Modal Symbolic Learning: Day 3

NATOPS: Interpretable gesture recognition

In [1]:
```
using Pkg
Pkg.activate(".")
Pkg.instantiate()
Pkg.update()
Pkg.status()
```

```
  Activating project at `~/Desktop/modal-symbolic-learning-course`
    Updating registry at `~/.julia/registries/General`
    Updating git-repo `https://github.com/JuliaRegistries/General.git`
  No Changes to `~/Desktop/modal-symbolic-learning-course/Project.toml`
  No Changes to `~/Desktop/modal-symbolic-learning-course/Manifest.toml`
Status `~/Desktop/modal-symbolic-learning-course/Project.toml`
  [a93c6f00] DataFrames v1.6.1
  [7806a523] DecisionTree v0.12.4
  [7073ff75] IJulia v1.24.2
⌅ [add582a8] MLJ v0.19.5
  [c6f25543] MLJDecisionTreeInterface v0.4.0
  [e54bda2e] ModalDecisionTrees v0.3.3
  [91a5bcdd] Plots v1.39.0
  [7b3b3b3f] Sole v0.3.1
  [b002da8f] SoleLogics v0.6.11
  [4249d9c7] SoleModels v0.5.3
  [2913bbd2] StatsBase v0.34.2
  [9a3f8284] Random
Info Packages marked with ⌅ have new versions available but compatibility co
nstraints restrict them from upgrading. To see why use `status --outdated`
```

In [2]:
```
# Import libraries for statistics & Machine Learning
using Random
using DataFrames
using MLJ
using Plots
using StatsBase
```

In [3]:
```
# Import the Sole framework
using Sole

# Load an example time-series classification dataset as a tuple (DataFrame,
X_df, y = Sole.load_arff_dataset("NATOPS");
```

In [4]:
```
countmap(y)
```

Out[4]:
```
Dict{CategoricalArrays.CategoricalValue{String, UInt32}, Int64} with 6 entr
ies:
  "Spread wings"   => 60
  "I have command" => 60
  "Not clear"      => 60
  "Lock wings"     => 60
  "All clear"      => 60
  "Fold wings"     => 60
```

In [5]:
```
X_df
```

Out[5]: 360×24 DataFrame *335 rows omitted*

Row	X[Hand tip l]	Y[Hand tip l]	Z[Hand tip l]	X[Hand tip r]	Y[Hand tip r]	Z[Hand tip r]	X[
	Array…	Array…	Array…	Array…	Array…	Array…	Ar
1	[-0.519771, -0.52758, -0.531415, -0.517159, -0.510312, -0.518154, -0.50362, -0.485176, -0.466677, -0.444535 … -0.45501, -0.458937, -0.465048, -0.471251, -0.470015, -0.464627, -0.462666, -0.460253, -0.459572, -0.456737]	[-2.14011, -2.18043, -2.18425, -2.16547, -2.16635, -2.18836, -2.17162, -2.15248, -2.08072, -2.00607 … -2.17597, -2.1638, -2.17779, -2.17766, -2.17848, -2.16689, -2.15667, -2.13474, -2.13435, -2.13855]	[-0.957224, -0.970778, -0.970232, -0.960666, -0.962437, -0.970488, -0.966847, -0.96441, -0.972943, -0.979085 … -1.04234, -1.03616, -1.03756, -1.03275, -1.02525, -1.03115, -1.02558, -1.01884, -1.01701, -1.01059]	[0.675893, 0.699281, 0.673774, 0.700096, 0.765257, 0.980454, 1.43803, 1.78334, 2.08495, 2.32037 … 0.755717, 0.778103, 0.755128, 0.751274, 0.742517, 0.743311, 0.786792, 0.730863, 0.730482, 0.732217]	[-2.31794, -2.36398, -2.48698, -2.3176, -2.34228, -2.34828, -2.24596, -1.8102, -1.28214, -0.703666 … -2.45044, -2.33026, -2.44767, -2.43509, -2.44371, -2.42475, -2.25219, -2.38539, -2.38603, -2.35704]	[-0.254602, -0.246883, -0.252635, -0.235782, -0.13363, 0.051243, 0.078424, 0.274688, 0.335957, 0.390646 … -0.210761, -0.181256, -0.213764, -0.206785, -0.222643, -0.214863, -0.169845, -0.20958, -0.202703, -0.201438]	[- -0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0 -0
2	[-0.489753, -0.48607, -0.484529, -0.492771, -0.492031, -0.493076, -0.491979, -0.493256, -0.493156, -0.487527 … -0.400825, -0.414617, -0.407231, -0.397206, -0.366296, -0.354333, -0.371938, -0.386065, -0.408146, -0.415736]	[-1.55293, -1.54966, -1.55206, -1.55821, -1.556, -1.56055, -1.55812, -1.5648, -1.56414, -1.56731 … -1.6062, -1.62319, -1.61939, -1.6173, -1.58341, -1.5697, -1.55188, -1.54089, -1.52865, -1.52388]	[-0.907814, -0.911305, -0.92587, -0.921268, -0.928352, -0.928697, -0.932141, -0.930564, -0.933592, -0.932622 … -0.989828, -0.990365, -0.998319, -0.994962, -0.994991, -0.983351, -0.976952, -0.975923, -0.963954, -0.953944]	[0.632831, 0.633167, 0.637368, 0.640823, 0.635858, 0.63401, 0.634496, 0.637154, 0.640618, 0.643018 … 0.558287, 0.447356, 0.452128, 0.525122, 0.651756, 0.77637, 0.948441, 1.09432, 1.30458, 1.42438]	[-1.61526, -1.61763, -1.62374, -1.61861, -1.62068, -1.62244, -1.62164, -1.6257, -1.62654, -1.62966 … 1.56275, 1.58349, 1.59581, 1.60302, 1.55387, 1.53016, 1.47453, 1.47069, 1.45205, 1.39396]	[-0.63772, -0.637168, -0.644338, -0.651686, -0.653233, -0.654332, -0.651011, -0.6489, -0.654768, -0.653883 … 0.526364, 0.534895, 0.553634, 0.564454, 0.478762, 0.47897, 0.444671, 0.328608, 0.29968, 0.242647]	[- -0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0 -0
3	[-0.521346, -0.518394, -0.522321, -0.519893, -0.521016, -0.521524, -0.523362, -0.511653, -0.512519, -0.511312 … -0.514448, -0.518708,	[-1.72326, -1.72407, -1.72326, -1.72352, -1.72479, -1.72389, -1.7244, -1.76782, -1.76903, -1.76877 … -1.79175, -1.77926,	[-0.581362, -0.578159, -0.586091, -0.582611, -0.583196, -0.582819, -0.580284, -0.57613, -0.576047, -0.575067 … -0.64696, -0.640021,	[0.480245, 0.413413, 0.425131, 0.420865, 0.481781, 0.483458, 0.415258, 0.429159, 0.449354, 0.476563 … 0.71045, 0.665733,	[-1.72509, -1.79325, -1.77693, -1.78382, -1.72083, -1.72458, -1.80616, -1.77722, -1.78057, -1.79041 … -1.57885, -1.64564,	[-0.749465, -0.814978, -0.79228, -0.801608, -0.754548, -0.74575, -0.806902, -0.788115, -0.775095, -0.768625 … -1.16744, -0.986366,	[- -0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0

Row	X[Hand tip l]	Y[Hand tip l]	Z[Hand tip l]	X[Hand tip r]	Y[Hand tip r]	Z[Hand tip r]	X[
	Array…	Array…	Array…	Array…	Array…	Array…	Ar
	-0.521672, -0.525064, -0.514835, -0.507935, -0.521132, -0.517193, -0.517363, -0.51327]	-1.77711, -1.77828, -1.77587, -1.76419, -1.77843, -1.77882, -1.77992, -1.77987]	-0.630712, -0.616814, -0.62093, -0.621969, -0.621552, -0.624645, -0.621028, -0.626299]	0.621122, 0.557295, 0.519791, 0.48524, 0.481703, 0.488414, 0.485208, 0.479489]	-1.68675, -1.81935, -1.76374, -1.76348, -1.78189, -1.7849, -1.78435, -1.78333]	-0.849024, -0.767521, -0.725116, -0.723884, -0.743611, -0.736042, -0.731239, -0.733958]	-0 -0 -0 -0 -0 -0 -0 -0
4	[-0.57022, -0.562064, -0.565967, -0.562913, -0.567557, -0.566175, -0.566748, -0.561748, -0.55966, -0.556271 … -0.530846, -0.535016, -0.537207, -0.533389, -0.530497, -0.532508, -0.522586, -0.53489, -0.534332, -0.54071]	[-1.91196, -1.90369, -1.90527, -1.90405, -1.90318, -1.90619, -1.90959, -1.89934, -1.8948, -1.89346 … -1.87427, -1.87535, -1.88059, -1.8954, -1.89976, -1.89333, -1.90898, -1.91169, -1.92236, -1.92444]	[-0.753404, -0.748702, -0.747062, -0.7541, -0.751551, -0.749891, -0.75006, -0.748899, -0.745352, -0.74102 … -0.704626, -0.713649, -0.720423, -0.721149, -0.720037, -0.727544, -0.718666, -0.731909, -0.73111, -0.727761]	[0.459493, 0.464525, 0.461903, 0.455969, 0.460419, 0.465137, 0.445696, 0.458416, 0.4603, 0.46256 … 2.09097, 1.91878, 1.58165, 1.21182, 0.941954, 0.708641, 0.537249, 0.464884, 0.459635, 0.46293]	[-1.90089, -1.87507, -1.89495, -1.89809, -1.87756, -1.87972, -1.9182, -1.88876, -1.8717, -1.86988 … -0.790038, -1.22432, -1.5668, -1.69141, -1.78663, -1.85248, -1.9406, -1.96856, -1.96701, -1.9625]	[-0.764456, -0.766048, -0.757716, -0.756718, -0.767963, -0.767328, -0.754985, -0.757794, -0.759314, -0.755372 … -0.437201, -0.560395, -0.625109, -0.675635, -0.628565, -0.654884, -0.668321, -0.737166, -0.740219, -0.737878]	[- -0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0
5	[-0.624417, -0.626031, -0.625388, -0.62798, -0.624838, -0.623534, -0.626624, -0.626658, -0.622853, -0.622373 … -0.606563, -0.611505, -0.614609, -0.607108, -0.598554, -0.621197, -0.625199, -0.644386, -0.657226, -0.663721]	[-1.84287, -1.84026, -1.84688, -1.84182, -1.84628, -1.84354, -1.84273, -1.83752, -1.83289, -1.83472 … -1.68283, -1.72178, -1.77294, -1.80126, -1.81198, -1.87223, -1.89073, -1.89526, -1.9043, -1.91686]	[-0.789348, -0.786501, -0.768675, -0.779753, -0.775049, -0.77593, -0.770693, -0.771605, -0.773377, -0.76946 … -0.831481, -0.841451, -0.848442, -0.851784, -0.846179, -0.850705, -0.837824, -0.816053, -0.801157, -0.795484]	[0.58095, 0.57809, 0.579865, 0.577963, 0.576101, 0.576345, 0.575145, 0.579263, 0.579383, 0.579958 … 2.07734, 2.11504, 2.1128, 1.91689, 1.5704, 1.18571, 0.803449, 0.617248, 0.555628, 0.519571]	[-1.83512, -1.83411, -1.83304, -1.83161, -1.82641, -1.82692, -1.82371, -1.81809, -1.81299, -1.81521 … 0.210206, -0.240879, -0.761203, -1.25598, -1.64153, -1.92075, -2.01471, -1.99813, -1.98928, -2.0021]	[-0.748908, -0.753321, -0.749488, -0.758251, -0.764208, -0.764563, -0.768688, -0.772309, -0.774509, -0.774836 … 0.212435, 0.104328, -0.043032, -0.286689, -0.430668, -0.572304, -0.638792, -0.682752, -0.718812, -0.761999]	[- -0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0
6	[-0.502501, -0.502525, -0.499415, -0.501144, -0.502677, -0.501937,	[-2.17556, -2.15613, -2.18516, -2.19291, -2.15844, -2.14539,	[-1.09413, -1.07683, -1.09008, -1.09044, -1.07624, -1.06987,	[0.631689, 0.624567, 0.638725, 0.640064, 0.617619, 0.609287,	[-2.39645, -2.35991, -2.39196, -2.35874, -2.39011, -2.38913,	[-0.174365, -0.166227, -0.164783, -0.171156, -0.171868, -0.168856,	[- -0 -0 -0 -0 -0

