CS 374A Midterm 1 Review

ACM @ UIUC

Hello :)

Feburary 21st, 2026

intro (end of the world) rahhh

- We are here as members of the ACM academic committee, rather than in our capacity as CAs. As a result, we'll only focus on **content** rather than exam-specific questions.
- Exam-specific questions are better suited for the class Edstem, Discord, office hours, or homework party.
- This review session is being recorded. Recordings and slides will be distributed on EdStem after the end.
- **Agenda:** We'll review all topics likely to be covered, then go through practice problems, then review individual topics by request.
- Please let us know if we're going too fast/slow, not speaking loud enough/speaking too loud, etc.
- If you have a question anytime during the review session, please ask! Someone else almost surely has a similar question.
- We'll provide a feedback form at the end of the session.

Content Review

breathin’

Induction

Template

Let x be an *arbitrary* string/integer/etc.

Inductive Hypothesis: Assume for all k s.t. k is shorter/smaller/etc. than x that $P(k)$ (what we're trying to prove) holds.

Base Case: If $x = 0, \epsilon,$ whatever your base case is, then . . . , so $P(x)$ holds.

Inductive Step: If $x \neq 0, \epsilon,$ whatever your base case is, then . . . , so by the inductive hypothesis, . . . , so $P(x)$ holds.

Thus, by the principle of induction, $P(x)$ holds.

- when $w = 0x$ $\qquad w = 00x$

  $w = 1x$

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Base Case: If $x = 0, \epsilon,$ whatever your base case is, then ..., so $P(x)$ holds.

Inductive Step: If $x \neq 0, \epsilon,$ whatever your base case is, then ..., so by the inductive hypothesis, ..., so $P(x)$ holds.

Thus, by the principle of induction, $P(x)$ holds.

Some tips:

- Always use strong induction.
- Write out your IH, base case, and inductive step out explicitly.
- Think about what you would like to know about your smaller/shorter numbers/strings.

Regular Languages/Expressions

- Built inductively on 3 operations:
  - + is the union operator. $L(r_1 + r_2) = L(r_1) \cup L(r_2)$
  - $*$ is the Kleene star. $L(r_1^*) = L(r_1)^*$
  - () are used to group expressions
  - (implicit) concatenation operator: $L(r_1 r_2) = \{xy : x \in L_1, y \in L_2\}$

$01 = 0 \cdot 1$

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  - You don't get to look back indefinitely.
  - If your language requires you to track a number or string indefinitely, it is not regular!

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  - You don't get to look back indefinitely.
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- **Regex Design Tips:**
  - What strings are in your language? Which ones aren't? Note edge cases (specifically check ϵ).
  - Look for patterns and substrings that you definitely need to include or repeat.

DFAs

- DFA $M = (Q, A, \Sigma, s, \delta)$
  - Q - FINITE set of states
  - $A \subseteq Q$ - accepting states
  - Σ - input alphabet, usually $\{0, 1\}$
  - $s \in Q$ - start state
  - $\delta : Q \times \Sigma \to Q$ - transition function

Strings with even # 0s

✓ product construction

$Q \times$ ___

count another DFA

$Q \vee$ ___

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- **Tips for Creating DFAs**:
  - Define your states exactly! What does it mean to be at each state?
  - Based on these definitions, when should you accept? Define A accordingly.
  - What state represents ϵ? Make that the start state. Make sure that if L accepts ϵ, you accept your start state.
  - How does reading in a 0 or 1 change each state? Define δ accordingly.

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  - How does reading in a 0 or 1 change each state? Define δ accordingly.
- Notes:
  - To go from L to its complement, just switch the accepting and non-accepting states.
  - Every DFA is automatically an NFA.
  - Every regular language can be represented by a DFA. Every DFA represents a regular language.

