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CS 325 – 400 F2019

November 11, 2019

Homework 6

1)

Software: LINDO

a) To find the shortest path from s to t in a weighted directed graph, I need to find all possible edges on the graph, so I can apply the pseudo code

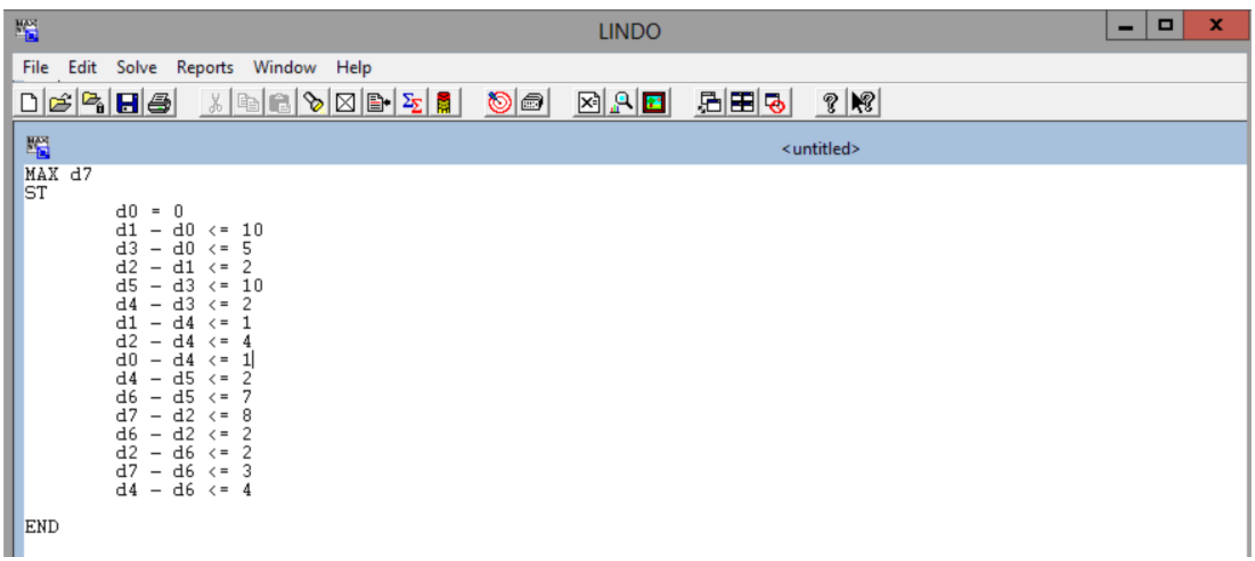
max dt

subject to

ds = 0

dv – du <= w(w, v) for all edges in the graph.

On LINDO software like a below screenshot:



Like the above screenshot, I put MAX d7 as a destination vertex in the graph as maximization problem and then ST (subject to) for expressing constraints of the finding path with bounds. For example, the d0 = 0 because vertex 0 to 0 is 0 as non-cycle. d1 – d0 <= 10 means that distance from vertex 0 to vertex 1 is 10 (weighted graph).

Therefore,

Objective function: Maximize d7

Constraints:

