

Model Documentation of the Underactuated Manipulator with Two Revolute Joints

1 Nomenclature

1.1 Nomenclature for Model Equations

s_i	distance from the joint to the center of gravity of link i , where $i = 1, 2$
m_i	mass of the link i , where $i = 1, 2$
J_i	moment of inertia of the link i , where $i = 1, 2$
l_1	length of link 1
τ	torque
q_1	angle between basis and link 1
p_1	angle between link 1 and link 2

1.2 Graphic of the Structure

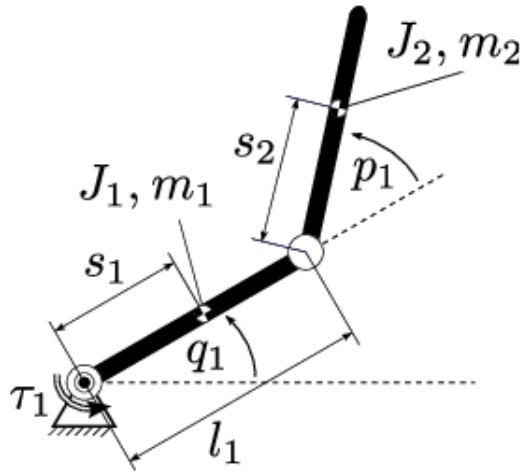


Figure 1: Structure of the manipulator.

Source: Knoll, Carsten/ Betrachtetes System: unteraktuierten Zweigelenkmanipulator.

2 Model Equations

State Vector and Input Vector:

$$\begin{aligned}\underline{x} &= (p_1 \ q_1 \ \dot{p}_1 \ \dot{q}_1)^T & &= (x_1 \ x_2 \ x_3 \ x_4)^T \\ \underline{u} &= (\tau \ 0)^T & &= (u_1 \ u_2)^T\end{aligned}$$

Kinetic Energy:

$$\begin{aligned}
T_{trans} &= \frac{1}{2}m_1s_1^2x_3^2\sin^2 x_1 + \frac{1}{2}m_1s_1^2x_3^2\cos^2 x_1 \\
&\quad + \frac{m_2}{2}(-l_1x_3\sin x_1 - s_2(x_3 + x_4)\sin(x_1 + x_2))^2 \\
&\quad + \frac{m_2}{2}(l_1x_3\cos x_1 + s_2(x_3 + x_4)\cos(x_1 + x_2))^2 \\
T_{rot} &= \frac{1}{2}J_1x_3^2 + \frac{1}{2}J_2(x_3 + x_4)^2
\end{aligned}$$

Potential Energy:

$$V = 0$$

Parameters: $s_1, s_2, m_1, m_2, J_1, J_2, l_1$

Outputs: \underline{x}

2.1 Assumptions

1. Friction is not taken into account.

2.2 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
distance from the joint to the center of gravity of link 1	s_1	0.1	m
distance from the joint to the center of gravity of link 2	s_2	0.25	m
mass of link 1	m_1	0.5	kg
mass of link 2	m_2	0.6	kg
moment of inertia of link 1	J_1	0.002	$kg \cdot m^2$
moment of inertia of link 2	J_2	0.001	$kg \cdot m^2$
length of link 1	l_1	0.2	m

3 Derivation and Explanation

The Lagrangian mechanics was used for the solution.

4 Simulation

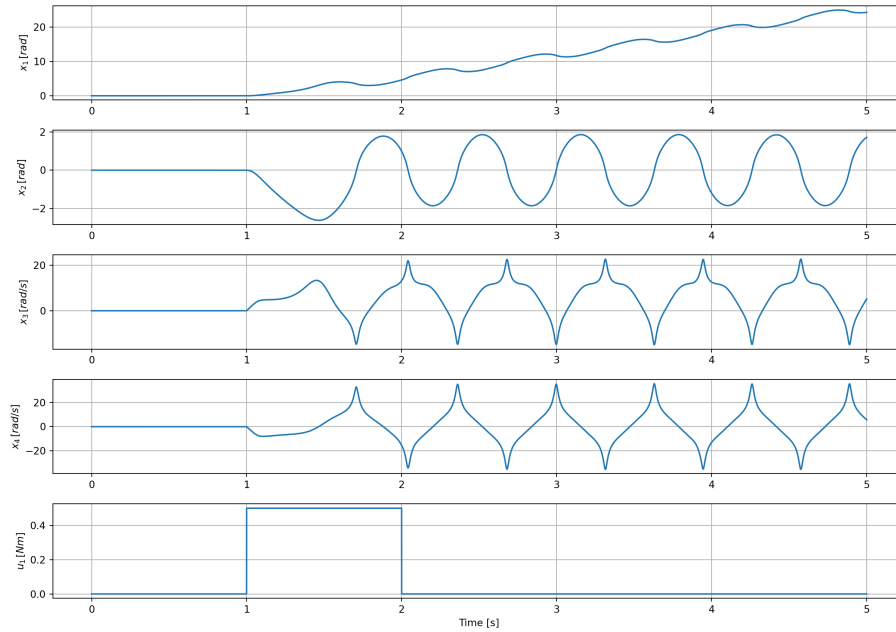


Figure 2: Simulation of the two bar manipulator.

References

- [1] Knoll, Carsten: *Betrachtetes System: unteraktuierten Zweigelenkmanipulator.*, Jupyter Notebook published 2016.
https://github.com/cknoll/beispiele/blob/master/zweigelenk_manipulator.ipynb
- [2] Wang, Yang: *Erstellung eines regelungstheoretischen Katalogs unteraktuierter mechanischer Systeme*, master thesis at the Institut of Control Theory TU Dresden, published 2016.
(not publicly accessible)