

# Model Documentation of the Four-Bar Linkage

## 1 Nomenclature

### 1.1 Nomenclature for Model Equations

$s_1$	center of gravity distance of first bar
$s_2$	center of gravity distance of second bar
$s_3$	center of gravity distance of third bar
$m_1$	mass of first bar
$m_2$	mass of second bar
$m_3$	mass of third bar
$J_1$	moment of inertia of first bar
$J_2$	moment of inertia of second bar
$J_3$	moment of inertia of third bar
$l_1$	length of first bar
$l_2$	length of second bar
$l_3$	length of third bar
$l_4$	length of fourth bar
$g$	acceleration due to gravity
$p_1$	angle of between th bars of the two link mainpulator
$p_2$	angle of one link manipulator
$p_2$	angle of two link manipulator
$u_1$	external force
$y$	array of angles
$\dot{y}$	array of angular velocities

## 2 Model Equations

DAE Variables and Input Vector:

$$\underline{x} = (x_1 \ x_2 \ x_3 \ \dot{x}_1 \ \dot{x}_2 \ \dot{x}_3 \ \lambda_1 \ \lambda_2)^T = (p_1 \ p_2 \ q_1 \ \dot{p}_1 \ \dot{p}_2 \ \dot{q}_1 \ \lambda_1 \ \lambda_2)^T$$
$$\underline{u} = u_1$$

Constraints:

$$l_1 \cos(x_3) + l_2 \cos(x_1 + x_3) - l_3 \cos(x_2) - l_4 = 0 \quad (1a)$$

$$l_1 \sin(x_3) + l_2 \sin(x_1 + x_3) - l_3 \sin(x_2) = 0 \quad (1b)$$

System Equations:

$$0 = J_2 \ddot{x}_1 + J_2 \ddot{x}_3 + g m_2 s_2 \cos(x_1 + x_3) + l_1 m_2 \ddot{x}_3 s_2 \cos(x_1) + l_1 m_2 \dot{x}_3^2 s_2 \sin(x_1) \\ + l_2 \lambda_1 \sin(x_1 + x_3) - l_2 \lambda_2 \cos(x_1 + x_3) + m_2 \ddot{x}_1 s_2^2 + m_2 \ddot{x}_3 s_2^2$$

$$0 = J_3 \ddot{x}_2 + g m_3 s_3 \cos(x_2) - l_3 \lambda_1 \sin(x_2) + l_3 \lambda_2 \cos(x_2) + m_3 \ddot{x}_2 s_3^2$$

$$0 = J_1 \ddot{x}_3 + J_2 \ddot{x}_1 + J_2 \ddot{x}_3 + g l_1 m_2 \cos(x_3) + g m_1 s_1 \cos(x_3) + g m_2 s_2 \cos(x_1 + x_3) \\ + l_1^2 m_2 \ddot{x}_3 + l_1 \lambda_1 \sin(x_3) - l_1 \lambda_2 \cos(x_3) + l_1 m_2 \ddot{x}_1 s_2 \cos(x_1) - l_1 m_2 \dot{x}_1^2 s_2 \sin(x_1) \\ - 2 l_1 m_2 \dot{x}_1 \dot{x}_3 s_2 \sin(x_1) + 2 l_1 m_2 \ddot{x}_3 s_2 \cos(x_1) + l_2 \lambda_1 \sin(x_1 + x_3) - l_2 \lambda_2 \cos(x_1 + x_3) \\ + m_1 \ddot{x}_3 s_1^2 + m_2 \ddot{x}_1 s_2^2 + m_2 \ddot{x}_3 s_2^2 - u_1$$

Parameters:  $s_1$   $s_2$   $s_3$   $m_1$   $m_2$   $m_3$   $J_1$   $J_2$   $J_3$   $l_1$   $l_2$   $l_3$   $l_4$   $g$

Outputs:  $y$   $\dot{y}$

## 2.1 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
center of gravity distance of first bar	$s_1$	0.5	m
center of gravity distance of second bar	$s_2$	0.5	m
center of gravity distance of third bar	$s_3$	0.5	m
mass of first bar	$m_1$	1	kg
mass of second bar	$m_2$	1	kg
mass of third bar	$m_3$	3	kg
moment of inertia	$J_1$	0.08333	$kg \cdot m^2$
moment of inertia	$J_2$	0.08333	$kg \cdot m^2$
moment of inertia	$J_3$	0.08333	$kg \cdot m^2$
length of first bar	$l_1$	0.8	m
length of second bar	$l_2$	1.5	m
length of third bar	$l_3$	1.5	m
length of fourth bar	$l_4$	2	m
acceleration due to gravity	$g$	9.81	$\frac{m}{s^2}$

## 3 Derivation and Explanation

*Not available*

## 4 Simulation

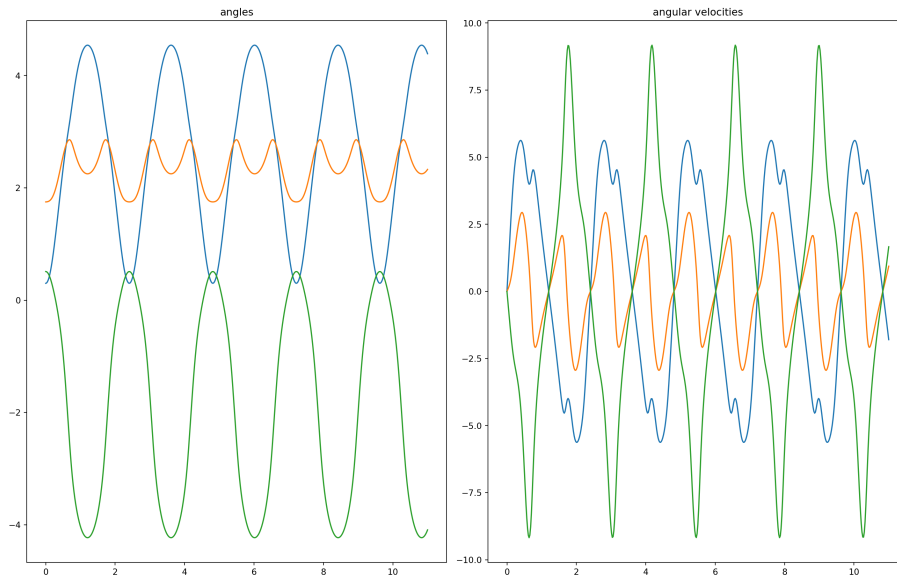


Figure 1: Simulation of the Four-bar Linkage.

## References

- [1] Knoll, Carsten: *Considered model: four-bar linkage (= two link manipulator + one link manipulator + rigid coupling)*, Jupyter Notebook published 2019