

# Model Documentation of the Loading Bridge

## 1 Nomenclature

### 1.1 Nomenclature for Model Equations

$x_m$	way of the load
$x_M$	way of the cart
$\varphi$	angle of deflection of the load in relation to the center of the cart
$m$	mass of the load
$M$	mass of the cart
$l$	rope length
$g$	acceleration due to gravitation
$f$	force that pushes the cart

## 2 Model Equations

State Vector and Input Vector:

$$\underline{x} = (x_1 \ x_2 \ x_3 \ x_4)^T = (x_M \ \varphi \ \dot{x}_M \ \dot{\varphi})^T$$
$$\underline{u} = f$$

System Equations:

$$\dot{x}_1 = x_3 \tag{1a}$$

$$\dot{x}_2 = x_4 \tag{1b}$$

$$\dot{x}_3 = \frac{u_1 + \frac{gm \sin(2x_2)}{2} + lmx_4^2 \sin(x_2)}{M + m \sin^2(x_2)} \tag{1c}$$

$$\dot{x}_4 = -\frac{g(M + m) \sin(x_2) + (u_1 + lmx_4^2 \sin(x_2)) \cos(x_2)}{l(M + m \sin^2(x_2))} \tag{1d}$$

Parameters:  $m, M, l, g$

Outputs:  $x_m, x_M$

### 2.1 Assumptions

1. The friction is neglected.
2. Mass of the load is a pointmass.
3. Mass of the cart is a pointmass.

## 2.2 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
mass of the load	$m$	0.25	kg
mass of the cart	$M$	1	kg
rope length	$l$	1	m
acceleration due to gravitation	$g$	9.81	$\frac{m}{s^2}$

## 3 Derivation and Explanation

The Lagrangian mechanics was used for the solution.

## 4 Simulation

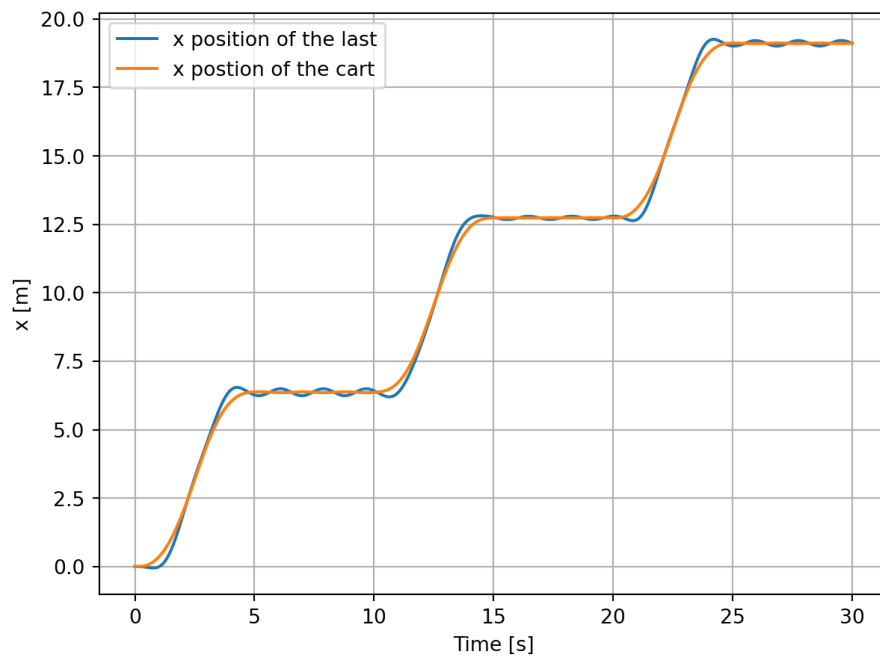


Figure 1: Simulation of the Loading Bridge.

## References

- [1] Institut für Regelungs- und Steuerungstheorie TU Dresden: *Regelungstechnik II, Übungsmaterial*, published in OPAL April 2020.  
(not publicly accessible)