Model Documentation of the Modular Multilevel Converter

1 Nomenclature

1.1 Nomenclature for Model Equations

e_{s0}	total stored energy (scaled by $\frac{2}{3}$)
e_{d0}	(vertical) difference between all upper and all lower arms (scaled by $\frac{2}{3}$)
$\underline{e}_s$	complex energy sum
$\underline{e}_d$	complex energy difference
θ	angle of rotating reference frame of $\underline{e}_s$ and $\underline{e}_d$
ω	angular speed of the rotating reference frame
v_{DC}	DC voltage of the MMC
v_{y0}	common-mode voltage
$\underline{v}_y$	complex output voltage
v_{x0}	voltage which drives the DC currents
$\underline{v}_x$	voltage which drives the internal currents
$\underline{v}_g$	grid voltage
$\underline{i}$	complex output current
i_{s0}	scaled version of DC current
$\underline{i}_s$	complex circulating current
L_z	arm inductance
M_z	mutual inductance
R	resistance of the load
L	inductance of the load

1.2 Nomenclature for Derivation

e_{z1}, ..., e_{z6}	arm energies
i_{z1}, ..., i_{z6}	arm currents
g_0, g_α, g_β	clark transform constants
$\underline{g}_{\alpha\beta}$	complex constant using g_α, g_β and θ

1.3 Circuit Graphic

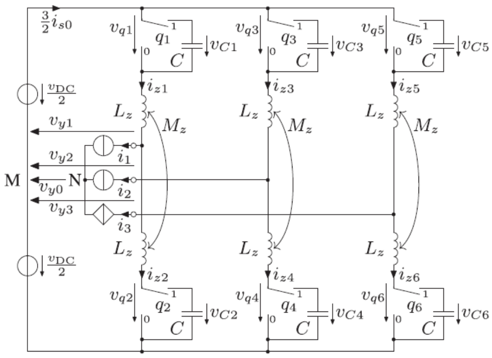

Figure 1: Circuit.
Source: Fehr, H.; Gensior, A./ Improved Energy Balancing of Grid-Side Modular Multilevel Converters by Optimized Feedforward Circulating Currents and Common-Mode Voltage

2 Model Equations

State Vector and Input Vector:

$$\underline{x} = (x_1\ x_2\ \underline{x}_3\ \underline{x}_4\ \underline{x}_5\ x_6\ \underline{x}_7\ x_8)^T = (e_{s0}\ e_{d0}\ \underline{e}_s\ \underline{e}_d\ \underline{i}_s\ i_{s0}\ \underline{i}\ \theta)^T$$
$$\underline{u} = (\underline{u}_1\ u_2\ \underline{u}_3\ u_4)^T = (\underline{v}_y\ v_{y0}\ \underline{v}_x\ v_{x0})^T$$

Model Equations:

$$\dot{x}_1 = v_{DC}\, x_6 - \mathrm{Re}(\underline{x}_7\, \underline{u}_1^*) \tag{1a}$$
$$\dot{x}_2 = -2u_2\, x_6 - \mathrm{Re}(\underline{x}_5^*\, \underline{v}_{y\Delta}) \tag{1b}$$
$$\dot{\underline{x}}_3 = v_{DC}\, \underline{x}_5 - e^{-3jx_8}\, \underline{u}_1^*\, \underline{x}_7^* - 2\underline{x}_7\, u_2 - j\omega\, \underline{x}_3 \tag{1c}$$
$$\dot{\underline{x}}_4 = v_{DC}\, \underline{x}_7 - e^{-3jx_8}\, \underline{x}_5^*\, \underline{v}_{y\Delta}^* - 2\underline{x}_5\, u_2 - 2x_6\, \underline{v}_{y\Delta} - j\omega \underline{x}_2 \tag{1d}$$
$$\dot{\underline{x}}_5 = \frac{1}{L_z + M_z}(\underline{u}_3 - j\omega(L_z + M_z)\, \underline{x}_5) \tag{1e}$$
$$\dot{x}_6 = \frac{1}{L_z + M_z} u_4 \tag{1f}$$
$$\dot{\underline{x}}_7 = \frac{1}{L}(\underline{u}_1 - (R + j\omega L)\underline{x}_7 - \underline{v}_g) \tag{1g}$$
$$\dot{x}_8 = \omega \tag{1h}$$

with

$$\underline{v}_{y\Delta} = \underline{u}_1 - M_z(j\omega \underline{i} + \dot{\underline{x}}_7) \tag{2}$$

Parameters: $v_{DC},\ \underline{v}_g,\ \omega,\ L_z,\ M_z,\ R, L$
Outputs: $\underline{e}_s,\ e_{s0},\ \underline{e}_d,\ e_{d0}$

2.1 Assumptions

1. The cells of the arm k = 1,2,...,6 are represented by one equivalent cell with the duty cycle $q_k \in$ [0,1] and a voltage v_Ck that accords the sum of the individual cells in the arm. This implies that the underlying problem of balancing the voltages within each arm has already been solved.

2. The load currents are assumed to be continuous, matched to the initial currents of the arm inductors, and satisfy the constraint $i_1 + i_2 + i_3 = 0$ caused by junction N.