Row	X[Hand tip l]	Y[Hand tip l]	Z[Hand tip l]	X[Hand tip r]	Y[Hand tip r]	Z[Hand tip r]	X[
	Array…	Array…	Array…	Array…	Array…	Array…	Ar
	-0.500699, -0.501717, -0.501963, -0.504734 … -0.43365, -0.436541, -0.447761, -0.456823, -0.460775, -0.467277, -0.464943, -0.469757, -0.468361, -0.469486]	-2.17194, -2.14695, -2.13512, -2.12584 … -2.05511, -2.07088, -2.07849, -2.09125, -2.10468, -2.12407, -2.13097, -2.16563, -2.1762, -2.19243]	-1.07743, -1.07267, -1.0659, -1.0586 … -0.994042, -1.00038, -1.00443, -1.00719, -1.01067, -1.02072, -1.02934, -1.0372, -1.04505, -1.04952]	0.623102, 0.614398, 0.60629, 0.591307 … 1.1434, 0.936468, 0.81356, 0.748232, 0.70633, 0.715475, 0.701832, 0.708491, 0.711467, 0.724143]	-2.4044, -2.3628, -2.33339, -2.39623 … -2.29177, -2.33778, -2.32335, -2.31428, -2.39301, -2.30433, -2.44085, -2.4951, -2.50935, -2.53032]	-0.167825, -0.166135, -0.168217, -0.164987 … 0.248703, 0.315796, 0.30116, 0.181476, 0.109893, 0.069707, 0.072417, 0.054247, 0.056288, 0.068829]	-0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0 -0
7	[-0.488461, -0.489463, -0.487539, -0.495673, -0.498767, -0.492156, -0.492845, -0.484968, -0.482085, -0.480355 … -0.493495, -0.492836, -0.49895, -0.503093, -0.509026, -0.513016, -0.515636, -0.523701, -0.519121, -0.512226]	[-2.17242, -2.18203, -2.18057, -2.18011, -2.16312, -2.16706, -2.1655, -2.16417, -2.16289, -2.16507 … -2.20638, -2.24703, -2.26315, -2.27173, -2.2962, -2.3206, -2.28483, -2.26507, -2.30022, -2.30896]	[-0.968068, -0.970886, -0.972168, -0.964309, -0.968031, -0.964959, -0.965357, -0.96689, -0.961591, -0.971308 … -0.946976, -0.954752, -0.957002, -0.954402, -0.956824, -0.959061, -0.951237, -0.939096, -0.95456, -0.970996]	[0.56396, 0.595508, 0.563289, 0.562872, 0.569912, 0.59887, 0.597455, 0.59935, 0.588259, 0.616466 … 1.90101, 1.68372, 1.45597, 1.22308, 1.03946, 0.896663, 0.789375, 0.750815, 0.748015, 0.753018]	[-2.39541, -2.32961, -2.40599, -2.4037, -2.38496, -2.30025, -2.29899, -2.29647, -2.34951, -2.34849 … -1.68622, -2.01573, -2.27071, -2.44548, -2.54653, -2.61704, -2.58102, -2.42942, -2.45797, -2.3098]	[-0.189166, -0.156892, -0.183036, -0.188968, -0.182562, -0.155315, -0.159691, -0.162753, -0.167263, -0.126656 … 0.428323, 0.353664, 0.283833, 0.227649, 0.192813, 0.144374, 0.100455, 0.056708, 0.020836, -0.010608]	[- -0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0 -0
8	[-0.468105, -0.410602, -0.473909, -0.475146, -0.465564, -0.459415, -0.408703, -0.407192, -0.406746, -0.471503 … -0.403425, -0.38908, -0.388014, -0.376936, -0.386189, -0.383457, -0.379303, -0.379167, -0.35105, -0.37192]	[-1.86535, -1.89011, -1.87105, -1.87014, -1.86305, -1.86513, -1.88585, -1.88192, -1.88197, -1.86255 … -1.70318, -1.71049, -1.71516, -1.72376, -1.72854, -1.75506, -1.78602, -1.83142, -1.82577, -1.85042]	[-0.697004, -0.708269, -0.681783, -0.685562, -0.700491, -0.698824, -0.712602, -0.714313, -0.709702, -0.683408 … -0.694703, -0.696871, -0.685235, -0.698665, -0.689877, -0.695257, -0.711887, -0.716584, -0.716449, -0.711233]	[0.51303, 0.535447, 0.526609, 0.529012, 0.516169, 0.514988, 0.515586, 0.504798, 0.504606, 0.531836 … 1.64002, 1.79941, 1.99117, 2.07635, 2.11726, 2.06097, 1.95247, 1.74608, 1.46703, 1.22156]	[-1.89671, -1.86846, -1.87776, -1.86998, -1.90863, -1.90658, -1.89158, -1.91002, -1.9078, -1.88441 … 0.980896, 0.775941, 0.411292, 0.105297, -0.240116, -0.606455, -0.99533, -1.35875, -1.59084, -1.78522]	[-0.72422, -0.706672, -0.716476, -0.716994, -0.720904, -0.726877, -0.722421, -0.730964, -0.734788, -0.696653 … -0.583728, -0.670627, -0.697702, -0.731501, -0.745341, -0.771108, -0.766031, -0.755091, -0.675288, -0.632553]	[- -0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0 -0

Row	X[Hand tip l]	Y[Hand tip l]	Z[Hand tip l]	X[Hand tip r]	Y[Hand tip r]	Z[Hand tip r]	X[
	Array…	Array…	Array…	Array…	Array…	Array…	Ar
9	[-0.568195, -0.572936, -0.571337, -0.577742, -0.562071, -0.563401, -0.56426, -0.56251, -0.567891, -0.568983 … -0.572375, -0.573012, -0.570897, -0.571727, -0.572124, -0.56665, -0.568694, -0.576087, -0.577822, -0.576184]	[-1.79059, -1.78162, -1.78303, -1.77291, -1.79029, -1.79301, -1.7906, -1.79415, -1.78852, -1.78192 … -1.71373, -1.712, -1.71488, -1.71719, -1.724, -1.73081, -1.73278, -1.74117, -1.74922, -1.7542]	[-0.629271, -0.631997, -0.629313, -0.636701, -0.635508, -0.634023, -0.639724, -0.637628, -0.638034, -0.642679 … -0.59198, -0.595464, -0.590844, -0.597421, -0.599602, -0.605654, -0.595905, -0.590594, -0.592675, -0.593041]	[0.574572, 0.572953, 0.58065, 0.576566, 0.576827, 0.576204, 0.578404, 0.575839, 0.571202, 0.578351 … 1.11163, 1.28867, 1.47037, 1.62137, 1.73751, 1.76605, 1.68489, 1.51916, 1.30435, 1.10295]	[-1.82092, -1.82256, -1.8185, -1.81975, -1.81878, -1.81891, -1.81806, -1.82086, -1.82313, -1.8185 … 1.05829, 0.920416, 0.710814, 0.407749, 0.04714, -0.35929, -0.776797, -1.13484, -1.41145, -1.55159]	[-0.617017, -0.615382, -0.612141, -0.614752, -0.611139, -0.616751, -0.615291, -0.615829, -0.618155, -0.617481 … -0.349956, -0.491946, -0.653769, -0.814871, -0.892301, -0.952159, -0.969561, -0.944127, -0.865372, -0.795585]	[-(-0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0 -0
10	[-0.517579, -0.515374, -0.517325, -0.516505, -0.514786, -0.513077, -0.518725, -0.520816, -0.519732, -0.521663 … -0.470503, -0.478416, -0.476322, -0.483407, -0.494542, -0.497023, -0.503007, -0.511892, -0.516592, -0.506665]	[-1.73887, -1.74072, -1.7397, -1.73405, -1.73453, -1.73561, -1.7393, -1.71604, -1.72597, -1.73219 … -1.72644, -1.73147, -1.73613, -1.75607, -1.76061, -1.78371, -1.80105, -1.82082, -1.83392, -1.84493]	[-0.693497, -0.691899, -0.687666, -0.692099, -0.686091, -0.6872, -0.693279, -0.690005, -0.68333, -0.693432 … -0.746989, -0.737197, -0.732215, -0.722234, -0.737401, -0.737225, -0.742772, -0.742642, -0.749105, -0.749828]	[0.514507, 0.526694, 0.549404, 0.506781, 0.550714, 0.548035, 0.542946, 0.536573, 0.54797, 0.515168 … 2.05311, 2.01937, 2.0615, 2.14439, 2.14045, 2.15822, 2.0852, 1.93771, 1.70293, 1.46727]	[-1.83433, -1.81353, -1.76559, -1.82393, -1.76616, -1.76641, -1.7819, -1.79043, -1.76682, -1.8396 … 0.671155, 0.405484, 0.170111, -0.095538, -0.391807, -0.683817, -0.992354, -1.32681, -1.56162, -1.79568]	[-0.711129, -0.706901, -0.695049, -0.725259, -0.690311, -0.68963, -0.702482, -0.705599, -0.698506, -0.715266 … -0.295706, -0.343539, -0.356921, -0.385912, -0.405628, -0.440152, -0.444337, -0.468822, -0.478764, -0.456275]	[-(-0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0 -0
11	[-0.631494, -0.629032, -0.630474, -0.628314, -0.625873, -0.620084, -0.622708, -0.615488, -0.604842, -0.609029 … -0.614349, -0.623253, -0.624818, -0.61368,	[-1.98071, -1.98581, -1.98407, -1.98487, -1.98305, -1.98431, -1.98929, -1.98398, -2.00128, -1.99469 … -1.96031, -1.97189, -1.97785, -1.98624,	[-0.747038, -0.74841, -0.747703, -0.75014, -0.754128, -0.759453, -0.75647, -0.766852, -0.762574, -0.767539 … -0.729413, -0.732674, -0.739823, -0.731681,	[0.856043, 0.857518, 0.851185, 0.859634, 0.858619, 0.854354, 0.905673, 1.16508, 1.42501, 1.71238 … 1.07754, 0.957434, 0.851525, 0.757334,	[-2.03258, -2.0345, -2.03402, -2.03329, -2.03401, -2.034, -2.02314, -1.8548, -1.60643, -0.912175 … -1.78783, -1.91655, -2.02219, -2.06767,	[-0.865939, -0.864099, -0.862266, -0.86024, -0.852186, -0.85945, -0.842965, -0.9574, -1.05019, -1.05552 … -0.972505, -0.913282, -0.893715, -0.822636, -0.784828,	[-(-0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0

