

Model Documentation of the 1D Wave Equation

1 Nomenclature

1.1 Nomenclature for Model Equations

t	time
z	space
c	propagation speed
$u(z, t)$	input function
$x(z, t)$	wanted function describing the material transport

2 Model Equations

System Equations:

$$\begin{aligned}\frac{1}{c^2}\ddot{x}(z, t) - x''(z, t) &= 0 & z \in (0, l], t > 0 \\ x(0, t) &= 0 & t > 0 \\ \dot{x}(l, t) &= u(t) & t > 0 \\ x(z, 0) &= 0 & z \in [0, l] \\ x'(z, 0) &= 0 & z \in [0, l]\end{aligned}$$

Parameters: c

2.1 Exemplary parameter values

Parameter Name	Symbol	Value
propagation speed of the wave	c	1

3 Derivation and Explanation

Weak form:

$$\begin{aligned}0 &= \frac{1}{c^2} \int_{z=0}^{z=l} \ddot{x}(z, t) \varphi(z) dz - \int_{z=0}^{z=l} x''(z, t) \varphi(z) dz \\ &\quad \text{with partial integration} \\ 0 &= \frac{1}{c^2} \int_0^l \ddot{x} \varphi dz - [x' \varphi]_0^l + \int_0^l x' \varphi' dz \\ 0 &= \frac{1}{c^2} \int_0^l \ddot{x} \varphi dz - u(t) \varphi(l) + x'(0, t) \varphi(0) + \int_0^l x' \varphi' dz\end{aligned}$$

References

- [1] Stefan Eklebe, Marcus Riesmeier:
<https://pyinduct.readthedocs.io/en/master/readme.html>