Kleene's Thm:

Regex $\Leftrightarrow$ DFA $\Leftrightarrow$ NFA

Product Constructions

- Say I can build DFA M_1 keeping track of one property and DFA M_2 keeping track of another property. What if I want a DFA M that keeps track of both properties?
- You can combine the information of both DFAs into one product DFA.
- The accept states in your new DFA define whether I only accept strings with both properties or strings with one or the other or some other logical operation.

$M_1 = (Q_1, s_1, A_1, \delta_1)$

$M_2 = (Q_2, s_2, A_2, \delta_2)$

state from M_1

$Q_1 \times Q_2$ ← M_2

(s_1, s_2)

$\delta((q_1, q_2), a) = (\delta_1(q_1, a), \delta_2(q_2, a))$

$A = \{(q_1, q_2) \in Q_1 \times Q_2 : q_1 \in A$ and / or $q_2 \in A\}$

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Template

1. Define (drawing/formal) your first DFA $M_1 = (Q_1, A_1, \Sigma, s_1, \delta_1)$.
2. Define (drawing/formal) your second DFA $M_2 = (Q_2, A_2, \Sigma, s_2, \delta_2)$.
3. Define the product DFA $M = (Q, A, \Sigma, s, \delta)$ as follows:
   - $Q = Q_1 \times Q_2 = \{(q_1, q_2) | q_1 \in Q_1, q_2 \in Q_2\}$ - each state is a tuple
   - $s = (s_1, s_2)$ - just the tuple containing both start states
   - $\delta((q_1, q_2), a) = (\delta_1(q_1, a), \delta_2(q_2, a))$ - apply the first transition function to the first state and the second transition function to the second state
   - $A = \{(q_1, q_2) | q_1 \in A_1$ **and/or/etc.** $q_2 \in A_2\}$ - check if the first state is accepted by the first DFA, check if the second state is accepted by the second DFA, and accept based on the problem statement

NFAs

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  - We've added epsilon transitions.
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- **Tips for Creating NFAs**:
  - Make sure that your transition function consistently leads to a SET of states (whether that set is empty, has 1 state, or multiple states).
  - Use epsilon transitions to jump from one state to another without reading anything (usually when you want to be able to transition from one phase to another).

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- Notes
  - Every NFA can be converted to a DFA through power set construction. The DFA states are the power set of the NFA states.

Fooling Sets

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Fooling Sets

max(fooling set size)
= min(DFA size)

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- **Tips and Tricks**
  - If L needs to keep track of a value with no bound, create a fooling set around the part you count up.
  - If you're using strings of the form $1^k, 0^p$, etc. when sampling elements of your fooling set a^i, a^j, you may assume WLOG that $i < j$.

Prove $\{0^n 1^n \mid n \geq 0\}$ is irregular.

Template adapted from *Fall 2025 Homework 3 solutions*.

Let $F = \left\{ \boxed{} \right\}$, which is infinite.

Let $x, y \in F$ with $x \neq y$. Thus, $x = \boxed{}$ and $y = \boxed{}$, where $\boxed{}$.

Let $z = \boxed{}$.

- Then, $xz \in L$ because $\boxed{}$.
- Then, $yz \notin L$ because $\boxed{}$.

Thus, z is a distinguishing suffix for x and y, so F is a fooling set for L.

Since F is infinite, L is not a regular language.

Note: We can also pick z so that $xz \notin L$ and $yz \in L$. Just do whichever is easier.

Language Transformations

- We have a language L we know is regular.
- We have a function f from strings to strings: $f(w) = x$.
- We define a transformed language in one of the following formats:
  1. $L' = \{f(w) | w \in L\}$
  2. $L' = \{w | f(w) \in L\}$
- We want to show that L' is regular by making a DFA/NFA that accepts L'.