Row	X[Hand tip l]	Y[Hand tip l]	Z[Hand tip l]	X[Hand tip r]	Y[Hand tip r]	Z[Hand tip r]	X[
	Array…	Array…	Array…	Array…	Array…	Array…	Ar
	-0.615468, -0.620129, -0.619546, -0.617533, -0.623453, -0.620921]	-1.98397, -1.97893, -1.97566, -1.97111, -1.96347, -1.97802]	-0.733116, -0.739864, -0.741084, -0.742643, -0.748294, -0.737384]	0.744572, 0.701962, 0.779988, 0.778182, 0.778619]	-2.06231, -2.04269, -2.01127, -1.73125, -1.7347, -1.73614]	-0.733635, -0.782048, -0.759991, -0.75973, -0.756357]	-0 -0 -0 -0 -0 -0
12	[-0.628575, -0.621757, -0.631781, -0.634901, -0.628957, -0.637745, -0.635805, -0.629445, -0.632091, -0.635745 … -0.591855, -0.582347, -0.588977, -0.598174, -0.599671, -0.619432, -0.621313, -0.628482, -0.633884, -0.627249]	[-1.64959, -1.65482, -1.6445, -1.65008, -1.65215, -1.64652, -1.64085, -1.65269, -1.64934, -1.64933 … -1.65049, -1.66718, -1.6695, -1.67345, -1.67695, -1.6821, -1.68837, -1.69011, -1.68936, -1.69211]	[-0.826129, -0.820825, -0.825739, -0.822362, -0.825972, -0.82397, -0.82843, -0.827873, -0.833039, -0.834004 … -0.850629, -0.836348, -0.83736, -0.830908, -0.825183, -0.81275, -0.804439, -0.805276, -0.816357, -0.813985]	[0.63093, 0.630534, 0.619773, 0.626157, 0.621542, 0.621468, 0.613933, 0.637572, 0.767764, 1.01477 … 1.33375, 1.03323, 0.808823, 0.693519, 0.658938, 0.589302, 0.627916, 0.67699, 0.70274, 0.72096]	[-1.73747, -1.73717, -1.7373, -1.73898, -1.7427, -1.74377, -1.73731, -1.74702, -1.75181, -1.73828 … -1.70125, -1.79771, -1.87956, -1.89704, -1.86995, -1.96147, -1.91128, -1.85101, -1.8577, -1.83568]	[-0.701359, -0.702162, -0.706464, -0.704693, -0.705164, -0.697885, -0.693803, -0.691132, -0.704814, -0.670501 … -0.380183, -0.407226, -0.445143, -0.422019, -0.475656, -0.479682, -0.47908, -0.507252, -0.51364, -0.539279]	[-(-0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0 -0
13	[-0.635968, -0.613303, -0.636138, -0.635273, -0.636906, -0.633466, -0.633318, -0.628543, -0.627045, -0.624422 … -0.61882, -0.614184, -0.626042, -0.633471, -0.634397, -0.639003, -0.645128, -0.654244, -0.651518, -0.649729]	[-1.96703, -1.98867, -1.96746, -1.9631, -1.96258, -1.96333, -1.96385, -1.9627, -1.96265, -1.96134 … -1.84131, -1.85966, -1.85407, -1.83433, -1.84256, -1.83536, -1.82502, -1.82088, -1.82251, -1.80934]	[-0.661007, -0.672837, -0.656478, -0.658991, -0.65884, -0.659275, -0.655378, -0.658712, -0.654345, -0.651957 … -0.577366, -0.578097, -0.565833, -0.580407, -0.577185, -0.581951, -0.598315, -0.612136, -0.617067, -0.631079]	[0.630916, 0.629885, 0.629919, 0.630639, 0.629542, 0.63015, 0.630143, 0.630747, 0.630963, 0.629108 … 2.06203, 2.03448, 1.9007, 1.64908, 1.37607, 1.14622, 0.918773, 0.727559, 0.594801, 0.538287]	[-2.01777, -2.01644, -2.01636, -2.01498, -2.01403, -2.01435, -2.01093, -2.01459, -2.01451, -2.01158 … -0.037665, -0.489951, -0.950234, -1.36542, -1.63791, -1.74946, -1.86018, -1.92242, -1.96731, -1.97118]	[-0.713973, -0.720726, -0.720042, -0.715272, -0.72197, -0.720044, -0.719547, -0.715168, -0.712008, -0.717159 … -0.679075, -0.752105, -0.784518, -0.804115, -0.727146, -0.627818, -0.604224, -0.602402, -0.59308, -0.614281]	[-(-0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0 -0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	
349	[-0.616689, -0.614287, -0.616339, -0.611394, -0.615198, -0.652625,	[-1.94958, -1.94994, -1.94992, -1.95348, -1.93972, -2.09082,	[-0.755485, -0.760594, -0.755267, -0.755398, -0.751462, -0.75515,	[0.664354, 0.657678, 0.663926, 0.653829, 0.672965, 0.729495,	[-2.00496, -2.03505, -2.01516, -2.05454, -2.06293, -2.0968,	[-0.547348, -0.552963, -0.548251, -0.563437, -0.571891, -0.557266,	[-(-0 -0 -0 -0 -0

Row	X[Hand tip l]	Y[Hand tip l]	Z[Hand tip l]	X[Hand tip r]	Y[Hand tip r]	Z[Hand tip r]	X[
	Array…	Array…	Array…	Array…	Array…	Array…	Ar
	-0.758759, -0.926013, -1.16653, -1.45786 … -0.634068, -0.638292, -0.638414, -0.633326, -0.629956, -0.624907, -0.628131, -0.623601, -0.621252, -0.619131]	-2.07774, -2.07786, -2.03293, -1.95337 … -2.01936, -2.01745, -2.01658, -2.01221, -2.01132, -2.00823, -1.99968, -1.99785, -1.99715, -1.99688]	-0.740546, -0.726338, -0.712166, -0.657644 … -0.700128, -0.702082, -0.69888, -0.70226, -0.701131, -0.708299, -0.705904, -0.705396, -0.706151, -0.706053]	0.949406, 1.34799, 1.67983, 1.83766 … 0.595432, 0.637802, 0.679586, 0.636176, 0.597256, 0.593704, 0.595663, 0.594957, 0.59227, 0.590825]	-1.96131, -1.51774, -0.880203, -0.12666 … -2.20064, -1.99626, -1.91715, -1.99687, -2.19972, -2.19755, -2.19723, -2.19504, -2.19353, -2.19285]	-0.713681, -0.837412, -0.903952, -0.789053 … -0.568336, -0.569397, -0.532565, -0.553386, -0.551853, -0.553435, -0.551775, -0.55471, -0.553742, -0.553014]	-0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0 -0
350	[-0.596139, -0.596019, -0.594476, -0.591199, -0.59016, -0.587625, -0.589226, -0.59427, -0.614432, -0.712729 … -0.575696, -0.57971, -0.582036, -0.585876, -0.576416, -0.57689, -0.578217, -0.57949, -0.578794, -0.575571]	[-1.67946, -1.68074, -1.67967, -1.68043, -1.68028, -1.67404, -1.68079, -1.67452, -1.66515, -1.63637 … -1.6783, -1.66726, -1.66698, -1.66767, -1.6741, -1.67276, -1.67021, -1.66882, -1.66735, -1.66801]	[-0.41818, -0.410148, -0.404465, -0.404289, -0.397016, -0.404511, -0.393309, -0.403948, -0.41825, -0.471417 … -0.358823, -0.375101, -0.372223, -0.371538, -0.370928, -0.37131, -0.378007, -0.373494, -0.373474, -0.379876]	[0.496632, 0.49866, 0.496747, 0.50127, 0.493619, 0.49165, 0.497987, 0.49167, 0.516722, 0.582021 … 0.430446, 0.429438, 0.434283, 0.434753, 0.433317, 0.432706, 0.429335, 0.431152, 0.432418, 0.429916]	[-1.6367, -1.63157, -1.63873, -1.63019, -1.63165, -1.63523, -1.62979, -1.63167, -1.59549, -1.48439 … -1.70408, -1.7138, -1.70738, -1.70491, -1.69697, -1.70152, -1.71372, -1.70201, -1.70447, -1.70052]	[-0.739211, -0.742347, -0.733166, -0.738768, -0.734405, -0.734872, -0.736452, -0.734485, -0.784128, -0.965686 … -0.707477, -0.698281, -0.703048, -0.703093, -0.706035, -0.702741, -0.694049, -0.696337, -0.688899, -0.695393]	[- -0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0 -0
351	[-0.937442, -1.02995, -0.985338, -0.828794, -0.719483, -0.764852, -0.790658, -0.906044, -1.06143, -1.24269 … -1.11567, -0.915111, -0.690979, -0.683455, -0.758403, -0.722961, -0.661048, -0.674346, -0.687475, -0.68439]	[-2.10495, -1.96835, -1.95082, -2.09426, -2.13928, -2.08863, -2.23229, -2.28766, -2.27454, -2.06905 … -1.9192, -1.99392, -1.94095, -2.02532, -2.04187, -2.00124, -1.94335, -1.91709, -1.91269, -1.90738]	[-0.445069, -0.419463, -0.494451, -0.546546, -0.592364, -0.584375, -0.554674, -0.464802, -0.479882, -0.533597 … -0.758341, -0.621436, -0.620878, -0.632757, -0.6223, -0.690714, -0.73709, -0.740402, -0.738781, -0.740575]	[0.593855, 0.611397, 0.602737, 0.584278, 0.563043, 0.581611, 0.588664, 0.595967, 0.858367, 1.31364 … 0.827272, 0.658031, 0.608874, 0.625905, 0.608621, 0.608801, 0.613776, 0.596233, 0.588538, 0.590146]	[-2.01059, -2.01198, -2.019, -2.02334, -2.07292, -2.11766, -2.14359, -2.12517, -2.2062, -2.03789 … -2.15671, -1.94299, -1.94304, -1.95421, -1.96479, -1.95365, -1.95082, -1.93788, -1.94164, -1.88977]	[-0.460162, -0.438999, -0.433038, -0.449091, -0.514482, -0.526686, -0.511987, -0.479525, -0.090289, 0.001343 … -0.13886, -0.521114, -0.532044, -0.553127, -0.572179, -0.632077, -0.637166, -0.669985, -0.66622, -0.659614]	[- -0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0 -0

