Automated Meta-Programming to Support High-Performance OCaml Codes

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Not Manual

B. Comp Dissertation

???

Automated Meta-Programming to Support High-Performance OCaml Codes

/səˈpɔːt/ verb bear all or part of the weight of

A Programming Language

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My Name

Fast!!

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My School

Meta-Programming

- Programs that manipulate other programs
 - "Code fragment objects" as data
 - Create new code
 - Modify existing code

- "Free" compiler from an interpreter
 - Partial evaluation of an Domain Specific Language (DSL) interpreter to a DSL program
 - Produces code in the interpreter's implementation language
 - Can then reuse the interpreter's implementation language's compiler

• "Free" compiler from an interpreter

```
(* if n = 0 then 1 else x * 200 *)
Ifz (Var "n", Int 1, Mul (Var "x", Int 200))
let rec eval e env =
  match e with
    Int i \rightarrow i
      Var s -> env s
      App (s,e2) -> (fenv s) (eval e2 env)
      Add (e1,e2) -> (eval e1 env fenv) + (eval e2 env)
     Sub (e1,e2) -> (eval e1 env) - (eval e2 env)
      Mul (e1,e2) -> (eval e1 env) * (eval e2 env)
      Div (e1,e2) -> (eval e1 env) / (eval e2 env)
      Ifz (e1,e2,e3) \rightarrow if (eval e1 env fenv) = 0
                           then (eval e2 env fenv)
                           else (eval e3 env fenv)
```

• "Free" compiler from an interpreter

"Free" compiler from an interpreter

• Results from Taha (2004)

| Program | Description of Interpreter | Fact10 | Fib20 |
|---------|----------------------------------|--------|-------------|
| (none) | OCaml implementations | 100% | 100% |
| eval1 | Simple | 1,570% | 1,736% |
| eval2 | Simple staged | 100% | 100% |
| eval3 | Error handling (EH) | 1,903% | $2,\!138\%$ |
| eval4 | EH staged | 417% | 482% |
| eval5 | CPS, EH | 2,470% | 2,814% |
| eval6 | CPS, EH, staged | 100% | 100% |
| eval7 | Inlining, staged | 87% | 85% |
| eval8 | Inlining, no duplication, staged | 97% | 97% |
| eval9 | Inlining, CPS, EH, staged | 90% | 85% |

"Free" compiler from an interpreter

• Results from Hermann and Langhammer (2006)

| | native base program | | bytecode base program | |
|--------------------|---------------------|----------------|-----------------------|----------------|
| configuration | t in sec. | speedup | t in sec. | speedup |
| none | 259.96 | 1.00× | 1 032.33 | 1.00× |
| simplify | 135.32 | 1.92× | 640.04 | 1.61× |
| staged* | 3.98 | $65.24 \times$ | 29.90 | $34.52 \times$ |
| simplify + staged* | 2.75 | $94.68 \times$ | 15.83 | 65.21× |

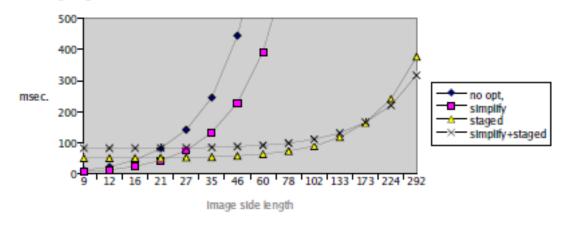
Staged run comprises code generation and .! application.

Table 4.2: Overall execution times for input image of size 1000×1000 .

