Honors Contract Report – PHY 432

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1 Abstract

In this project, I created a simulation of the phenomenon of Compton scattering, in which gamma rays produced by a radioactive source collide with the electrons in a target mass. This effect serves as proof of a fundamental aspect of modern physics: the quantisation of light. According to the classical interpretation of physics, light is a wave and therefore massless, but that view cannot account for the collisions between it and electrons seen in reality. My model demonstrates this unique behavior by simulating the interactions between electrons and photons emitted by decaying cesium-137 and barium-133. Since momentum is known to be a conserved quantity in this context, we can find the mass of the electrons and check it against the accepted value to ensure that the energy part of the simulation is accurate. To check the accuracy of the intensities, an R^2 analysis using the probability density function was performed on the data. Ultimately, my results were highly consistent with my expectations (0.5106 MeV vs. 0.5110 MeV (NIST) for the energies and $R^2 \approx 1$ for the intensities), so we can then conclude that my model does indeed describe reality accurately.

2 Introduction

Physically speaking, the simulation can be visualised as a radioactive source mass some arbitrary distance away from a target mass. For the sake of simplicity, the beam of gamma rays the source produces was treated as collimated and the target mass as a point mass, rendering the system effectively one-dimensional.

As per the laws of conservation of energy and momentum, the scattering angle and photon wavelengths (incident and final) are related by the equation below (Daw):

$$\lambda' - \lambda = \lambda_c(1 - \cos(\theta)), \tag{1}$$

where λ is the incident photon wavelength, λ' is the final wavelength, θ is the scattering angle, and λ_c is the Compton wavelength, 2.426 pm (NIST). The following well-known equations can then be used to find the scattered photon's energy:

$$f' = \frac{c}{\lambda'}, \tag{2}$$

where f' is the scattered frequency and c is the speed of light, and

$$E' = hf', \tag{3}$$

where E' is the energy in question and h is Planck's constant. The initial gamma ray energies are 0.6617 MeV for cesium-137 (Bowens-Rubin) and 0.3560 MeV for barium (UWM).

The incident photons scatter off the electrons at a random angle, as governed by the Klein-Nishina formula:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \lambda_c^2}{8\pi^2} \left(\frac{\lambda}{\lambda'}\right)^2 \left(\frac{\lambda}{\lambda'} + \frac{\lambda'}{\lambda} - \sin(\theta)^2\right), \tag{4}$$

where the variables are as before and α is the fine-structure constant ($\frac{1}{137.04}$). Eq. 4 gives a differential cross-section – i.e. the probability that the scattered photon will be found within an infinitesimally-small angular range – that, for our purposes, can be treated as a probability density function relating incident wavelength and scattering angle. (Final wavelength is itself related to those two by Eq. 1, written as above for the sake of readability.)

The scattered photon energy gives rise to the mass of the electron (measured according to its energy equivalent, in MeV) by the following equation (Bowens-Rubin):

$$\frac{1}{E'} - \frac{1}{E} = \frac{1}{m_e c^2}(1 - \cos(\theta)), \tag{5}$$

where E is the incident photon energy, E' is the scattered energy, and $m_e c^2$ is the mass of the electron.

Fundamental objective: To create a program that accurately models the behavior of photons undergoing Compton scattering off of electrons, demonstrated in the following ways:

- By representing photon energy as a function of scattering angle, plotted on a graph;
- And by also representing intensity as a function of scattering angle, plotted on a separate graph.

3 Methods

I made ubiquitous use of the NumPy library throughout my code for general mathematical purposes, mostly for its trigonometric functions. My results were plotted using Matplotlib, and I also used the 'random' and 'sklearn' libraries as part of my implementation of Eq. 4.

I modelled the collisions for 10000 individual photons, generating the scattering angles for each of them according to the Klein-Nishina formula (Eq. 4). The range of possible angles was bound between 0 and π, since the system's inherent symmetry would make all the angles from π to 2π (or $-\pi$ to 0) redundant to include. The $d\theta$ I chose to use was 1e-3 radians, corresponding to a range of 1000 angles. I fed this set of angles into the PDF defined by Eq. 4, generating a set of weights that was then used with the 'random' library's 'choices' function to produce random scattering angles. Intensities were found in terms of photon count, summing the total number of photons to fall within a bin of 6 $d\theta$s. The scattered energies and intensities were plotted against θ to produce the graphs displayed in the next section.

I found the value of the mass of an electron by performing linear regression on Eq. 5, the x-axis being $1 - \cos(\theta)$ and the y-axis being $1/E$. NumPy's 'polyfit' function (with degree 1) expedited this process significantly, and the resultant plots are also shown in the next section. The R^2 analysis on the intensities was performed using the 'r2_score' function from the 'sklearn' library.

Code can be found in the GitHub repository at
https://github.com/aclewis242/PHY432HonorsContract.

4 Results

4.1 Post-collision energy as a function of scattering angle

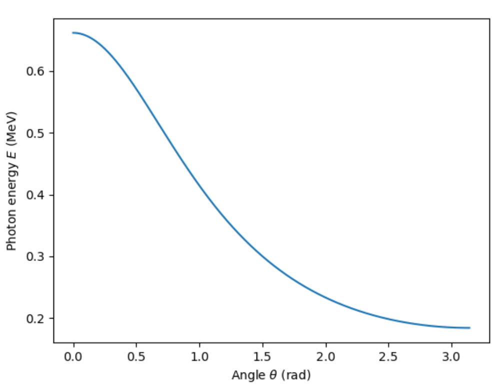

Figure 1.1, Scattering energy by angle, ^{137}Cs.