Subject to

d0 = 0

d1 - d0 <= 10

d3 - d0 <= 5

d2 - d1 <= 2

d5 - d3 <= 10

d4 - d3 <= 2

d1 - d4 <= 1

d2 - d4 <= 4

d0 - d4 <= 1

d4 - d5 <= 2

d6 - d5 <= 7

d7 - d2 <= 8

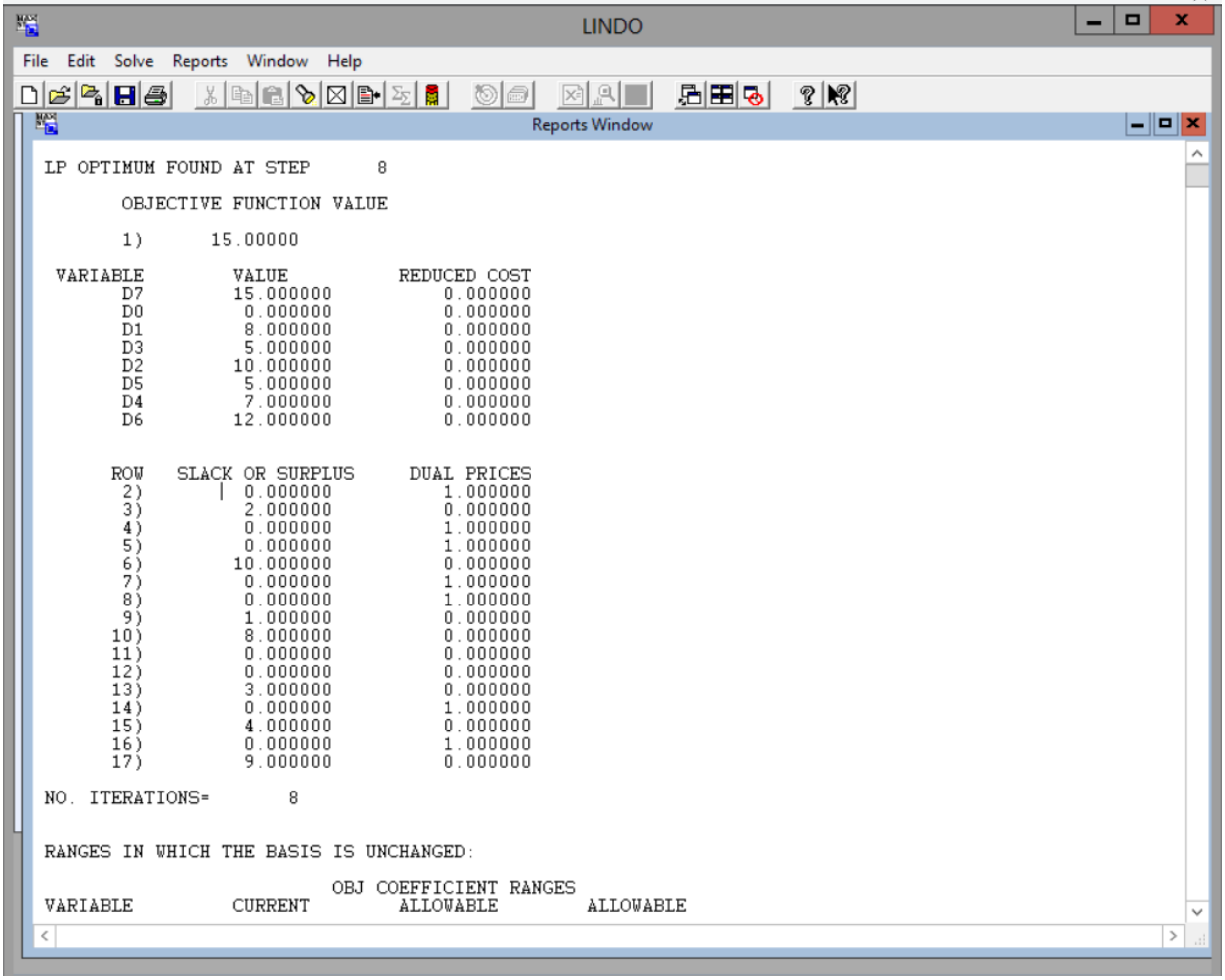
d6 - d2 <= 2

d2 - d6 <= 2

d7 - d6 <= 3

d4 - d6 <= 4

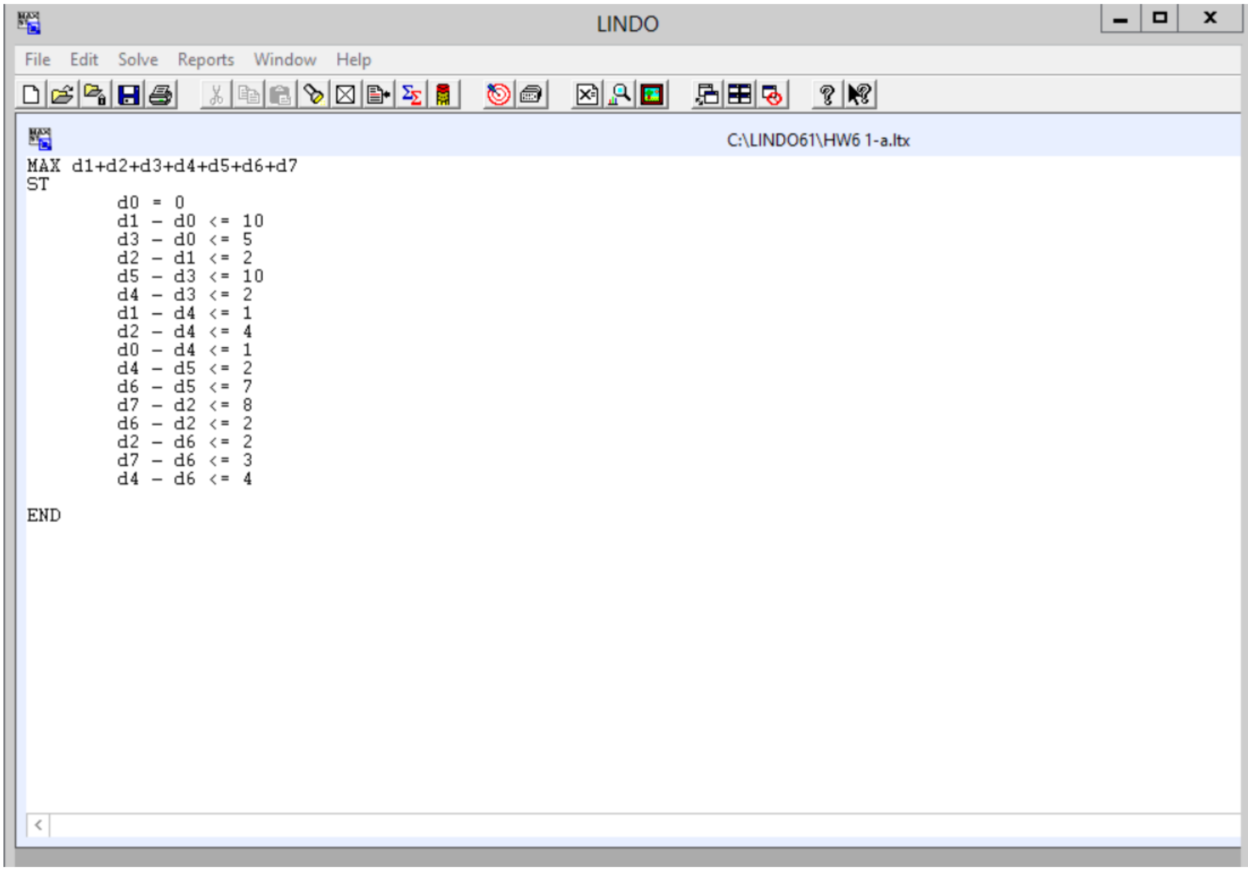
Output log:




```
LP OPTIMUM FOUND AT STEP      8

       OBJECTIVE FUNCTION VALUE

       1)      15.00000

 VARIABLE        VALUE          REDUCED COST
       D7        15.000000          0.000000
       D0         0.000000          0.000000
       D1         8.000000          0.000000
       D3         5.000000          0.000000
       D2        10.000000          0.000000
       D5         5.000000          0.000000
       D4         7.000000          0.000000
       D6        12.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          1.000000
       3)         2.000000          0.000000
       4)         0.000000          1.000000
       5)         0.000000          1.000000
       6)        10.000000          0.000000
       7)         0.000000          1.000000
       8)         0.000000          1.000000
       9)         1.000000          0.000000
      10)         8.000000          0.000000
      11)         0.000000          0.000000
      12)         0.000000          0.000000
      13)         3.000000          0.000000
      14)         0.000000          1.000000
      15)         4.000000          0.000000
      16)         0.000000          1.000000
      17)         9.000000          0.000000

NO. ITERATIONS=       8


RANGES IN WHICH THE BASIS IS UNCHANGED:

                             OBJ COEFFICIENT RANGES
VARIABLE          CURRENT        ALLOWABLE         ALLOWABLE
```

The objective function value is 15. Therefore, the distance of the shortest path from vertex 0 to 7 is 15. (path: 0 > 3 > 4 > 1 > 2 > 6 > 7)

b) To find the shortest paths from vertex 0 to all other vertices in LINDO, I should change the objective function of the linear programming model like below screenshot:

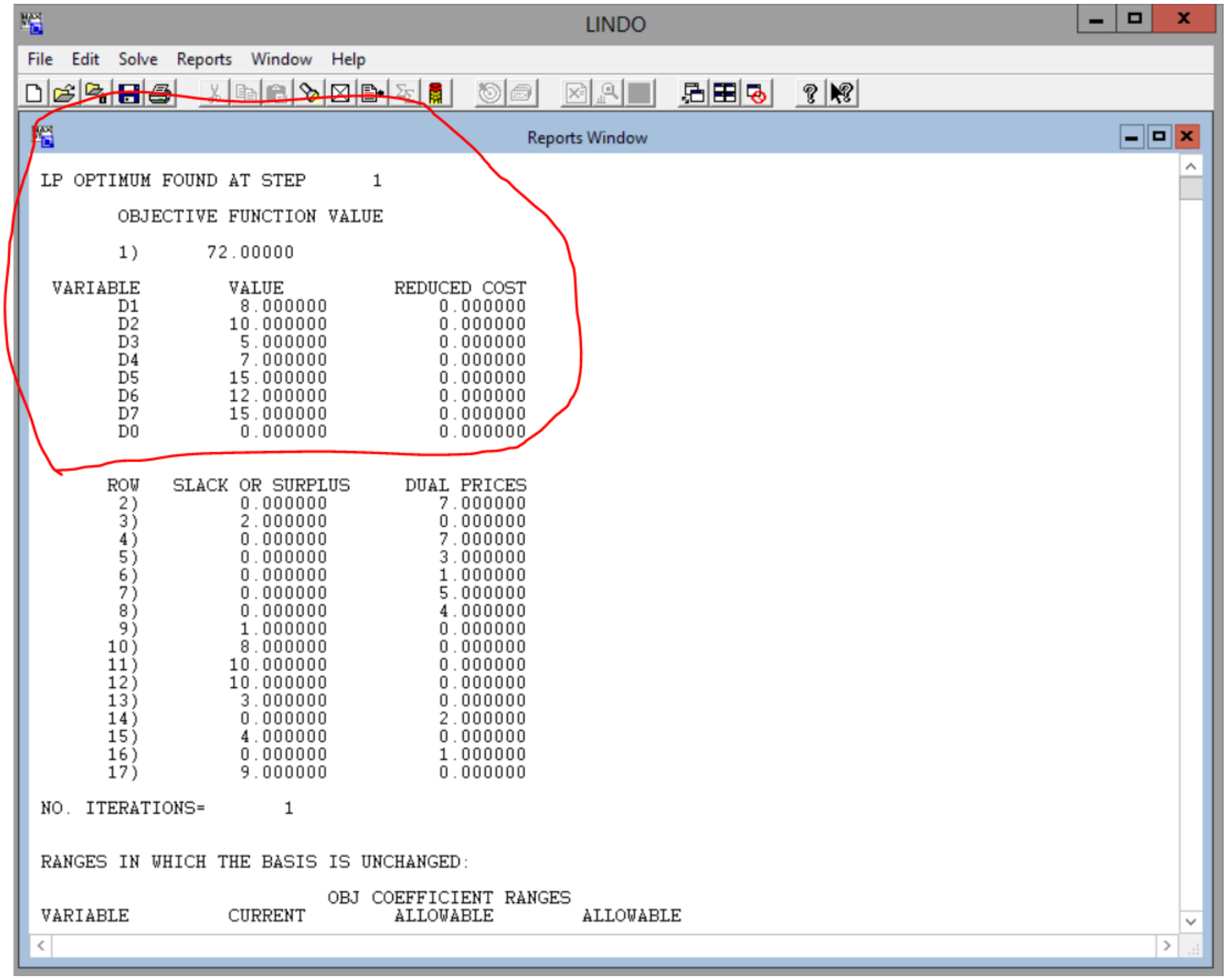


Objective function: Maximize d1+d2+d3+d4+d5+d6+d7

Constraints: same with Q1 a)

Because it maximizes distances that are between vertex 0 and all other vertices.

Output log:




```
LP OPTIMUM FOUND AT STEP      1

       OBJECTIVE FUNCTION VALUE

       1)      72.00000

 VARIABLE        VALUE          REDUCED COST
       D1         8.000000          0.000000
       D2        10.000000          0.000000
       D3         5.000000          0.000000
       D4         7.000000          0.000000
       D5        15.000000          0.000000
       D6        12.000000          0.000000
       D7        15.000000          0.000000
       D0         0.000000          0.000000

      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          7.000000
       3)         2.000000          0.000000
       4)         0.000000          7.000000
       5)         0.000000          3.000000
       6)         0.000000          1.000000
       7)         0.000000          5.000000
       8)         0.000000          4.000000
       9)         1.000000          0.000000
      10)         8.000000          0.000000
      11)        10.000000          0.000000
      12)        10.000000          0.000000
      13)         3.000000          0.000000
      14)         0.000000          2.000000
      15)         4.000000          0.000000
      16)         0.000000          1.000000
      17)         9.000000          0.000000

NO. ITERATIONS=       1


RANGES IN WHICH THE BASIS IS UNCHANGED:

                          OBJ COEFFICIENT RANGES
VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
```

From above screenshot, I can check the shortest paths from vertex 0 to others on “objective function value”

Vertex 0 to 1: 8

Vertex 0 to 2: 10

Vertex 0 to 3: 5

Vertex 0 to 4: 7

Vertex 0 to 5: 15

Vertex 0 to 6: 12

Vertex 0 to 7: 15

2) As the goal is to maximize profit (profit per tie = selling price – labor cost (0.75) – material cost), I can formulate the problem as a linear program like this:

Objective function: Maximize (6.7 – 0.75 – (20 * 0.125)) s + (3.55- 0.75 – (6 * 0.08)) p + (4.31 – 0.75 – (0.05 * 6 + 0.05 * 9)) b + (4.81 – 0.74 – (0.03 * 6 + 0.07 * 9)) c

=> Maximize 3.45s + 2.32p + 2.81b + 3.25c

Subject to

#Capacity constraints

0.125s <= 1000 (silk)

0.08p + 0.05b + 0.03c <= 2000 (polyester)

0.05b + 0.07c <= 1250 (cotton)

#Production capacity constraints

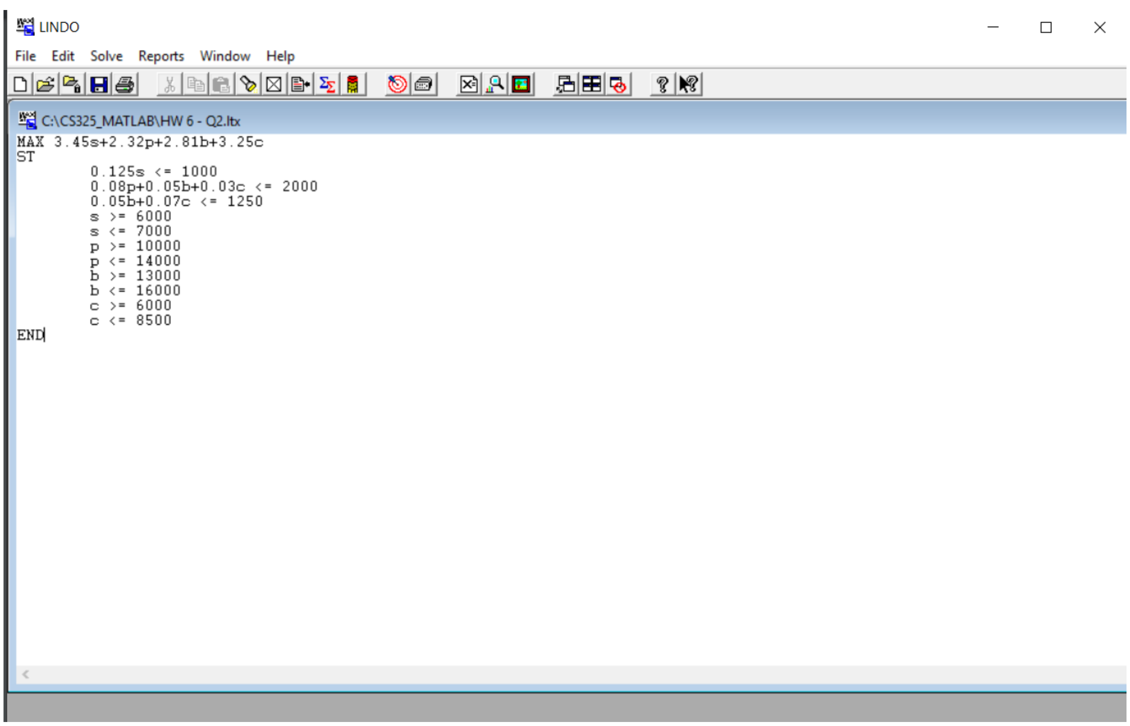
6000 <= s <= 7000 (silk)