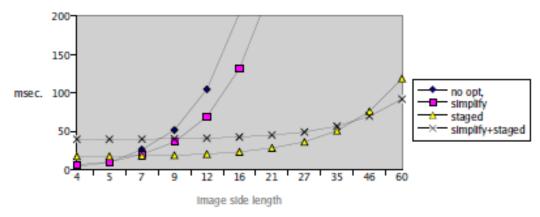
"Free" compiler from an interpreter

• Results from Hermann and Langhammer (2006)

Native base program:



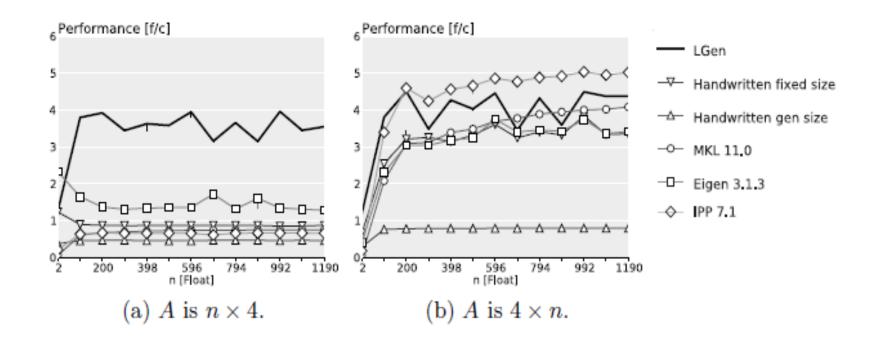
Bytecode base program:



- Efficient genericity and high level code
 - Adaptable code that optimizes to the problem being solved
 - input size
 - input/output type
 - Automatic optimization to platform
 - Detection and utilization of underlying hardware
 - Unrolling according to cache size
 - SSE
 - Distributed/Parallel Architecture

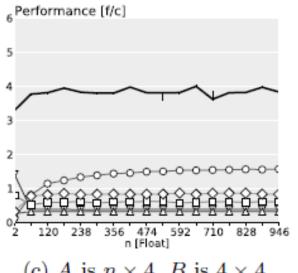
Efficient genericity and high level code

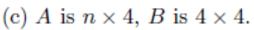
Results from Spampinato and Puschel (2014)

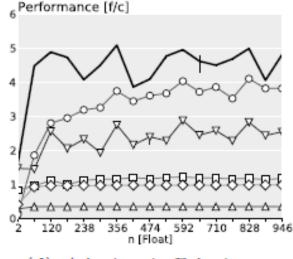


Efficient genericity and high level code

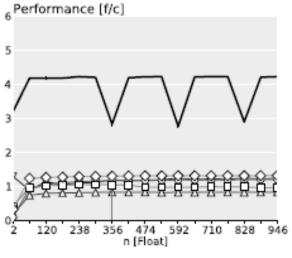
Results from Spampinato and Puschel (2014)



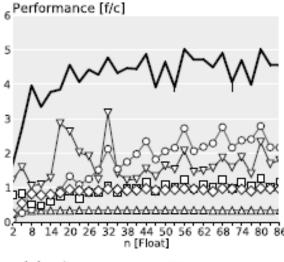




(d) A is 4×4 , B is $4 \times n$.



(e) A is $4 \times n$, B is $n \times 4$.



(f) A is $n \times 4$, B is $4 \times n$.

Meta-Programming, how to do it?

- MetaOCaml
 - Multi-staged programming language
 - Allows the creation of delayed computations in the form of code
 - The delayed computations can produce further delayed computations
 - Based on OCaml

Meta-Programming, how to do it?

MetaOCaml

- Bracket .< ... >.
 - Delay the computation inside it

```
let plus2 x = .<x + 2>.;;
# plus2 3;;
- : int code = .<3 + 2>.
```

- Escape .~
 - Runs code pointed by it to produce code to be spliced

```
# .<.~(plus2 3) * .~(plus2 4)>.;;
- : int code = .<(3 + 2) * (4 + 2)>.
```

- Run !.
 - Executes a delayed computation

```
# !. .<.~(plus2 3) * .~(plus2 4)>.;;
- : int = 30
```

Meta-Programming, how to do it?

