Model Documentation of the Four-Bar Linkage

1 Nomenclature

1.1 Nomenclature for Model Equations

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s_i	distance from the joint to the center of gravity of link i, where $i = 1, 2, 3$
m_i	mass of link i, where $i = 1, 2, 3$
J_i	moment of inertia of link i, where $i = 1, 2, 3$
l_i	length (distance between joints) of link i, where $i = 1, 2, 3, 4$
g	acceleration due to gravity
p_1	angle between basis and link 1 (joint 1)
p_2	angle between link 1 and link 2 (joint 2)
q_1	angle between basis and link 3 (joint 4)
u_1	external torgue applied to joint 1
y	array of angles
$\dot{y}$	array of angular velocities

1.2 Graphic of the Structure
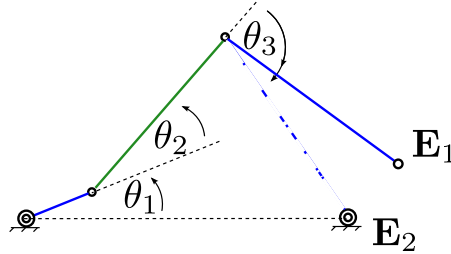
Figure 1: Structure of the Four-Bar Linkage.
Source: Knoll, Carsten/ private illustration

2 Model Equations

DAE Variables and Input Vector:

$$\underline{x} = (p_1 \ p_2 \ q_1 \ \dot{p}_1 \ \dot{p}_2 \ \dot{q}_1 \ \lambda_1 \ \lambda_2)^T \qquad = (x_1 \ x_2 \ x_3 \ \dot{x}_1 \ \dot{x}_2 \ \dot{x}_3 \ \lambda_1 \ \lambda_2)^T$$

$$\underline{u} = u_1$$

Constraints:

$$l_1 \cos(x_3) + l_2 \cos(x_1 + x_3) - l_3 \cos(x_2) - l_4 \qquad (1a)$$

$$l_1 \sin(x_3) + l_2 \sin(x_1 + x_3) - l_3 \sin(x_2) \qquad (1b)$$

System Equations:

$$0 = J_2 \ddot{x}_1 + J_2 \ddot{x}_3 + g m_2 s_2 \cos(x_1 + x_3) + l_1 m_2 \ddot{x}_3 s_2 \cos(x_1) + l_1 m_2 \dot{x}_3^2 s_2 \sin(x_1)$$
$$+ l_2 \lambda_1 \sin(x_1 + x_3) - l_2 \lambda_2 \cos(x_1 + x_3) + m_2 \ddot{x}_1 s_2^2 + m_2 \ddot{x}_3 s_2^2$$

$$0 = J_3 \ddot{x}_2 + g m_3 s_3 \cos(x_2) - l_3 \lambda_1 \sin(x_2) + l_3 \lambda_2 \cos(x_2) + m_3 \ddot{x}_2 s_3^2$$

$$0 = J_1 \ddot{x}_3 + J_2 \ddot{x}_1 + J_2 \ddot{x}_3 + g l_1 m_2 \cos(x_3) + g m_1 s_1 \cos(x_3) + g m_2 s_2 \cos(x_1 + x_3)$$
$$+ l_1^2 m_2 \ddot{x}_3 + l_1 \lambda_1 \sin(x_3) - l_1 \lambda_2 \cos(x_3) + l_1 m_2 \ddot{x}_1 s_2 \cos(x_1) - l_1 m_2 \dot{x}_1^2 s_2 \sin(x_1)$$
$$- 2 l_1 m_2 \dot{x}_1 \dot{x}_3 s_2 \sin(x_1) + 2 l_1 m_2 \ddot{x}_3 s_2 \cos(x_1) + l_2 \lambda_1 \sin(x_1 + x_3) - l2 \lambda_2 \cos(x_1 + x_3)$$
$$+ m_1 \ddot{x}_3 s_1^2 + m_2 \ddot{x}_1 s_2^2 + m_2 \ddot{x}_3 s_2^2 - u_1$$

Parameters: s_1, s_2, s_3, m_1, m_2, m_3, J_1, J_2, J_3, l_1, l_2, l_3, l_4, g
Outputs: y, $\dot{y}$

2.1 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
distance from the joint to the center of gravity of link 1	s_1	0.5	m
distance from the joint to the center of gravity of link 2	s_2	0.5	m
distance from the joint to the center of gravity of link 3	s_3	0.5	m
mass of link 1	m_1	1	kg
mass of link 2	m_2	1	kg
mass of link 3	m_3	3	kg
moment of inertia of link 1	J_1	0.08333	$kg \cdot m^2$
moment of inertia of link 2	J_2	0.08333	$kg \cdot m^2$
moment of inertia of link 3	J_3	0.08333	$kg \cdot m^2$
length of link 1	l_1	0.8	m
length of link 2	l_2	1.5	m
length of link 3	l_3	1.5	m
length of link 4	l_4	2	m
acceleration due to gravity	g	9.81	$\frac{m}{s^2}$

3 Derivation and Explanation

The Lagrangian mechanics was used for the solution.
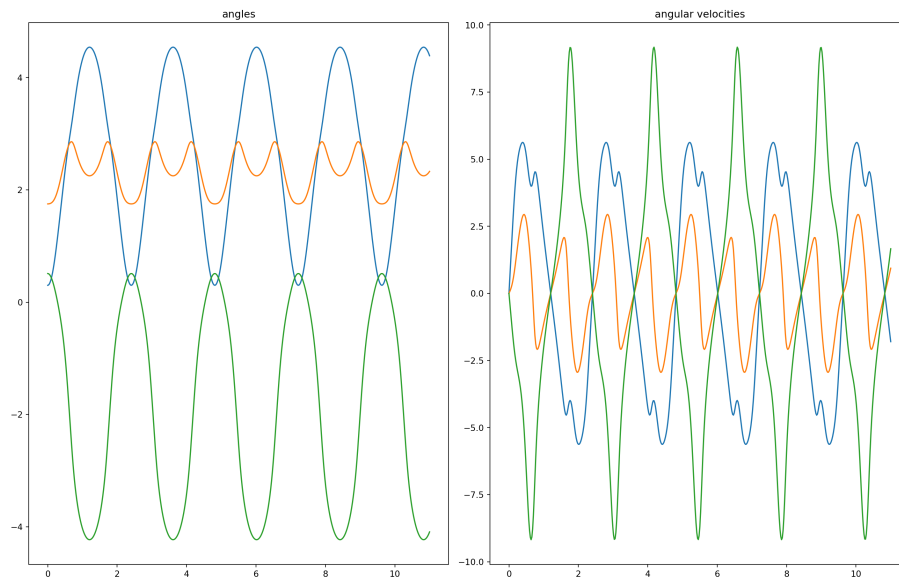
4 Simulation

Figure 2: Simulation of the Four-bar Linkage.

References

[1] Knoll, Carsten: *Considered model: four-bar linkage (= two link manipulator + one link manipulator + rigid coupling)*, Jupyter Notebook published 2019.
`https://github.com/TUD-RST/symbtools/blob/main/docs/demo_`
`notebooks/modeltools/four-bar_linkage_model_and_simulation.`
`ipynb`