10000 <= p <= 14000 (polyester)

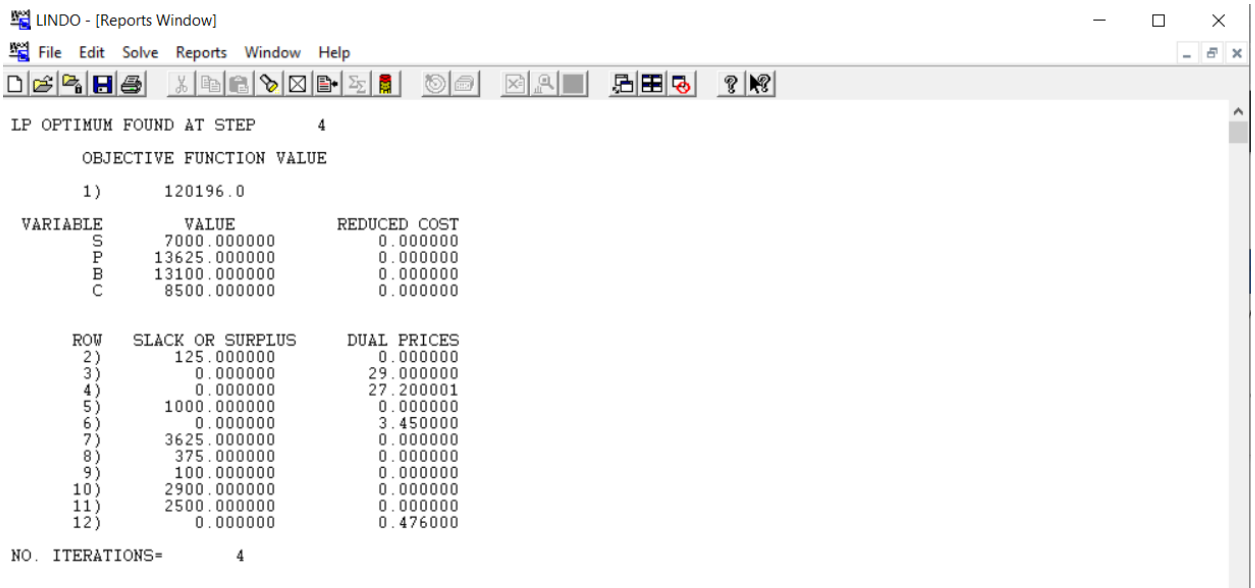
13000 <= b <= 16000 (blend 1)

6000 <= c <= 8500 (blend 2)

Implement on LINDO software:



Output log:




```
 LP OPTIMUM FOUND AT STEP      4

        OBJECTIVE FUNCTION VALUE

        1)      120196.0

  VARIABLE        VALUE          REDUCED COST
         S      7000.000000          0.000000
         P     13625.000000          0.000000
         B     13100.000000          0.000000
         C      8500.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)       125.000000          0.000000
        3)         0.000000         29.000000
        4)         0.000000         27.200001
        5)      1000.000000          0.000000
        6)         0.000000          3.450000
        7)      3625.000000          0.000000
        8)       375.000000          0.000000
        9)       100.000000          0.000000
       10)      2900.000000          0.000000
       11)      2500.000000          0.000000
       12)         0.000000          0.476000

 NO. ITERATIONS=       4
```

As a result of compiling the formulated problem on LINDO, the maximum profit is 120,196 $ by producing 7000 silk ties, 13625 polyester tiles, 13100 blend 1 ties, and 8500 blend 2 ties.

3)

Part A)

i) To formulate the problem (minimize calories) as a linear program with an objective function and all constraints, I should check the objective condition on the problem description and all requirements on the script:

When tomato = t, lettuce = l, spinach = s, carrot = c, sunflower seeds = ss, smoked tofu = st, chickpeas = cp, and oil = o, then

#Objective function

Goal: minimize calories

→ Minimize 21t + 16l + 40s + 41c + 585ss + 120st + 164cp + 884o

Each salad must contain:

- At least 15 grams of protein
- At least 2 and at most 8 grams of fat
- At least 4 grams of carbohydrates
- At most 200 milligrams of sodium

• At least 40% leafy greens by mass.

Based on the nutrition table for each salad ingredients,

Constraints can be described like this:

#At least 15 grams of protein

$\rightarrow 0.85t + 1.62l + 2.86s + 0.93c + 23.4ss + 16st + 9cp + 0o >= 15$

#At least 2 and at most 8 grams of fat

$\rightarrow 2 <= 0.33t + 0.2l + 0.39s + 0.24c + 48.7ss + 5st + 2.6cp + 100o <= 8$

