Model Documentation of the Distillation Column

1 Nomenclature

1.1 Nomenclature for Model Equations

K_{R1}, T_{N1}	parameters of the first PI controller
K_{R2}, T_{N2}	parameters of the second PI controller
K_1, K_2, K_3, K_4, T_1	parameters of the model, equilibrium point
x_S	filling level
x_T	temperature on the bottom
z_{ii}	malfunctions for i = 1, 2
fb_i	feedback for i = 1, 2
w	supply of heat steam, equivalent to M_H

1.2 Signal Flowchart

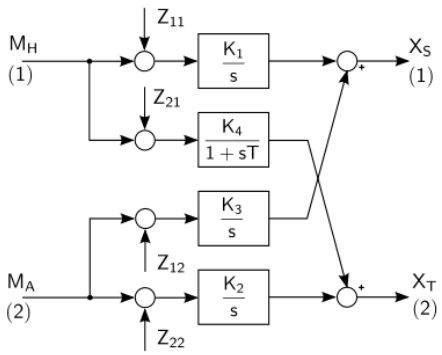

Figure 1: Signal Flowchart

2 Model Equations

State Vector and Input Vector:

$$\underline{x} = (x_S \; x_T)^T \qquad = (x_1 \; x_2)^T$$
$$\underline{u} = (fb_1 \; fb_2 \; w \; z_{11} \; z_{21} \; z_{12} \; z_{22}) \qquad = (u_1 \; u_2 \; u_3 \; u_4 \; u_5 \; u_6 \; u_7)^T$$

Transfer Functions:

$$G_{R_{11}} = K_{R1}\left(1 + \frac{1}{sT_{N1}}\right) \tag{1a}$$

$$G_{R_{22}} = K_{R2}\left(1 + \frac{1}{sT_{N1}}\right) \tag{1b}$$

$$G_{P_{11}} = \frac{K_1}{s} \tag{1c}$$

$$G_{P_{12}} = \frac{K_4}{1 + sT} \tag{1d}$$

$$G_{P_{21}} = \frac{K_3}{s} \tag{1e}$$

$$G_{P_{22}} = \frac{K_2}{s} \tag{1f}$$

Parameters: K_{R1}, T_{N1}, K_{R2}, T_{N2}, T_1, K_1, K_2, K_3, K_4
Outputs: x_1, x_2

2.1 Exemplary parameter values

Symbol	Value
K_{R1}	1.7
T_{N1}	1.29
K_{R2}	0.57
T_{N2}	1.29
T_1	1
K_1	0.4
K_2	1.2
K_3	−0.8
K_4	−0.2

3 Derivation and Explanation

A rough analysis of the column behavior leads to the following approaches for the four subtransfer functions:

$$\frac{X_S}{M_H} = \frac{K_1}{s};\ \frac{X_T}{M_A} = \frac{K_2}{s};\ \frac{X_S}{M_A} = \frac{K_3}{s};\ \frac{X_T}{M_H} = \frac{K_4}{1 + sT}.$$

M_H is standing for the supply of the heat steam and M_A represents the drain of the product. The system model is based on the signal flowchart.

4 Simulation

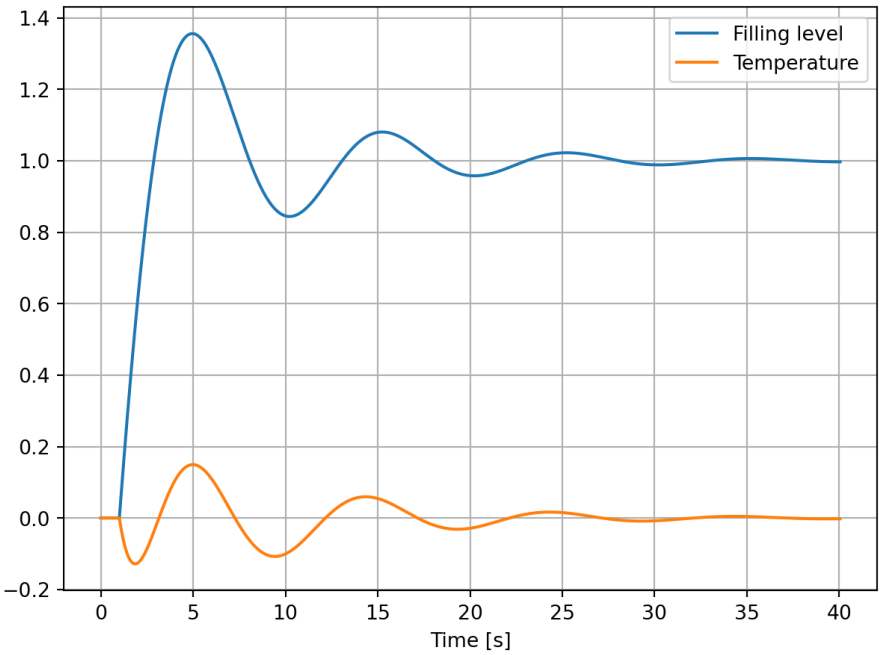

Figure 2: Simulation of the distillation column.

References

[1] Institut of Control Theory TU Dresden: *Regelungstechnikpratikum, Praktikumsanleitung*, published in OPAL April 2022.
(not publicly accessible)

[2] Knoll, Carsten: *Example2 : linear system consiting of various blocks*, Python Script published 2019.
`https://github.com/TUD-RST/pyblocksim/blob/master/examples/example2.py`