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Template

- Since L is regular, there is a DFA $M = (Q, A, \Sigma, s, \delta)$ that accepts it.
- Now, we create a DFA/NFA $M' = (Q', A', \Sigma, s', \delta')$ that accepts L'
  1. $L' = \{f(w) : w \in L\}$ - M' reads $f(w)$; we need to check if the original string w is accepted by M
  2. $L' = \{w : f(w) \in L\}$ - M' reads w; we need to check if $f(w)$ is accepted by M
- Define the states Q' by thinking about what information you need to interpret the next letter you read - usually $Q \times \{\text{Information needed to convert } x \text{ to } w \text{ or } w \text{ to } x\}$
- Define the transition function δ so it passes w or x back to the original DFA - potentially using nondeterminism or epsilon transitions

Context-Free Languages/Grammars

- Nonterminals and productions: e.g.,

$$0^n 00^* 1^n$$

$$S \rightarrow G \mid L \mid 0S1 \qquad + \, 0^n 11^* 1^n$$

$$G \rightarrow 0G \mid 0 \qquad 00^*$$

$$L \rightarrow L1 \mid 1 \qquad 11^*$$

- Closed under union, concatenation, and Kleene star (Every regular language is a CFL)
- **Not closed** under complement or intersection

Context-Free Languages/Grammars

- Nonterminals and productions: e.g.,

$$\begin{aligned} S &\rightarrow G \mid L \mid 0S1 \\ G &\rightarrow 0G \mid 0 \\ L &\rightarrow L1 \mid 1 \end{aligned}$$

- Closed under union, concatenation, and Kleene star (Every regular language is a CFL)
- **Not closed** under complement or intersection
- Tips for constructing CFGs:
  - Think recursively! Given a language, how can strings in the language be broken down?
  - Match outer structure first
  - For languages like $0^n 1^n$, generate matched symbols together
  - Split into cases as needed (in the example above, we split into greater and less than cases)

Practice Problems

main thing

Short Answer T/F (1)

For each of the following, determine if the statement is **true** or **false**, and give a one-sentence explanation of your answer. (These are intentionally tricky)

(a) For all languages L, if L is irregular, then L has a finite fooling set. True

$\emptyset$ is always fooling set

(b) If M is a minimal DFA that decides a language L, and running M on strings x and y result in states q and q', respectively, where $q \neq q'$, then there exists a distinguishing suffix between x and y in L. True

(c) The language $L = \{0^i 1^j 0^k : i = j \text{ and } k \equiv i \pmod{374}\}$ is context-free. True

(d) For context-free languages L_1, L_2, the language $L = (L_1^* L_2) \cup (L_1 L_2^*)$ is context-free.

True

(e) **(Fall 2024)** For **all** regular languages L_R and context free languages L_C, $L_R \setminus L_C$ is context free.

False CFLS not closed under intersection

(f) Suppose $L_1, L_2, \ldots$ is an infinite sequence of regular languages, where $L_i \supseteq L_{i+1}$ for all $i \geq 1$. Then, $\bigcup_{i=1}^{\infty} L_i$ is regular.

True!

$L_1 \supseteq L_2 \supseteq L_3 \ldots \Rightarrow \bigcup_{i=1}^{\infty} L_i = L_1$ which is regular

$\{0^i 1^j 0^k : i \neq j \text{ and } k \equiv i \pmod{374}\}$

$S \to A_0 B_0 \mid A_1 B_1 \mid \dots \mid A_{373} B_{373}$

$A_0 \to \varepsilon \mid 0 A_{373} \mid$

$A_1 \to 0 A_0 \mid$

$\vdots$

$A_{373} \to 0 A_{372} \mid$

$B_0 \to \varepsilon \mid 0 B_{373}$

$B_1 \to 0 B_0$

$\vdots$

$B_{373} \to 0 B_{37}$

Short Answer T/F (2)

For each of the following, determine if the statement is **true** or **false**, and give a one-sentence explanation of your answer. (These are intentionally tricky)

$x=\varepsilon$ 0110 1010 11 $x=\varepsilon$ 0 1 1 1 0 1 1 1 0

y

(g) The language $\{xx^Ry : x, y \in \{0,1\}^*\}$ is regular.