Row	X[Hand tip l]	Y[Hand tip l]	Z[Hand tip l]	X[Hand tip r]	Y[Hand tip r]	Z[Hand tip r]	X[
	Array…	Array…	Array…	Array…	Array…	Array…	Ar
352	[-0.595625, -0.593605, -0.588495, -0.584801, -0.59971, -0.595148, -0.59403, -0.591524, -0.591834, -0.591661 … -0.631753, -0.613123, -0.604525, -0.578524, -0.539126, -0.486437, -0.4675, -0.453359, -0.445844, -0.454189]	[-1.93897, -1.93641, -1.93761, -1.91826, -1.97002, -1.97009, -1.966, -1.94677, -1.94209, -1.93865 … -2.05116, -2.02755, -1.97911, -1.93749, -1.94224, -1.95526, -1.95181, -1.94552, -1.94973, -1.96401]	[-0.683665, -0.694508, -0.682787, -0.701766, -0.747765, -0.744019, -0.744061, -0.72129, -0.711405, -0.707665 … -0.40507, -0.479012, -0.569021, -0.649807, -0.70869, -0.75495, -0.780949, -0.791049, -0.774547, -0.751589]	[0.676463, 0.682937, 0.682541, 0.685074, 0.686095, 0.681547, 0.680452, 0.681045, 0.684939, 0.676113 … 0.660947, 0.637387, 0.590117, 0.57134, 0.574835, 0.595796, 0.619714, 0.625665, 0.638661, 0.6339]	[-1.8211, -1.82324, -1.82337, -1.82025, -1.87301, -1.86164, -1.8572, -1.83805, -1.83235, -1.82618 … -1.90219, -1.88858, -1.87824, -1.87888, -1.88442, -1.89448, -1.90057, -1.90804, -1.90383, -1.91231]	[-0.693638, -0.693881, -0.685873, -0.687236, -0.726534, -0.726996, -0.728512, -0.706081, -0.696437, -0.698008 … -0.650799, -0.709445, -0.729737, -0.740974, -0.743894, -0.735067, -0.731009, -0.713037, -0.708452, -0.695166]	[-0 -0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0 -0
353	[-0.466582, -0.469372, -0.43454, -0.432809, -0.480658, -0.500828, -0.532364, -0.552448, -0.453876, -0.439888 … -0.715931, -0.734048, -0.696927, -0.643663, -0.575416, -0.492768, -0.422277, -0.389059, -0.394768, -0.411497]	[-1.71874, -1.72861, -1.58691, -1.58583, -1.70418, -1.64664, -1.61578, -1.6056, -1.52025, -1.30873 … -1.81162, -1.98329, -1.972, -1.93991, -1.8876, -1.85807, -1.83981, -1.81063, -1.75395, -1.65299]	[-1.03662, -1.03794, -0.949864, -0.954459, -1.04864, -1.078, -1.1047, -1.15505, -1.40861, -1.56395 … -0.820564, -0.976599, -1.03893, -1.08675, -1.09675, -1.09772, -1.07486, -1.03899, -1.04458, -1.0622]	[0.59468, 0.594129, 0.593619, 0.602656, 0.641599, 0.735446, 0.882214, 1.15693, 1.48269, 1.78098 … 0.40946, 0.618935, 0.637069, 0.640586, 0.635214, 0.636883, 0.636803, 0.632184, 0.609569, 0.601727]	[-1.84908, -1.84819, -1.85054, -1.85237, -1.85198, -1.90322, -1.86753, -1.73999, -1.51691, -1.05561 … -1.63182, -1.88926, -1.90622, -1.89375, -1.86071, -1.83122, -1.80992, -1.7971, -1.7638, -1.75888]	[-0.365797, -0.368545, -0.357051, -0.344981, -0.34665, -0.378178, -0.396243, -0.545071, -0.604943, -0.635584 … -0.569574, -0.114756, -0.139989, -0.165661, -0.214669, -0.267517, -0.306533, -0.329637, -0.362193, -0.385993]	[-0 -0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0 -0
354	[-0.500404, -0.502824, -0.504771, -0.505733, -0.505021, -0.510656, -0.477738, -0.455724, -0.450382, -0.43636 … -0.62565, -0.617651, -0.588326, -0.552376, -0.529874,	[-1.89209, -1.88986, -1.89188, -1.88755, -1.88602, -1.92025, -1.89595, -1.88366, -1.85715, -1.8259 … -1.95207, -1.95965, -1.95948, -1.98069, -1.98885,	[-0.667846, -0.671957, -0.666312, -0.67298, -0.671848, -0.673591, -0.705428, -0.734485, -0.761981, -0.783711 … -0.707524, -0.715147, -0.718432, -0.710816,	[0.604001, 0.608896, 0.619072, 0.620853, 0.622834, 0.630271, 0.618775, 0.598823, 0.585954, 0.59969 … 0.562149, 0.525271, 0.513633, 0.528847, 0.554589,	[-1.73447, -1.74005, -1.74104, -1.73596, -1.73738, -1.7684, -1.76486, -1.75967, -1.71397, -1.62112 … -1.75231, -1.76314, -1.76987, -1.79338,	[-0.852622, -0.847307, -0.850225, -0.849992, -0.843751, -0.848163, -0.854097, -0.872344, -0.9205, -0.982214 … -0.955112, -0.961629, -0.94402, -0.937481,	[-0 -0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0

Row	X[Hand tip l]	Y[Hand tip l]	Z[Hand tip l]	X[Hand tip r]	Y[Hand tip r]	Z[Hand tip r]	X[
	Array…	Array…	Array…	Array…	Array…	Array…	Ar
	-0.553498, -0.572458, -0.588936, -0.604985, -0.611643]	-1.97063, -1.99709, -2.00647, -2.00631, -2.00625]	-0.693616, -0.713655, -0.704543, -0.704291, -0.710759, -0.712378]	0.577905, 0.581103, 0.576899, 0.575532, 0.577343]	-1.80963, -1.81876, -1.82579, -1.82698, -1.80892, -1.808]	-0.934921, -0.935206, -0.925465, -0.917304, -0.927874, -0.930375]	-0 -0 -0 -0 -0 -0
355	[-0.686893, -0.690966, -0.710514, -0.771405, -0.865657, -1.10153, -1.28476, -1.3701, -1.17664, -0.759236 … -0.590119, -0.586947, -0.58872, -0.586027, -0.586456, -0.587513, -0.590223, -0.588903, -0.591829, -0.591138]	[-2.04375, -2.05011, -2.07035, -2.08768, -2.11075, -2.08723, -2.01236, -1.89968, -1.62723, -1.3901 … -2.01395, -2.0175, -2.01779, -2.00809, -2.00814, -2.00896, -2.01059, -2.01857, -2.0202, -2.01936]	[-0.763731, -0.739648, -0.709582, -0.698519, -0.695876, -0.734551, -0.780351, -0.911202, -1.30088, -1.56614 … -0.750251, -0.749495, -0.755412, -0.75954, -0.761369, -0.763274, -0.76296, -0.76822, -0.767053, -0.773619]	[0.619419, 0.641105, 0.677359, 0.750174, 0.969919, 1.55344, 1.8599, 2.083, 1.98471, 1.68652 … 0.736513, 0.739569, 0.743421, 0.743809, 0.735055, 0.766925, 0.749379, 0.748856, 0.740557, 0.744968]	[-2.08314, -2.08161, -2.08674, -2.08547, -2.02839, -1.70898, -1.29341, -0.711759, 0.182653, 0.622182 … -1.97317, -1.97663, -1.97623, -1.97845, -2.14337, -2.07043, -1.98182, -1.9928, -2.13751, -2.12822]	[-0.63965, -0.621357, -0.591876, -0.534126, -0.468331, -0.174424, -0.017619, 0.10863, 0.149411, 0.097924 … -0.510329, -0.509272, -0.517616, -0.512647, -0.534151, -0.5225, -0.516237, -0.519779, -0.529337, -0.530785]	[- -0 -0 -0 -0 -0 -0 -1 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0
356	[-0.525938, -0.516073, -0.5177, -0.516002, -0.517101, -0.531324, -0.598619, -0.632816, -0.641388, -0.624213 … -0.506511, -0.53477, -0.536864, -0.537779, -0.545924, -0.540783, -0.544388, -0.550136, -0.546228, -0.545995]	[-1.69259, -1.70519, -1.70824, -1.72114, -1.72487, -1.74597, -1.73602, -1.70956, -1.66064, -1.55737 … -1.7578, -1.67215, -1.66308, -1.6627, -1.64937, -1.66342, -1.66382, -1.66711, -1.67062, -1.66981]	[-0.514372, -0.521267, -0.51823, -0.507091, -0.489786, -0.500235, -0.559973, -0.659927, -0.761855, -0.891289 … -0.536724, -0.524299, -0.517065, -0.522152, -0.517777, -0.526417, -0.521008, -0.531884, -0.52938, -0.531231]	[0.385693, 0.391897, 0.387426, 0.391813, 0.416009, 0.49509, 0.833951, 1.08184, 1.34254, 1.6102 … 0.473226, 0.439757, 0.435853, 0.444632, 0.418755, 0.410171, 0.416042, 0.413618, 0.409138, 0.410571]	[-1.71975, -1.72145, -1.72375, -1.72402, -1.73314, -1.73277, -1.60902, -1.50969, -1.29886, -0.787043 … -1.7279, -1.71693, -1.71674, -1.7201, -1.72104, -1.72351, -1.74602, -1.75108, -1.74972, -1.74926]	[-0.715464, -0.711937, -0.712161, -0.714993, -0.70302, -0.716358, -0.748026, -0.79191, -0.858587, -0.998692 … -0.702175, -0.710662, -0.711929, -0.700321, -0.716329, -0.719247, -0.716009, -0.72007, -0.723947, -0.720406]	[- -0 -0 -0 -0 -0 -0 -0 -0 -0 -0 -0 -0 -0 -0 -0 -0 -0 -0
357	[-0.440887, -0.452221, -0.447185, -0.451468, -0.451852, -0.460196, -0.454885, -0.456595,	[-1.90444, -1.91917, -1.91652, -1.92718, -1.92486, -1.93992, -1.93326, -1.931,	[-1.07556, -1.08468, -1.08069, -1.08455, -1.08098, -1.08955, -1.08634, -1.08236,	[0.601969, 0.607711, 0.600509, 0.620354, 0.619002, 0.601562, 0.59883, 0.585152,	[-2.15266, -2.17978, -2.19577, -2.043, -2.03998, -2.21929, -2.21141, -2.21772,	[-0.432073, -0.435872, -0.42879, -0.406766, -0.410107, -0.432601, -0.431501, -0.438689,	[- -0 -0 -0 -0 -0 -0 -0