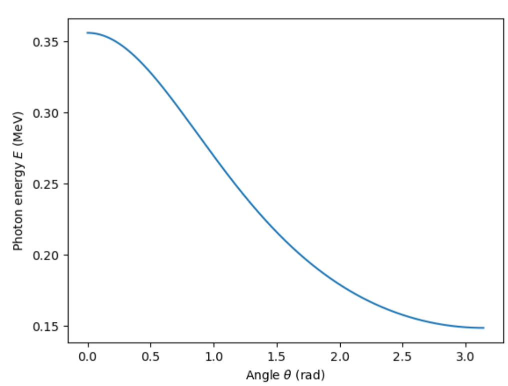

Figure 1.2, Scattering energy by angle, ^{133}Ba.

4.2 Electron mass linear regression

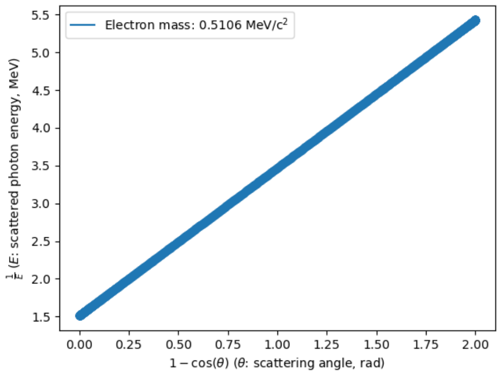

Figure 2.1, Electron mass, ^{137}Cs.

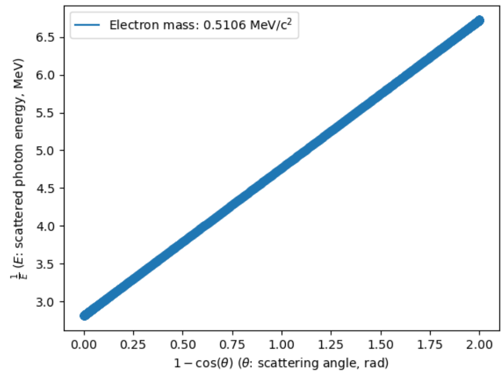

Figure 2.2, Electron mass, ^{133}Ba.

4.3 Intensity as a function of scattering angle

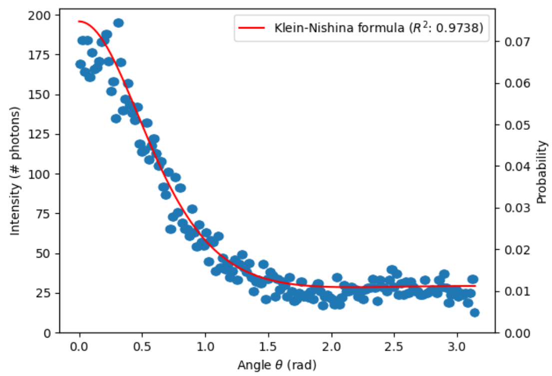

Figure 3.1, Intensity by angle, ^{137}Cs.

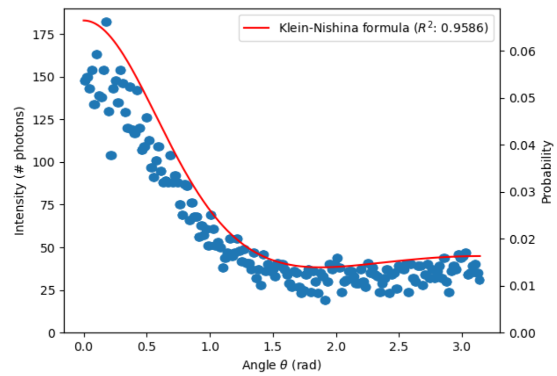

Figure 3.2, Intensity by angle, ^{133}Ba.

5 Discussion

The graphs for post-collision energy peak at $\theta = 0$ and then gently curve downwards to their minima at $\theta = \pi$. The peaks of cesium-137 and barium-133 are at 0.6617 MeV and 0.3560 MeV respectively, the energies of their incident gamma rays. This is consistent with our understanding of the physical phenomenon of Compton scattering: when the photon experiences little to no deflection by the electron, i.e. has a very low scattering angle, it transfers none of its energy to the electron and carries on largely as it had before.

The accuracy of the data can be more rigorously checked by comparing the mass of an electron found via a linear regression analysis of the data using Eq. 5 to the accepted value. I found it to be 0.5106 MeV in both cases, which is in very good agreement with its accepted value of 0.5110 MeV (NIST).

As for the intensities, the graphs my code produces show individual data-points for θ bins with their corresponding photon counts (scaled as per the left axis) and the curve of the probability density function given by the Klein-Nishina formula (scaled as per the right axis). A standard R^2 analysis comparing the two data sets gives values of $R^2 = 0.9738$ and $R^2 = 0.9586$ for cesium and barium respectively, which are both within an acceptable distance from $R^2 = 1$ (the ideal). This verifies the accuracy of my results.

Therefore, we can conclude that my model is an accurate simulation of the physical phenomenon of Compton scattering.

6 References

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4. R. Bowens-Rubin (Jan 2010), "Compton Scattering and Attenuation," *MIT Department of Physics.* Retrieved from `http://web.mit.edu/rbowru/Public/Jlab/BowensRubin_Compton.pdf`. Accessed May 2022.

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