MetaOCaml

Project Objective

To automate the staging process, enabling users to reap the benefits of multi-staged meta-programming without having to deal with the complexities of manually staging source programs

Overview

- Automated Staging of OCaml Codes
 - Translation Rules
 - Source Code Annotation
 - Source Code Preprocessing
- Results
- Conclusions

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Automated Staging of OCaml Codes

- Annotate OCaml Code
 - What function to stage
 - What static information is available
- Preprocess the annotated OCaml code
 - Extract information in annotations
 - Analyze and transform
 - Produce staged MetaOCaml code

Overview

- Automated Staging of OCaml Codes
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- Simple functions with no control flows or recursions
 - let plus x y = x + y
 - If y is always 2
 - \Rightarrow let plus x = x + 2
 - How to stage?
 - => let plus_staged y = .< let plus x = x + y in plus >.
 - Sample Run: # plus_staged 2;;
 - -: (int -> int) code = .< let plus x = x + 2 in plus >.

```
let plus x y = x + y \Rightarrow let plus_staged y = .< let plus x = x + y in plus >.
```

```
<FunBody> => <FunBodyS>
             let <FunName> <Args> = <FunBody>
                           =><sub>toMeta</sub>
let <FunName>_staged <StaticArgs> =
  .< let <FunName> <DynArgs> = .~(<FunBodyS>) in <FunName> >.
```

```
<FunBody> => .< <FunBody> >.
```

Function with control flow

```
• let f x y = if x > 0 then x + y else x - y
• If y is static
    \Rightarrow let f_staged y = .< let f x = if x > 0 then x + y else x - y in f >.
• If x is static?
    \Rightarrow let f_staged x = .< let f y = if x > 0 then x + y else x - y in f >.
    # f_staged 1;;
    -: (int -> int) code = .< let f y = if 1 > 0 then 0 + y else 0 - y in f >.
```

- Function with control flow
 - let f x y = if x > 0 then x + y else x y
 - If x is static

```
=> let f_staged x =
  let aux x y = if x > 0 then .< 0 + .~y >. else .< 0 - .~y >.
  in .< let f y = .~(aux x .<y>.) in f >.

# f_staged 1;;
- : (int -> int) code = .< let f y = 1 + y in f >.
```

- Function with control flow
 - Nested control structures?
 - let $f \times y = if \times x = 0$ then (if y = 0 then true else false) else false
 - If y is static

```
=> let f_staged y =
   let aux x y = if y = 0 then .<true>. else .<false>. in
   .< let f = if x = 0 then .~(aux x y) else false in f</pre>
```

- Function with control flow
 - Nested control structures?
 - let $f \times y = if \times x = 0$ then (if y = 0 then true else false) else false
 - If both x and y are static

```
=> let f_staged x y =
let aux_2 x y = if y = 0 then .< true >. else .< false >. In
let aux_1 x y = if x = 0 then .~(aux_2 x y) else .< false >. In
.< let f = .~(aux_1 x y) in f >.
```

```
<FunBody> => ([], .< <FunBody> >.)
```

```
\langle FunBody \rangle = \rangle ([\langle Aux_1 \rangle, ..., \langle Aux_n \rangle], \langle FunBodyS \rangle)
                    let <FunName> <Args> = <FunBody>
                                         =><sub>toMeta</sub>
let <FunName>_staged <StaticArgs> =
   let \langle AuxName_1 \rangle = \langle AuxBody_1 \rangle
   and ...
   and \langle AuxName_n \rangle = \langle AuxBody_n \rangle in
   .< let <FunName> <DynArgs> = .~(<FunBodyS>) in <FunName> >.
```

For if-then-else structure with statically computable condition

```
isStatic(CondExp)
  if <CondExp> then <ThenBody> else <ElseBody>
                                  =>
( ThenAuxList @ ElseAuxList @
 [if <CondExp>
                                     , .< .~(aux <Args[DynArg->.<DynArg>.]>) >.
   then .< .~(<ThenBodyS[DynArg->.~DynArg]>) >.
   else .< .~(<ElseBodyS[DynArg->.~DynArg]>) >.]
```