Row	X[Hand tip l]	Y[Hand tip l]	Z[Hand tip l]	X[Hand tip r]	Y[Hand tip r]	Z[Hand tip r]	X[
	Array…	Array…	Array…	Array…	Array…	Array…	Ar
	-0.453205, -0.469354 … 0.181187, -0.131194, -0.541423, -0.777776, -0.953652, -1.04438, -0.984894, -0.891926, -0.788926, -0.698249]	-1.89117, -1.90504 … -1.18755, -1.39716, -1.36595, -1.59362, -1.67972, -1.74415, -1.83087, -1.86974, -1.92518, -1.86742]	-1.07372, -1.08595 … -1.38186, -1.4358, -1.35391, -1.36938, -1.34895, -1.24551, -1.14865, -1.07423, -1.03839, -1.01313]	0.58419, 0.583967 … 1.82576, 2.01202, 1.99811, 2.14004, 1.9395, 1.61937, 1.20757, 0.874289, 0.691181, 0.562766]	-2.21474, -2.23002 … 0.350891, 0.023507, -0.423043, -0.812878, -1.24957, -1.70558, -2.05941, -2.229, -2.24742, -2.19937]	-0.431665, -0.434617 … -0.097338, -0.017904, 0.118915, 0.111467, 0.145836, 0.084069, 0.021073, -0.070298, -0.066992, -0.291861]	-0 -0 … -0 -0 -0 -0 -0 -0 -0 -0
358	[-0.647672, -0.653511, -0.642305, -0.6383, -0.637352, -0.641916, -0.641393, -0.641784, -0.644001, -0.646141 … -0.224099, -0.500692, -0.682501, -0.758069, -0.765982, -0.72369, -0.645501, -0.624031, -0.62637, -0.628032]	[-1.6173, -1.61051, -1.60491, -1.59955, -1.60063, -1.60052, -1.60037, -1.60233, -1.61603, -1.62083 … -1.3884, -1.479, -1.63708, -1.65932, -1.664, -1.6762, -1.69198, -1.6986, -1.69814, -1.69324]	[-0.505743, -0.499972, -0.497074, -0.500385, -0.494025, -0.496815, -0.49508, -0.503109, -0.513043, -0.515419 … -0.97989, -0.827817, -0.673086, -0.504513, -0.400298, -0.340097, -0.315575, -0.317331, -0.321467, -0.327659]	[0.500621, 0.496023, 0.498986, 0.498838, 0.504341, 0.500749, 0.503862, 0.504645, 0.505147, 0.504536 … 0.293131, 0.443811, 0.581705, 0.631393, 0.575838, 0.527186, 0.534738, 0.529456, 0.518023, 0.517005]	[-1.61356, -1.61504, -1.61717, -1.61682, -1.61745, -1.61724, -1.6164, -1.6166, -1.61653, -1.61602 … -0.727617, -1.1854, -1.55158, -1.64122, -1.65639, -1.67336, -1.67886, -1.68227, -1.6856, -1.68569]	[-0.682743, -0.680679, -0.674725, -0.676847, -0.673253, -0.674156, -0.676122, -0.674299, -0.673238, -0.675257 … -1.2196, -1.11147, -0.889893, -0.700757, -0.638817, -0.619261, -0.614219, -0.615918, -0.62318, -0.623297]	[- -0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0 -0
359	[-0.476117, -0.4705, -0.474443, -0.475678, -0.475745, -0.476801, -0.466286, -0.470162, -0.466426, -0.456026 … -0.310213, -0.498413, -0.660566, -0.792202, -0.859166, -0.837351, -0.769369, -0.684986, -0.545003, -0.53447]	[-1.70846, -1.7099, -1.70912, -1.70614, -1.70891, -1.70891, -1.70919, -1.70437, -1.71174, -1.71172 … -1.01014, -1.24986, -1.4634, -1.57394, -1.71292, -1.80086, -1.85209, -1.84221, -1.8798, -1.78985]	[-0.5028, -0.509458, -0.508224, -0.509525, -0.508446, -0.505009, -0.504267, -0.49798, -0.500755, -0.489398 … -1.27009, -1.1431, -1.00582, -0.793894, -0.569638, -0.399036, -0.319151, -0.279739, -0.295245, -0.332945]	[0.379579, 0.380022, 0.381541, 0.37959, 0.387175, 0.383584, 0.39284, 0.397443, 0.395351, 0.395874 … 0.733999, 0.871185, 0.973005, 0.972278, 0.867967, 0.770433, 0.654358, 0.552439, 0.431852, 0.378889]	[-1.62798, -1.63086, -1.63214, -1.63089, -1.62406, -1.63349, -1.62449, -1.60594, -1.59181, -1.58358 … 0.25072, -0.203643, -0.645084, -1.02025, -1.43843, -1.60047, -1.74759, -1.79286, -1.89259, -1.82064]	[-0.840636, -0.834709, -0.83162, -0.833223, -0.83019, -0.823642, -0.817607, -0.812333, -0.80052, -0.792545 … -1.65719, -1.75717, -1.73933, -1.55679, -1.24865, -1.06737, -0.932671, -0.84789, -0.79027, -0.771634]	[- -0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0 -0

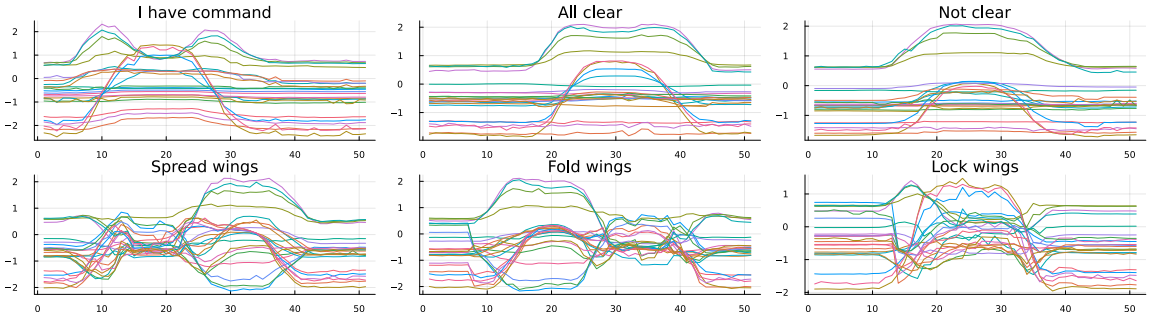
Row	X[Hand tip l]	Y[Hand tip l]	Z[Hand tip l]	X[Hand tip r]	Y[Hand tip r]	Z[Hand tip r]	X[
	Array…	Array…	Array…	Array…	Array…	Array…	Ar
360	[-0.553245, -0.551704, -0.548044, -0.544929, -0.546446, -0.546651, -0.554797, -0.580788, -0.634431, -0.679319 … -0.605506, -0.606554, -0.662612, -0.709257, -0.727722, -0.721847, -0.687693, -0.655425, -0.627145, -0.613324]	[-1.69493, -1.69349, -1.69951, -1.7085, -1.71117, -1.71164, -1.70638, -1.68895, -1.66512, -1.63018 … -1.76228, -1.75493, -1.74816, -1.73299, -1.72764, -1.72671, -1.725, -1.72086, -1.71081, -1.70149]	[-0.756008, -0.762863, -0.760376, -0.756736, -0.756973, -0.758584, -0.766189, -0.790081, -0.83163, -0.879984 … -0.568478, -0.590416, -0.601258, -0.618041, -0.641417, -0.648827, -0.660898, -0.674366, -0.687214, -0.689185]	[0.569726, 0.570003, 0.573506, 0.572164, 0.621714, 0.696098, 0.876118, 1.12203, 1.47091, 1.64837 … 0.503982, 0.51867, 0.550847, 0.56638, 0.586101, 0.585824, 0.581762, 0.575169, 0.572896, 0.568447]	[-1.8093, -1.80937, -1.80944, -1.80935, -1.8126, -1.85915, -1.8061, -1.67266, -1.36509, -1.061 … -1.84879, -1.81557, -1.82067, -1.82043, -1.8186, -1.8199, -1.81979, -1.8188, -1.81805, -1.81838]	[-0.448892, -0.45103, -0.445198, -0.449493, -0.440023, -0.397585, -0.392426, -0.419542, -0.481797, -0.566341 … -0.39616, -0.374028, -0.372259, -0.369527, -0.37349, -0.374199, -0.375562, -0.382328, -0.383967, -0.382463]	[- -0 -0 -0 -0 -0 -0 -0 -0 -0 … -0 -0 -0 -0 -0 -0 -0 -0 -0 -0

