

Model Documentation of the Linear Transport System

1 Nomenclature

1.1 Nomenclature for Model Equations

t	time
z	space
v	velocity (constant)
$u(z, t)$	input function
$x(z, t)$	wanted function describing the material transport

2 Model Equations

System Equations:

$$\begin{aligned}\dot{x}(z, t) + vx'(z, t) &= 0 & z \in (0, l], t > 0 \\ x(z, 0) &= x_0(z) & z \in [0, l] \\ x(0, t) &= u(t) & t > 0\end{aligned}$$

Parameters: v

Outputs: $x(l, t)$

2.1 Assumptions

1. $x_0(z) = 0$

2.2 Exemplary parameter values

Parameter Name	Symbol	Value
velocity-constant	v	4

3 Derivation and Explanation

Weak formulation approach with weight function $\varphi(z)$:

$$\begin{aligned}0 &\stackrel{!}{=} \int_{z=0}^{z=l} [\dot{x}(z, t) + vx'(z, t)] \varphi(z) dz \\ 0 &= \int_{z=0}^{z=l} \dot{x}(z, t) \varphi(z) dz + v \int_{z=0}^{z=l} x'(z, t) \varphi(z) dz \\ &\quad \text{with partial integration} \\ 0 &= \int_{z=0}^{z=l} \dot{x}(z, t) \varphi(z) dz + v[x\varphi]_{z=0}^{z=l} - v \int_{z=0}^{z=l} x(z, t) \varphi'(z) dz \\ 0 &= \int_{z=0}^{z=l} \dot{x}(z, t) \varphi(z) dz + vx(l) \varphi(l) - v\varphi(0)u(t) - v \int_{z=0}^{z=l} x(z, t) \varphi'(z) dz\end{aligned}$$

4 Simulation

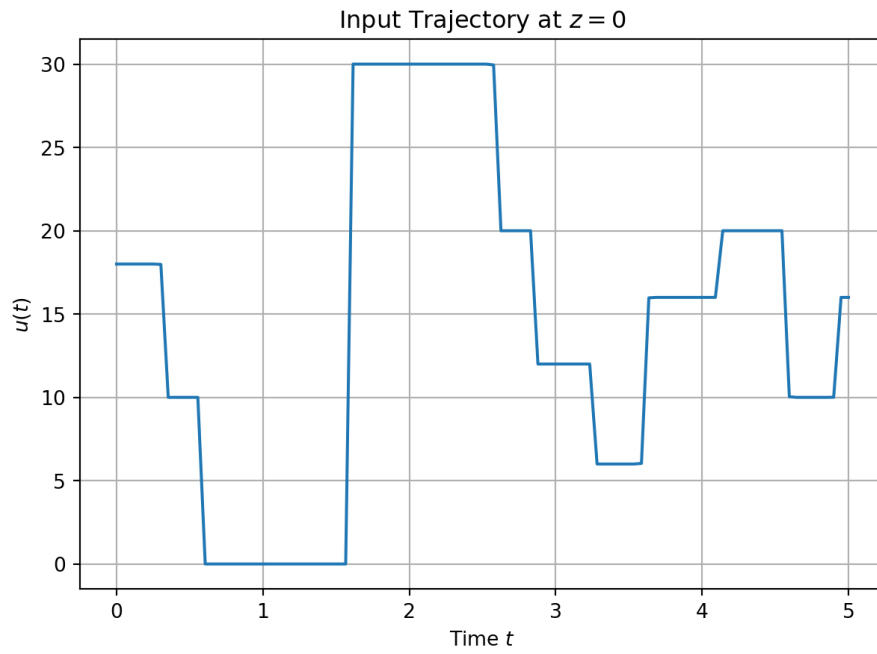


Figure 1: Simulation of the linear transport system.

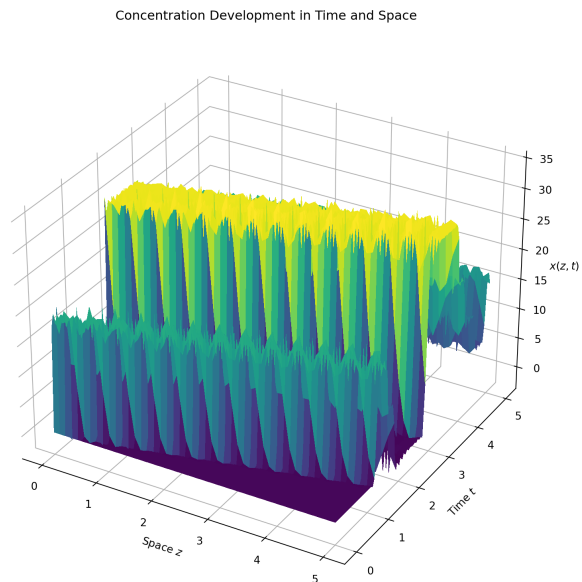


Figure 2: Simulation of the linear transport system.

References

- [1] Stefan Eklebe, Marcus Riesmeier:
https://pyinduct.readthedocs.io/en/master/examples/transport_system.html