For if-then-else structure with non-statically computable condition

```
!isStatic(CondExp) <CondBody> => (CondAuxList, <CondBodyS>)
if <CondExp> then <ThenBody> else <ElseBody>
                              =>
                                     if .~(<CondExpS>)
 CondAuxList @ ThenAuxList @ ElseAuxList
                                       then .< .~(<ThenBodyS>) >.
                                       else .< .~(<ElseBodyS>) >.
```

Function with recursion

- let rec pow x n = if n = 0 then 1 else x * pow x (n 1)
- Change recursive call to the optimized function call
 - Omitted here for brevity

- Function that uses other staged function
 - let rec ff y m =

```
if y = 0 then 0 else (pow y m) + (ff (y - 1) m)
```

• If m is static and pow is staged over its second argument

```
=> let ff_staged m =
    let pow = !. .< .~(pow_staged m) >. in
    .< let rec ff y =
        if y = 0 then 0 else (pow y) + (ff (y - 1)) in ff >.
```

Function that uses other staged function

```
=> let ff_staged m =
     let pow = !. .< .~(pow_staged m) >. in
     .< let rec ff y =</pre>
          if y = 0 then 0 else (pow y) + (ff (y - 1)) in ff >.
# ffstaged 3;;
- : (int -> int) code =
      .< let rec ff y =</pre>
           if y = 0 then 0 else ((* CSP pow *) y) + (ff (y - 1))
         in ff >.
```

Function that uses other staged function

```
=> let ff_staged m =
    let pow = !. .< .~(pow_staged m) >. in
    .< let rec ff y =
        if y = 0 then 0 else (pow y) + (ff (y - 1)) in ff >.
```

- CSP = Cross-Stage Persistence
 - Values/functions from the code generator environment used in the produced code
 - Code generator environment needs to be available to the produced code
 - Assume ff_staged always called before the produced specialized code is used

```
\langle FunBody \rangle = \rangle ([\langle Aux_1 \rangle, ..., \langle Aux_n \rangle], \langle FunBodyS \rangle)
                              let <FunName> <Args> = <FunBody>
                                                 =><sub>toMeta</sub>
let <FunName>_staged <StaticArgs> =
  let <StagedFun> = !. .< .~(<StagedFun>_staged <StaticArgs>) >. in
   let \langle AuxName_1 \rangle = \langle AuxBody_1 \rangle
  and ...
   and \langle AuxName_n \rangle = \langle AuxBody_n \rangle in
   .< let <FunName> <DynArgs> = .~(<FunBodyS>) in <FunName> >.
```

- Now we know how to systematically translate
 OCaml code into a staged MetaOCaml code
 - Given the static information available to them
 - Staged code produces optimized function

Overview

- Automated Staging of OCaml Codes
 - Translation Rules
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 - Source Code Preprocessing
- Results
- Conclusions

- How to indicate in the source program what is static?
- OCaml Attributes
 - "decorations" attachable to OCaml AST
 - [@id payload] or [@@id payload] or [@@@id payload]
 - (1 + 2) [@doSomething]
 - let x = 2 [@@doSomething [data1;data2]]

Available since OCaml 4.02.1

- How to indicate in the source program what is static?
 - [@@static [statVar₁; ...; statVar_n]

```
let rec pow x n =
   if n = 0 then 1 else x * (pow x (n - 1))
[@@static [n]]
```

- How to indicate in the source program what is static?
 - [@@static [statVar₁; ...; statVar_n]

```
let rec pow x n =
  if n = 0 then 1 else x * (pow x (n - 1))

[@@static []] [@@static [x]] [@@static [n]] [@@static [x;n]]
```

- How to indicate in the source program if we use other (automatically) staged function?
 - [@static.use]
 let rec ff x n =
 if x = 0 then 0 else pow x n [@static.use] + ff (x 1) n
 [@@static [n]]

