

# Model Documentation of the Inertia Wheel Pendulum

## 1 Nomenclature

### 1.1 Nomenclature for Model Equations

$m_1$	mass of the pendulum
$m_2$	mass of the wheel
$l_1$	length of the pendulum
$s_1$	distance of the center of gravity
$J_1$	moment of inertia of the pendulum
$J_2$	moment of inertia of the wheel
$g$	acceleration due to gravity
$\tau$	torque
$q_1$	angle between the vertical rest position and the pendulum
$q_2$	angel of the wheel

### 1.2 Graphic of the Structure

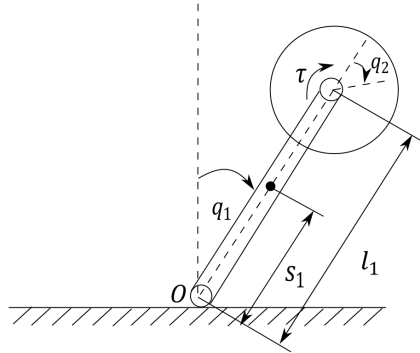


Figure 1: Structure of the IWP.

Source: Wang, Yang/Erstellung eines regelungstheoretischen Katalogs unteraktuierter mechanischer Systeme

## 2 Model Equations

State Vector and Input Vector:

$$\underline{x} = (q_1 \ q_2 \ \dot{q}_1 \ \dot{q}_2)^T = (x_1 \ x_2 \ x_3 \ x_4)^T$$

$$u = \tau$$

System Equations:

$$\dot{x}_1 = x_3 \quad (1a)$$

$$\dot{x}_2 = x_4 \quad (1b)$$

$$\dot{x}_3 = \frac{J_2(m_1 s_1 + m_2 l_1)g \sin x_1 - J_2 u}{J_2(m_1 s_1^2 + m_2 l_1^2 + J_1)} \quad (1c)$$

$$\dot{x}_4 = \frac{-J_2(m_1 s_1 + m_2 l_1)g \sin x_1 + (m_1 s_1^2 + m_2 l_1^2 + J_1 + J_2)u}{J_2(m_1 s_1^2 + m_2 l_1^2 + J_1)} \quad (1d)$$

Parameters:  $m_1, m_2, l_1, s_1, J_1, J_2, g$

Outputs:  $\underline{x}$

## 2.1 Assumptions

1. The dissipative forces are not taken into account.

## 2.2 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
mass of the pendulum	$m_1$	0.5	kg
mass of the wheel	$m_2$	1	kg
length of the pendulum	$l_1$	0.5	m
distance of the center of gravity	$s_1$	0.25	m
moment of inertia of the pendulum	$J_1$	0.02	$kg \cdot m^2$
moment of inertia of the wheel	$J_2$	0.002	$kg \cdot m^2$
acceleration due to gravity	$g$	10	$\frac{m}{s^2}$

## 3 Derivation and Explanation

The Lagrangian mechanics was used for the solution.

Kinetic Energy:

$$T = \frac{1}{2}(m_1 s_1^2 + m_2 l_1^2 + J_1 + J_2)x_3^2 + J_2 x_3 x_4 + \frac{1}{2}J_2 x_4^2 \quad (2)$$

Potential Energy:

$$V = (m_1 s_1 - m_2 l_1)g(\cos x_1 - 1) \quad (3)$$

## 4 Simulation

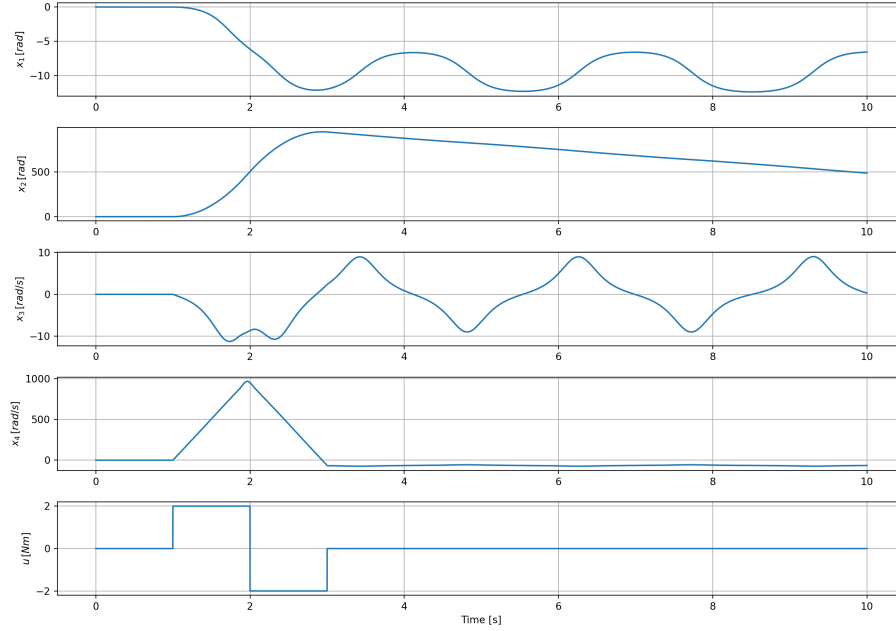


Figure 2: Simulation of the iwp.

## References

- [1] Wang, Yang: *Erstellung eines regelungstheoretischen Katalogs unteraktuierter mechanischer Systeme*, master thesis at the Institut of Control Theory TU Dresden, published 2016.  
(not publicly accessible)