MATH 2413 Differential Calculus

Take Home Quiz 4

Due : 04/30, Wednesday, 11:59 pm

Term : Spring 2025

[First Name] [Last Name] [Net ID] [Lab Section]

Instructions:

1. Print this document, complete your solutions in the spaces provided, and scan the completed work. Submit the scanned document as a single PDF file on eLearning. Alternatively, you may download this PDF and write your solutions using a writing tablet. However, the submission must still be in a single PDF file.
2. To receive full credit, you must clearly show all your work. Correct answers provided without adequate supporting work will result in no credit or reduced credit.
3. This is a take-home quiz. Your work must be entirely your own. Collaboration with other students or receiving assistance from any other individual or source is strictly prohibited. The use of AI tools is also not permitted.

1. Evaluate the following limits using L'Hôpital's Rule, if applicable:

   a. $\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x^3}$

   b. $\lim_{x\to 1} \frac{c^{1-x} - 1}{1 - x}$ where c is a constant and $c > 1$

   c. $\lim_{x\to \pi} (x - \pi)\cot x$

   d. $\lim_{x\to 0^+} (\sin\ x)^{\tan(x)}$

2. Consider the function $f(x) = \frac{x+1}{\sqrt{x^2+1}}$
    a. Find any intercepts.
    b. Determine the horizontal and vertical asymptotes, if any.
    c. Find all critical points.
    d. Determine the intervals where $f(x)$ is increasing and decreasing.
    e. Identify all local extrema using the First Derivative Test.
    f. Find all inflection points and determine concavity intervals (concave up and down)
    g. Sketch the graph based on your findings.

3. For what values of the constants a and b is (1,3) a point of inflection of the curve $y = ax^3 + bx^2$?

4. The revenue function of a company is given by $R(x) = 50x - x^2$, and the cost function is $C(x) = 20x + 100$. Find the number of items x that will maximize the company's profit.

5. A company manufactures a cylindrical can with a fixed volume of 200 cm^3. The cost of the material for the sides is \$0.02 per cm^2, and the cost for the top and bottom is \$0.03 per cm^2. What should the radius and height be to minimize the cost of materials?

6. A right circular cylinder is inscribed in a cone with height h and base radius r. Find the largest possible volume of such a cylinder.

7. Find f such that.

a) $f''(\mathrm{x}) = 24x^2 + 6x + 4, \quad f(0) = 3,\ f'(0) = 1$

b) $f'(\mathrm{x}) = 4 - 3(1 + x^2)^{-1},\ f(1) = 0$

8. Approximate the area under the graph of $f(x) = x + 3\ln x$ on $[1,\ 6]$ using five approximating rectangles and left-hand endpoints.

9. Express the integral as a limit of Riemann sums. Evaluate the limit.

$$\int_1^4 (\,3x^2 + 2x - 2)dx$$

10. Evaluate the integral by interpreting it in terms of areas.

a) $\int_{-3}^{5} (4 - 2x)\, dx$

b) $\int_{-3}^{0} \left(2 + \sqrt{9 - x^2}\right) dx$

11. Integrate:

a) $\int \frac{4}{x^2+1} + e^{x+3} - \csc^2 x \ dx$

b) $\int 2x^3 \left(-3\sqrt[6]{x} - e + \frac{2}{x^4} - \sqrt[7]{x^3}\right) dx$

c) $\int \frac{x^3 + x^2 + x - 2}{x^2 + 1} \ dx$

$d) \int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx$

$e) \int \frac{\sin\theta + \sin\theta \tan^2\theta}{\sec^2\theta} d\theta$