Model Documentation of the Permanent Magnet DC Motor

1 Nomenclature

1.1 Nomenclature for Model Equations

$c\phi$	motor constant
J	moment of inertia
L_A	armature inductance
R_A	armature resistance
i_A	armature current
ω	angular velocity
u_A	armature voltage
ξ_L	load moment

2 Model Equations

State Vector and Input Vector:

$$\begin{aligned} \underline{x} &= (\omega \; i_A)^T && = (x_1 \; x_2)^T \\ \underline{u} &= (u_A \; \xi_L)^T && = (u_1 \; u_2)^T \end{aligned}$$

System Equations:

$$\dot{x}_1 = \frac{c\phi}{J} x_2 - \frac{1}{J} u_2 \tag{1a}$$

$$\dot{x}_2 = -\frac{R_A}{L_A} x_2 - \frac{c\phi}{L_A} x_1 + \frac{1}{L_A} u_1 \tag{1b}$$

Parameters: $c\phi \; J \; L_A \; R_A$
Output: ω

2.1 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
motor constant	c_ϕ	0.169	Vs
moment of inertia	J	0.0017	Ws^3
armature inductance	La	0.0256	H
armature resistance	Ra	3.2	Ω

3 Derivation and Explanation

The function of the electrical motor is based on the interaction between the electromechanical power law and Faraday's law of induction.

$$\xi(t) = c\phi i_A(t) \tag{2}$$

$$u(t) = c\phi\omega(t) \tag{3}$$

The following applies to the voltage drops in the electrical armature circuit:

$$u_A(t) = u(t) + R_A i_A(t) + L_A \frac{di_A(t)}{dt}. \tag{4}$$

For the mechanical system, Newton's law for rotational motion provides

$$\frac{\omega(t)}{dt} = \frac{1}{J}\xi_b(t). \tag{5}$$

Furthermore:

$$\xi_b(t) = \xi(t) - \xi_L(t). \tag{6}$$

4 Simulation

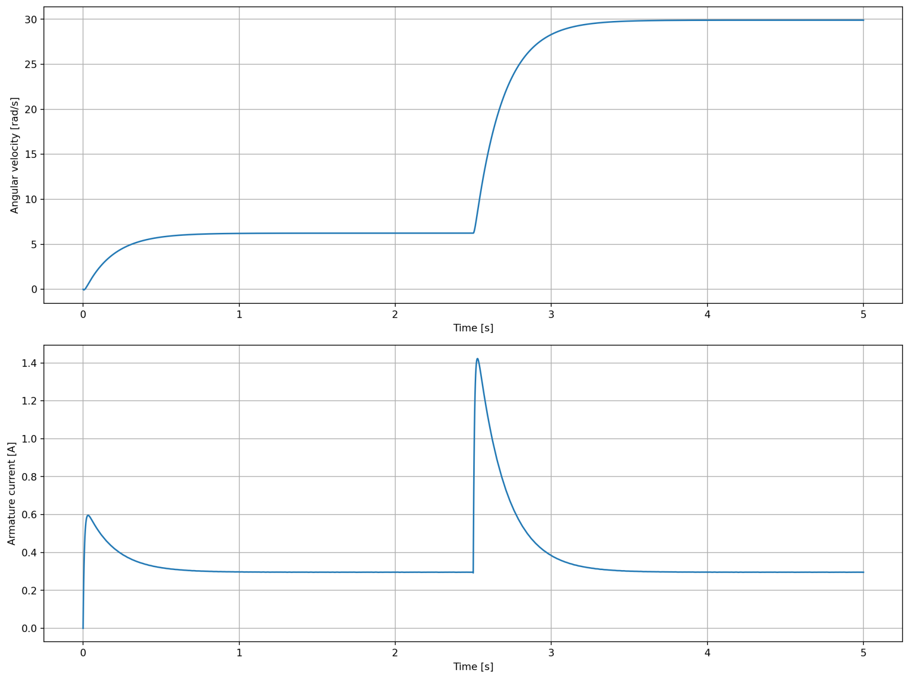

Figure 1: Simulation of the permanent magnet DC motor.

References

[1] TU Dresden – Institut für Regelungs- und Steuerungstheorie: *Regelungstechnikpraktikum, Praktikumsanleitung*, published on OPAL April 2022. (not publicly accessible)