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# Testing the Conditional CAPM

Or something like that

## Introduction

The unconditional CAPM assumes investors have time-invariant beliefs, and therefore estimates of betas and the market risk premium are constant over time. It is given by

$$E[R_i] = R_f + \beta_i\left(E[R_m] - R_f\right). \qquad (1)$$

Numerous works, including Fama and Macbeth (1973) and Fama and French (1993), have largely disproven the CAPM's ability to explain the cross section of (stock) returns. However, the CAPM may not hold in hindsight (unconditionally) over long time series, it may still hold if the betas and risk premia are estimated conditionally on information available at a given point in time. We call such a state-dependent model the conditional CAPM, and define it precisely as

$$E_t[R_i] = R_{f,t} + \beta_{i,t}\left(E_t\left[R_{m,t}\right] - R_{f,t}\right). \qquad (2)$$

This model is of great interest to asset pricing theory, but it is difficult to test as we cannot observe the full state space investors use to periodically estimate the inputs to the model. A simple test that gets around this issue of assuming a state space is that proposed by Lewellen and Nagel (2006), which suggest short window estimates of alphas and betas are good unbiased estimates of their conditional counterparts. It is has been observed betas are fairly stable over short windows, say one month or quarter, and therefore this test is justifiably valid. I replicate a similar test to this using daily and monthly CRSP and Fama-French data to estimate conditional alphas and betas. I explain in more detail the methodology, data and results of this study in the following sections.

## Methodology

I start by estimating monthly betas from daily excess returns and market risk premia. These monthly betas are merged with monthly stock returns and factor returns. We then, in each month, form 25 portfolios by doubling sorting stocks on size (lagged market cap) and betas (estimated from daily returns). The portfolios essentially differ by their quintile rank in either beta or size, and their returns are computed as lagged market cap-weighted averages. These portfolio sorts are useful for a variety of reasons in testing the conditional CAPM. Firstly, betas estimated on individual stocks, especially on small samples, can be very noisy and returns are very idiosyncratic. Portfolio returns eliminate idiosyncratic risks among individual stocks and their beta estimates are better measured. Secondly, we can test whether the CAPM explains cross sectional within each of the five size bins, and if portfolios with the same beta vary in returns across size quintiles.

We can then conduct a test of the conditional CAPM by regressing all 25 portfolio excess returns on their corresponding market risk premia for each month in the sample. This gives us a time-series of alphas, with which we can conduct the test statistic

$$t_{\alpha} = \frac{\bar{\alpha}}{\sigma_{\widehat{\alpha}}/\sqrt{T}}. \qquad (3)$$

Similarly, we can regress each portfolio's time series of excess returns on their corresponding risk premia to test the conditional CAPM. Finally, we measure the conditional alphas of the following four factor model

$$E_t[R_i] = R_{f,t} + \beta_{i,t}^{mkt}\left(E_t\left[R_{m,t}\right] - R_{f,t}\right) + \beta_{i,t}^{SMB} SMB_{i,t} + \beta_{i,t}^{HML} HML_{i,t} + \beta_{i,t}^{UMD} UMD_{i,t} \qquad (4)$$

to see whether or not other anomalies (momentum, value, etc.) can better explain the cross section of stock returns.

## Data

The dataset spans from 1926 to 2024 and comes from the CRSP daily and monthly stock files (dsf and msf), as well as from the Fama and French daily and monthly factor estimates (factors_daily and factors_monthly). We also restrict the universe to common stocks in the CRSP stock files.

## Results

The conditional alphas and beta estimated on the above-mentioned portfolio returns are plotted in **Figure 1**, while their corresponding summary statistics are given in **Table 1**. We find monthly alphas to be nearly 70 basis points on average, but can reach magnitudes of 30% and -30% in our sample. The more extreme values occur earlier in the sample prior to 1950. Looking at the conditional betas, they vary wildly from the period of time prior to 1940, but are otherwise fairly stable most years. Looking at the test statics reported in **Table 1**, we can see both the conditional alphas and betas are statistically significantly different from zero, leading me to therefore reject the conditional CAPM. Interestingly, however, I fail to reject the unconditional CAPM. In fact it seems to perform quite well (**Table 3**). Looking at the betas in panel B of **Table 2**, betas are monotonically decreasing in size within each quintile, however the sign of the difference in alphas between the highest and lowest size cut is not consistent across beta cuts.

