HKN x acm

CS/ECE374~~A~~B Midterm 1

Regular languages and context free languages

Disclaimer and logistics

- This is being recorded. Recording will be on HKN website.
- Slides will be available on EdStem and HKN website later.
- Some of us are CAs, but we have not seen the exam. We have no idea what the questions are. However, we've reviewed the practice exams, and taken the course, so we have *suspicions* as to what the questions will be like.
- Slides that contain information that's not on your cheatsheet will be indicated as such!

Regular expressions

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- Precedence: first $()$, then *, then concatenation, then $+$
- Regular expressions must use a *finite* number of these operators

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- Explain, in English, each part of your regular expression with a short sentence. Does the explanation match the language?

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0110

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  $(01 + 10 + \cancel{1010 + 0101 + 0110 + 1001})^*(\epsilon + 0 + 1)$ ✓

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  - The first is wrong with counterexmaple 001. The second is correct. $1010, 0101, 0110, 1001$ are all the strings of length 4 in $(01 + 10)^*$

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- Final answer: $0^*((11)^*10^+)^* + (11)^*1$

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  - Union ($\cup$), intersection ($\cap$), concatenation ($\cdot$), kleene star (*), complement (^{C}), set difference ($\setminus$), and reverse (^{R})
- How do we know this? Think of NFA/DFA transformations from Thompson's algorithm and product construction (we did a special case of set difference earlier!)
- Regular languages are closed only under *finite* applications of these operations - this stems from the fact that a DFA must have a finite number of states, so we can't do an infinite product construction.
  - Note: sometimes an infinite sequence of these operations *will* result in a language that's still regular, but *not always*!

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010000

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0000010000

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  - Note that we can set $y = \epsilon$ so then $L = \{xz : x, z \in \Sigma^*\} = \Sigma^*$ so L is regular!

Short answer regular languages 2

- If L is not regular, then is L^* also not regular?

$L = 0^p$ p prime $\quad L^* = 00^*$, reg.

$L = 1 + 0 + \underbrace{L}_{\text{bad}} \rightarrow L^* = (1+0)^*$

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  - Yes! If L was regular, then L^* would be regular.
- If L_1 is not regular and L_2 is regular, then must L_1L_2 be irregular?
  - No. Consider $L_1 = \{0^{2^n} : n \geq 1\} \cup \{\epsilon\}$ and $L_2 = 0^*$. Then, $L_1L_2 = 0^*$, which is regular.

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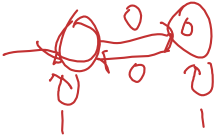
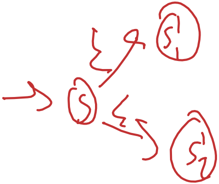
- Validate your DFA/NFA. If asked, make sure to define **all five** elements of the tuple $(Q, \Sigma, \delta, s, A)$! This is an easy way to lose points!





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  - And, make sure your set of states is finite!

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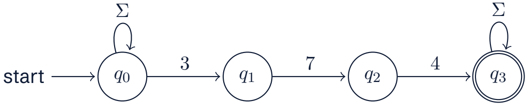

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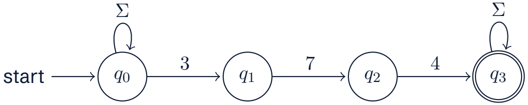

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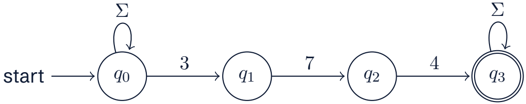

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bacon

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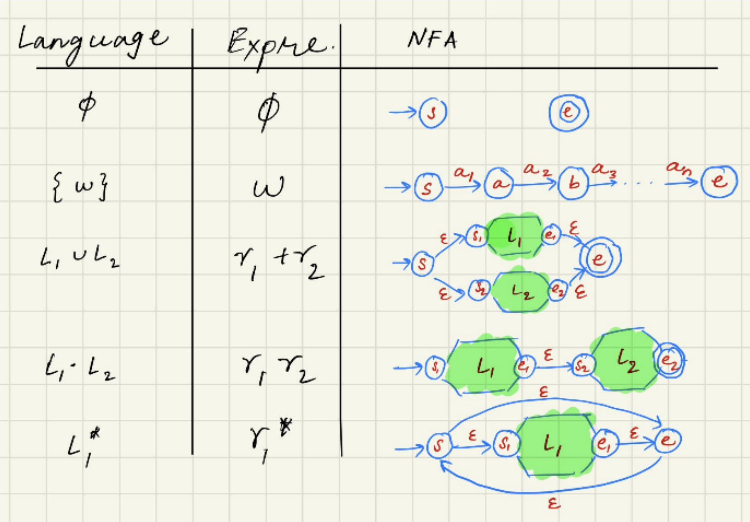
Combine with Thompson's Algorithm

Note: not all of this is not on your cheatsheet! You should make sure you remember it, just in case.



