Bachelor of Science

Semester :- 5

Group theory

Group Theory

- ## Groupoid or Binary Algebra Definition
  - A non-empty set G equipped with one binary operation is called groupoid
    - i.e. G is a Groupoid is closed for * .

    It is denated by (G,*).

    For Example :- (N,+), (Z, -), (Q, x) etc.

    Note :- Groupoid is also called Quasi Group.

- ## Semi Group :- Definition
  - An Algebraic Structure (G, *) is called a semi Group if the binary operation * satisfy associative Property

i.e. :- [G] $(a * b) * c = a * (b * c), \forall a \in G$

Ex.1

The algebraic structures (N,+), (Z,+), (Z, x), (Q, x) are semi groups but the structure (Z, -) is not so because subtraction, (-) is associative.

Ex. 2

The structures (p(s), ∪) and (P(s), ∩) where P(s) is the power set of a set S are Semi Groups as both the Operations Union (U) an intersection (∩) are associative.

Monoid : Definition

- A Semi Group is called monoid if there exist an identity element 'e' in G such that

$$[G2]\quad e * a = a * e = a, \forall a \in \mathbf{G}$$

Ex.1

The Semi Group (N, x) is Monoid because 1 is the identity for the multiplication. But the Semi Group (N, +) is not because 0 is the identity for addition is not in N.

Ex. 2

The Semi Group (P(s), ∪) and (P(s), ∩) are monoid because Φ and S are the identity respectively for union (∪) and (∩) in P(s).

Group : Definition

- An algebraic structure of set G and a binary operation * defined in G i.e. (G, *) is called a group if * satisfies the following postulate :

[G1] Closure :

$$a \in G, b \in G \Rightarrow a * b \in G, \ \forall a, b \in G$$

[G2] Associativity :

- The composition * is associative in G
  i.e. $(a * b) * c = a * (b * c), \ \forall a, b, c \in G$

[G3] Existence of Identity :

- There exist an identity element e in G such that

$$e * a = a * e = a, \ \forall a \in G$$

[G4] Existence of Inverse :

- Each element of G is invertible, i.e., for every $a \in G$, there exist a^{-1}in G such that
$$a * a^{-1} = a^{-1} * a = \mathrm{e} \text{ (Identity)}$$
- Thus group (G, *) is a monoid in which each of its element is ...

Abelian group or Commutative group : Definition

- A group $(G,*)$ is said to be abelian or commutative if * is commutative also. A group $(G,*)$ is an abelian group, if

[G4] Commutativity :

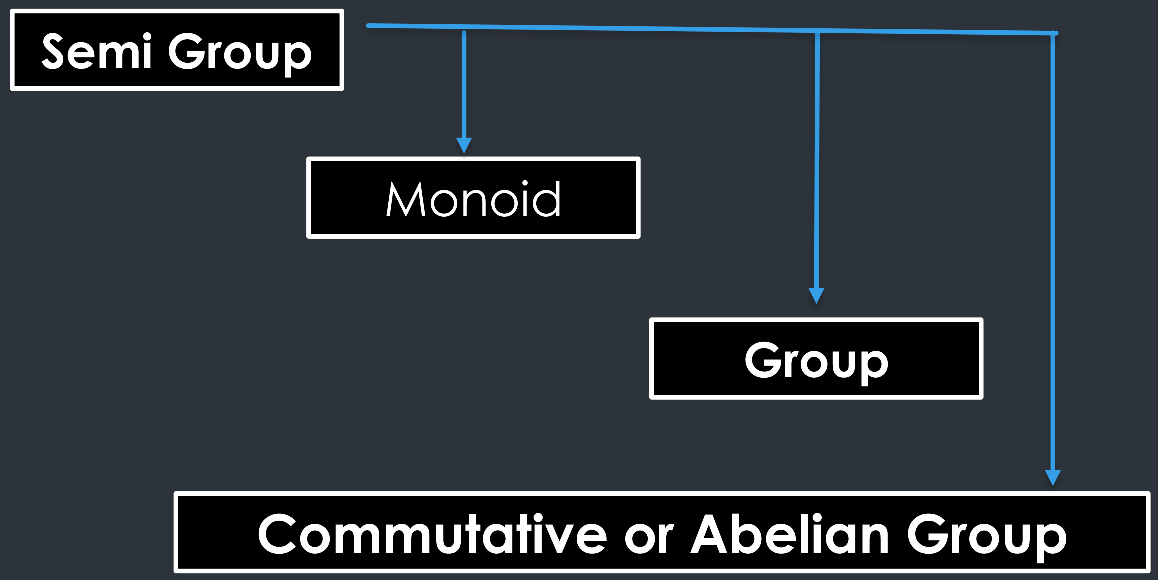
$$a * b = b * a, \ \forall a, b \in G$$

G1. Associativity

G2. Existence of Identity

G3. Existence of Inverse

G4. Commutativity



Finite and Infinite groups :

- A group (G, *) is said to be finite if its underlying set G is a finite set and a group which is not finite is called an infinite group.

Order of a group : Definition

- The number of elements in a finite group is called the order of the group. It is denoted by **O(G)**.
  If (G, *) is an infinite group, then it is said to be of infinite order.