True - let $x = \varepsilon$, then $y = \Sigma^*$

(h) If L is regular, then $\textsc{SelfFold}(L) := \{a_1 a_n a_2 a_{n-1} \cdots a_{\lceil \frac{n}{2} \rceil} : a_1 a_2 \cdots a_n \in L\}$ is regular.

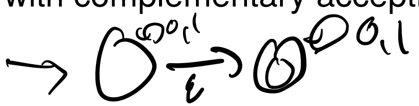
True $M \times M^R \times \{0,1\}$ ← some subtlety

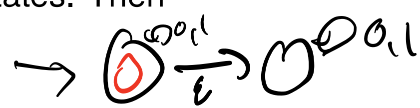
(i) Consider the language $L = \{1^x 2^y 3^z : y = x + z\}$. There exists a distinguishing suffix between the strings 1112222223 and 2223.

False - can only add 3s ⇒ sum always the same

(j) Let M_1, M_2 be arbitrary NFAs with identical alphabets, states, starting states, and transition functions, but with complementary accepting states. Then $L(M_1) \cap L(M_2) = \emptyset$.

False ⇐





(k) Consider an infinite sequence of regular languages $L_1, L_2, \ldots$ s.t. $L_i \subseteq L_{i+1}$ for all $i \geq 1$. The language $\cup_{i=1}^{\infty} L_i$ is context free.

False $L_i = \{0^j 1^j 2^j : j \leq i\}$ $\bigcup_{i=1}^{\infty} L_i = \{0^n 1^n 2^n : n \geq 1\}$

Regular or Not?

$L_1 = \{012\}$ ✓

$L_2 = \{012, 001122\}$ ✓

$L_3 = \{012, 001122, 000111222\}$

For each of the following languages, either *prove* that the language is regular, or *prove* that it is not regular (**Hint:** exactly two of the four languages are regular). For all questions, $\Sigma = \{0, 1\}$.

- $\{1xyx \mid x, y \in \Sigma^*\}$
- $\{x1xy \mid x, y \in \Sigma^*\}$
- $\{w \in \Sigma^* \colon |w| \geq 374$ and last 374 characters of w have equal number of 0s and 1s$\}$
- **(Fall 2021 Conflict)** $\{0^p 1^q 0^r \mid r = p + q\}$

Language Transformations $(\Sigma = \{0, 1\})$

(Spring 2025) For a language $L \subseteq \Sigma^*$, we define operation BYE:

$$\text{BYE}(L) := \{uw : uvw \in L \text{ and } u, v, w \in \Sigma^*, |v| \geq 2\}$$

For example, if $L = \{0110, 01, 101\}$, then

$$\text{BYE}(L) = \{\varepsilon, 0, 1, 00, 01, 10\}$$

Intuitively, BYE(L) is the set of all strings obtained by deleting a contiguous substring of length ≥ 2 from some string in L. **Prove** that if L is regular, then BYE(L) is also regular.

Given arbitrary DFA M that accepts L. We're gonna create NFA M' to show Bye(L) is regular.

$M' = (Q', \Sigma, \delta', s', A')$

$Q' = Q \times \{inU, inV, inW\} \times \{0,1,2\}$

$s' = (s, inU, 0)$

Q = original states

$\{inU, inV, inW\}$ = where we are in string

$\{0,1,2\}$ = how many chars we've added to v

Language Transformations $(\Sigma = \{0, 1\})$

$$\text{inU}\begin{cases} \delta'((q, \text{inU}, 0), c) = \{(\delta(q, c), \text{inU}, 0)\} \\ \delta'((q, \text{inU}, 0), \varepsilon) = \{(q, \text{inV}, 0)\} \end{cases}$$

$$\text{inV}\begin{cases} \delta'((q, \text{inV}, i), \varepsilon) = \begin{cases} (\delta(q, 0), \text{inV}, j) \\ (\delta(q, 1), \text{inV}, j) \end{cases}, \quad j = \min(2, i+1) \\ \delta'((q, \text{inV}, 2), \varepsilon) = \{(q, \text{inW}, 2)\} \end{cases}$$

