

Model Documentation of the Four-Bar Linkage

1 Nomenclature

1.1 Nomenclature for Model Equations

s_1	center of gravity distance of first bar
s_2	center of gravity distance of second bar
s_3	center of gravity distance of third bar
m_1	mass of first bar
m_2	mass of second bar
m_3	mass of third bar
J_1	moment of inertia of first bar
J_2	moment of inertia of second bar
J_3	moment of inertia of third bar
l_1	length of first bar
l_2	length of second bar
l_3	length of third bar
l_4	length of fourth bar
g	acceleration due to gravity
p_1	angle of between th bars of the two link mainpulator
p_2	angle of one link manipulator
p_2	angle of two link manipulator
u_1	external force
y	array of angles
\dot{y}	array of angular velocities

2 Model Equations

DAE Variables and Input Vector:

$$\underline{x} = (x_1 \ x_2 \ x_3 \ \dot{x}_1 \ \dot{x}_2 \ \dot{x}_3 \ \lambda_1 \ \lambda_2)^T = (p_1 \ p_2 \ q_1 \ \dot{p}_1 \ \dot{p}_2 \ \dot{q}_1 \ \lambda_1 \ \lambda_2)^T$$
$$\underline{u} = u_1$$

Constraints:

$$l_1 \cos(x_3) + l_2 \cos(x_1 + x_3) - l_3 \cos(x_2) - l_4 = 0 \quad (1a)$$

$$l_1 \sin(x_3) + l_2 \sin(x_1 + x_3) - l_3 \sin(x_2) = 0 \quad (1b)$$

System Equations:

$$0 = J_2 \ddot{x}_1 + J_2 \ddot{x}_3 + g m_2 s_2 \cos(x_1 + x_3) + l_1 m_2 \ddot{x}_3 s_2 \cos(x_1) + l_1 m_2 \dot{x}_3^2 s_2 \sin(x_1) \\ + l_2 \lambda_1 \sin(x_1 + x_3) - l_2 \lambda_2 \cos(x_1 + x_3) + m_2 \dot{x}_1 s_2^2 + m_2 \ddot{x}_3 s_2^2$$

$$0 = J_3 \ddot{x}_2 + g m_3 s_3 \cos(x_2) - l_3 \lambda_1 \sin(x_2) + l_3 \lambda_2 \cos(x_2) + m_3 \ddot{x}_2 s_3^2$$

$$0 = J_1 \ddot{x}_3 + J_2 \ddot{x}_1 + J_2 \ddot{x}_3 + g l_1 m_2 \cos(x_3) + g m_1 s_1 \cos(x_3) + g m_2 s_2 \cos(x_1 + x_3) \\ + l_1^2 m_2 \ddot{x}_3 + l_1 \lambda_1 \sin(x_3) - l_1 \lambda_2 \cos(x_3) + l_1 m_2 \dot{x}_1 s_2 \cos(x_1) - l_1 m_2 \dot{x}_1^2 s_2 \sin(x_1) \\ - 2 l_1 m_2 \dot{x}_1 \dot{x}_3 s_2 \sin(x_1) + 2 l_1 m_2 \ddot{x}_3 s_2 \cos(x_1) + l_2 \lambda_1 \sin(x_1 + x_3) - l_2 \lambda_2 \cos(x_1 + x_3) \\ + m_1 \ddot{x}_3 s_1^2 + m_2 \dot{x}_1 s_2^2 + m_2 \ddot{x}_3 s_2^2 - u_1$$

Parameters: s_1 s_2 s_3 m_1 m_2 m_3 J_1 J_2 J_3 l_1 l_2 l_3 l_4 g

Outputs: y \dot{y}

2.1 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
center of gravity distance of first bar	s_1	0.5	m
center of gravity distance of second bar	s_2	0.5	m
center of gravity distance of third bar	s_3	0.5	m
mass of first bar	m_1	1	kg
mass of second bar	m_2	1	kg
mass of third bar	m_3	3	kg
moment of inertia	J_1	0.08333333333333333	$\frac{kg}{m^2}$
moment of inertia	J_2	0.08333333333333333	$\frac{kg}{m^2}$
moment of inertia	J_3	0.08333333333333333	$\frac{kg}{m^2}$
length of first bar	l_1	0.8	m
length of second bar	l_2	1.5	m
length of third bar	l_3	1.5	m
length of fourth bar	l_4	2	m
acceleration due to gravity	g	9.81	$\frac{m}{s^2}$

3 Derivation and Explanation

Not available

References

- [1] Knoll, Carsten: *Considered model: four-bar linkage (= two link manipulator + one link manipulator + rigid coupling)*, Jupyter Notebook published 2019