Necessary because sometimes we don't want to use the static version

Overview

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- How to process the annotated code?
- ppx Preprocessor
 - Mapper from OCaml AST to OCaml AST
 - Default mapper
 - Do deep identity mapping
 - Can be overridden at points where it takes in different OCaml AST nodes

- Hooking into the default mapper
 - let plus x y = x + y

```
structure
  structure_item (foo.ml[1,0+0]..[1,0+20])
   Pstr value Nonrec
   (def>)
       pattern (foo.ml[1,0+4]..[1,0+8])
         Ppat var "plus" (foo.ml[1,0+4]..[1,0+8])
       expression (foo.ml[1,0+9]..[1,0+20]) ghost
          Pexp fun ""
         None
          pattern (foo.ml[1,0+9]..[1,0+10])
            Ppat var "x" (foo.ml[1,0+9]..[1,0+10])
         expression (foo.ml[1,0+11]..[1,0+20]) ghost
            Pexp fun ""
            None
            pattern (foo.ml[1,0+11]..[1,0+12])
              Ppat var "y" (foo.ml[1,0+11]..[1,0+12])
```

```
expression (foo.ml[1,0+15]..[1,0+20])
  Pexp_apply
  expression (foo.ml[1,0+17]..[1,0+18])
    Pexp_ident "+" (foo.ml[1,0+17]..[1,0+18])
[ <label> ""
        expression (foo.ml[1,0+15]..[1,0+16])
            Pexp_ident "x" (foo.ml[1,0+15]..[1,0+16])
        <label> ""
        expression (foo.ml[1,0+19]..[1,0+20])
            Pexp_ident "y" (foo.ml[1,0+19]..[1,0+20])
]
```

Hooking into the default mapper

```
let toMeta_mapper argv =
    { default_mapper with
    structure = fun mapper structure_item_list -> ... }
```

Hooking into the default mapper

```
match structure_item with
  {pstr_desc = Pstr_value (_, _)} ->
    if hasToMetaAnnot structure_item
        then (* do generation of staged code *)
        else [default_mapper.structure_item mapper structure_item]
        | _ -> [default_mapper.structure_item mapper structure_item]
```

- Extracting information and annotations
 - let f x = x + 1 [@@static [x]]

```
[{pstr_desc =
   Pstr value (Nonrecursive,
   [{pvb_pat = {ppat_desc = Ppat_var {txt = "f"}};
      pvb_expr =
       {pexp desc =
         Pexp_fun ("", None, {ppat_desc = Ppat_var {txt = "x"}},
           {pexp_desc =
            Pexp apply ({pexp desc = Pexp ident {txt = Lident "+"}},
             [("", {pexp_desc = Pexp_ident {txt = Lident "x"}});
              ("", {pexp desc = Pexp constant (Const int 1)})])});
     pvb_attributes =
     [({txt = "static"},
       PStr [{pstr_desc =
        Pstr_eval
         ({pexp desc =
             Pexp_construct ({txt = Lident "::"},
             Some {pexp desc =
              Pexp tuple
                [{pexp desc = Pexp ident {txt = Lident "x"}};
                 {pexp desc = Pexp construct ({txt = Lident "[]"}, None)}]})},
...)}])]}]
```

- Extracting information and annotations
 - OCaml Structure item (let-bound function defs): Pstr_value
 - Control structures: Pexp_ifthenelse, Pexp_match
 - Function application: Pexp_apply
 - Annotations: pvb_attributes, pexp_attributes

