

# Model Documentation of the: Kapitza's Pendulum

## 1 Nomenclature

### 1.1 Nomenclature for Model Equations

$\gamma$	Dampening factor
$l$	length of the pendulum
$g$	acceleration due to gravity
$\varphi$	angle of deflection from the equilibrium position
$a$	magnitude of the harmonic oscillation of the suspension point
$\omega$	frequency of the harmonic oscillation of the suspension point

### 1.2 Nomenclature for Derivation

## 2 Model Equations

State Vector: State Vector and Input Vector:

$$\underline{x} = (x_1 \ x_2)^T = (\varphi \ \dot{\varphi})^T$$
$$\underline{u} = \emptyset$$

System Equations:

$$\dot{x}_1 = x_2 \tag{1a}$$

$$\dot{x}_2 = -2\gamma x_2 - \left(\frac{g}{l} - \frac{a}{l}\omega^2 \cos(\omega t)\right) \sin(x_1) \tag{1b}$$

Parameters:  $\omega$ ,  $a$ ,  $l$ ,  $g$ ,  $\gamma$

Outputs:  $\varphi$

### 2.1 Assumptions

1. Mass of the pendulum is a pointmass

### 2.2 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
Pendulum length	$l$	0.3	cm
acceleration due to gravitation	$g$	9.81	$\frac{m}{s^2}$
Amplitude of Oscillation	$a$	0.2 l	cm
Frequency of Oscillation	$\omega$	$16\omega_0$	Hz
Dampening Factor	$\gamma$	$0.1\omega_0$	Hz

with  $\omega_0 = \sqrt{\frac{g}{l}}$

### 3 Derivation and Explanation

#### References

- [1] Butikov, E. I.: *Kapitza's Pendulum: A Physically Transparent Simple Treatment*, published 2021