#At least 4 grams of carbohydrates

$\rightarrow 4.64t + 2.37l + 3.63s + 9.58c + 15ss + 3st + 27cp +0o >= 4$

#At most 200 milligrams of sodium

$\rightarrow 9t + 28l + 65s + 69c + 3.8ss + 120st + 78cp + 0o <= 200$

#At least 40% leafy greens (lettuce and spinach) by mass

$l+s / (t + l + s + c + ss +st + cp +o) * 100 >= 40$

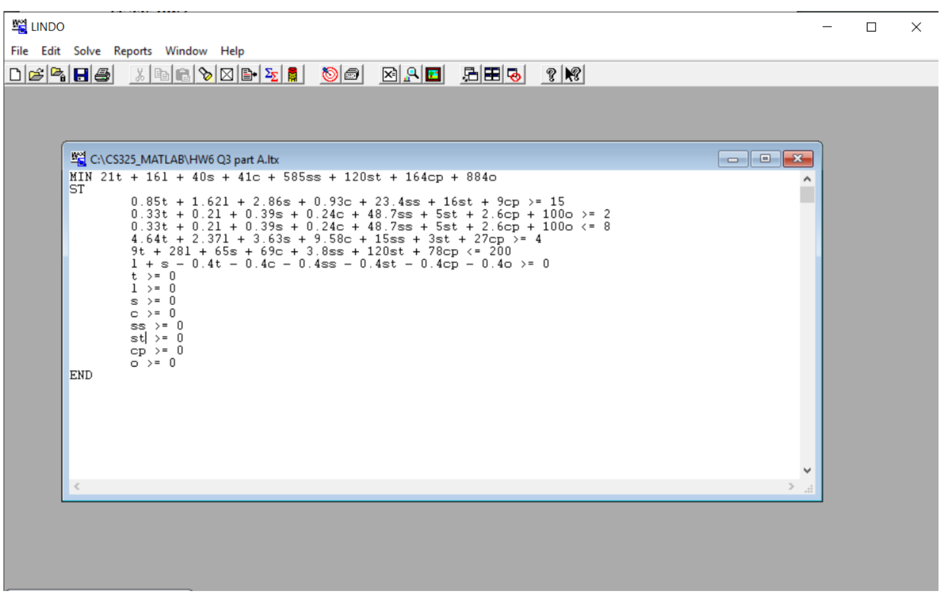
$\rightarrow l + s – 0.4(t + c+ ss + st + cp + o) >= 0$

Finally, all values of ingredients should be positive integer or 0:

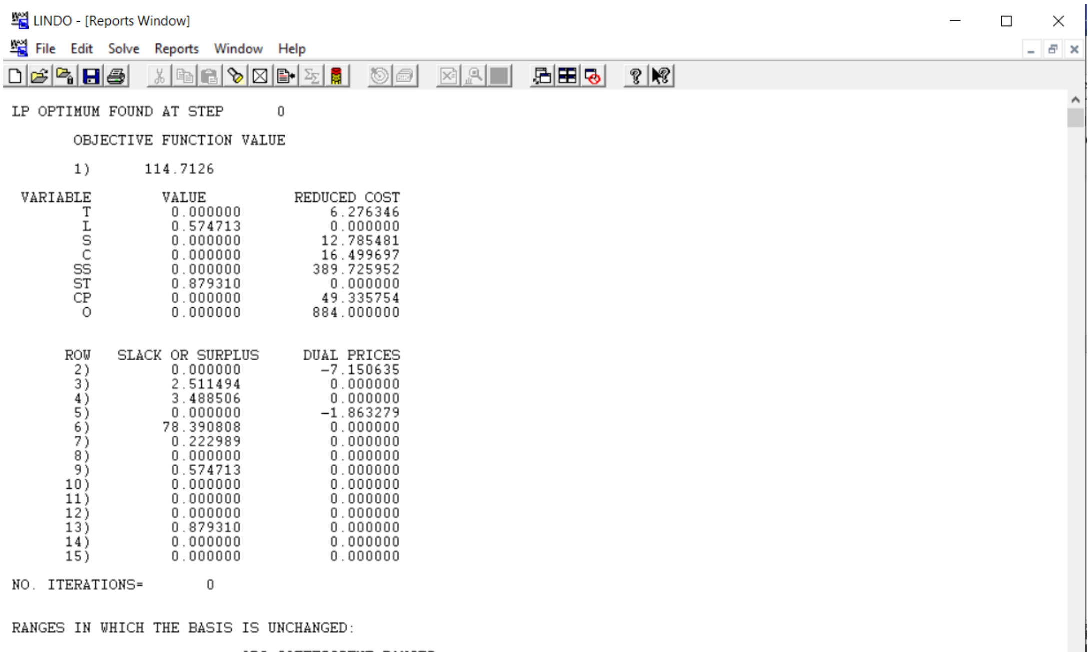
$t, l, s, c, ss, st, cp, o >= 0$

ii)

Implement i)'s formulated problem on LINDO software:



Output log:



```
LINDO - [Reports Window]
File  Edit  Solve  Reports  Window  Help

 LP OPTIMUM FOUND AT STEP      0

        OBJECTIVE FUNCTION VALUE

        1)      114.7126

  VARIABLE        VALUE          REDUCED COST
         T         0.000000          6.276346
         L         0.574713          0.000000
         S         0.000000         12.785481
         C         0.000000         16.499697
        SS         0.000000        389.725952
        ST         0.879310          0.000000
        CP         0.000000         49.335754
         O         0.000000        884.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -7.150635
        3)         2.511494          0.000000
        4)         3.488506          0.000000
        5)         0.000000         -1.863279
        6)        78.390808          0.000000
        7)         0.222989          0.000000
        8)         0.000000          0.000000
        9)         0.574713          0.000000
       10)         0.000000          0.000000
       11)         0.000000          0.000000
       12)         0.000000          0.000000
       13)         0.879310          0.000000
       14)         0.000000          0.000000
       15)         0.000000          0.000000

 NO. ITERATIONS=       0

 RANGES IN WHICH THE BASIS IS UNCHANGED:
```

As a result of compiling the formulated problem on LINDO, the minimum calories are 114.7126 calories by 57.4713 grams of lettuce and 87.931 grams of smoked tofu.

iii) The cost of low calories salad is (lettuce, 0.75 $ * (57.4713/100)) + (smoked tofu, 2.15$ * (87.931/100)) = 2.32155125 $

Part B)

i) Like part A, I can formulate the above problem which minimizes cost like this:

When tomato = t, lettuce = l, spinach = s, carrot = c, sunflower seeds = ss, smoked tofu = st, chickpeas = cp, and oil = o, then

#Objective function

Goal: minimize cost for salad

→ Minimize t + 0.75l + 0.5s + 0.5c + 0.45ss + 2.15st + 0.95cp + 2o

Based on the nutrition table for each salad ingredients,

Constraints can be described like this:

