Bachelor of Science

Semester :- 5

Group theory

Group Theory

o ## Groupoid or Binary Algebra Definition

➢ **A non-empty set G equipped with one binary operation is called groupoid**

 ➢ **i.e. G is a Groupoid is closed for * .**

It is denated by (G,*).

For Example :- (N,+), (Z, -), (Q, x) etc.

Note :- Groupoid is also called Quasi Group.

o ## Semi Group :- Definition

➢ **An Algebraic Structure (G, *) is called a semi Group if the binary operation * satisfy associative Property**

i.e. :- [G] (a * b) * c = a * (b * c), $\forall a \in G$

Ex.1

     The algebraic structures (N,+), (Z,+), (Z, x), (Q, x) are semi groups but the structure (Z, -) is not so because subtraction, (-) is associative.

Ex. 2

     The structures (p(s), ∪) and (P(s), ∩) where P(s) is the power set of a set S are Semi Groups as both the Operations Union (U) an intersection (∩) are associative.

Monoid : Definition

➢ A Semi Group is called monoid if there exist an identity element 'e' in G such that

[G2] e * a = a * e = a, ∀ a ∈ **G**

Ex.1

The Semi Group (N, x) is Monoid because 1 is the identity for the multiplication. But the Semi Group (N, +) is not because 0 is the identity for addition is not in N.

Ex. 2

The Semi Group (P(s), ∪) and (P(s), ∩) are monoid because Φ and S are the identity respectively for union (∪) and (∩) in P(s).