In [6]:
```
names(X_df)
```

Out[6]:
```
24-element Vector{String}:
 "X[Hand tip l]"
 "Y[Hand tip l]"
 "Z[Hand tip l]"
 "X[Hand tip r]"
 "Y[Hand tip r]"
 "Z[Hand tip r]"
 "X[Elbow l]"
 "Y[Elbow l]"
 "Z[Elbow l]"
 "X[Elbow r]"
 "Y[Elbow r]"
 "Z[Elbow r]"
 "X[Wrist l]"
 "Y[Wrist l]"
 "Z[Wrist l]"
 "X[Wrist r]"
 "Y[Wrist r]"
 "Z[Wrist r]"
 "X[Thumb l]"
 "Y[Thumb l]"
 "Z[Thumb l]"
 "X[Thumb r]"
 "Y[Thumb r]"
 "Z[Thumb r]"
```

In [7]:
```
# Let's inspect an instance for each class.
plot(map(i->plot(collect(X_df[i,:]), labels=nothing,title=y[i]), 1:30:180)..
```

Out[7]:



In [8]:
```
X = scalarlogiset(X_df)
println(X)
```

```
SupportedLogiset with 1 supports (343.08 MBs)
├ worldtype:                    SoleLogics.Interval{Int64}
├ featvaltype:                  Float64
├ featuretype:                  SoleModels.AbstractUnivariateFeature
├ frametype:                    SoleLogics.FullDimensionalFrame{1, SoleLogic
s.Interval{Int64}}
├ # instances:                  360
├ usesfullmemo:                 true
├[BASE] UniformFullDimensionalLogiset of channel size (51,) (342.91 MBs)
│ ├ size × eltype:              (51, 51, 360, 48) × Float64
│ └ features:                   48 -> SoleModels.AbstractUnivariateFeature
["max[V1]", "min[V1]", "max[V2]", "min[V2]", "...", "min[V22]", "max[V23]",
"min[V23]", "max[V24]", "min[V24]"]
└[SUPPORT 1] FullMemoset (0 memoized values, 174.42 KBs))
```

In [9]:
```
# Accessibles on multirelation frames
# Get the structure ("frame") of the first instance. It is a "dimensional" f
fr = SoleLogics.frame(X, 1)
```

Out[9]:
```
SoleLogics.FullDimensionalFrame{1, SoleLogics.Interval{Int64}}((51,))
```

In [10]:
```
# Enumerate all worlds
collect(allworlds(fr))
```

Out[10]:
```
1326-element Vector{SoleLogics.Interval{Int64}}:
 SoleLogics.Interval{Int64}(1, 2)
 SoleLogics.Interval{Int64}(1, 3)
 SoleLogics.Interval{Int64}(2, 3)
 SoleLogics.Interval{Int64}(1, 4)
 SoleLogics.Interval{Int64}(2, 4)
 SoleLogics.Interval{Int64}(3, 4)
 SoleLogics.Interval{Int64}(1, 5)
 SoleLogics.Interval{Int64}(2, 5)
 SoleLogics.Interval{Int64}(3, 5)
 SoleLogics.Interval{Int64}(4, 5)
 SoleLogics.Interval{Int64}(1, 6)
 SoleLogics.Interval{Int64}(2, 6)
 SoleLogics.Interval{Int64}(3, 6)
 ⋮
 SoleLogics.Interval{Int64}(40, 52)
 SoleLogics.Interval{Int64}(41, 52)
 SoleLogics.Interval{Int64}(42, 52)
 SoleLogics.Interval{Int64}(43, 52)
 SoleLogics.Interval{Int64}(44, 52)
 SoleLogics.Interval{Int64}(45, 52)
 SoleLogics.Interval{Int64}(46, 52)
 SoleLogics.Interval{Int64}(47, 52)
 SoleLogics.Interval{Int64}(48, 52)
 SoleLogics.Interval{Int64}(49, 52)
 SoleLogics.Interval{Int64}(50, 52)
 SoleLogics.Interval{Int64}(51, 52)
```

In [11]:
```
using SoleLogics: Interval
# Enumerate the intervals that are "Later" than [1,10]
accessibles(fr, Interval(1,10), IA_L) |> collect
```

Out[11]:
```
861-element Vector{Interval{Int64}}:
 Interval{Int64}(11, 12)
 Interval{Int64}(11, 13)
 Interval{Int64}(12, 13)
 Interval{Int64}(11, 14)
 Interval{Int64}(12, 14)
 Interval{Int64}(13, 14)
 Interval{Int64}(11, 15)
 Interval{Int64}(12, 15)
 Interval{Int64}(13, 15)
 Interval{Int64}(14, 15)
 Interval{Int64}(11, 16)
 Interval{Int64}(12, 16)
 Interval{Int64}(13, 16)
 ⋮
 Interval{Int64}(40, 52)
 Interval{Int64}(41, 52)
 Interval{Int64}(42, 52)
 Interval{Int64}(43, 52)
 Interval{Int64}(44, 52)
 Interval{Int64}(45, 52)
 Interval{Int64}(46, 52)
 Interval{Int64}(47, 52)
 Interval{Int64}(48, 52)
 Interval{Int64}(49, 52)
 Interval{Int64}(50, 52)
 Interval{Int64}(51, 52)
```

In [12]:
```
# Remember that features are computed on each world
# Let's compute the minimum of the first variable on an arbitrary interval,
feature = UnivariateMin(1)
Sole.featvalue(feature, X, 1, Interval(10,30))
```

Out[12]:
```
-0.444535
```

In [13]:
```
# Remember that atoms are *scalar conditions on features*
# Let's check one on an interval of the first instance
p = Atom(ScalarCondition(feature, >, -0.5))
check(p, X, 1, Interval(10,30))
```

Out[13]:
```
true
```

In [14]:
```
# I can check any formula
p = Atom(ScalarCondition(UnivariateMin(1), >, -0.5))
q = Atom(ScalarCondition(UnivariateMin(2), <=, 10))
φ = p ∨ q
check(φ, X, 1, Interval(10,30))
```

Out[14]:
```
true
```

In [40]:
```
# Generate a random HS formula with scalar conditions on features, and check
features = [UnivariateMin(i_variable) for i_variable in 1:ncol(X_df)]
alpha = [Atom(ScalarCondition(feat, >, thresh)) for feat in features for thr

HS_connectives = SoleLogics.diamondsandboxes(SoleLogics.IARelations)
```

```
propo_connectives = SoleLogics.BASE_PROPOSITIONAL_CONNECTIVES

println("Propositional connectives: $(join(syntaxstring.(propo_connectives),
println("HS connectives: $(join(syntaxstring.(HS_connectives), ", "))")

propo_weights = fill(1/length(propo_connectives), length(propo_connectives))
HS_weights = fill(1/length(HS_connectives), length(HS_connectives))

connectives = vcat(propo_connectives, HS_connectives)

opweights = vcat(propo_weights, HS_weights)

treeheight = 3
φ2 = randformula(Random.MersenneTwister(30), treeheight, alpha, connectives;
println()
println("Random formula:")
println(syntaxstring(φ2))

check(φ2, X, 1, Interval(10,30))
```

```
Propositional connectives: ¬, ∧, ∨, →
HS connectives: ⟨A⟩, [A], ⟨L⟩, [L], ⟨B⟩, [B], ⟨E⟩, [E], ⟨D⟩, [D], ⟨O⟩, [O],
⟨Ā⟩, [Ā], ⟨L̄⟩, [L̄], ⟨B̄⟩, [B̄], ⟨Ē⟩, [Ē], ⟨D̄⟩, [D̄], ⟨Ō⟩, [Ō]

Random formula:
¬((min[V8] > 0.3 → min[V13] > 0.2) → (min[V18] > 0.6 ∨ min[V16] > 0.2))
```

Out[40]: false

In [41]:
```
# Let's check a formula on all the instances
check_mask = check(φ2, X, Interval(10,30))
```

Out[41]:
```
360-element Vector{Bool}:
 0
 0
 0
 0
 0
 0
 0
 0
 0
 0
 0
 0
 0
 ⋮
 0
 1
 0
 0
 0
 1
 0
 0
 0
 0
 0
 0
```

In [42]:
```
# It holds on part of the instances
sum(check_mask)
```

Out[42]:
```
98
```

In [43]:
```
# Let's ask whether the formula holds *all* intervals, instead of checking i
println("Applying the universal global operator: ", SoleLogics.globalbox)
println()

universal_φ = globalbox(φ2)
println("Formula: ", syntaxstring(universal_φ))
check_mask = check(universal_φ, X)

# It holds on no instance... Too restrictive!
sum(check_mask)
```

```
Applying the universal global operator: [G]

Formula: [G]¬((min[V8] > 0.3 → min[V13] > 0.2) → (min[V18] > 0.6 ∨ min[V16]
> 0.2))
```

Out[43]:
```
0
```

In [44]:
```
# Let's ask whether there exists any interval where the formula holds
println("Applying the existential global operator: ", SoleLogics.globaldiamo
println()

existential_φ = globaldiamond(φ2)
```

```
println("Formula: ", syntaxstring(existential_φ))
check_mask = check(existential_φ, X)

# It holds on more instances
sum(check_mask)
```

```
Applying the existential global operator: ⟨G⟩

Formula: ⟨G⟩¬((min[V8] > 0.3 → min[V13] > 0.2) → (min[V18] > 0.6 ∨ min[V16]
> 0.2))
```

Out[44]: 127

In [45]:
```
# Question: does it lead to a good rule?

println(syntaxstring(existential_φ))

println()
println(SoleLogics.experimentals.formula2natlang(existential_φ))
```

```
⟨G⟩¬((min[V8] > 0.3 → min[V13] > 0.2) → (min[V18] > 0.6 ∨ min[V16] > 0.2))

∃ interval where (¬(whenever whenever min[V8] > 0.3 holds, also min[V13] >
0.2 holds, also min[V18] > 0.6 ∨ min[V16] > 0.2))
```

In [46]:
```
neg_existential_φ = normalize(¬ existential_φ;
    profile = :readability,
    remove_implications = true,
    allow_atom_flipping = true
)

syntaxstring(neg_existential_φ; remove_redundant_parentheses = false)
```

Out[46]: "[G]((min[V16] > 0.2) ∨ ((min[V18] > 0.6) ∨ ((min[V8] > 0.3) ∧ (min[V13] ≤ 0.2))))"

In [47]:
```
(SoleLogics.precedence(∧), SoleLogics.precedence(∨))
```

Out[47]: (12, 11)