## Conclusion

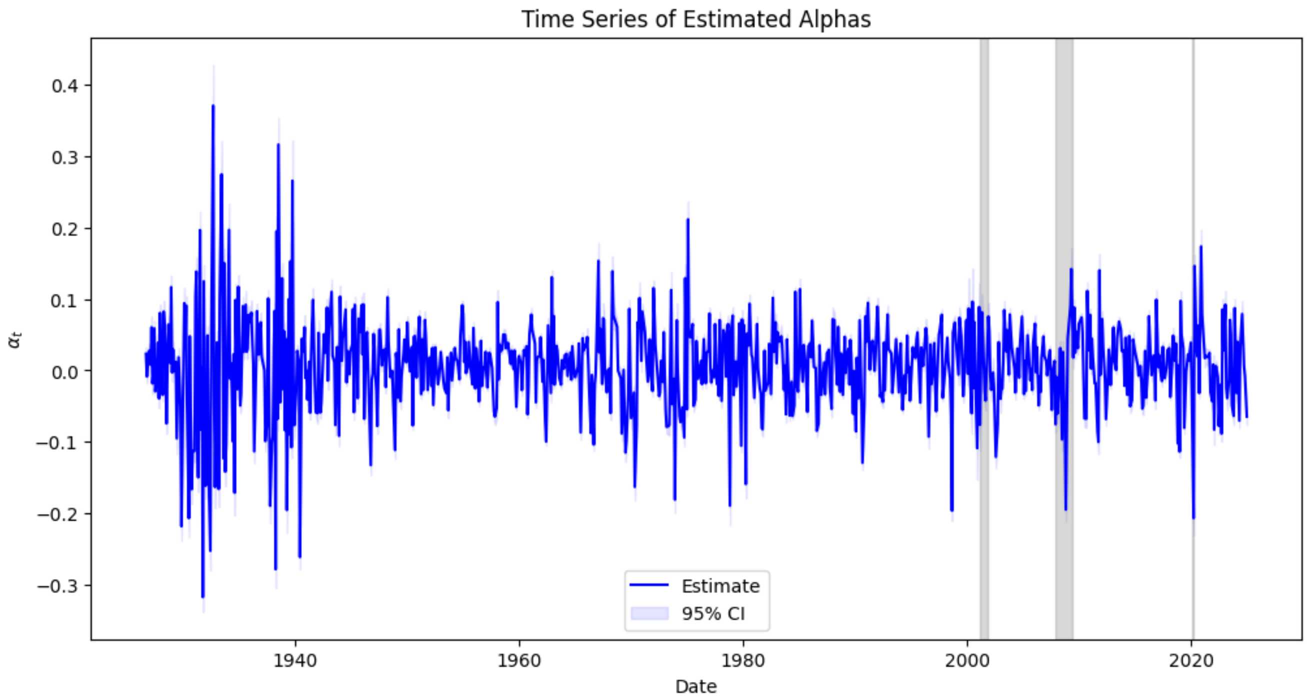
I find significant evidence to reject the conditional CAPM based on short window (monthly) regressions on CRSP stock level data. Conditional alphas are significant and on the order of 70 basis points per month.

## Figures and Tables

**Figure 1**

Conditional (monthly) alphas (above) and betas (below) estimated.

