Model Documentation of the Linear Transport System

1 Nomenclature

1.1 Nomenclature for Model Equations

t	time
z	space
v	velocity (constant)
$u(z,t)$	input function
$x(z,t)$	wanted function describing the material transport

2 Model Equations

System Equations:

$$\begin{aligned} \dot{x}(z,t) + vx'(z,t) &= 0 && z \in (0,l], t > 0 \\ x(z,0) &= x_0(z) && z \in [0,l] \\ x(0,t) &= u(t) && t > 0 \end{aligned}$$

Parameters: v
Outputs: $x(l,t)$

2.1 Assumptions

1. $x_0(z) = 0$

2.2 Exemplary parameter values

Parameter Name	Symbol	Value
velocity-constant	v	4

3 Derivation and Explanation

Weak formulation approach with weight function $\varphi(z)$:

$$0 \overset{!}{=} \int_{z=0}^{z=l} [\dot{x}(z,t) + vx'(z,t)]\varphi(z)\,dz$$

$$0 = \int_{z=0}^{z=l} \dot{x}(z,t)\varphi(z)\,dz + v\int_{z=0}^{z=l} x'(z,t)\varphi(z)\,dz$$

with partial integration

$$0 = \int_{z=0}^{z=l} \dot{x}(z,t)\varphi(z)\,dz + v[x\varphi]_{z=0}^{z=l} - v\int_{z=0}^{z=l} x(z,t)\varphi'(z)\,dz$$

$$0 = \int_{z=0}^{z=l} \dot{x}(z,t)\varphi(z)\,dz + vx(l)\varphi(l) - v\varphi(0)u(t) - v\int_{z=0}^{z=l} x(z,t)\varphi'(z)\,dz$$

References

[1] Stefan Ecklebe, Marcus Riesmeier:
https://pyinduct.readthedocs.io/en/master/examples/transport_system.html