

# Model Documentation of the Permanent Magnet DC Motor

## 1 Nomenclature

### 1.1 Nomenclature for Model Equations

$c\phi$	motor constant
$J$	moment of inertia
$L_A$	armature inductance
$R_A$	armature resistance
$i_A$	armature current
$\omega$	angular velocity
$u_A$	armature voltage
$\xi_L$	load moment

### 1.2 Graphic of the Structure

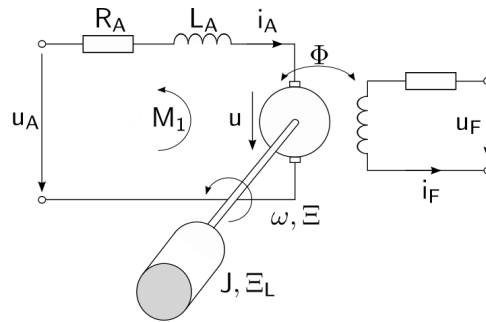


Figure 1: Structure of the Permanent Magnet DC Motor Model.  
Source: Institut of Control Theory TU Dresden: Regelungstechnikpraktikum,  
Praktikumsanleitung

## 2 Model Equations

State Vector and Input Vector:

$$\begin{aligned}\underline{x} &= (\omega \ i_A)^T & &= (x_1 \ x_2)^T \\ \underline{u} &= (u_A \ \xi_L)^T & &= (u_1 \ u_2)^T\end{aligned}$$

System Equations:

$$\dot{x}_1 = \frac{c\phi}{J}x_2 - \frac{1}{J}u_2 \quad (1a)$$

$$\dot{x}_2 = -\frac{R_A}{L_A}x_2 - \frac{c\phi}{L_A}x_1 + \frac{1}{L_A}u_1 \quad (1b)$$

Parameters:  $c\phi$ ,  $J$ ,  $L_A$ ,  $R_A$   
Output:  $\omega$

## 2.1 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
motor constant	$c\phi$	0.169	Vs
moment of inertia	$J$	0.0017	$W s^3$
armature inductance	$L_A$	0.0256	H
armature resistance	$R_A$	3.2	$\Omega$

## 3 Derivation and Explanation

The function of the electrical motor is based on the interaction between the electromechanical power law and Faraday's law of induction.

$$\xi(t) = c\phi i_A(t) \quad (2)$$

$$u(t) = c\phi\omega(t) \quad (3)$$

The following applies to the voltage drops in the electrical armature circuit:

$$u_A(t) = u(t) + R_A i_A(t) + L_A \frac{di_A(t)}{dt}. \quad (4)$$

For the mechanical system, Newton's law for rotational motion provides

$$\frac{\omega(t)}{dt} = \frac{1}{J} \xi_b(t). \quad (5)$$

Furthermore:

$$\xi_b(t) = \xi(t) - \xi_L(t). \quad (6)$$

## 4 Simulation

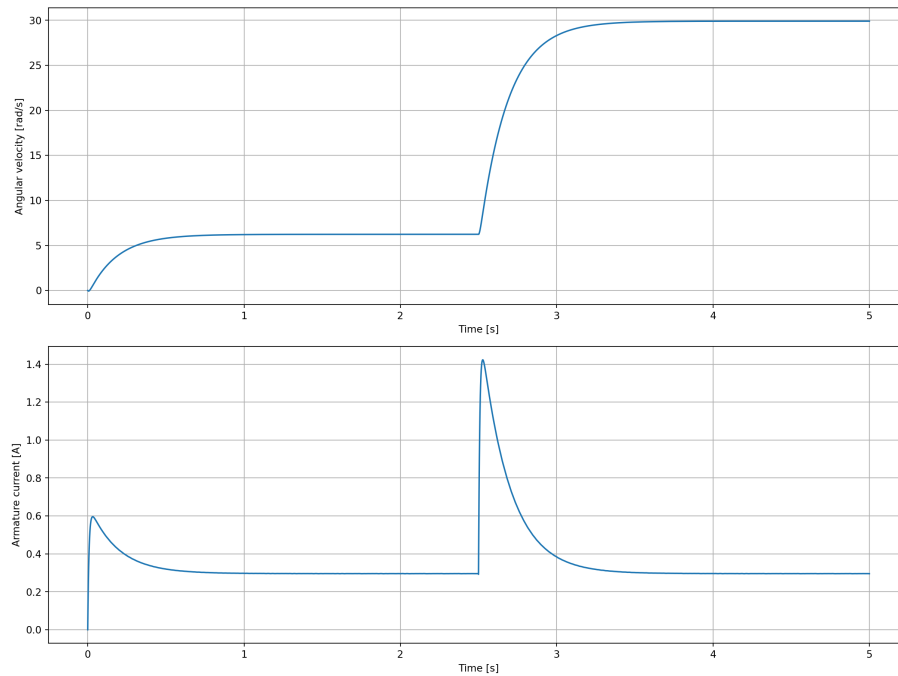


Figure 2: Simulation of the permanent magnet DC motor.

## References

- [1] Institut of Control Theory TU Dresden: *Regelungstechnikpraktikum, Praktikumsanleitung*, published on OPAL April 2022.  
(not publicly accessible)