Model Documentation of the Loading Bridge

1 Nomenclature

1.1 Nomenclature for Model Equations

x_m	way of the load
x_M	way of the cart
φ	angle of deflection of the load in relation to the center of the cart
m	mass of the load
M	mass of the cart
l	rope length
g	acceleration due to gravitation
f	force that pushes the cart

1.2 Graphic of the Structure

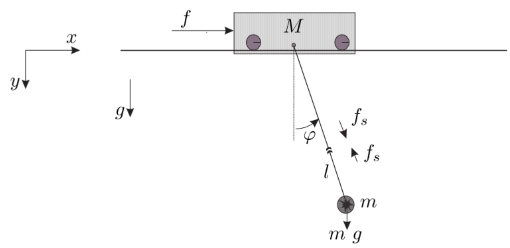

Figure 1: Structure of the Loading Bridge Model.
Source: Institut of Control Theory TU Dresden/Regelungstechnik II, Übungsmaterial

2 Model Equations

State Vector and Input Vector:

$$\underline{x} = (x_1\ x_2\ x_3\ x_4)^T = (x_M\ \varphi\ \dot{x}_M\ \dot{\varphi})^T$$
$$\underline{u} = f$$

System Equations:

$$\dot{x}_1 = x_3 \tag{1a}$$

$$\dot{x}_2 = x_4 \tag{1b}$$

$$\dot{x}_3 = \frac{u_1 + \frac{gm\sin(2x_2)}{2} + lmx_4 2\sin(x_2)}{M + m\sin^2(x_2)} \tag{1c}$$

$$\dot{x}_4 = -\frac{g(M+m)\sin(x_2) + (u_1 + lmx_4^2\sin(x_2))\cos(x_2)}{l(M + m\sin^2(x_2))} \tag{1d}$$

Parameters: m, M, l, g
Outputs: x_m, x_M

2.1 Assumptions

1. The friction is neglected.
2. Mass of the load is a pointmass.
3. Mass of the cart is a pointmass.

2.2 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
mass of the load	m	0.25	kg
mass of the cart	M	1	kg
rope length	l	1	m
acceleration due to gravitation	g	9.81	$\frac{m}{s^2}$

3 Derivation and Explanation

The Lagrangian mechanics was used for the solution.

4 Simulation

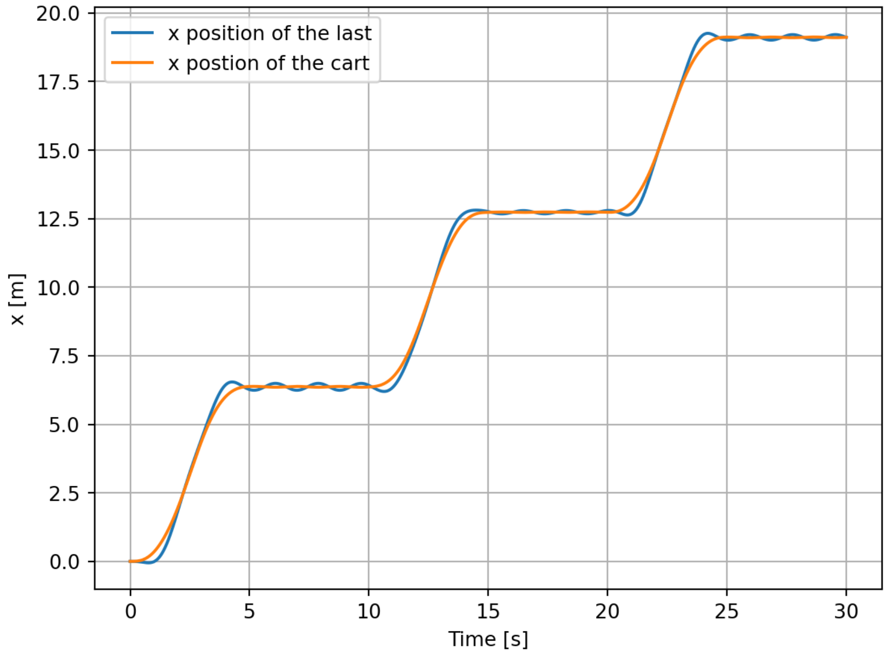

Figure 2: Simulation of the Loading Bridge.

References

[1] Institut of Control Theory TU Dresden: *Regelungstechnik II, Übungsmaterial*, published in OPAL April 2020.
(not publicly accessible)