

# Model Documentation of the Acrobot

## 1 Nomenclature

### 1.1 Nomenclature for Model Equations

$s_i$	center of gravity distance of the bar for $i = 1, 2$
$m_i$	mass of the bar for $i = 1, 2$
$J_i$	moment of inertia of the bar for $i = 1, 2$
$l_1$	length of the first bar
$g$	acceleration due to gravity
$p_1$	angle between the vertically downwards rest position and the first bar
$q_1$	angle between the first and the second bar
$\dot{p}_1$	angle velocity of the first bar
$\dot{q}_1$	angle velocity of the second bar
$\tau_1$	input force at the joint

### 1.2 Graphic of the Structure

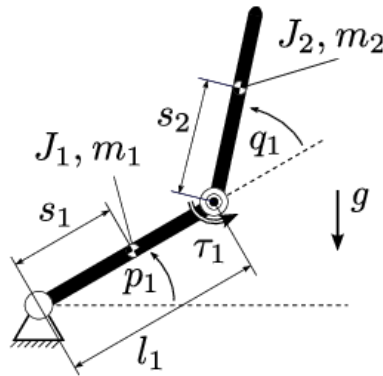


Figure 1: Acrobot

Source: Knoll, Carsten/Acrobot (=unteraktuierter Zweigelenkmanipulator, Stellglied im Ellenbogengelenk)

## 2 Model Equations

State Vector and Input Vector:

$$\begin{aligned} \underline{x} &= (p_1 \ q_1 \ \dot{p}_1 \ \dot{q}_1)^T & &= (x_1 \ x_2 \ x_3 \ x_4)^T \\ \underline{u} &= \tau_1 & &= u_1 \end{aligned}$$

Kinetic Energy:

$$T = \frac{J_1 x_3^2}{2} + \frac{J_2 (x_3 + x_4)^2}{2} + \frac{m_1 x_3^2 s_1^2 \sin(x_1)^2}{2} + \frac{m_1 x_3^2 s_1^2 \cos(x_1)^2}{2} \\ + m_2 (-l_1 x_3 \sin(x_1) - \frac{s_2 (x_3 + x_4) \sin(x_1 + x_2)}{2})^2 + m_2 (l_1 x_3 \cos(x_1) \\ + \frac{s_2 (x_3 + x_4) \cos(x_1 + x_2)}{2})^2$$

Potential Energy:

$$V = g m_1 s_1 \sin(x_1) + g m_2 (l_1 \sin(x_1) + s_2 \sin(x_1 + x_2))$$

Parameters:  $s_1, s_2, m_1, m_2, J_1, J_2, l_1, g$

Outputs:  $\underline{x}$

## 2.1 Assumptions

1. The rest position of the acrobot is vertically downward.

## 2.2 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
center of gravity distance of first bar	$s_1$	0.25	m
center of gravity distance of second bar	$s_2$	0.25	m
mass of first bar	$m_1$	1	kg
mass of second bar	$m_2$	1	kg
moment of inertia of first bar	$J_1$	0.006667	$kg \cdot m^2$
moment of inertia of second bar	$J_2$	0.01333	$kg \cdot m^2$
length of first bar	$l_1$	0.5	m
acceleration due to gravity	$g$	9.81	$\frac{m}{s^2}$

## 3 Derivation and Explanation

The Lagrangian mechanics was used for the solution.

## 4 Simulation

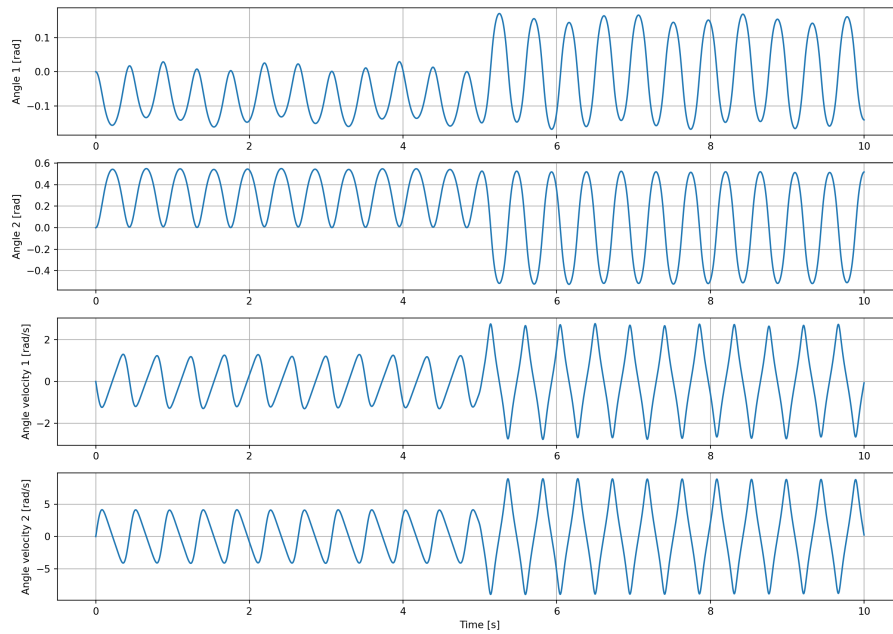


Figure 2: Simulation of the acrobot.

## References

- [1] Knoll, Carsten: *Acrobot (=unteraktuierter Zweigelenkmanipulator, Stellglied im Ellenbogengelenk)*, Jupyter Notebook published 2017.  
[https://github.com/cknoll/beispiele/blob/master/acrobot\\_rwa.ipynb](https://github.com/cknoll/beispiele/blob/master/acrobot_rwa.ipynb)