Model Documentation of the Triple Pendulum on a Cart

1 Nomenclature

1.1 Nomenclature for Model Equations

m_0	mass of the cart
m_i	mass of link i, where $i = 1, 2, 3$
J_i	moment of inertia i, where $i = 1, 2, 3$
l_i	length (distance between joints) of link i, where $i = 1, 2, 3, 4$
a_i	distance from the joint to the center of gravity of link i, where $i = 1, 2, 3$
g	acceleration due to gravity
p_i	angle φ_i, where $i = 1, 2, 3$
q_1	distance x_0
F	force on the cart

1.2 Graphic of the Structure

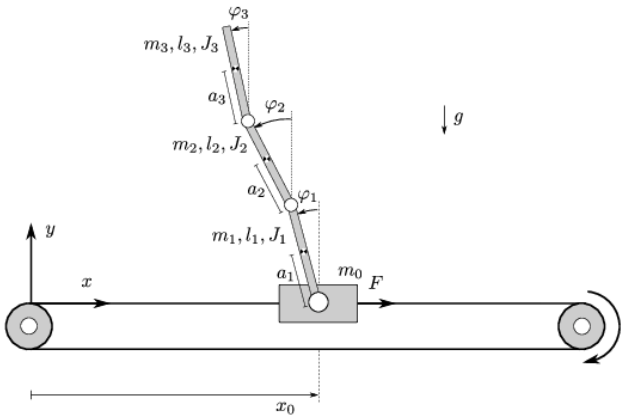

Figure 1: Triple Pendulum
Source: Knoll, Carsten/Triple Pendulum on a Cart: Derivation of Equations of Motion and Simulation

2 Model Equations

State Vector and Input Vector:

$$\underline{x} = (p_1\ p_2\ p_3\ q_1\ \dot{p}_1\ \dot{p}_2\ \dot{p}_3\ \dot{q}_1)^T = (x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8)^T$$
$$\underline{u} = (0, 0, 0, F)^T = (u_1, u_2, u_3, u_4)^T$$

Kinetic Energy:

$$T = \frac{1}{2}J_1x_5^2 + \frac{1}{2}J_2x_6^2 + \frac{1}{2}J_3x_7^2 + \frac{1}{2}m_0x_8^2 + \frac{1}{2}m_1(a_1^2x_5^2\sin x_1^2 + (-a_1x_5\cos x_1 + x_8)^2)$$
$$+ \frac{1}{2}m_2((-a_2x_6\sin x_2 - l_1x_5\sin x1)^2 + (-a_2x_6\cos x2 - l_1x_5\cos x_1 + x_8)^2)$$
$$+ \frac{1}{2}m_3((-a_2x_7\sin x_3 - l_1x_5\sin x_1 - l_2x_6\sin x_2)^2 + (-a_2x_7\cos x_3 - l_1x_5\cos x_1 - l_2x_6\cos x_2 + x_8)^2)$$

Potential Energy:

$$V = g(a_1m_1\cos x1 + m_2(a_2\cos x_2 + l_1\cos x_1) + m_3(a_2\cos x_3 + l_1\cos x_1 + l_2\cos x_2))$$

Parameters: m_0, m_1, m_2, m_3, J_1, J_2, J_3, l_1, l_2, l_3, a_1, a_2, a_3, g
Outputs: $\underline{x}$

2.1 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
mass of the cart	m_0	3.34	kg
mass of link 1	m_1	0.8512	kg
mass of link 2	m_2	0.8973	kg
mass of link 3	m_3	0.5519	kg
moment of inertia of link 1	J_1	0.0198	$kg \cdot m^2$
moment of inertia of link 2	J_2	0.02105	$kg \cdot m^2$
moment of inertia of link 3	J_3	0.01819	$kg \cdot m^2$
length of link 1	l_1	0.32	m
length of link 2	l_2	0.419	m
length of link 3	l_3	0.485	m
distance from the joint to the center of gravity of link 1	a_1	0.2	m
distance from the joint to the center of gravity of link 2	a_2	0.2689	m
distance from the joint to the center of gravity of link 3	a_3	0.2167	m
acceleration due to gravity	g	9.81	$\frac{m}{s^2}$

3 Derivation and Explanation

The Lagrangian mechanics was used for the solution.

4 Simulation

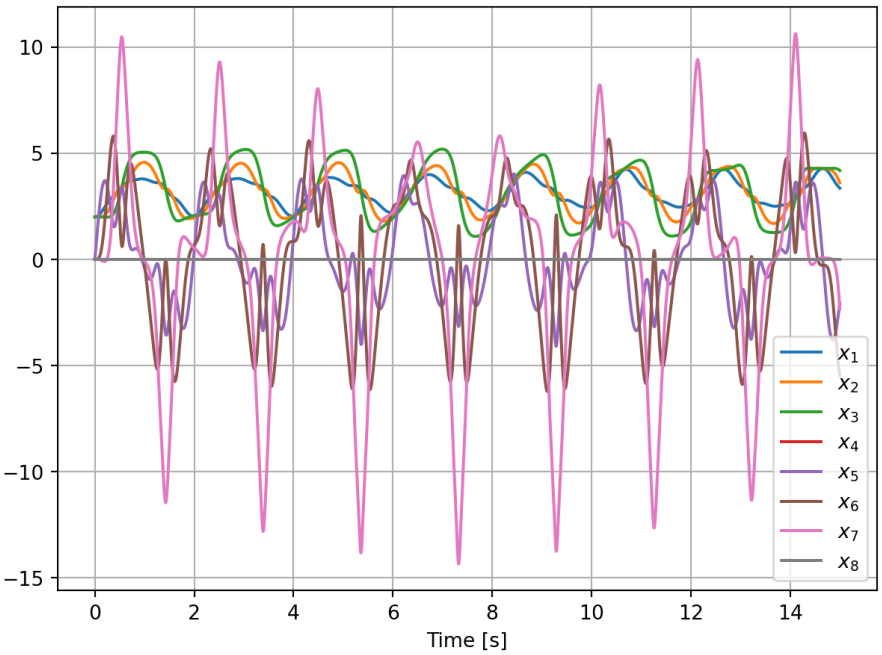

Figure 2: Simulation of the triple pendulum.

References

[1] Knoll, Carsten: *Triple Pendulum on a Cart: Derivation of Equations of Motion and Simulation*, Jupyter Notebook published 2021. `https://github.com/cknoll/demo-material/blob/main/underactuated_systems/triple_pendulum_with_modeltools_plus_simulation-en.ipynb`