A: alphas




B: betas

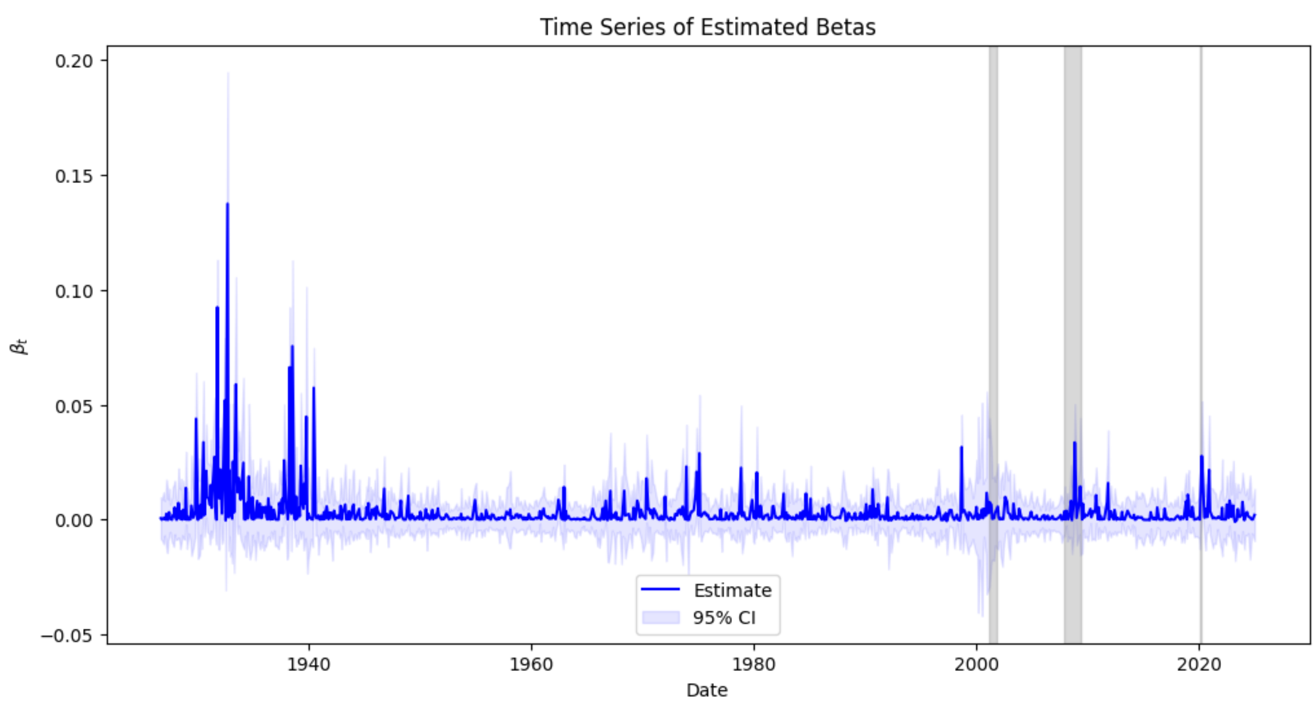


**Table 1**

Summary statistics of the conditional CAPM test.

| | alpha | beta_mkt |
|---|---|---|
| **Cond. CAPM** | | |
| count | 860.000000 | 860.000000 |
| mean | 0.006770 | 0.003433 |
| std | 0.066245 | 0.008802 |
| min | -0.317366 | -0.000975 |
| 25% | -0.027443 | 0.000231 |
| 50% | 0.012761 | 0.001010 |
| 75% | 0.043465 | 0.003240 |
| max | 0.370912 | 0.137460 |
| t_stat | 2.997128 | 11.438865 |
| std_err | 0.002259 | 0.000300 |

**Table 2**

Unconditional alphas (above) and betas (below).

**Panel A**

Unconditional alphas

| beta_rank | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **size_rank** | | | | | |
| 1 | -0.000944 | 0.003029 | 0.002068 | -0.000130 | -0.006498 |
| 2 | 0.001203 | 0.002019 | 0.000866 | -0.000569 | -0.005395 |
| 3 | 0.000765 | 0.002095 | 0.001009 | -0.000909 | -0.004098 |
| 4 | 0.000892 | 0.001055 | 0.000673 | -0.000367 | -0.004441 |
| 5 | 0.001331 | 0.000163 | 0.000139 | -0.000029 | -0.001818 |

**Panel B**

Unconditional betas

| beta_rank | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **size_rank** | | | | | |
| 1 | 1.152216 | 1.161126 | 1.306890 | 1.419863 | 1.624254 |
| 2 | 1.006845 | 1.177596 | 1.281460 | 1.482351 | 1.684898 |
| 3 | 0.896566 | 1.085169 | 1.199569 | 1.394794 | 1.698430 |
| 4 | 0.862639 | 0.973337 | 1.100399 | 1.267859 | 1.601558 |
| 5 | 0.665151 | 0.789907 | 0.895289 | 1.096482 | 1.304313 |

**Table 3**

Summary statistics of the unconditional CAPM test.

| | alpha | beta_mkt |
|---|---|---|
| **Uncond. CAPM** | | |
| count | 25.000000 | 25.000000 |
| mean | -0.000316 | 1.205158 |
| std | 0.002422 | 0.281622 |
| min | -0.006498 | 0.665151 |
| 25% | -0.000909 | 1.006845 |
| 50% | 0.000163 | 1.177596 |
| 75% | 0.001055 | 1.394794 |
| max | 0.003029 | 1.698430 |
| t_stat | -0.651790 | 21.396741 |
| std_err | 0.000484 | 0.056324 |

**Table 4**

Summary statistics of the four factor model test.

| | alpha | beta_mkt | beta_smb | beta_hml | beta_umd |
|---|---|---|---|---|---|
| count | 856.000000 | 856.000000 | 856.000000 | 856.000000 | 856.000000 |
| mean | 0.006452 | 0.003354 | 0.001212 | 0.000600 | -0.001003 |
| std | 0.064820 | 0.008025 | 0.004412 | 0.004984 | 0.007451 |
| min | -0.313062 | -0.000964 | -0.009986 | -0.016828 | -0.142259 |
| 25% | -0.027474 | 0.000231 | -0.000070 | -0.000380 | -0.000715 |
| 50% | 0.012471 | 0.001014 | 0.000221 | 0.000020 | 0.000000 |
| 75% | 0.043473 | 0.003234 | 0.001286 | 0.000659 | 0.000612 |
| max | 0.298426 | 0.101290 | 0.086214 | 0.093582 | 0.015765 |
| t_stat | 2.912366 | 12.227094 | 8.034850 | 3.522742 | -3.937280 |
| std_err | 0.002215 | 0.000274 | 0.000151 | 0.000170 | 0.000255 |

## References

Fama, Eugene F., and James D. MacBeth. “Risk, Return, and Equilibrium: Empirical Tests.” Journal of Political Economy, vol. 81, no. 3, 1973, pp. 607–36. JSTOR, http://www.jstor.org/stable/1831028. Accessed 2 May 2025.

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