CS 325 - Homework 7

1. *(6 pts)* Let X and Y be two decision problems. Suppose we know that X reduces to Y in polynomial time. Which of the following can we infer? Explain.

   a. If Y is NP-complete then so is X.

   b. If X is NP-complete then so is Y.

   c. If Y is NP-complete and X is in NP then X is NP-complete.

   d. If X is NP-complete and Y is in NP then Y is NP-complete.

   e. If X is in P, then Y is in P.

   f. If Y is in P, then X is in P.

2. *(4 pts)* Consider the problem COMPOSITE: given an integer *y*, does *y* have any factors other than one and itself? For this exercise, you may assume that COMPOSITE is in NP, and you will be comparing it to the well-known NP-complete problem SUBSET-SUM: given a set S of *n* integers and an integer target *t*, is there a subset of *S* whose sum is exactly *t*? Clearly explain whether or not each of the following statements follows from that fact that COMPOSITE is in NP and SUBSET-SUM is NP-complete:

   a. SUBSET-SUM ≤p COMPOSITE.

   b. If there is an $O(n^3)$ algorithm for SUBSET-SUM, then there is a polynomial time algorithm for COMPOSITE.

   c. If there is a polynomial algorithm for COMPOSITE, then P = NP.

   d. If P $\neq$ NP, then ***no*** problem in NP can be solved in polynomial time.

3. *(8 pts)* A Hamiltonian path in a graph is a simple path that visits every vertex exactly once. Prove that HAM-PATH = { (G, u, v): there is a Hamiltonian path from u to v in G } is NP-complete. You may use the fact that HAM-CYCLE is NP-complete.

4. *(12 pts)* K-COLOR. Given a graph G = (V,E), a k-coloring is a function c: V -> {1, 2, … , k} such that c(u) $\neq$ c(v) for every edge (u,v) $\in$ E. In other words the number 1, 2, .., k represent the k colors and adjacent vertices must have different colors. The decision problem K-COLOR asks if a graph can be colored with at most K colors.

   a. The 2-COLOR decision problem is in P. Describe an efficient algorithm to determine if a graph has a 2-coloring. What is the running time of your algorithm?

   b. It is known that the 3-COLOR decision problem is NP-complete by using a reduction from SAT. Use the fact that 3-COLOR is NP-complete to prove that 4-COLOR is NP-complete.