Model Documentation of the Kapitza's Pendulum

1 Nomenclature

1.1 Nomenclature for Model Equations

γ	Dampening factor
l	length of the pendulum
g	acceleration due to gravity
φ	angle of deflection from the equilibrium position
a	magnitude of the harmonic oscillation of the suspension point
ω	frequency of the harmonic oscillation of the suspension point

1.2 Graphic of the Structure

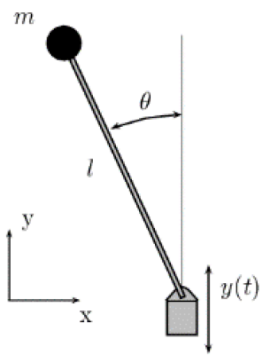

Figure 1: Structure of the Pendulum.
Source: Bello, Thomas; Huang, Emily; Lopez, Fabian; Rumsey, Kellin; Tao, Tao / Pendulum With Vibrating Base

2 Model Equations

State Vector and Input Vector:

$$\underline{x} = (x_1 \; x_2)^T = (\varphi \; \dot{\varphi})^T$$
$$\underline{u} = \emptyset$$

System Equations:

$$\dot{x}_1 = x_2 \tag{1a}$$
$$\dot{x}_2 = -2\gamma x_2 - \left(\frac{g}{l} - \frac{a}{l}\omega^2 \cos(\omega t)\right) \sin(x_1) \tag{1b}$$

Parameters: $\omega, \; a, \; l, \; g, \; \gamma$
Outputs: φ

2.1 Assumptions

1. Mass of the pendulum is a pointmass.

2.2 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
Pendulum length	l	0.3	cm
acceleration due to gravitation	g	9.81	$\frac{m}{s^2}$
Amplitude of Oscillation	a	$0.2l$	cm
Frequency of Oscillation	ω	$16\omega_0$	Hz
Dampening Factor	γ	$0.1\omega_0$	Hz

with $\omega_0 = \sqrt{\frac{g}{l}}$

3 Derivation and Explanation

The Lagrangian mechanics was used for the solution.

4 Simulation

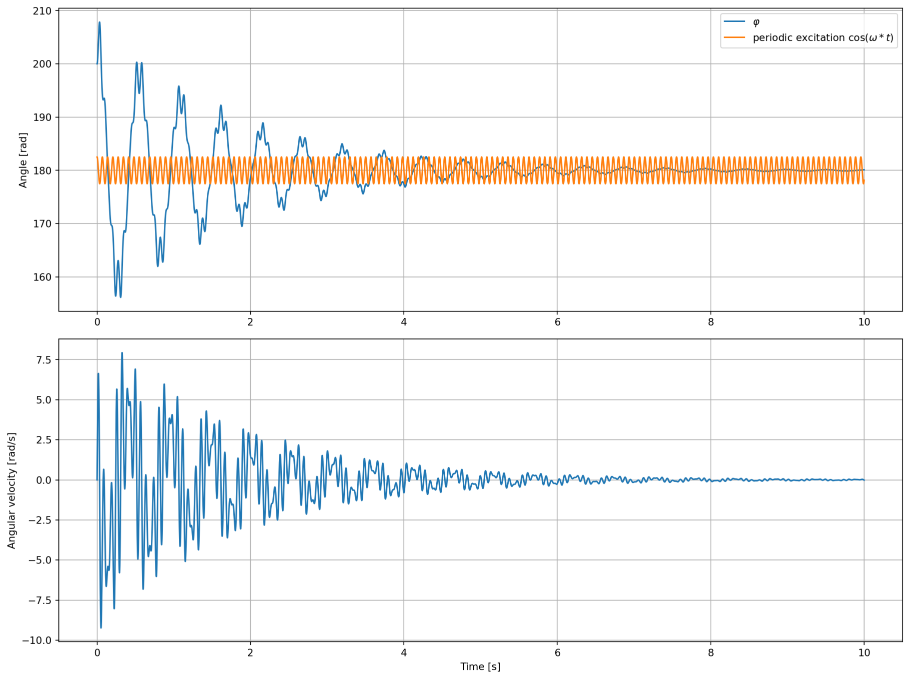

Figure 2: Simulation of the Kapitza's Pendulum.

References

[1] Butikov, E. I.: *Kapitza's Pendulum: A Physically Transparent Simple Treatment*, published 2017.

[2] Bello, Thomas; Huang, Emily; Lopez, Fabian; Rumsey, Kellin; Tao Tao: *Pendulum With Vibrating Base*, published 2014.