In [48]:
```
countmap(y)
```

Out[48]:
```
Dict{CategoricalArrays.CategoricalValue{String, UInt32}, Int64} with 6 entr
ies:
  "Spread wings"   => 60
  "I have command" => 60
  "Not clear"      => 60
  "Lock wings"     => 60
  "All clear"      => 60
  "Fold wings"     => 60
```

In [49]:
```
println(existential_φ)
countmap(y[check_mask])
```

```
SyntaxBranch{DiamondRelationalConnective{GlobalRel}}: ⟨G⟩¬((min[V8] > 0.3 →
min[V13] > 0.2) → (min[V18] > 0.6 ∨ min[V16] > 0.2))
```

Out[49]:
```
Dict{CategoricalArrays.CategoricalValue{String, UInt32}, Int64} with 5 entr
ies:
  "Spread wings" => 58
  "Lock wings"   => 10
  "Not clear"    => 1
  "Fold wings"   => 57
  "All clear"    => 1
```

In [50]:
```
println(neg_existential_φ)
countmap(y[(!).(check_mask)])
```

```
SyntaxBranch{BoxRelationalConnective{GlobalRel}}: [G](min[V16] > 0.2 ∨ min[V
18] > 0.6 ∨ min[V8] > 0.3 ∧ min[V13] ≤ 0.2)
```

Out[50]:
```
Dict{CategoricalArrays.CategoricalValue{String, UInt32}, Int64} with 6 entr
ies:
  "Spread wings"   => 2
  "I have command" => 60
  "Not clear"      => 59
  "Lock wings"     => 50
  "All clear"      => 59
  "Fold wings"     => 3
```

In [51]:
```
branch = Branch(neg_existential_φ, "I have command", "Spread wings")
```

Out[51]:
```
▣ [G]((min[V16] > 0.2) ∨ (min[V18] > 0.6) ∨ (min[V8] > 0.3) ∧ (min[V13] ≤
0.2))
├✔ I have command
└✘ Spread wings
```

In [52]:
```
y_preds = SoleModels.apply(branch, X)
println("Accuracy of this branch: $(sum(y .== y_preds)/length(y))")
println("Random chanc e: $(60/length(y))")
```

```
Accuracy of this branch: 0.3277777777777778
Random chanc e: 0.16666666666666666
```

In [53]:
```
# Some values during checking were memoized
println(X)
```

```
SupportedLogiset with 1 supports (417.29 MBs)
├ worldtype:                    Interval{Int64}
├ featvaltype:                  Float64
├ featuretype:                  SoleModels.AbstractUnivariateFeature
├ frametype:                    SoleLogics.FullDimensionalFrame{1, Interval{I
nt64}}
├ # instances:                  360
├ usesfullmemo:                 true
├[BASE] UniformFullDimensionalLogiset of channel size (51,) (342.91 MBs)
│ ├ size × eltype:              (51, 51, 360, 48) × Float64
│ └ features:                   48 -> SoleModels.AbstractUnivariateFeature
["max[V1]", "min[V1]", "max[V2]", "min[V2]", "...", "min[V22]", "max[V23]",
"min[V23]", "max[V24]", "min[V24]"]
└[SUPPORT 1] FullMemoset (7923 memoized values, 74.38 MBs))
```

In [54]:
```
# Randomly split the data: 20% training, 80% testing
N = nrow(X_df)
perm = randperm(Random.MersenneTwister(1), N)
train_idxs, test_idxs = perm[1:round(Int, N*.2)], perm[round(Int, N*.2)+1:en
println("Using $(length(train_idxs)) instances for training")
println("Using $(length(test_idxs)) instances for testing")
```

```
Using 72 instances for training
Using 288 instances for testing
```

In [55]:
```
using ModalDecisionTrees

# Bind a machine learning algorithm to logiset & labels
mach = machine(ModalDecisionTree(; relations = :IA7, features = [minimum]),

# Train!
@time fit!(mach; rows=train_idxs);

# Compute accuracy
yhat = predict_mode(mach; rows=test_idxs)
MLJ.accuracy(yhat, y[test_idxs])
```

```
[ Info: Precomputing logiset...
[ Info: Training machine(ModalDecisionTree(max_depth = nothing, …), …).
 35.403512 seconds (431.26 M allocations: 21.343 GiB, 18.47% gc time)
```

Out[55]:
```
0.7847222222222222
```

In [56]:
```
# Show the restricted MDT learnt
printmodel(report(mach).rawmodel_full; hidemodality = true)
```

```
{1} SimpleDecision(⟨G⟩min[V1] ≥ 0.428173)                    All clear : 16/
72 (conf = 0.2222)
✔ {1} SimpleDecision(⟨G⟩min[V13] < -1.536833)                Lock wings : 1
4/34 (conf = 0.4118)
│✔ {1} SimpleDecision(⟨Ā Ō⟩min[V1] ≥ 0.428173)               Fold wings : 1
2/20 (conf = 0.6000)
│✔ Spread wings : 8/8 (conf = 1.0000)
│✘ Fold wings : 12/12 (conf = 1.0000)
✘ Lock wings : 14/14 (conf = 1.0000)
✘ {1} SimpleDecision(⟨G⟩min[V5] ≥ 0.847021)                  All clear : 16/
38 (conf = 0.4211)
 ✔ {1} SimpleDecision(⟨=⟩min[V2] ≥ -1.668041)                I have command
: 13/14 (conf = 0.9286)
 │✔ I have command : 3/4 (conf = 0.7500)
 │✘ I have command : 10/10 (conf = 1.0000)
 ✘ {1} SimpleDecision(⟨G⟩min[V3] ≥ -0.62357)                 All clear : 15/
24 (conf = 0.6250)
  ✔ {1} SimpleDecision(⟨=⟩min[V5] < -1.850843)               Not clear : 8/1
1 (conf = 0.7273)
  │✔ All clear : 3/4 (conf = 0.7500)
  │✘ Not clear : 7/7 (conf = 1.0000)
  ✘ {1} SimpleDecision(⟨G⟩min[V1] ≥ -0.413014)               All clear : 12/
13 (conf = 0.9231)
   ✔ All clear : 3/4 (conf = 0.7500)
   ✘ All clear : 9/9 (conf = 1.0000)
```

In [57]:
```
# Show its *pure* version
printmodel(report(mach).solemodel_full; show_metrics = true, hidemodality =
```

```
▣ ⟨G⟩(min[V1] ≥ 0.428173)
├✔ ⟨G⟩((min[V1] ≥ 0.428173) ∧ ⟨G⟩(min[V13] < -1.536833))
│├✔ ⟨G⟩((min[V1] ≥ 0.428173) ∧ ⟨G⟩((min[V13] < -1.536833) ∧ ⟨Ā Ō⟩(min[V1] ≥
0.428173)))
│├✔ Spread wings : (ninstances = 8, confidence = 1.0, coverage = 1.0)
│└✘ Fold wings : (ninstances = 12, confidence = 1.0, coverage = 1.0)
└✘ Lock wings : (ninstances = 14, confidence = 1.0, coverage = 1.0)
 ├✔ ⟨G⟩((min[V5] ≥ 0.847021) ∧ (min[V2] ≥ -1.668041))
 │├✔ I have command : (ninstances = 4, confidence = 0.75, coverage = 1.0)
 │└✘ I have command : (ninstances = 10, confidence = 1.0, coverage = 1.0)
 └✘ ⟨G⟩(min[V3] ≥ -0.62357)
  ├✔ ⟨G⟩((min[V3] ≥ -0.62357) ∧ (min[V5] < -1.850843))
  │├✔ All clear : (ninstances = 4, confidence = 0.75, coverage = 1.0)
  │└✘ Not clear : (ninstances = 7, confidence = 1.0, coverage = 1.0)
  └✘ ⟨G⟩(min[V1] ≥ -0.413014)
   ├✔ All clear : (ninstances = 4, confidence = 0.75, coverage = 1.0)
   └✘ All clear : (ninstances = 9, confidence = 1.0, coverage = 1.0)
```

In [58]:
```
simplified_restricted_tree = ModalDecisionTrees.prune(report(mach).rawmodel_
printmodel(simplified_restricted_tree)

println()
println("# Leaves: ", nleaves(simplified_restricted_tree))
```

```
{1} SimpleDecision(⟨G⟩min[V1] ≥ 0.428173)                    All clear : 16/
72 (conf = 0.2222)
✔ {1} SimpleDecision(⟨G⟩min[V13] < -1.536833)                Lock wings : 1
4/34 (conf = 0.4118)
│✔ {1} SimpleDecision(⟨Ā Ō⟩min[V1] ≥ 0.428173)               Fold wings : 1
2/20 (conf = 0.6000)
│├✔ Spread wings : 8/8 (conf = 1.0000)
│├✘ Fold wings : 12/12 (conf = 1.0000)
│✘ Lock wings : 14/14 (conf = 1.0000)
✘ {1} SimpleDecision(⟨G⟩min[V5] ≥ 0.847021)                  All clear : 16/
38 (conf = 0.4211)
 ✔ I have command : 13/14 (conf = 0.9286)
 ✘ {1} SimpleDecision(⟨G⟩min[V3] ≥ -0.62357)                 All clear : 15/
24 (conf = 0.6250)
  ✔ {1} SimpleDecision(⟨=⟩min[V5] < -1.850843)               Not clear : 8/1
1 (conf = 0.7273)
  │✔ All clear : 3/4 (conf = 0.7500)
  │✘ Not clear : 7/7 (conf = 1.0000)
  ✘ All clear : 12/13 (conf = 0.9231)

# Leaves: 7
```

In [59]:
```
solemodel = ModalDecisionTrees.translate(simplified_restricted_tree)
```

Out[59]:
```
▣ {1}(⟨G⟩(min[V1] ≥ 0.428173))
├✔ {1}(⟨G⟩((min[V1] ≥ 0.428173) ∧ ⟨G⟩(min[V13] < -1.536833)))
│├✔ {1}(⟨G⟩((min[V1] ≥ 0.428173) ∧ ⟨G⟩((min[V13] < -1.536833) ∧ ⟨AO⟩(min[V
1] ≥ 0.428173))))
│├✔ Spread wings
│└✘ Fold wings
│└✘ Lock wings
└✘ {1}(⟨G⟩(min[V5] ≥ 0.847021))
 ├✔ I have command
 └✘ {1}(⟨G⟩(min[V3] ≥ -0.62357))
  ├✔ {1}(⟨G⟩((min[V3] ≥ -0.62357) ∧ (min[V5] < -1.850843)))
  │├✔ All clear
  │└✘ Not clear
  └✘ All clear
```