#At least 15 grams of protein

→ 0.85t + 1.62l + 2.86s + 0.93c + 23.4ss + 16st + 9cp + 0o >= 15

#At least 2 and at most 8 grams of fat

→ 2 <= 0.33t + 0.2l + 0.39s + 0.24c + 48.7ss + 5st + 2.6cp + 100o <= 8

#At least 4 grams of carbohydrates

→ 4.64t + 2.37l + 3.63s + 9.58c + 15ss + 3st + 27cp +0o >= 4

#At most 200 milligrams of sodium

→ 9t + 28l + 65s + 69c + 3.8ss + 120st + 78cp + 0o <= 200

#At least 40% leafy greens (lettuce and spinach) by mass

l+s / (t + l + s + c + ss +st + cp +o) * 100 >= 40

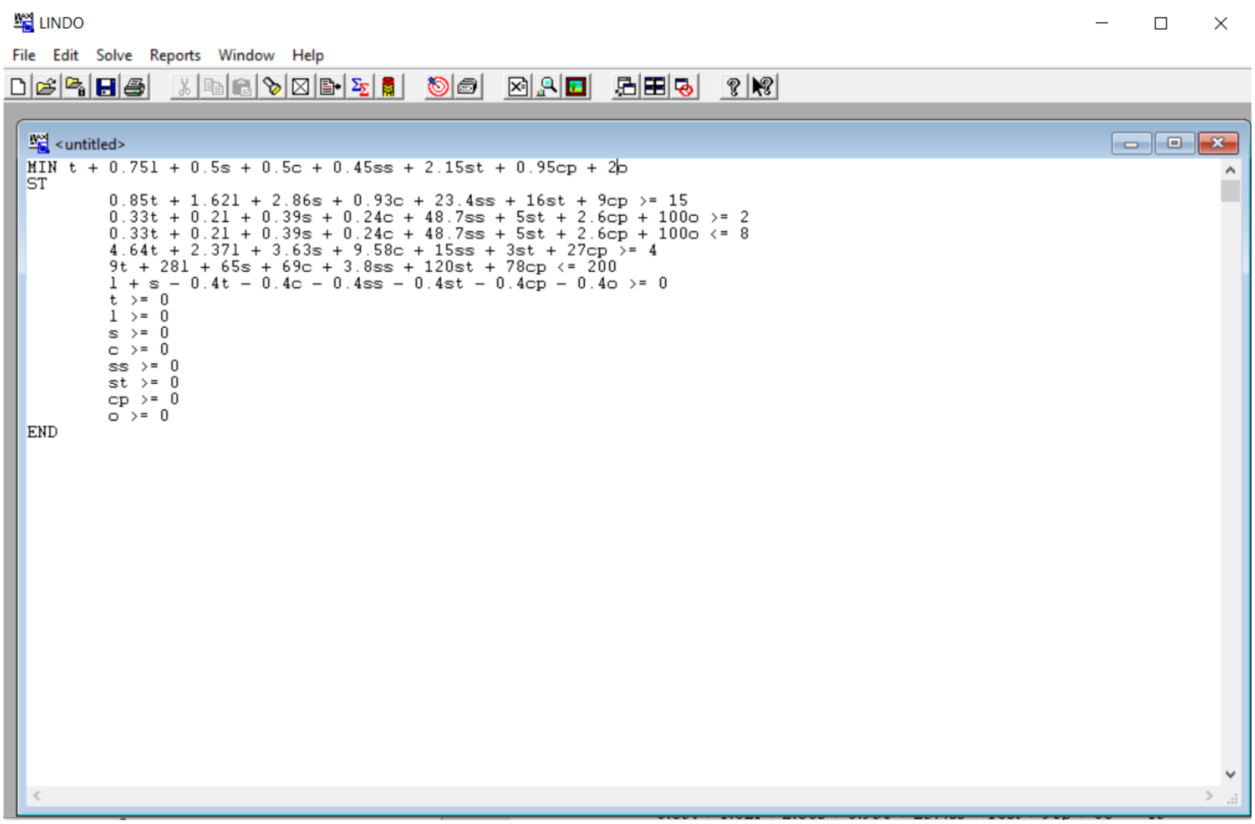
→ l + s – 0.4(t + c+ ss + st + cp + o) >= 0

Finally, all values of ingredients should be positive integer or 0:

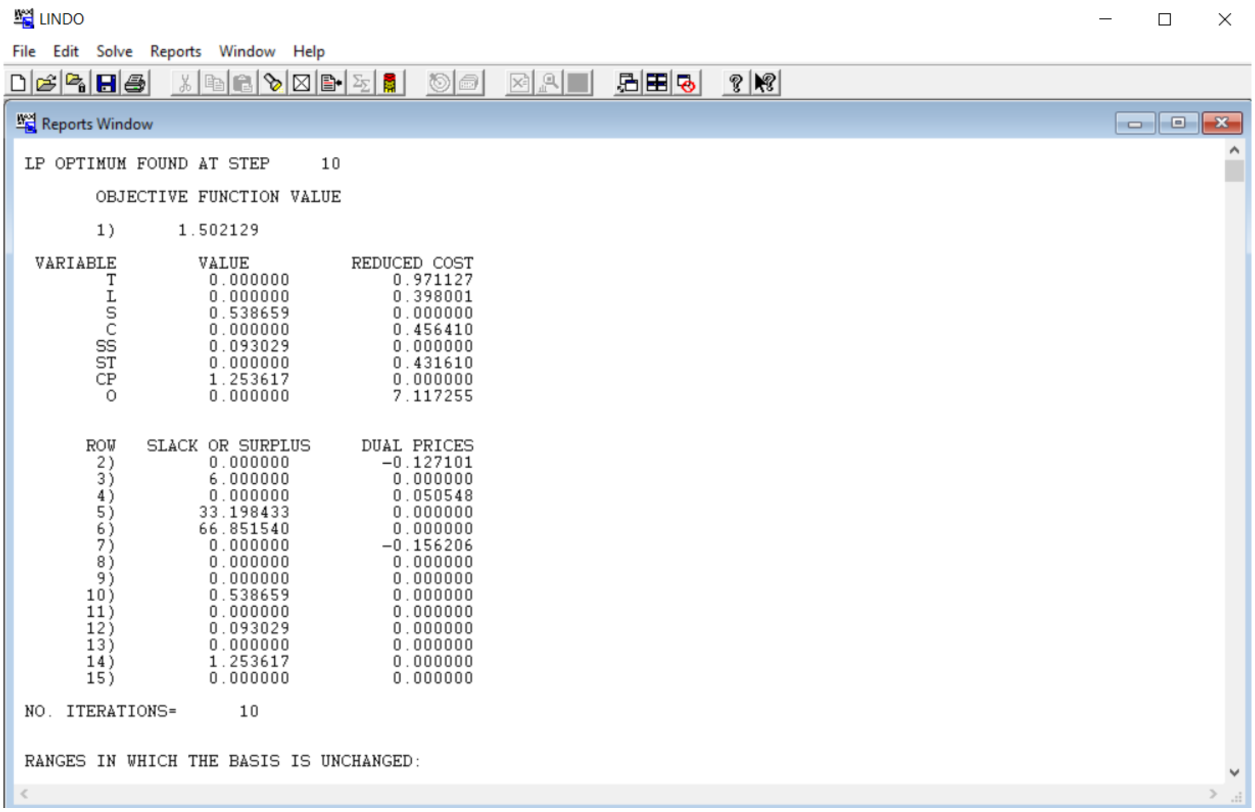
t, l, s, c, ss, st, cp, o >= 0

ii)

Implement i)'s formulated problem on LINDO software:



