other parameters of the motion

Bibliography

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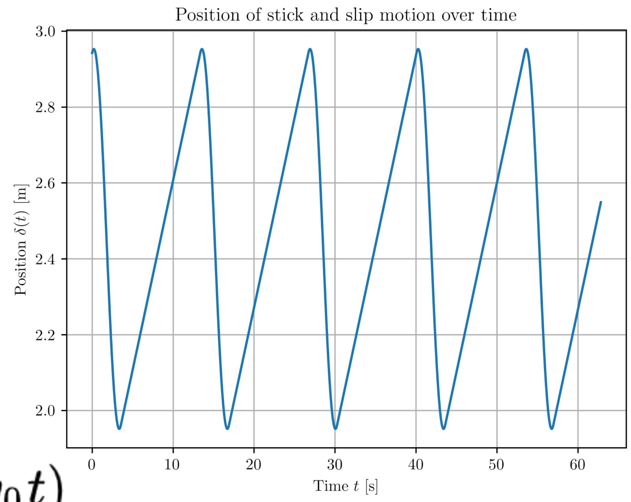
Calculations



$$u - \dot{\delta}(t) \geq 0 \,\forall t,\ \delta(0) = \frac{\mu_s g}{\omega_0^2},\ \dot{\delta}(0) = u$$

$$\Rightarrow \delta_k(t) = \frac{\mu_k g}{\omega_0^2} + \frac{u}{\omega_0}\sin(\omega_0 t) + \frac{(\mu_s - \mu_k)g}{\omega_0^2}\cos(\omega_0 t)$$

$$\Rightarrow \delta_s(t) = \frac{(2\mu_k - \mu_s)g}{u\omega_0^2} + ut$$

$$T_1 = \frac{1}{u}\left(\frac{\mu_s g}{\omega_0^2} - \delta_s(0)\right)$$

$$\Delta x = uT_2 - \Delta\delta = uT \Rightarrow \Delta E = \mu_k m g u T$$

$$E(t) = E_0 - n\Delta E = E_0\left(1 - n\frac{\Delta E}{E_0}\right)$$

assuming $\exists p : E \propto u^p$

$$\approx E_0 \exp\left(-\frac{\Delta E}{E_0} n\right) \Rightarrow u = u_0 \exp(-\alpha n)$$

Paper on its side