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  - For example, L is the set of strings in L_1 and not in L_2
- Create a DFA/NFA for L using the following *rough* format:
  - $Q = Q_1 \times \cdots \times Q_n$
  - $\delta'(q_1, \ldots, q_n) = (\delta_1(q_1), \ldots, \delta_2(q_2))$
  - $s = (s_1, \ldots, s_n)$
  - $A' = \{\text{convert } f \text{ into a set expression}\}$

Practice product construction

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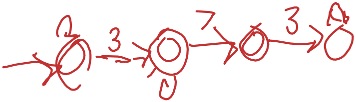

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$A' = (q_1 \in A_{373}$ and $q_2 \in A_{374})$
or
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- Intuition: when you need to keep track of unbounded information (i.e. occurrences of a character), the language is probably irregular

Practice fooling sets

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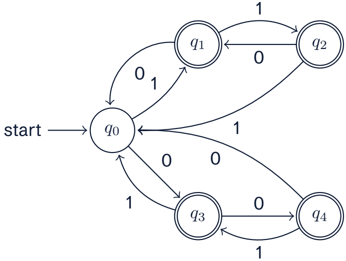
  - We can write a DFA for L_2 with 5 states, so $|F| \leq 5$



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$S \to AB$

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- Intuition 2: a CFL "peels back" strings, starting from a chosen pivot point.

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  - Example $L = \{0^i 1^{2i} : i > 0\}$. To build 001111, we only need the production rule $S \to 0S11|\epsilon$. We start with $0S11$, then replace the S with $0S11$ to get $00S1111$ and then take the other case, ϵ
- Intuition 2: a CFL "peels back" strings, starting from a chosen pivot point.
  - Example: follow the above example, but in reverse

Context-free languages

- A context-free language L is generated by a context-free grammar $G = (V, T, P, S)$
- What does context-free mean? Concatenation, union, *and recursion*
  - This means every regular language is context free, but not every context free language is regular
- CFLs only closed under union, kleene star, and concatenation
- Intuition 1: a CFL "builds" strings, going from the outside in, you can choose any rule to add characters at the left / right
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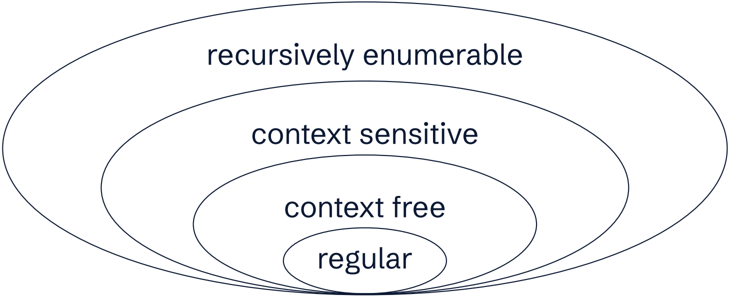
- Intuition 2: a CFL "peels back" strings, starting from a chosen pivot point.
  - Example: follow the above example, but in reverse
- Both ways to think about CFLs are completely fine!

The Chomsky Hierarchy

- Remember, all regular languages are context-free!

The Chomsky Hierarchy

- Remember, all regular languages are context-free!
- You'll learn about other classes of languages later in the course.



Practice with CFLs

- Let $L = \{0^j 1^i 0^k : i + j \neq k\}$

$i+j < k$

$i+j > k$

Practice with CFLs

$i \neq k - j$

CFLs are NOT closed under negation or intersection

- Let $L = \{0^j 1^i 0^k : i + j \neq k\}$
- First try:

$$S \to 0S0 | N$$
$$N \to 1N0 | 1 | 0$$

$A^m B^m C^n \leftarrow$ CFL

$A^m B^n C^n \leftarrow$

$A^n B^n C^n \leftarrow$ NOT CFL

Practice with CFLs

- Let $L = \{0^j 1^i 0^k : i + j \neq k\}$
- First try:

$$S \to 0S0|N$$
$$N \to 1N0|1|0$$

- What's wrong with this?

Practice with CFLs

- Let $L = \{0^j 1^i 0^k : i + j \neq k\}$
- First try:

11111

$$S \to 0S0 | N$$
$$N \to 1N0 | 1 | 0$$

1 . . . 1 1 0 . . . 0
0

- What's wrong with this? Forces $j = k$. How can we solve this?

Practice with CFLs

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- Casework!

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Practice with CFLs

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- What's wrong with this? Forces $j = k$. How can we solve this?
- Casework! Create production rules L and G for when $j < k$ and $j > k$, respectively:

$$S \to 0S0|N|L0|0G$$
$$L \to L0|N0$$
$$G \to 0G|0N$$
$$N \to 1N0|1|0$$

0 1111110 i+j+

$\{0^j 1^i 0^k : i+j \leq k\}$

i+j ≥ k

$0^j 1^i 0^m 0^k$

$0^j 1^i 1^m 0^k$

$0^j 0^m 1^i 1^k$

$S \to (N)S \mid \varepsilon$

$0^m 1^n$ where $m<n$

$0^m 1^{k+m}$

$N \to 0N1 \mid N1 \mid 1$

$\{(0+1)^* \mid$ every run of 0's is followed by a longer run of 1's.$\}$

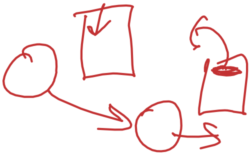
$S \to \varepsilon \mid NS$

$N \to 1 \mid 0N1$

$\underbrace{0011111}_{N}\underbrace{011}_{N}\underbrace{000111}_{N}\underbrace{1}_{N}$

