Some relationships between homotopy and homology exponents

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Abstract

We investigate the integral homology of spaces having a homotopy exponent. The main examples considered here are simply connected Postnikov pieces with finite homotopy groups and we would like to know if it is possible for them to have a homology exponent.

By the work of H. Cartan it is well known that the answer is "no" for simply connected Eilenberg-MacLane spaces associated to finite groups since their integral homology groups contain torsion elements of arbitrarily high order. Therefore any finite product of such spaces has the same property.

The situation is less trivial when the Postnikov invariants of a Postnikov piece are non-trivial. In some cases the space retracts onto an Eilenberg-MacLane and thus the question has an obvious *topological* answer. We shall exhibit more complicated spaces having elements of arbitrarily high order in integral homology by using such *algebraic* tools as Cartan's work and the Bockstein spectral sequence.