In [60]:
```
# Print leaf rules + their training performances
ruleset = listrules(solemodel)
printmodel.(ruleset; show_metrics = true, threshold_digits = 2, variable_nam
```

```
▣ ⟨G⟩(min[X[Hand tip l]] ≥ 0.43 ∧ ⟨G⟩(min[X[Wrist l]] < -1.54 ∧ ⟨AO⟩min[X[Ha
nd tip l]] ≥ 0.43))  ➡  Spread wings : (ninstances = 8, confidence = 1.0, co
verage = 0.11)
▣ ⟨G⟩(min[X[Hand tip l]] ≥ 0.43 ∧ ⟨G⟩min[X[Wrist l]] < -1.54) ∧ [G](min[X[Ha
nd tip l]] ≥ 0.43 → ([G](min[X[Wrist l]] < -1.54 → ([AO]min[X[Hand tip l]] <
0.43))))  ➡  Fold wings : (ninstances = 12, confidence = 1.0, coverage = 0.1
7)
▣ ⟨G⟩min[X[Hand tip l]] ≥ 0.43 ∧ [G](min[X[Hand tip l]] ≥ 0.43 → ([G]min[X[W
rist l]] ≥ -1.54))  ➡  Lock wings : (ninstances = 14, confidence = 1.0, cove
rage = 0.19)
▣ ⟨G⟩min[Y[Hand tip r]] ≥ 0.85 ∧ [G]min[X[Hand tip l]] < 0.43  ➡  I have com
mand : (ninstances = 14, confidence = 0.93, coverage = 0.19)
▣ ⟨G⟩(min[Z[Hand tip l]] ≥ -0.62 ∧ min[Y[Hand tip r]] < -1.85) ∧ [G]min[X[Ha
nd tip l]] < 0.43 ∧ [G]min[Y[Hand tip r]] < 0.85  ➡  All clear : (ninstances
= 4, confidence = 0.75, coverage = 0.06)
▣ ⟨G⟩min[Z[Hand tip l]] ≥ -0.62 ∧ [G]min[X[Hand tip l]] < 0.43 ∧ [G]min[Y[Ha
nd tip r]] < 0.85 ∧ [G](min[Z[Hand tip l]] ≥ -0.62 → min[Y[Hand tip r]] ≥ -
1.85)  ➡  Not clear : (ninstances = 7, confidence = 1.0, coverage = 0.1)
▣ [G]min[X[Hand tip l]] < 0.43 ∧ [G]min[Y[Hand tip r]] < 0.85 ∧ [G]min[Z[Han
d tip l]] < -0.62  ➡  All clear : (ninstances = 13, confidence = 0.92, cover
age = 0.18)
```

In [61]:
```
last_rule = ruleset[end]
last_antd = antecedent(last_rule)

println("First formula, translated:")
println(SoleLogics.experimentals.formula2natlang(last_antd; threshold_digits


for (i_rule, rule) in enumerate(ruleset)
    println()
    println("[$i_rule]")
    antd = antecedent(rule)
    println(SoleLogics.experimentals.formula2natlang(antd; threshold_digits
end
```

```
First formula, translated:
((∀ intervals (V1 ≱ 0.43)) and (∀ intervals (V5 ≱ 0.85))) and (∀ intervals
(V3 ≱ -0.62))

[1]
∃ interval where ((V1 ≧ 0.43) and (∃ interval where ((V13 ≱ -1.54) and (∃ pr
eceding, partially overlapping interval where (V1 ≧ 0.43)))))


[2]
(∃ interval where ((V1 ≧ 0.43) and (∃ interval where (V13 ≱ -1.54)))) and (∀
intervals (whenever V1 ≧ 0.43 holds, also ∀ intervals (whenever V13 ≱ -1.54
holds, also ∀ preceding, partially overlapping intervals (V1 ≱ 0.43))))


[3]
(∃ interval where (V1 ≧ 0.43)) and (∀ intervals (whenever V1 ≧ 0.43 holds, a
lso ∀ intervals (V13 ≧ -1.54)))


[4]
(∃ interval where (V5 ≧ 0.85)) and (∀ intervals (V1 ≱ 0.43))


[5]
((∃ interval where ((V3 ≧ -0.62) and (V5 ≱ -1.85))) and (∀ intervals (V1 ≱
0.43))) and (∀ intervals (V5 ≱ 0.85))


[6]
(((∃ interval where (V3 ≧ -0.62)) and (∀ intervals (V1 ≱ 0.43))) and (∀ inte
rvals (V5 ≱ 0.85))) and (∀ intervals (whenever V3 ≧ -0.62 holds, also V5 ≧
-1.85))


[7]
((∀ intervals (V1 ≱ 0.43)) and (∀ intervals (V5 ≱ 0.85))) and (∀ intervals
(V3 ≱ -0.62))
```

In [62]:

```
# Print rules + their *test* performances

# Sprinkle the model with the test instances!
predictions, tree_test = report(mach).sprinkle(X_df[test_idxs,:], y[test_idx

# Extract ruleset and print its metrics
ruleset_test = listrules(tree_test);

# printmodel.(ruleset_test; show_metrics = true, threshold_digits = 2, varia
printmodel.(ruleset_test; show_metrics = true, threshold_digits = 2, parenth
```

```
Applying tree... 100%|████████████████████████████████| Time: 0:00:02
```

```
▣ ⟨G⟩(min[V1] ≥ 0.43 ∧ ⟨G⟩(min[V13] < -1.54 ∧ ⟨AO⟩min[V1] ≥ 0.43))  ➡  Sprea
d wings : (ninstances = 8, confidence = 1.0, coverage = 0.11)
▣ ⟨G⟩(min[V1] ≥ 0.43 ∧ ⟨G⟩min[V13] < -1.54) ∧ [G](min[V1] ≥ 0.43 → ([G](min
[V13] < -1.54 → ([AO]min[V1] < 0.43))))  ➡  Fold wings : (ninstances = 12, c
onfidence = 1.0, coverage = 0.17)
▣ ⟨G⟩min[V1] ≥ 0.43 ∧ [G](min[V1] ≥ 0.43 → ([G]min[V13] ≥ -1.54))  ➡  Lock w
ings : (ninstances = 14, confidence = 1.0, coverage = 0.19)
▣ ⟨G⟩min[V5] ≥ 0.85 ∧ [G]min[V1] < 0.43  ➡  I have command : (ninstances = 1
4, confidence = 0.93, coverage = 0.19)
▣ ⟨G⟩(min[V3] ≥ -0.62 ∧ min[V5] < -1.85) ∧ [G]min[V1] < 0.43 ∧ [G]min[V5] <
0.85  ➡  All clear : (ninstances = 4, confidence = 0.75, coverage = 0.06)
▣ ⟨G⟩min[V3] ≥ -0.62 ∧ [G]min[V1] < 0.43 ∧ [G]min[V5] < 0.85 ∧ [G](min[V3] ≥
-0.62 → min[V5] ≥ -1.85)  ➡  Not clear : (ninstances = 7, confidence = 1.0,
coverage = 0.1)
▣ [G]min[V1] < 0.43 ∧ [G]min[V5] < 0.85 ∧ [G]min[V3] < -0.62  ➡  All clear :
(ninstances = 13, confidence = 0.92, coverage = 0.18)
```

In [63]:
```
println("IF\n\t", SoleLogics.experimentals.formula2natlang(antecedent(rulese
println("THEN\n\t", consequent(ruleset_test[4]))
```

```
IF
        (∃ interval where (min[V5] ≥ 0.847021)) and (∀ intervals (min[V1] <
0.428173))
THEN
        ▣ I have command
```

In [64]:
```
# Obtain class rules & show their *test* metrics
condensed_ruleset_test = joinrules(ruleset_test)
printmodel.(condensed_ruleset_test; show_metrics = true, threshold_digits =
```

```
▣ (⟨G⟩(min[V1] ≥ 0.43 ∧ ⟨G⟩(min[V13] < -1.54 ∧ ⟨AO⟩min[V1] ≥ 0.43)))  ➡  Spr
ead wings : (ninstances = 8, confidence = 1.0, coverage = 0.11)
▣ (⟨G⟩(min[V1] ≥ 0.43 ∧ ⟨G⟩min[V13] < -1.54) ∧ [G](min[V1] ≥ 0.43 → ([G](min
[V13] < -1.54 → ([AO]min[V1] < 0.43)))))  ➡  Fold wings : (ninstances = 12,
confidence = 1.0, coverage = 0.17)
▣ (⟨G⟩min[V1] ≥ 0.43 ∧ [G](min[V1] ≥ 0.43 → ([G]min[V13] ≥ -1.54)))  ➡  Lock
wings : (ninstances = 14, confidence = 1.0, coverage = 0.19)
▣ (⟨G⟩min[V5] ≥ 0.85 ∧ [G]min[V1] < 0.43)  ➡  I have command : (ninstances =
14, confidence = 0.93, coverage = 0.19)
▣ (⟨G⟩(min[V3] ≥ -0.62 ∧ min[V5] < -1.85) ∧ [G]min[V1] < 0.43 ∧ [G]min[V5] <
0.85) ∨ ([G]min[V1] < 0.43 ∧ [G]min[V5] < 0.85 ∧ [G]min[V3] < -0.62)  ➡  All
clear : (ninstances = 17, confidence = 0.88, coverage = 0.12)
▣ (⟨G⟩min[V3] ≥ -0.62 ∧ [G]min[V1] < 0.43 ∧ [G]min[V5] < 0.85 ∧ [G](min[V3]
≥ -0.62 → min[V5] ≥ -1.85))  ➡  Not clear : (ninstances = 7, confidence = 1.
0, coverage = 0.1)
```

Exercise

`ModalDecisionTrees.jl` can also handle images! In which case, they use a 2D logic of rectangles instead of a 1D logic of intervals.

Apply ModalDecisionTrees to Indian Pines, a benchmark dataset for Land Cover Classification with 16 classes. The dataset consists of a hyperspectral image

(i.e., 200 color channels instead of the typical 3 RGB channels) where many pixels have been labelled as belonging to one of the classes.

Sketch of the idea:

- Load the image and the ground truths. The package MAT.jl can be helpful;
- From the 145×145 image provided, sample a (small) number m of 3×3 patches for each class, and label each patch with the class label for the central pixel;
- Gather the ground truths into a vector of strings `y`, and the samples into a Julia `DataFrame` `X` with 200 columns, $16m$ rows and 3×3 *matrices* in the cells;
- Use MLJ to train a ModalDecisionTree on `X` and `y`, similarly to the above case.

Suggestion: since the formulas are desirably rotation-invariant, ask the algorithm to use *topological* relations instead of *directional* relations. Rrefer to the doc to know more.