Building the staged function

```
match exp with
                           {pexp desc = Pexp_ifthenelse(condExp, thenExp, elseExpOpt)} ->
                             let (thenAux, thenExp') = buildStagedBody thenExp ...
                             in let (elseAux, elseExpOpt') =
                               begin match elseExpOpt with
                                 None -> ([], None)
                                 | Some elseExp ->
                                     let (aux, e) = buildStagedBody elseExp ...
                                     in (aux, Some e)
                               end in
                             let body = Exp.ifthenelse ~loc ~attrs
                                         condExp thenExp' elseExpOpt'
                             in (thenAux @ elseAux, body)
match exp with
  {pexp_desc = Pexp_ifthenelse(condExp, thenExp, elseExpOpt)} ->
      if isStaticExp condExp statVars
        then
          let auxName = fresh "aux" in
          let (auxAux, auxBody) = buildAuxBody
          in let auxCall = buildAuxCall ...
          in (auxAux @ [(auxName, auxBody)], auxCall)
        else
          let (condAux, condExp') = stage condExp in
          let (thenAux, thenExp') = stage thenExp in
          let (elseAux, elseExpOpt') =
            begin match elseExpOpt with
              None -> ([], None)
               | Some elseExp ->
                  let (aux, e) = stage elseExp in
                    (aux, Some (applyEsc e))
            end in
          let body = Exp.ifthenelse ~loc ~attrs (applyEsc condExp')
                        (applyEsc thenExp') elseExpOpt'
    in (condAux @ thenAux @ elseAux, applyBracket body)
```

Adding MetaOCaml Constructs

```
    .< ... >. -> [@metaocaml.bracket]
    .~ -> [@metaocaml.escape]
    !. -> Pexp_apply ({pexp_desc = Pexp_ident {txt = Lident "!."}},
    ...
```

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```
let rec pow x n = if n = 0 then 1 else x * (pow x (n - 1))
[@@static []] [@@static [x]] [@@static [n]] [@@static [x; n]]
```

Staged MetaOCaml code:

```
let pow =
  .<let rec pow x n = if n = 0 then 1 else x * (pow x (n - 1))
    in pow >.
let pow x x =
  .<let rec pow n = if n = 0 then 1 else x * (pow (n - 1))
    in pow >.
let pow n n =
  let rec aux 8 x n =
    if n = 0 then .< 1 >. else .< .~x * .~(aux_8 x (n - 1)) >. in
  .< let pow x = .\sim(aux_8 .< x >. n) in pow >.
let pow_xn x n =
  let rec aux 7 x n =
    if n = 0 then .< 1 >. else .< x * .~(aux 7 x (n - 1)) >. in
\cdot< let pow = \cdot~(aux 7 x n) in pow >.
```

Sample runs:

```
let _ = pow_x 3
- : (int -> int) code =
.<let rec pow_62 n_63 =
    if n_63 = 0 then 1 else 3 * (pow_62 (n_63 - 1)) in pow_62>.

let _ = pow_n 3
- : (int -> int) code =
.<let pow_65 x_64 = x_64 * (x_64 * (x_64 * 1)) in pow_65>.

let _ = pow_xn 3 3
- : int code = .<let pow_66 = 3 * (3 * (3 * 1)) in pow_66>.
```

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- System that automates the staging process
 - Only by annotating the static information
 - No need to worry about staging constructs and levels
 - Get optimized code

However...

```
let rec f x n = if n = 0 then 1 else x * (f (x+1) (n-1))

[@@static [x]]

let f_staged x =
    .< let f = if n = 0 then 1 else x * (f (n-1)) in f >.
```

- However...
 - Anonymous functions?
 - Curried functions?
 - Let-binding?

However...

```
let plus_xn x n = .< let plus = x + n in plus >.
# plus_xn 3 5;;
- : int code = .< let plus = 3 + 5 in plus>.
let plus_xn x n = let r = 3 + 5 in .< let plus = r in plus >.
# plus_xn 3 5;;
- : int code = .< let plus = 8 in plus>.
```

However...

```
.< let ff m =
     ((* CSP pow *) m) + ((* CSP pow *) m) + 0
in ff >.
```

- System that automates the staging process
 - Only by annotating the static information
 - No need to worry about staging constructs and levels
 - Get optimized code
- However the success is still limited
 - There are various limitations which warrants further research



Any Questions?

Appendices

• Omitted slides for reference

- Function with recursion
 - let rec pow x n = if n = 0 then 1 else x * pow x (n 1)
 - If x is static

```
=> let rec pow_staged x =
    .< let pow n = if n = 0 then 1 else x * pow (n - 1) in pow >.