$$\text{inW}\begin{cases} \delta'((q, \text{inW}, 2), c) = \{(\delta(q, c), \text{inW}, 2)\} \end{cases}$$

$$A' = A \times \{\text{inW}\} \times \{2\}$$

Language Transformations $(\Sigma = \{0, 1\})$

For a language $L \subseteq \{0,1\}^*$, define:

$$\textsc{Warm}(L) := \{x1^k y : xy \in L, k \geq 1\}$$

That is, $\textsc{Warm}(L)$ inserts exactly one substring of 1s into each string from L.

For example, Let $L = \{0, 01\}$.

$$\textsc{Warm}(L) = \{10, 011, 0111, 110, 0111, 1011, \ldots\}$$

Prove that if L is regular, then $\textsc{Warm}(L)$ is also regular.

$Q' = Q \times \{\text{before}, \text{during}, \text{after}\}$

$\delta'((q, \text{before}), 0) = \{(\delta(q, 0), \text{before})\}$

$\delta'((q, \text{before}), 1) = \{(\delta(q, 1), \text{before}), (q, \text{during})\}$

$\delta'((q, \text{during}), 0) = \{(\delta(q, 0), \text{after})\}$

$\delta'((q, \text{during}), 1) = \{(q, \text{during}), (\delta(q, 1), \text{after})\}$

$\delta'((q, \text{after}), a) = \{(\delta(q, a), \text{after})\}$

$s' = (s, \text{before})$

$A' = A \times \{\text{after}, \text{during}\}$

Language Transformations $(\Sigma = \{0, 1\})$

Language Transformations $(\Sigma = \{0, 1\})$

(Fall 2024 HW) For two strings $s, t \in \Sigma^*$, we define *YesAnd* to be the set of strings:

$$\textit{YesAnd}(s,t) = \begin{cases} \{s\} & \text{if } t = \varepsilon \\ \{t\} & \text{if } s = \varepsilon \\ \{a \cdot w \mid w \in \textit{YesAnd}(s', t)\} \cup \{b \cdot w \mid w \in \textit{YesAnd}(s, t')\} & \text{if } s = as',\ t = bt' \end{cases}$$

For example, let $s = 00$, and $t = 111$. Then:

$$\textit{YesAnd}(00, 111) = \{00111, 01011, 01101, 01110, 10011, 10101, 10110, 11001, 11010, 11100\}$$

Intuitively, *YesAnd*(s, t) is the set of all strings formed by interleaving the characters of s and t, preserving the order within each string.

For two regular languages $L_1, L_2 \subseteq \Sigma^*$, **prove** that

$$\textsc{SideToSide}(L_1, L_2) := \{w \in \textit{YesAnd}(s,t) : s \in L_1, t \in L_2\}$$

is also regular.

01101

0111

010110111

Since L_1 & $\bar{L_2}$ are reg, we have 2 DFAs $M_1 = (Q_1, \delta_1, A_1, s_1)$ and $M_2 = (Q_2, \delta_2, A_2, s_2)$ which accept them.

$Q' = Q_1 \times Q_2$

$s' = (s_1, s_2)$

$\delta((\overset{x}{q_1}, \overset{y}{q_2}), a) = \{(\delta_1(q_1, a), q_2), (q_1, \delta_2(q_2, a))\} \; \forall q_1 \in Q_1, q_2 \in Q_2, a \in \Sigma$

$A' = \{(q_1, q_2) : q_1 \in A_1 \text{ and } q_2 \in A_2\}$

Language Transformations $(\Sigma = \{0, 1\})$

DFAs/NFAs/Regexes

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(c) All strings that do not contain 010 as a substring.

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(c) $\{0^x 1^y 2^z : x - y = z\}$

Feedback (thank u, next)

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