# Model Documentation of the 1D Wave Equation

#### 1 Nomenclature

#### 1.1 Nomenclature for Model Equations

t time

z space

c propagation speed

u(z,t) input function

x(z,t) wanted function describing the material transport

### 2 Model Equations

System Equations:

$$\frac{1}{c^2}\ddot{x}(z,t) - x''(z,t) = 0 \qquad z \in (0,l], t > 0$$

$$x(0,t) = 0 \qquad t > 0$$

$$\dot{x}(l,t) = u(t) \qquad t > 0$$

$$x(z,0) = 0 \qquad z \in [0,l]$$

$$x'(z,0) = 0 \qquad z \in [0,l]$$

Parameters: c

#### 2.1 Exemplary parameter values

Parameter Name	Symbol	Value
propagation speed of the wave	c	1

## 3 Derivation and Explanation

Weak form:

$$0 = \frac{1}{c^2} \int_{z=0}^{z=l} \ddot{x}(z,t)\varphi(z) dz - \int_{z=0}^{z=l} x''(z,t)\varphi(z) dz$$

with partial integration

$$0 = \frac{1}{c^2} \int_0^l \ddot{x}\varphi \, dz - [x'\varphi]_0^l + \int_0^l x'\varphi' \, dz$$
$$0 = \frac{1}{c^2} \int_0^l \ddot{x}\varphi \, dz - u(t)\varphi(l) + x'(0,t)\varphi(0) + \int_0^l x'\varphi' \, dz$$

## 4 Simulation

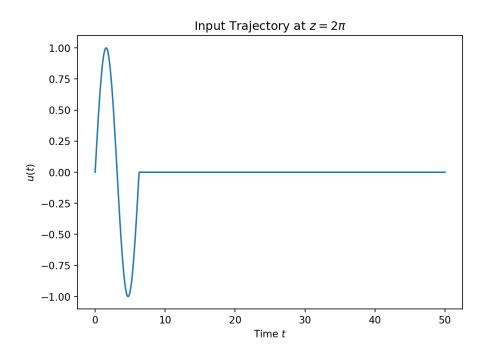


Figure 1: Simulation of the Wave Equation 1D.

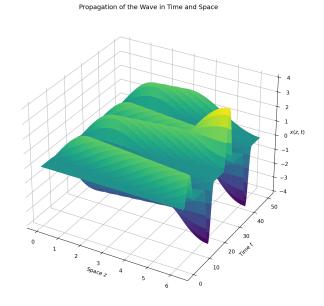


Figure 2: Simulation of the Wave Equation 1D.

## References

[1] Stefan Ecklebe, Marcus Riesmeier: https://pyinduct.readthedocs.io/en/master/readme.html