Model Documentation of the PVTOL with 2 Forces

1 Nomenclature

1.1 Nomenclature for Model Equations

x	horizontal displacement
y	vertical displacement
θ	roll angle
F_1, F_2	Forces on the left and right site of the PVTOL
g	acceleration due to gravity
l	distance between mass center of PVTOL and target point of the forces
m	mass of the PVTOL
J	moment of inertia of the PVTOL

2 Model Equations

State Vector and Input Vector:

$$\underline{x} = (x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6)^T = (x \ \dot{x} \ y \ \dot{y} \ \theta \ \dot{\theta})^T$$
$$\underline{u} = (u_1 \ u_2)^T = (F_1 \ F_2)^T$$

Model Equations:

$$\dot{x}_1 = x_2 \tag{1a}$$
$$\dot{x}_2 = -\frac{\sin(x_5)}{m}(u_1 + u_2) \tag{1b}$$
$$\dot{x}_3 = x_4 \tag{1c}$$
$$\dot{x}_4 = \frac{\cos(x_5)}{m}(u_1 + u_2) - g \tag{1d}$$
$$\dot{x}_5 = x_6 \tag{1e}$$
$$\dot{x}_6 = \frac{l}{J}(u_2 - u_1) \tag{1f}$$

Parameters: $m, \ J, \ l, \ g$
Outputs: $x, \ y, \ \theta$

2.1 Assumptions

1. Forces target the body of the PVTOL in a 90° angle.

2.2 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
acceleration due to gravity	g	9.81	$\frac{\mathrm{m}}{\mathrm{s}^2}$
distance of forces to mass center	l	0.1	m
mass	m	0.25	kg
moment of inertia	J	0.00076	$\mathrm{kg} \cdot \mathrm{m}^2$

3 Derivation and Explanation

The Lagrangian mechanics was used for the solution.

4 Simulation

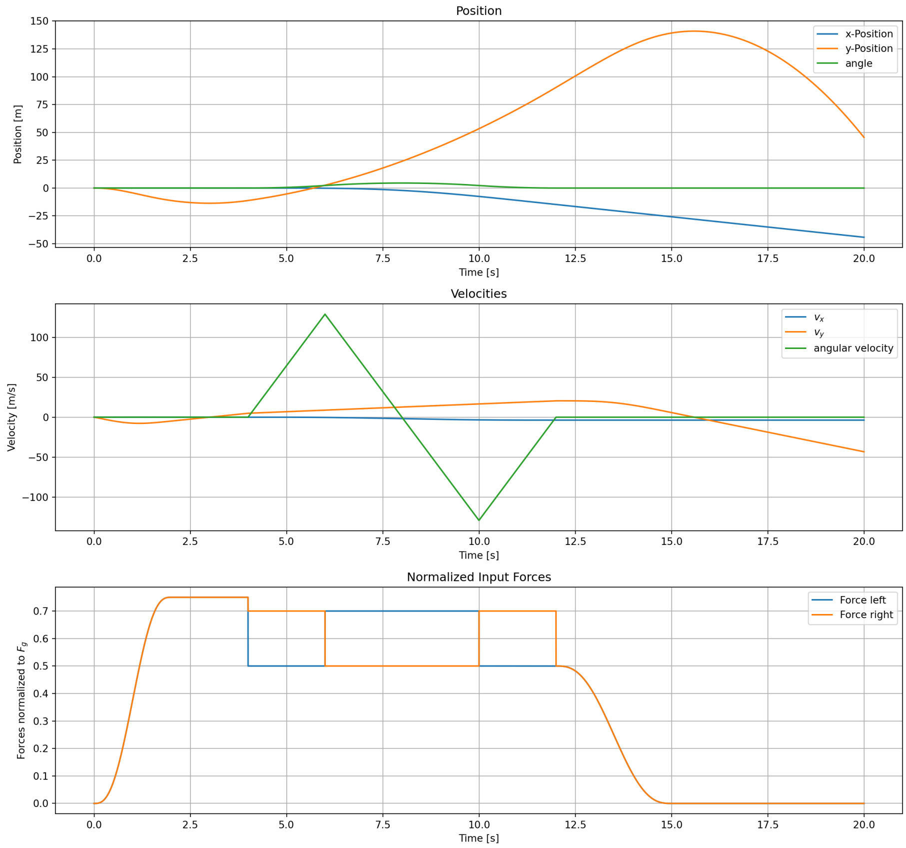

Figure 1: Simulation of the PVTOL with 2 forces.

References

[1] Knoll, C: *Regelungstheoretische Analyse- und Entwurfsansätze für unteraktuierte mechanische Systeme*, p. 169, TU Dresden, 2016.