Good Covers of Stratified Spaces

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0 Theorem

Theorem 0.1. *Every stratified manifold has a good cover.*

0.1 Riemannian Metrics

We do not claim that this is the most general structure that deserves the name 'Riemannian metric' on a stratified space. This is why we opt to add the adjective 'tame' to the name. Whatever that most general definition of a Riemannian metric is it should certainly include, as an example, the structure that we consider here. For the purposes of defining good covers this restricted structure is enough.

Remark 0.2. Recall that, by ...., every stratum of a stratified manifold is a smooth manifold. Furthermore, by ...., every stratum has a tubular neighborhood in the stratified manifold that is characterized by the link.

As a preliminary, we consider the construction of a Riemannian metric on a stratified manifold of depth 1. The general construction, that we will describe in ......, proceeds by induction on this case.

Definition 0.3. Let M be a stratified manifold of depth 1, so that M_0 is its lower stratum, M_1 is its higher one, and the link is $M_0 \twoheadleftarrow \pi L \overset{\iota}{\hookrightarrow} M_1$. A *tame Riemannian metric* g on M is prescribed by Riemannian metrics g_0 and g_1 on the respective strata such that under the canonical isomorphism

$$\mathsf{T}L \cong \iota^*\mathsf{T}M_1 \oplus \mathsf{T}M_0 \oplus \underline{\mathbb{R}} \tag{1}$$

the pullback metric $\iota^* g_1$ on the link decomposes as

$$(r^2 g_1, g_0, dr^2). \tag{2}$$

Definition 0.4. A tame Riemannian metric on a stratified manifold is ...

Example 0.5. If a stratified manifold M is such that it has an underlying smooth manifold $\bar{M}$, like in the case of stratified manifolds created through a filtration of closed subsets, then a Riemannian metric $g_{\bar{M}}$ on $\bar{M}$ provides the data of a tame Riemannian metric on the stratified manifold M, as in definition Definition 0.4. Namely, ...

Definition 0.6. Let $\gamma : [0,1] \to M$ be a curve in the stratified manifold $M \to P$, and let M be equipped with a tame Riemannian metric. The length of γ is defined to be

$$\mathrm{len}_g(\gamma) := \sum_{p \in P} \mathrm{len}_{g_p}(\gamma|_{M_p}), \tag{3}$$

the sum of the lengths in each stratum. If the metric is clear from context we will drop the subscript notation.

Lemma 0.7. *Let (M, g) be a stratified manifold with a tame Riemannian metric. Then M can be made into a length space, by equipping it with the intrinsic length metric d_g, so that the distance between two points is the infimum of the lengths of admissible curves joining them.*

Proposition 0.8. *Let (M, g) be a connected stratified manifold with a tame Riemannian metric. Then every two points of M can be connected by a minimizing curve.*

Proof. By Lemma 0.7, (M, d_g) is a length space, where d_g is the intrinsic metric generated by the tame Riemannian metric g. By virtue of being a stratified manifold it is also a locally compact topological space. Then using the Hopf–Rinow–Cohn-Vossen theorem .... for a locally compact length space, and its immediate corollaries, we know that what we need to show is that every closed and bounded subset of M is compact......... □