2.2 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
DC voltage	v_{DC}	300	V
grid voltage	v_g	235	V
angular speed	ω	10π	Hz
arm inductance	L_z	0.15	mH
mutual inductance	M_z	0.094	mH
load resistance	R	26	Ω
load inductance	L	0.3	mH

3 Derivation and Explanation

The six arm energies e_{z1}, ..., e_{z6} transformed into

$$e_{s0} = 2g_0 \left[(e_{z1}, e_{z3}, e_{z5})^T + (e_{z2}, e_{z4}, e_{z6})^T\right] \tag{3a}$$

$$e_{d0} = 2g_0 \left[(e_{z1}, e_{z3}, e_{z5})^T - (e_{z2}, e_{z4}, e_{z6})^T\right] \tag{3b}$$

$$\underline{e}_s = 2\underline{g}_{\alpha\beta} \left[(e_{z1}, e_{z3}, e_{z5})^T + (e_{z2}, e_{z4}, e_{z6})^T\right] \tag{3c}$$

$$\underline{e}_d = 2\underline{g}_{\alpha\beta} \left[(e_{z1}, e_{z3}, e_{z5})^T - (e_{z2}, e_{z4}, e_{z6})^T\right] \tag{3d}$$

with use of the Clark Transform

$$T_{0\alpha\beta} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ 0 & \sqrt{3} & -\sqrt{3} \end{pmatrix} = \begin{pmatrix} g_0 \\ g_\alpha \\ g_\beta \end{pmatrix} \tag{4}$$

and $\underline{g}_{\alpha\beta} = e^{-j\theta}(g_\alpha + jg_\beta)$.
The currents can be transformed as

$$i_{s0} = 2g_0 \left[(i_{z1}, i_{z3}, i_{z5})^T + (i_{z2}, i_{z4}, i_{z6})^T\right] \tag{5a}$$

$$0 = g_0(i_1, i_2, i_3)^T \tag{5b}$$

$$\underline{i}_s = \underline{g}_{\alpha\beta} \left[(i_{z1}, i_{z3}, i_{z5})^T + (i_{z2}, i_{z4}, i_{z6})^T\right] \tag{5c}$$

$$\underline{i} = \underline{g}_{\alpha\beta}\,(i_1, i_2, i_3)^T. \tag{5d}$$

The voltages are transformed as

$$
\begin{aligned}
v_{x0} &= g_0 \left[v_{DC}(1,1,1)^T - (v_{q1}, v_{q3}, v_{q5})^T - (v_{q2}, v_4, v_{q6})^T\right] && \text{(6a)}\\
v_{y0} &= g_0 \left(v_{y1}, v_{y2}, v_{y3}\right)^T && \text{(6b)}\\
\underline{v}_x &= \underline{g}_{\alpha\beta} \left[v_{DC}(1,1,1)^T - (v_{q1}, v_{q3}, v_{q5})^T - (v_{q2}, v_4, v_{q6})^T\right] && \text{(6c)}\\
\underline{v}_y &= \underline{g}_{\alpha\beta} \left(v_{y1}, v_{y2}, v_{y3}\right)^T && \text{(6d)}\\
&= \underline{g}_{\alpha\beta} \left[(v_{q2}, v_4, v_{q6})^T - (v_{q1}, v_{q3}, v_{q5})^T - (L_z - M_z)\frac{d}{dt}(i_1, i_2, i_3)^T\right]. && \text{(6e)}
\end{aligned}
$$

4 Simulation

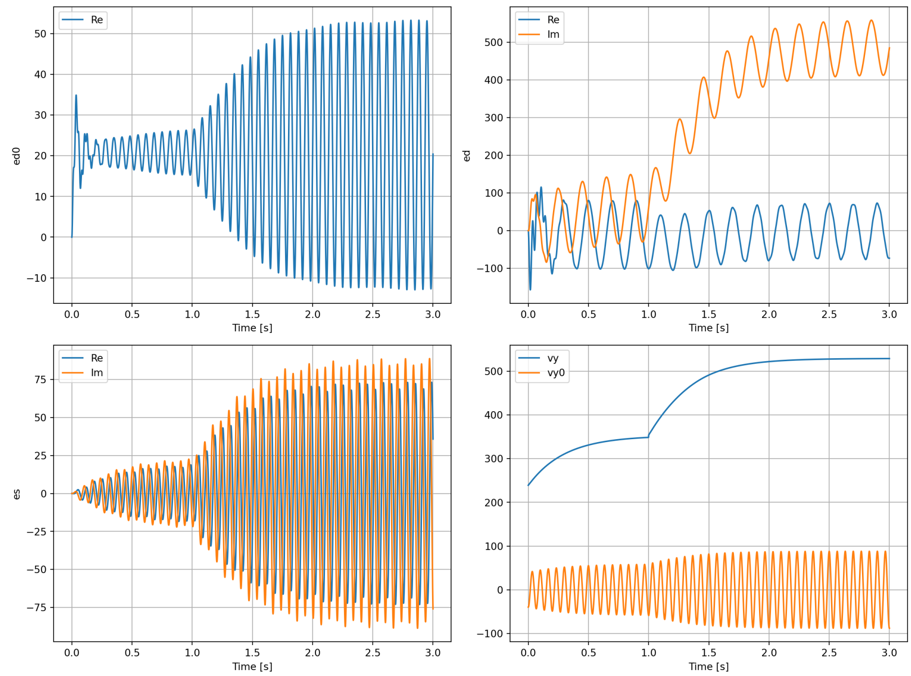

Figure 2: Simulation of the modular multilevel converter (MMC).

References

[1] Fehr, H.; Gensior, A.: *Improved Energy Balancing of Grid-Side Modular Multilevel Converters by Optimized Feedforward Circulating Currents and Common-Mode Voltage*, IEEE 2018.