Output log:



```
LP OPTIMUM FOUND AT STEP     10

       OBJECTIVE FUNCTION VALUE

       1)      1.502129

 VARIABLE        VALUE          REDUCED COST
        T         0.000000          0.971127
        L         0.000000          0.398001
        S         0.538659          0.000000
        C         0.000000          0.456410
       SS         0.093029          0.000000
       ST         0.000000          0.431610
       CP         1.253617          0.000000
        O         0.000000          7.117255


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000         -0.127101
       3)         6.000000          0.000000
       4)         0.000000          0.050548
       5)        33.198433          0.000000
       6)        66.851540          0.000000
       7)         0.000000         -0.156206
       8)         0.000000          0.000000
       9)         0.000000          0.000000
      10)         0.538659          0.000000
      11)         0.000000          0.000000
      12)         0.093029          0.000000
      13)         0.000000          0.000000
      14)         1.253617          0.000000
      15)         0.000000          0.000000

NO. ITERATIONS=      10


RANGES IN WHICH THE BASIS IS UNCHANGED:
```

As a result of compiling the formulated problem on LINDO, the minimum cost for salad is 1.502129 $ by 53.8659 grams of spinach, 9.3029 grams of sunflower seeds, and 125.3617 grams of chickpeas.

iii) The calories of minimum cost salad can be calculated like this:

(spinach, 40 * 0.538659) + (sunflower seeds, 585 * 0.093029) + (chickpeas, 164 * 1.253617) =

281.5611513 calories

4)

At first, to solve this problem, I should formulate the problem as a linear program with an objective function and all constraints.

From the problem prompt, I can find the objective function indicator (goal sentences) is “What are the optimal shipping routes and minimum cost?”. Therefore, I can set up the objective function like this:

When all edges of shipping costs can be described like below:

#Edges of shipping costs

P1 to W1 = $10 pw11, P1 to W2 = $15 pw12, P1 to W3 = X,

P2 to W1 = $11 pw21, P2 to W2 = $8 pw22, P2 to W3 = X,

P3 to W1 = $13 pw31, P3 to W2 = $8 pw32, P3 to W3 = $9 pw33

P4 to W1 = X, P4 to W2 = $14 pw42, P4 to W3 = $8 pw43

W1 to R1 = $5 wr11, W1 to R2 = $6 wr12, W1 to R3 = $7 wr13, W1 to R4 = $10 wr14,

W2 to R3 = $12 wr23, W2 to R4 = $8 wr24, W2 to R5 = $10 wr25, W2 to R6 = $14 wr26

W3 to R4 = $14 wr34, W3 to R5 = $12 wr35, W3 to R6 = $12 wr36, W3 to R7 = $6 wr37

#Objective function

Minimize 10pw11 + 15pw12 + 11pw21 + 8pw22 + 13pw31 + 8pw32 + 9pw33 + 14pw42 + 8pw43 + 5wr11 + 6wr12 + 7wr13 + 10wr14 + 12wr23 + 8wr24 + 10wr25 + 14wr26 + 14wr34 + 12wr35 + 12wr36 + 6wr37

To formulate all constraints, I should check the supply and demand table between plants and retailers.

From supply table:

#Supply constraints

P1: P1 to W1 + P1 to W2 <= 150

P2: P2 to W1 + P2 to W2 <= 450

P3: P3 to W1 + P3 to W2 + P3 to W3 <= 250

P4: P4 to W2 + P4 to W3 <= 150

From demand table:

#Demand constraints

W1 to R1 >= 100 as demand, so it can be described like at least 100

W1 to R2 >= 150

W1 to R3 + W2 to R3 >= 100

W1 to R4 + W2 to R4 + W3 to R4 >= 200

W2 to R5 + W3 to R5 >= 200

W2 to R6 + W3 to R6 >= 150

W3 to R7 >= 100

Lastly, all flowers in the refrigerators should be shipped-in to the warehouse as much or same as much the amount of flowers which are shipped-out to the retailers.

Therefore, I can formulate constraints of leftover flower amount like this:

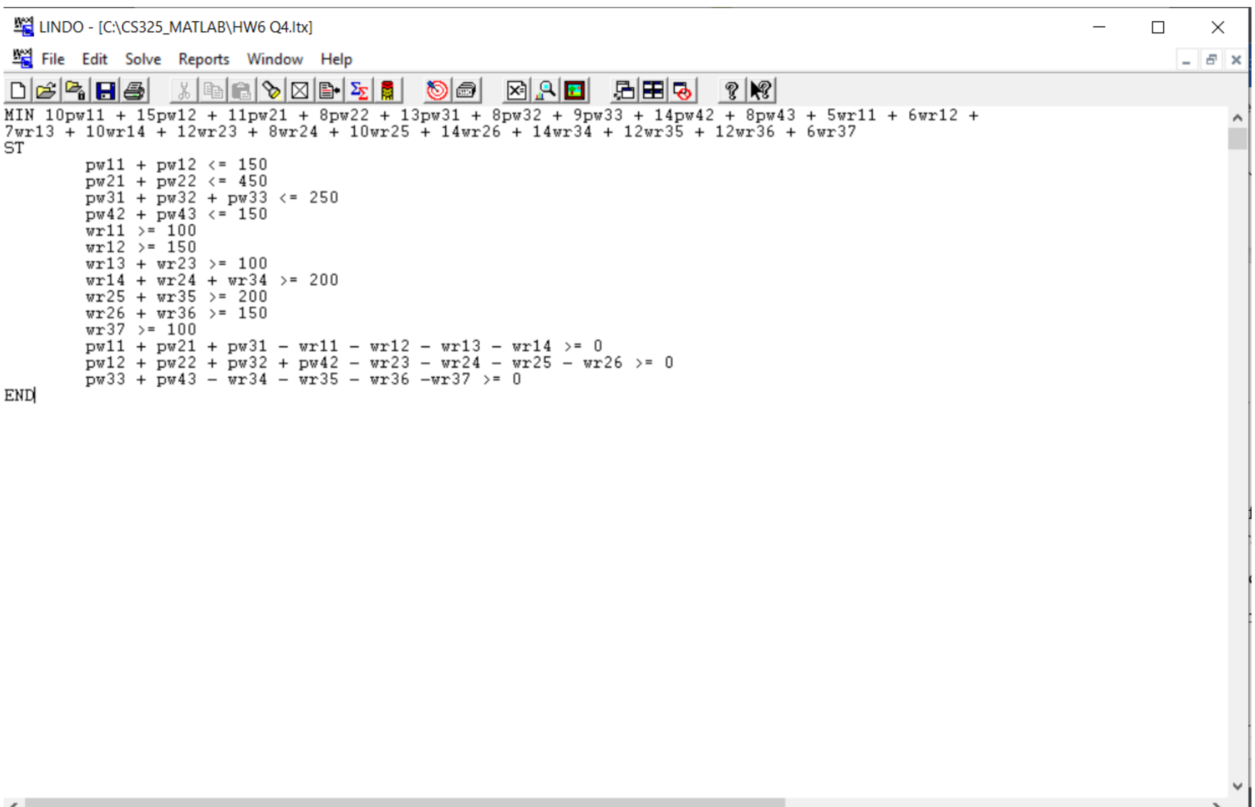
#Leftover flowers constraints

P1 to W1 + P2 to W1 + P3 to W1 – (W1 to R1, R2, R3, R4) >= 0

P1 to W2 + P2 to W2 + P3 to W2 + P4 to W2 – (W2 to R3, R4, R5, R6) >= 0

P3 to W3 + P4 to W3 – (W3 to R4, R5, R6, R7) >= 0

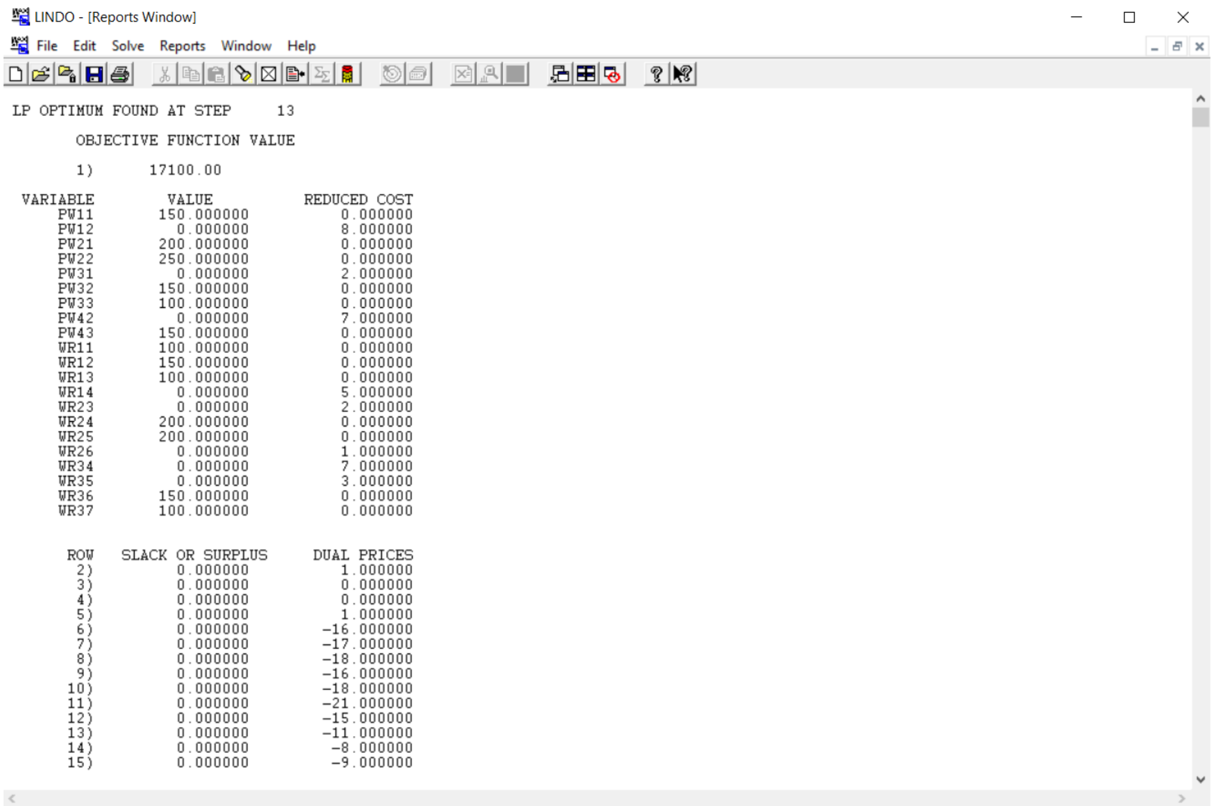
Implement of formulated problem on LINDO software:


LINDO - [C:\CS325_MATLAB\HW6 Q4.ltx]

File Edit Solve Reports Window Help

```
MIN 10pw11 + 15pw12 + 11pw21 + 8pw22 + 13pw31 + 8pw32 + 9pw33 + 14pw42 + 8pw43 + 5wr11 + 6wr12 +
7wr13 + 10wr14 + 12wr23 + 8wr24 + 10wr25 + 14wr26 + 14wr34 + 12wr35 + 12wr36 + 6wr37
ST
        pw11 + pw12 <= 150
        pw21 + pw22 <= 450
        pw31 + pw32 + pw33 <= 250
        pw42 + pw43 <= 150
        wr11 >= 100
        wr12 >= 150
        wr13 + wr23 >= 100
        wr14 + wr24 + wr34 >= 200
        wr25 + wr35 >= 200
        wr26 + wr36 >= 150
        wr37 >= 100
        pw11 + pw21 + pw31 - wr11 - wr12 - wr13 - wr14 >= 0
        pw12 + pw22 + pw32 + pw42 - wr23 - wr24 - wr25 - wr26 >= 0
        pw33 + pw43 - wr34 - wr35 - wr36 -wr37 >= 0
END
```

Output log:




```
 LP OPTIMUM FOUND AT STEP     13

        OBJECTIVE FUNCTION VALUE

        1)      17100.00

  VARIABLE        VALUE          REDUCED COST
      PW11       150.000000          0.000000
      PW12         0.000000          8.000000
      PW21       200.000000          0.000000
      PW22       250.000000          0.000000
      PW31         0.000000          2.000000
      PW32       150.000000          0.000000
      PW33       100.000000          0.000000
      PW42         0.000000          7.000000
      PW43       150.000000          0.000000
      WR11       100.000000          0.000000
      WR12       150.000000          0.000000
      WR13       100.000000          0.000000
      WR14         0.000000          5.000000
      WR23         0.000000          2.000000
      WR24       200.000000          0.000000
      WR25       200.000000          0.000000
      WR26         0.000000          1.000000
      WR34         0.000000          7.000000
      WR35         0.000000          3.000000
      WR36       150.000000          0.000000
      WR37       100.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          1.000000
        3)         0.000000          0.000000
        4)         0.000000          0.000000
        5)         0.000000          1.000000
        6)         0.000000        -16.000000
        7)         0.000000        -17.000000
        8)         0.000000        -18.000000
        9)         0.000000        -16.000000
       10)         0.000000        -18.000000
       11)         0.000000        -21.000000
       12)         0.000000        -15.000000
       13)         0.000000        -11.000000
       14)         0.000000         -8.000000
       15)         0.000000         -9.000000
```

As the result of compiling the formulated problem on LINDO, the minimum cost of shipping is 17100 $ and the optimal shipping routes are

shipped-in to Warehouse 1

P1 to W1: 150

P2 to W1: 200

shipped out from Warehouse 1

W1 to R1: 100

W1 to R2: 150

W1 to R3: 100

shipped-in to Warehouse 2

P2 to W2: 250

P3 to W2: 150

shipped out from Warehouse 2

W2 to R4: 200

W2 to R5: 200

shipped-in to Warehouse 3

P3 to W3: 100

P4 to W3: 150

shipped out from Warehouse 3

W3 to R6: 150

W3 to R7: 100