# pow_staged 5;;
- : (int -> int) code =
    .< let pow n = if n = 0 then 1 else 5 * pow (n - 1) in pow >.
```

- Function with recursion
 - let rec pow x n = if n = 0 then 1 else x * pow x (n 1)
 - If n is static

```
=> let rec pow_staged n =
    let rec aux x n = if n = 0 then .< 1 >. else .< .~x * .~(aux x (n-1)) >.
    in .< let pow x = .~(aux .<x>. n) in pow >.

# pow_staged 3;;
- : (int -> int) code = .< let pow x = (x * (x * (x * 1))) in pow >.
```

For recursive function call in auxiliary function

```
isRecursiveCall() isInAux()
          <Args> => (ArgsAuxList, <ArgsS>)
      <ArgsS'> = <ArgsS[DynArgS->.<DynArgS>.]>
                  <FunName> <Args>
                         =>
( ArgsAuxList, .< <FirstAuxName> .~(<ArgsS'>) >. ))
```

For recursive function call in main function body

```
isRecursiveCall() !isInAux()
    <DynArgs> => (DynArgsAuxList, <DynArgsS>)
                 <FunName> <Args>
( DynArgsAuxList, .< <FunName> .~(<DynArgsS>) >. )
```

For all other function application

```
<Args> => (ArgsAuxList, <ArgsS>)
            <FunName> <Args>
                   =>
ArgsAuxList, .< <FunName> .~(<ArgsS>) >. )
```

- Function that uses other staged function
 - let rec ff y m = if y = 0 then 0 else (pow y m) + (ff (y 1) m)
 - If y is static and pow staged over first argument

```
=> let ff_staged y =
    let rec aux y m =
        if y = 0 then .< 0 >. else .< (pow .~m) + .~(aux (y - 1) m) >.
        in .< let pow = .~(pow_staged y) in
        let ff m = .~(aux y .<m>.) in ff) >.
```

For staged function application

```
isPrevStagedFun()
    <DynArgs> => (DynArgsAuxList, <DynArgsS>)
                 <FunName> <Args>
                        =>
( DynArgsAuxList, .< <FunName> .~(<DynArgsS>) >. )
```

.< let double = .~(aux_11 xs) in double >.

```
let rec double xs = match xs with
                                 | [] -> []
                                | x::xs -> (2 * x) :: (double xs)
[@@static []][@@static [xs]]
    Staged MetaOCaml code:
    let double_ =
                                                                       Sample runs:
      .<let rec double xs =</pre>
         match xs with | [] \rightarrow [] | x::xs \rightarrow (2*x)::(double xs) in
                                                                       let _ = double_xs [3;5;9]
        double >.
                                                                       - : int list code =
    let double_xs xs =
                                                                               .<let double_29 =</pre>
      let rec aux_11 xs =
                                                                                   [2 * 3; 2 * 5; 2 * 9]
        match xs with
        | [] -> .< [] >.
                                                                                 in double_29>.
        | x::xs \rightarrow .< (2 * x) :: .~(aux_11 xs) >. in
```

