Integral Cohomology of Finite Postnikov Towers

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Abstract

By the work of H. Cartan, it is well known that one can find elements of arbitrarilly high torsion in the integral (co)homology groups of an Eilenberg-MacLane space *K(G,n)*, where *G* is a non-trivial abelian group and *n>1*.

The main goal of this work is to extend this result to H-spaces having more than one non-trivial homotopy groups.

In order to have an accurate hold on H. Cartan's result, we start by studying the duality between homology and cohomology of 2-local Eilenberg-MacLane spaces of finite type. This leads us to some improvements of H. Cartan's methods in this particular case.

Our main result can be stated as follows. Let *X* be an H-space with two non-vanishing finite 2-torsion homotopy groups. Then *X* does not admit any exponent for its reduced integral graded (co)homology group.

We construct a wide class of examples for which this result is a simple consequence of a topological feature, namely the existence of a weak retract $X \to K(G,n)$ for some abelian group *G* and *n>1*.

We also generalize our main result to more complicated stable two stage Postnikov systems, using the Eilenberg-Moore spectral sequence and analytic methods involving Betti numbers and their asymptotic behaviour.

Finally, we investigate some guesses on the non-existence of homology exponents for finite Postnikov towers. We conjecture that Postnikov pieces do not admit any (co)homology exponent.

This work also includes the presentation of the "Eilenberg-MacLane machine", a C++ program designed to compute explicitely all integral homology groups of Eilenberg-MacLane spaces.