

# Model Documentation of the Ball in the Tube

## 1 Nomenclature

### 1.1 Nomenclature for Model Equations

$A_B$	ball cross-sectional area
$A_{SP}$	air gap cross-sectional area
$m$	mass of the styrofoam ball
$g$	acceleration due to gravitation
$T_M$	time constant of the motor model
$k_M$	amplification of the motor model
$k_V$	proportional factor between the volume flow of the fan and the absolute speed rotation
$k_L$	parameter, describes the relation between the air gap velocity and the flow resistance force
$\eta$	rotation speed
$\eta_0$	basic rotation speed
$h$	height of the ball
$\dot{h}$	velocity of the ball
$u_{PWM}$	motor control voltage

## 2 Model Equations

State Vector and Input Vector:

$$\begin{aligned}\underline{x} &= (\eta \ h \ \dot{h})^T &= (x_1 \ x_2 \ x_3)^T \\ \underline{u} &= u_{PWM} &= u\end{aligned}$$

System Equations:

$$\dot{x}_1 = -\frac{1}{T_M}x_1 + \frac{k_M}{T_M}u \quad (1a)$$

$$\dot{x}_2 = x_3 \quad (1b)$$

$$\dot{x}_3 = \frac{k_L}{m} \left( \frac{k_V(x_1 + \eta_0) - A_B \dot{h}}{A_{SP}} \right)^2 - g \quad (1c)$$

Parameters:  $A_B, A_{SP}, m, g, T_M, k_M, k_V, k_L, \eta_0$

Outputs:  $h$

### 2.1 Assumptions

1. The behavior of the ball does not influence the air flow provided by the fan.

## 2.2 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
ball cross-sectional area	$A_B$	0.002827	$m^2$
air gap cross-sectional area	$A_{SP}$	0.0004299	$m^2$
mass of the ball	$m$	0.0028	kg
acceleration due to gravitation	$g$	9.81	$\frac{m}{s^2}$
time constant	$T_M$	0.369	s
amplification	$k_M$	0.273	$s^{-1}$
proportional factor	$k_V$	0.00012	$m^3$
parameter	$k_L$	0.0002823	$\frac{kg}{\frac{m^3}{s}}$
basic rotation speed	$n_0$	456	$\frac{min}{min}$

## 3 Derivation and Explanation

The model of the system consists of two submodels. The first one is the model of the ball. Essentially there two forces acting on the ball, the weight force  $F_g$  and the force exerted by the flow resistance  $F_w$ .

$$F_g = mg \quad (2)$$

$$F_w = \frac{1}{2} c_w \varrho_L A_B v_{SP}^2. \quad (3)$$

After inserting the equation for the relative velocity between the sphere and the medium flowing around it  $v_{SP}$  and using the proportional factor  $k_V$  we get

$$F_w = \frac{1}{2} c_w \varrho_L A_B \left( \frac{k_V(\eta + \eta_0) - A_B \dot{h}}{A_{SP}} \right)^2. \quad (4)$$

If we now introduce the parameter  $k_L$

$$F_w = k_L \left( \frac{k_V(\eta + \eta_0) - A_B \dot{h}}{A_{SP}} \right)^2 \quad (5)$$

results. Using the vertical force balance, the following equation can be set up.

$$m\ddot{h} = k_L \left( \frac{k_V(\eta + \eta_0) - A_B \dot{h}}{A_{SP}} \right)^2 - mg \quad (6)$$

The second model is the fan. We assume an ordinary fan with pulse width modulation (PWM) control. We are using a model with  $PT_1$ -behaviour with the time constant  $T_M$  and the amplification  $k_M$ .

$$T_M \dot{\eta} + \eta = k_M u_{PWM} \quad (7)$$

## 4 Simulation

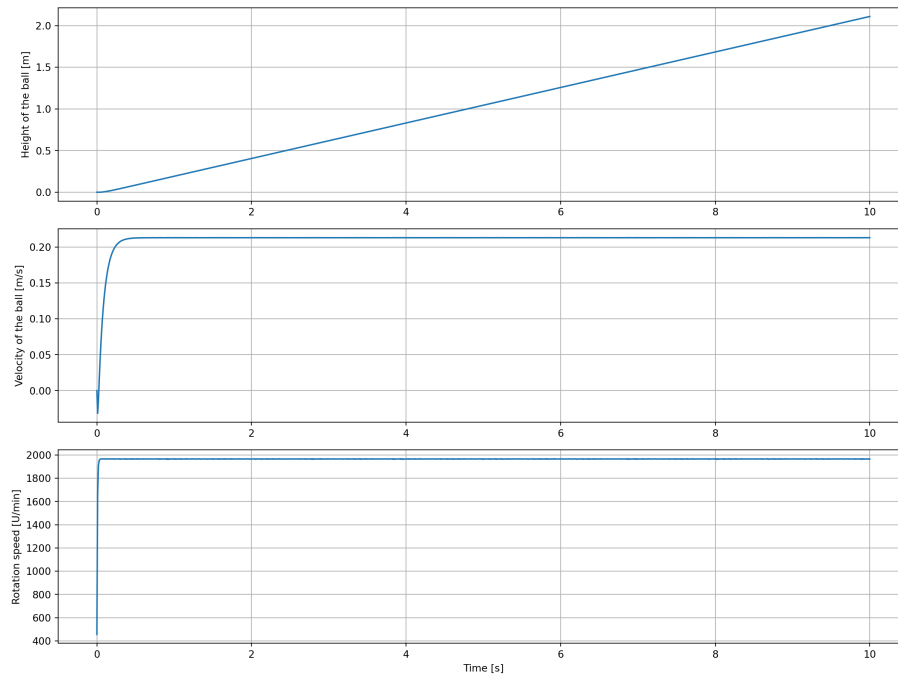


Figure 1: Simulation of the ball in tube.

## References

- [1] TU Dresden – Institut für Regelungs- und Steuerungstheorie: *Regelungstechnikpraktikum, Praktikumsanleitung*, published on OPAL April 2022. (not publicly accessible)