```
let nestedBranch x y =
   if x = 1 then (match y with | 0 -> "x0" | 1 -> "x1" | _ -> "xy") else "_"
   [@@static []][@@static [x]][@@static [y]][@@static [x; y]]
 Staged MetaOCaml code:
                                                                       let nestedBranch y y =
 let nestedBranch =
                                                                         let rec aux 3 x y =
   .<let nestedBranch x y =
                                                                           match y with
       if x = 1
                                                                           \mid 0 \rightarrow . \langle "x0" \rightarrow . \mid 1 \rightarrow . \langle "x1" \rightarrow . \mid - \rangle . \langle "xy" \rightarrow . in
       then match y with | 0 -> "x0" | 1 -> "x1" | -> "xy"
                                                                         .<let nestedBranch x =</pre>
       else " " in nestedBranch >.
                                                                            if x = 1 then .~(aux 3 .< x >. y) else " " in
                                                                           nestedBranch >.
 let nestedBranch x x =
   let rec aux 4 x y =
                                                                       let nestedBranch xy x y =
     if x = 1
                                                                         let rec aux 2 x y =
     then .< match .~y with | 0 -> "x0" | 1 -> "x1" | -> "xy" >.
                                                                           match y with | 0 -> .< "x0" >. | 1 -> .< "x1" >. | -> .< "xy" >.
     else .< "_" >. in
                                                                         and aux_1 x y = if x = 1 then aux_2 x y else .< "_" >. in
   .< let nestedBranch y = .~(aux_4 x .< y >.) in nestedBranch >.
```

.< let nestedBranch = .~(aux 1 x y) in nestedBranch >.

```
let nestedBranch x y = if x = 1 then (match y with | 0 -> "x0" | 1 -> "x1" | _ -> "xy") else "_" [@@static []][@@static [x]][@@static [y]][@@static [x; y]]
```

Sample runs:

```
let _ = nestedBranch_x 0
- : (int -> string) code =
.<let nestedBranch_5 y_4 = "_" in nestedBranch_5>.

let _ = nestedBranch_x 1
- : (int -> string) code =
.<let nestedBranch_7 y_6 =
    match y_6 with | 0 -> "x0" | 1 -> "x1" | _ -> "xy" in nestedBranch_7>.
```

```
let _ = nestedBranch_y 0
- : (int -> string) code =
.<let nestedBranch_9 x_8 =
    if x_8 = 1 then "x0" else "_" in nestedBranch_9>.

let _ = nestedBranch_y 5
- : (int -> string) code =
.<let nestedBranch_11 x_10 =
    if x_10 = 1 then "xy" else "_" in nestedBranch_11>.

let _ = nestedBranch_xy 1 2
- : string code = .<let nestedBranch_12 = "xy" in nestedBranch_12>.
```

```
let rec ff y m =
   if y = 0 then 0 else ((pow y m)[@static.use ]) + (ff (y - 1) m)
[@@static []][@@static [y]][@@static [m]][@@static [y; m]]
```

Staged MetaOCaml code:

```
let ff_ =
  let pow = Runcode.run .< .~pow_ >. in
  .<let rec ff y m =
      if y = 0 then 0 else (pow y m) + (ff (y - 1) m) in
      ff >.

let ff_y y =
  let pow = Runcode.run .< .~(pow_x y) >. in
  let rec aux_21 y m =
    if y=0 then .< 0 >. else .< (pow .~m) + .~(aux_21 (y - 1) m) >. in
  .< let ff m = .~(aux_21 y .< m >.) in ff >.
```

```
let ff_m m =
  let pow = Runcode.run .< .~(pow_n m) >. in
  .<let rec ff y = if y = 0 then 0 else (pow y) + (ff (y - 1)) in
    ff >.

let ff_ym y m =
  let pow = Runcode.run .< .~(pow_xn y m) >. in
  let rec aux_20 y m =
    if y = 0 then .< 0 >. else .< pow + .~(aux_20 (y - 1) m) >. in
  .< let ff = .~(aux_20 y m) in ff >.
```

```
let rec ff y m =
   if y = 0 then 0 else ((pow y m)[@static.use ]) + (ff (y - 1) m)
[@@static []][@@static [y]][@@static [m]][@@static [y; m]]
```

```
let _ = ff_m 3
- : (int -> int) code =
.<let rec ff_104 y_105 =
    if y_105 = 0
    then 0
    else ((* CSP pow *) y_105) + (ff_104 (y_105 - 1)) in
    ff_104>.

let _ = ff_ym 3 3
- : int code = .<let ff 107 = 27 + (27 + (27 + 0)) in ff 107>.
```