Push Down Automata

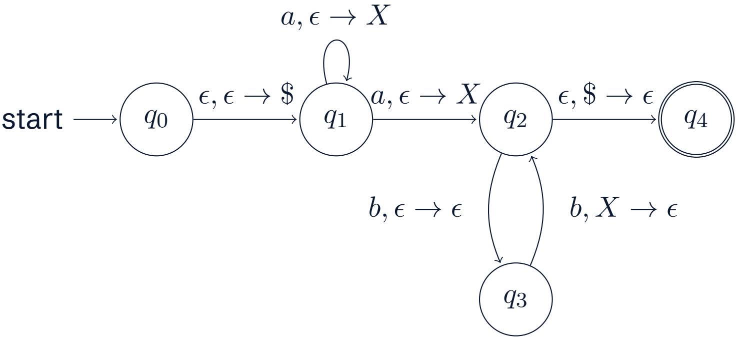
DFA + stack



- Unlikey to be on exam, alternative to using a CFG to prove a language is context free.
- Adds a stack to a FSM, making it able to decide context free grammars.
- Each transition can interact with the stack by popping or pushing a character from the stack alphabet.
- In order to take a transition, the top of the stack and the input must match the transition rule. Unless they are ϵ.
- Formally defined as a 6-Tuple: $(Q, \Sigma, \Gamma, s, A, \delta)$

Practice with PDAs

- What language does this PDA accept?



Practice with PDAs

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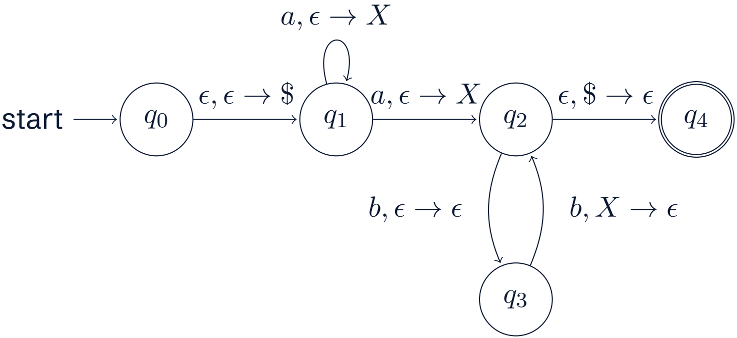

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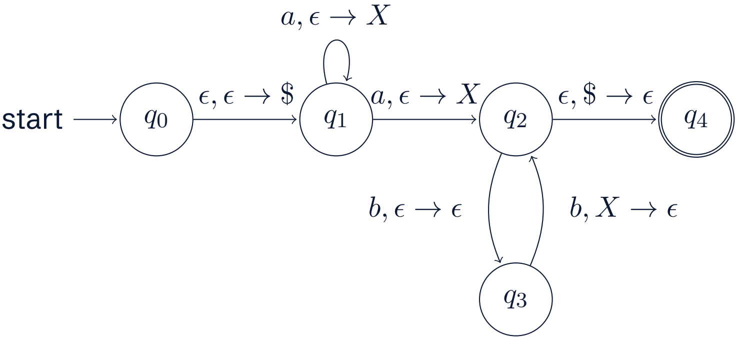

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Practice with PDAs

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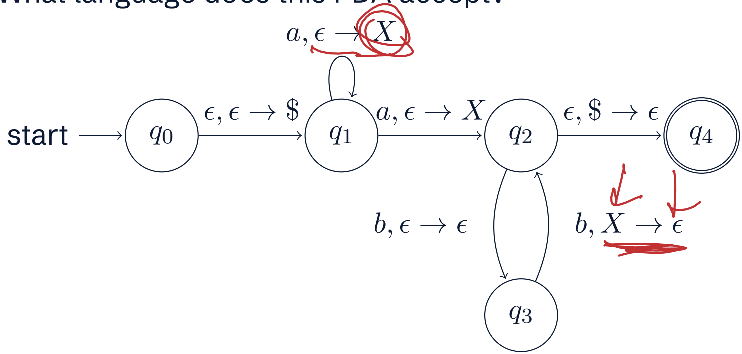

- $L = \{a^n b^{2n} \mid n \geq 1\}$
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- $L = \{a^n b^{2n} \mid n \geq 0\}$

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- When you see recursive definitions, you should almost always use induction!
- Writing something is better than nothing
- Go easiest to hardest
- Sanity check your answers (consider edge cases: strings of length 1 or 0)
- Practice until you could explain the concept to a friend

Closing Remarks

- Good luck on Midterm 1!
- Please fill out feedback form for this review session.
- Worksheet links and feedback form: