Model Documentation of the Four-Bar Linkage

1 Nomenclature

1.1 Nomenclature for Model Equations

s_1	center of gravity distance of first bar
s_2	center of gravity distance of second bar
s_3	center of gravity distance of third bar
m_1	mass of first bar
m_2	mass of second bar
m_3	mass of third bar
J_1	moment of inertia of first bar
J_2	moment of inertia of second bar
J_3	moment of inertia of third bar
l_1	length of first bar
l_2	length of second bar
l_3	length of third bar
l_4	length of fourth bar
g	acceleration due to gravity
p_1	angle of between th bars of the two link mainpulator
p_2	angle of one link manipulator
p_2	angle of two link manipulator
u_1	external force
y	array of angles
$\dot{y}$	array of angular velocities

2 Model Equations

DAE Variables and Input Vector:

$$\underline{x} = (x_1\ x_2\ x_3\ \dot{x}_1\ \dot{x}_2\ \dot{x}_3\ \lambda_1\ \lambda_2)^T \qquad = (p_1\ p_2\ q_1\ \dot{p}_1\ \dot{p}_2\ \dot{q}_1\ \lambda_1\ \lambda_2)^T$$
$$\underline{u} = u_1$$

Constraints:

$$l_1 \cos(x_3) + l_2 \cos(x_1 + x_3) - l_3 \cos(x_2) - l_4 \tag{1a}$$
$$l_1 \sin(x_3) + l_2 \sin(x_1 + x_3) - l_3 \sin(x_2) \tag{1b}$$

System Equations:

$$J_2\ddot{x}_1 + J_2\ddot{x}_3 + gm_2s_2\cos(x_1 + x_3) + l_1m_2\ddot{x}_3s_2\cos(x_1) + l_1m_2\dot{x}_3^2s_2\sin(x_1)$$
$$+l_2\lambda_1\sin(x_1 + x_3) - l_2\lambda_2\cos(x_1 + x_3) + m_2\ddot{x}_1s_2^2 + m_2\ddot{x}_3s_2^2$$

$$J_3\ddot{x}_2 + gm_3s_3\cos(x_2) - l_3\lambda_1\sin(x_2) + l_3\lambda_2\cos(x_2) + m_3\ddot{x}_2s_3^2$$

$$J_1\ddot{x}_3 + J_2\ddot{x}_1 + J_2\ddot{x}_3 + gl_1m_2\cos(x_3) + gm_1s_1\cos(x_3) + gm_2s_2\cos(x_1 + x_3)$$
$$+l_1^2m_2\ddot{x}_3 + l_1\lambda_1\sin(x_3) - l_1\lambda_2\cos(x_3) + l_1m_2\ddot{x}_1s_2\cos(x_1) - l_1m_2\dot{x}_1^2s_2\sin(x_1)$$
$$-2l_1m_2\dot{x}_1\dot{x}_3s_2\sin(x_1) + 2l_1m_2\ddot{x}_3s_2\cos(x_1) + l_2\lambda_1\sin(x_1 + x_3) - l2\lambda_2\cos(x_1 + x_3)$$
$$+m_1\ddot{x}_3s_1^2 + m_2\ddot{x}_1s_2^2 + m_2\ddot{x}_3s_2^2 - u_1$$

Parameters: s_1 s_2 s_3 m_1 m_2 m_3 J_1 J_2 J_3 l_1 l_2 l_3 l_4 g
Outputs: y $\dot{y}$

2.1 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
center of gravity distance of first bar	s_1	0.5	m
center of gravity distance of second bar	s_2	0.5	m
center of gravity distance of third bar	s_3	0.5	m
mass of first bar	m_1	1	kg
mass of second bar	m_2	1	kg
mass of third bar	m_3	3	kg
moment of inertia	J_1	0.0833333333333333	$\frac{kg}{m^2}$
moment of inertia	J_2	0.0833333333333333	$\frac{kg}{m^2}$
moment of inertia	J_3	0.0833333333333333	$\frac{kg}{m^2}$
length of first bar	l_1	0.8	m
length of second bar	l_2	1.5	m
length of third bar	l_3	1.5	m
length of fourth bar	l_4	2	m
acceleration due to gravity	g	9.81	$\frac{m}{s^2}$

3 Derivation and Explanation

Not available

References

[1] Knoll, Carsten: *Considered model: four-bar linkage (= two link manipulator + one link manipulator + rigid coupling)*, Jupyter Notebook published 2019