Model Documentation of the Acrobot

1 Nomenclature

1.1 Nomenclature for Model Equations

s_i	center of gravity distance of the bar for i = 1, 2
m_i	mass of the bar for i = 1, 2
J_i	moment of inertia of the bar for i = 1, 2
l_1	length of the first bar
g	acceleration due to gravity
p_1	angle between the vertically downwards rest position and the first bar
q_1	angle between the first and the second bar
$\dot{p_1}$	angle velocity of the first bar
$\dot{q_1}$	angle velocity of the second bar
τ_1	input force at the joint

2 Model Equations

State Vector and Input Vector:

$$\begin{aligned} \underline{x} &= (p_1\ q_1\ \dot{p_1}\ \dot{q_1})^T &&= (x_1\ x_2\ x_3\ x_4)^T \\ \underline{u} &= \tau_1 &&= u_1 \end{aligned}$$

Kinetic Energy:

$$\begin{aligned} T = &\frac{J_1 x_3^2}{2} + \frac{J_2(x_3+x_4)^2}{2} + \frac{m_1 x_3^2 s_1^2 \sin(x_1)^2}{2} + \frac{m_1 x_3^2 s_1^2 \cos(x_1)^2}{2} \\ &+ m_2(-l_1 x_3 \sin(x_1) - \frac{s_2(x_3+x_4)\sin(x_1+x_2))^2}{2} + m_2(l_1 x_3 \cos(x_1) \\ &+ \frac{s_2(x_3+x_4)\cos(x_1+x_2))^2}{2} \end{aligned}$$

Potential Energy:

$$V = g m_1 s_1 \sin(x_1) + g m_2(l_1 \sin(x_1) + s_2 \sin(x_1+x_2))$$

Parameters: $s_1\ s_2\ m_1\ m_2\ J_1\ J_2\ l_1\ g$
Outputs: $\underline{x}$

2.1 Assumptions

1. The rest position of the acrobot is vertically downward.

2.2 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
center of gravity distance of first bar	s_1	0.25	m
center of gravity distance of second bar	s_2	0.25	m
mass of first bar	m_1	1	kg
mass of second bar	m_2	1	kg
moment of inertia of first bar	J_1	0.006667	$\frac{kg}{m^2}$
moment of inertia of second bar	J_2	0.01333	$\frac{kg}{m^2}$
length of first bar	l_1	0.5	m
acceleration due to gravity	g	9.81	$\frac{m}{s^2}$

3 Derivation and Explanation

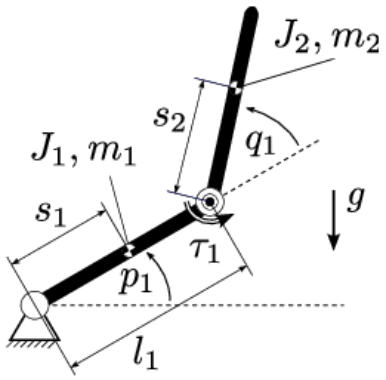

Figure 1: Acrobot

4 Simulation

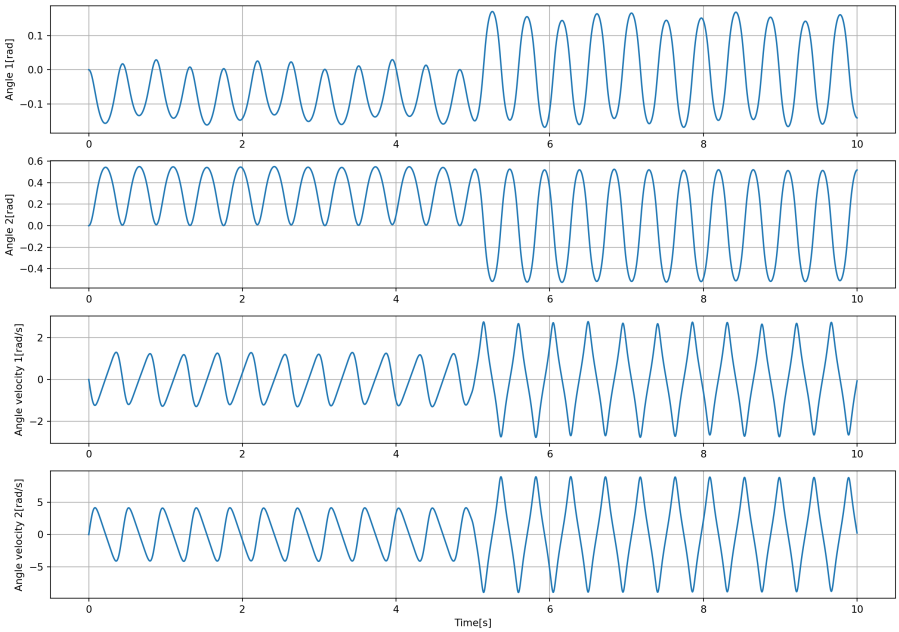

Figure 2: Simulation of the acrobot.

References

[1] Knoll, Carsten: *Acrobot (=unteraktuierter Zweigelenkmanipulator, Stellglied im Ellenbogengelenk)*, Jupyter Notebook published 2017.